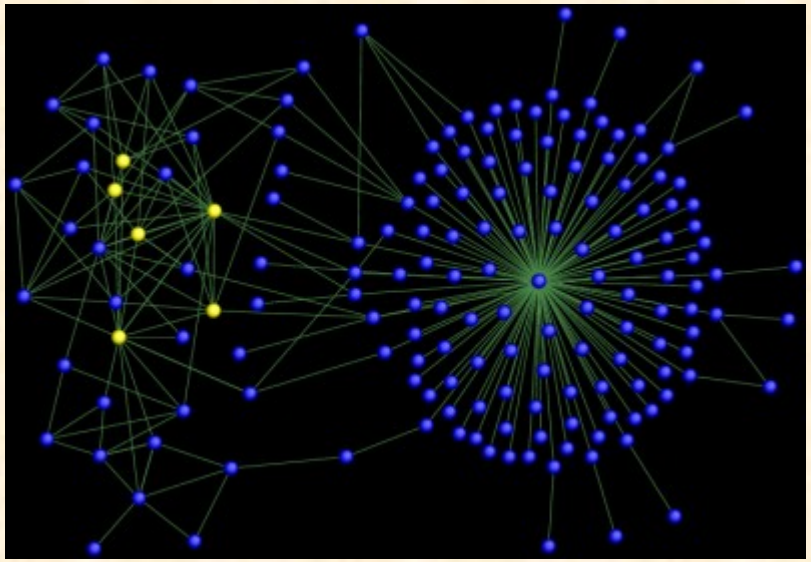
IWCSN 2009, Bristol UK

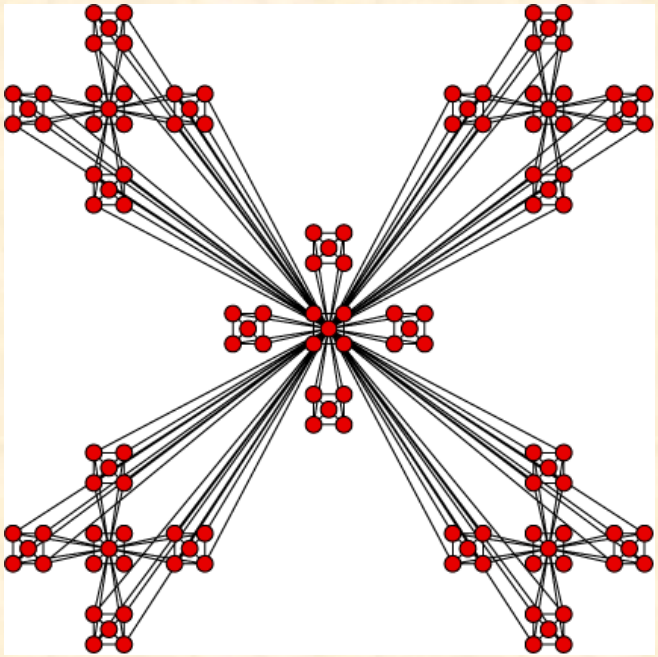
Introduction to Complex Networks



G Ron Chen
Centre for Chaos and Complex Networks

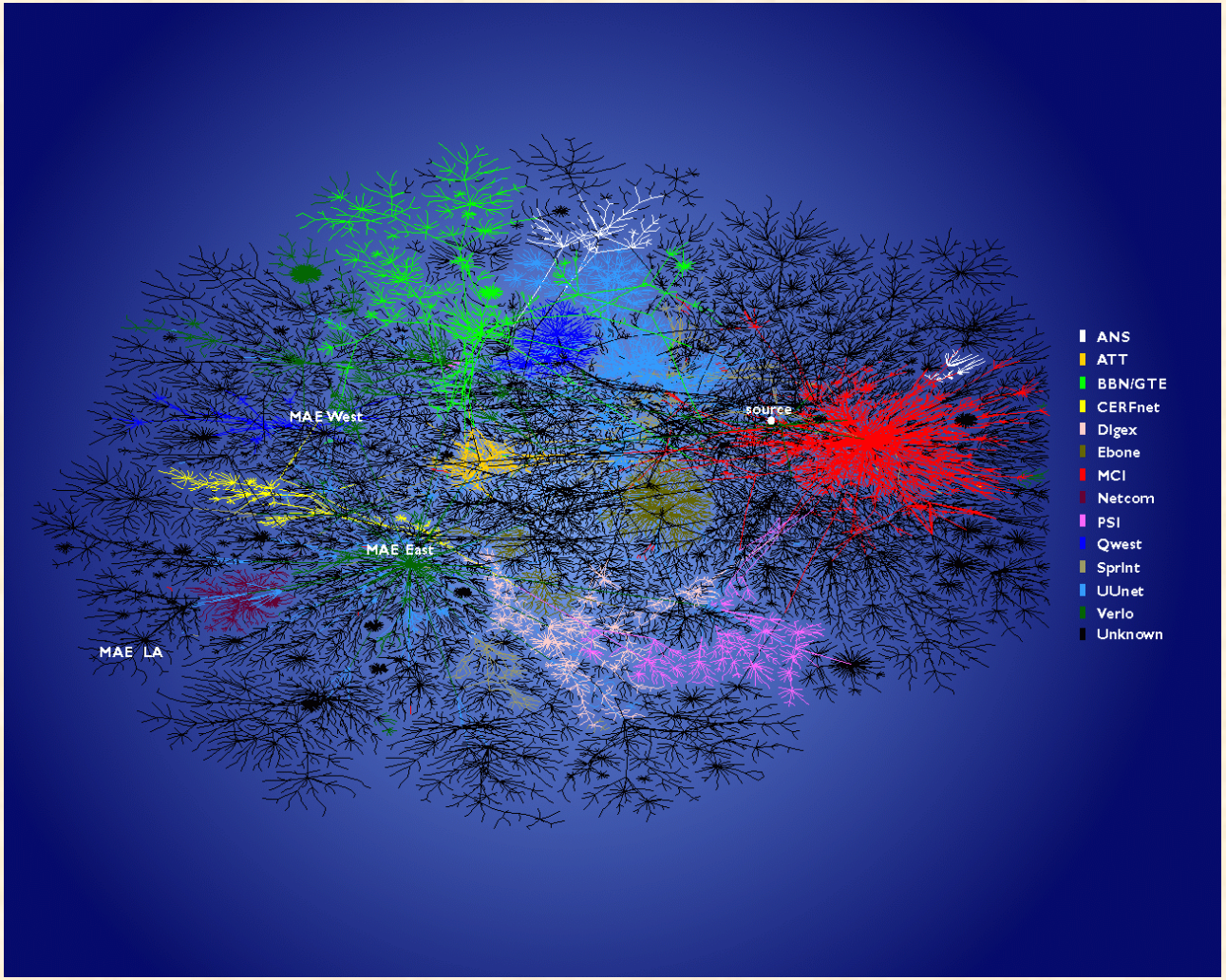
Complex Networks:

Some Typical Examples



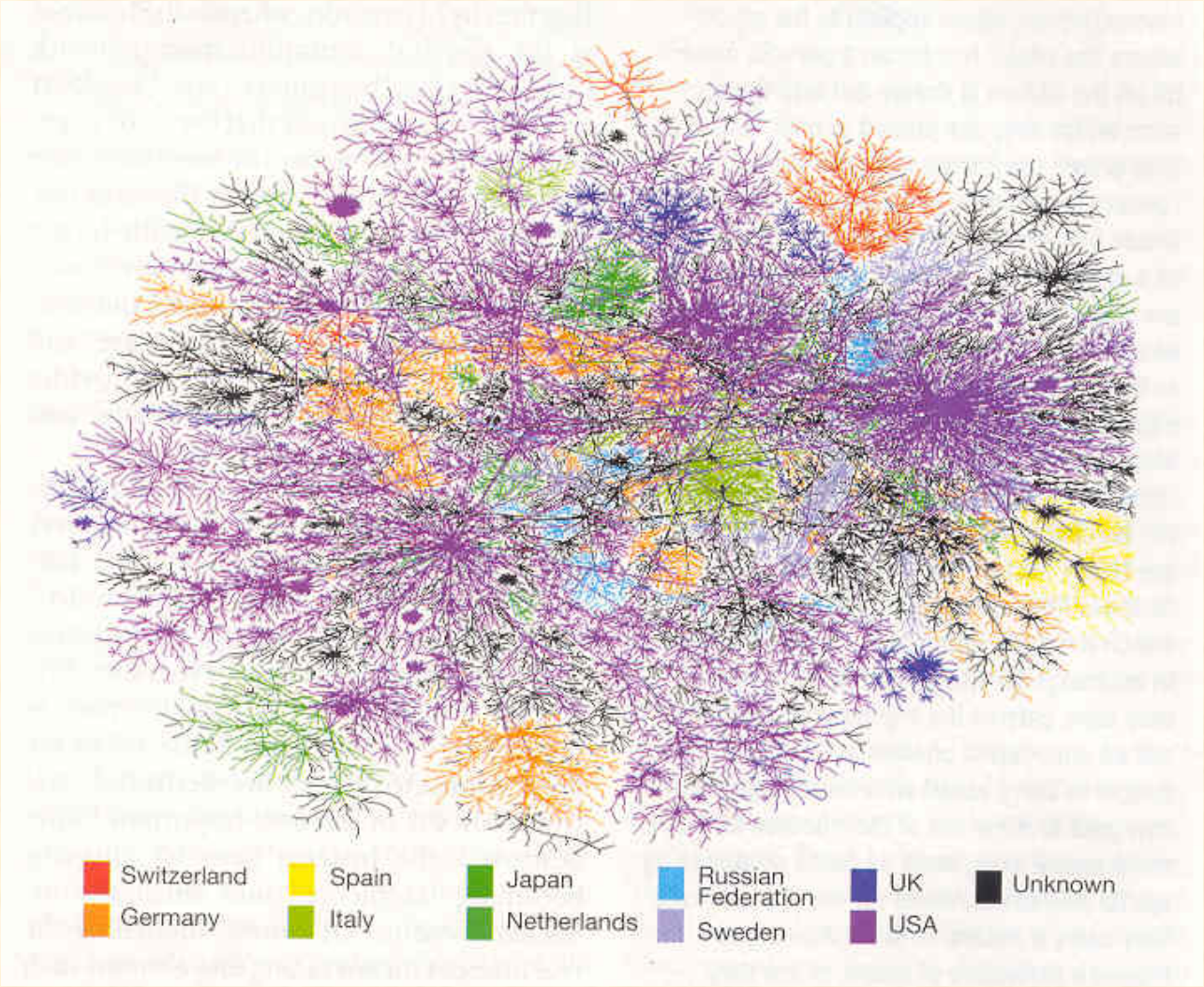
Complex Network Example: Internet

(K. C. Claffy)

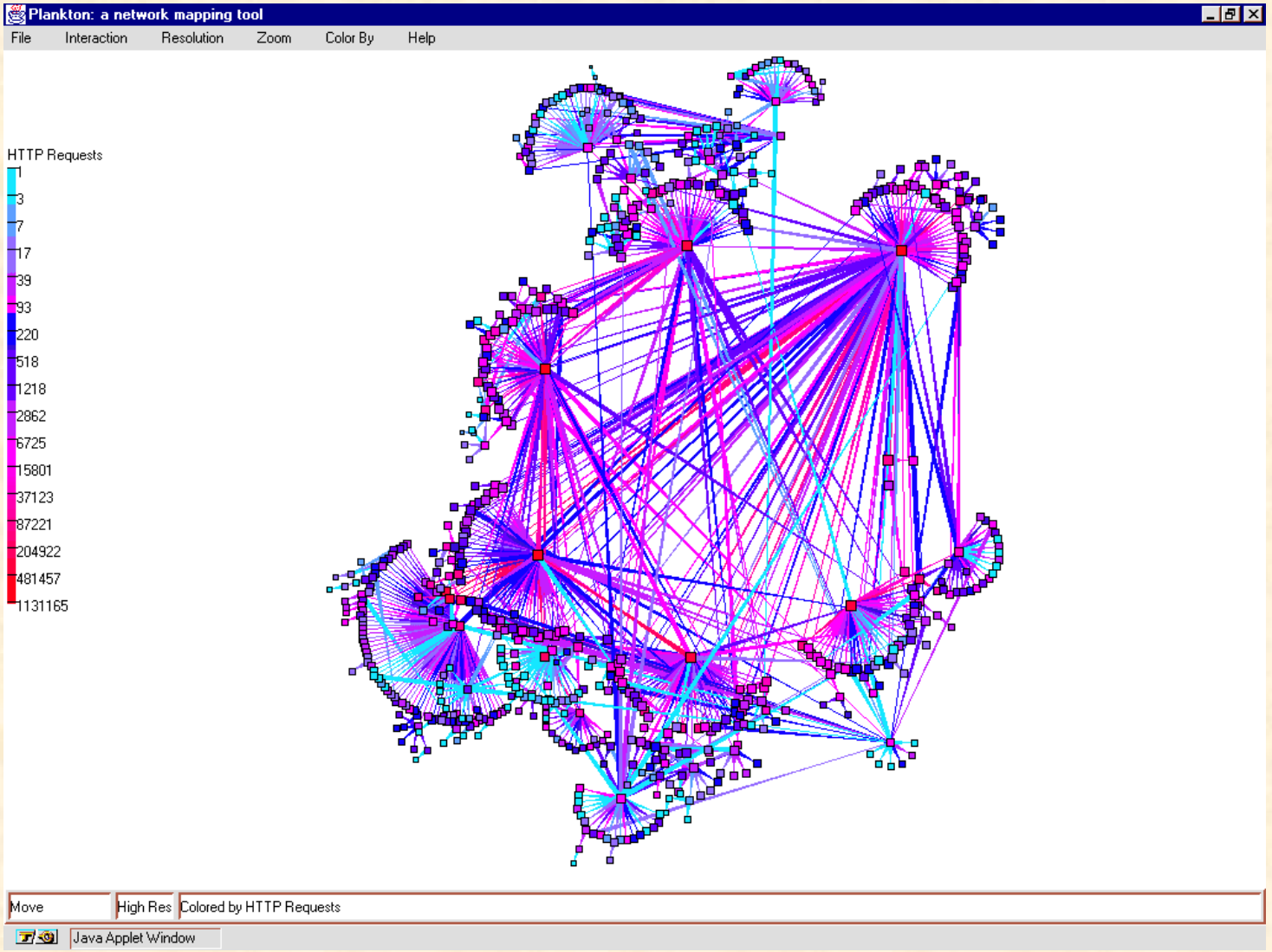


Complex Network Example: WWW

(William R. Cheswick)



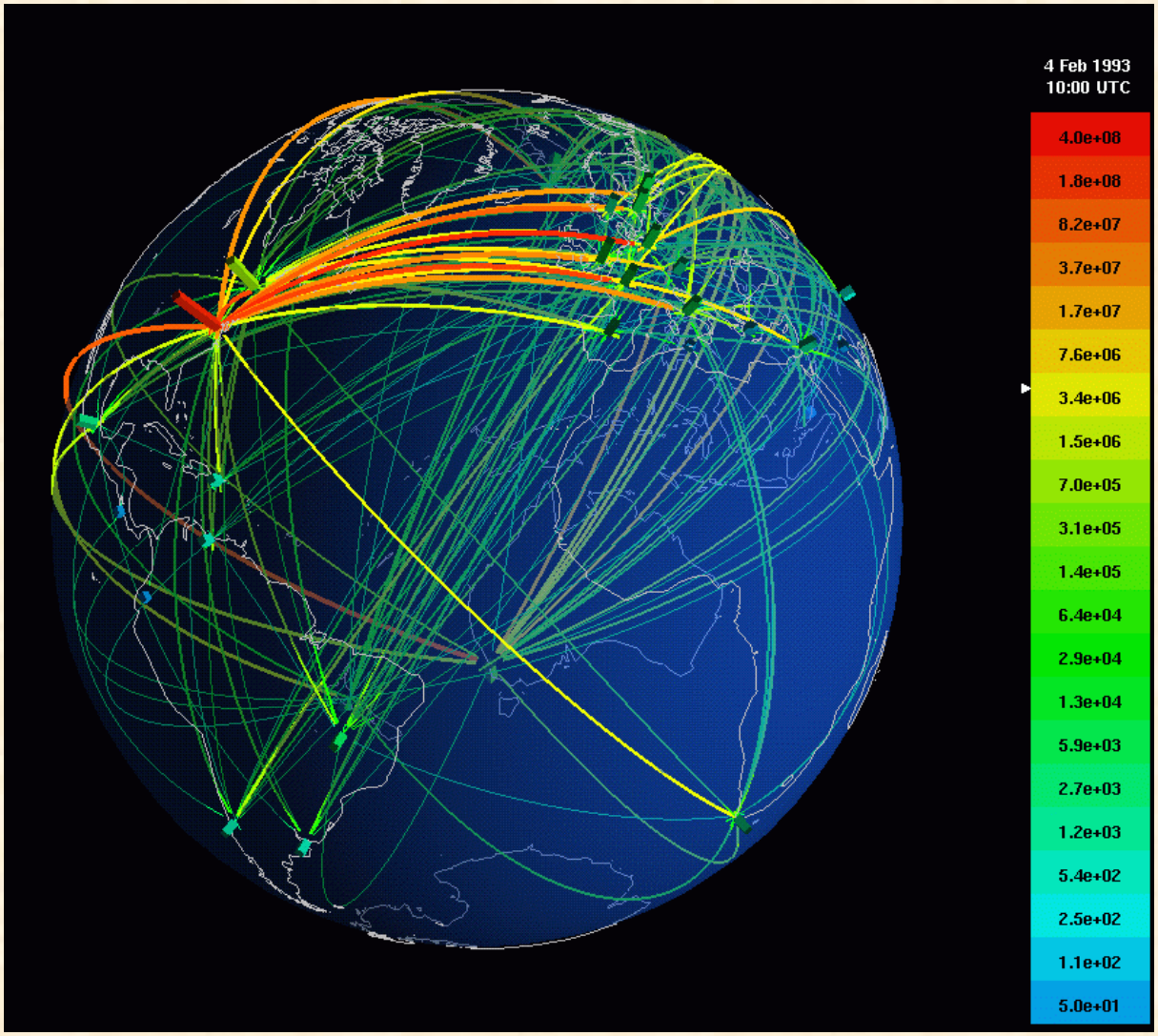
Complex Network Example: HTTP



(Bradley Huffaker)

Complex Network Example: Telecomm Networks

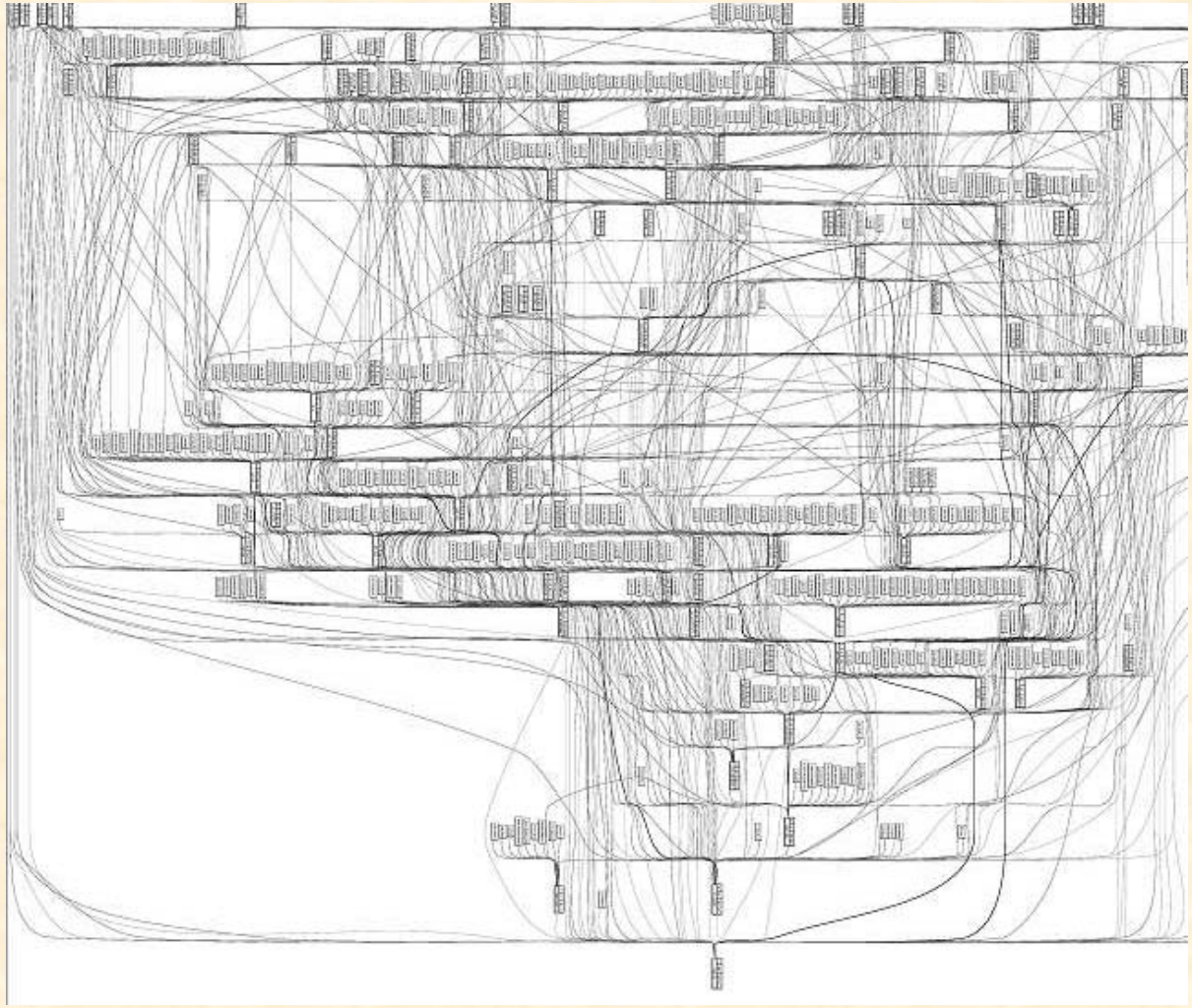
(Stephen G. Eick)



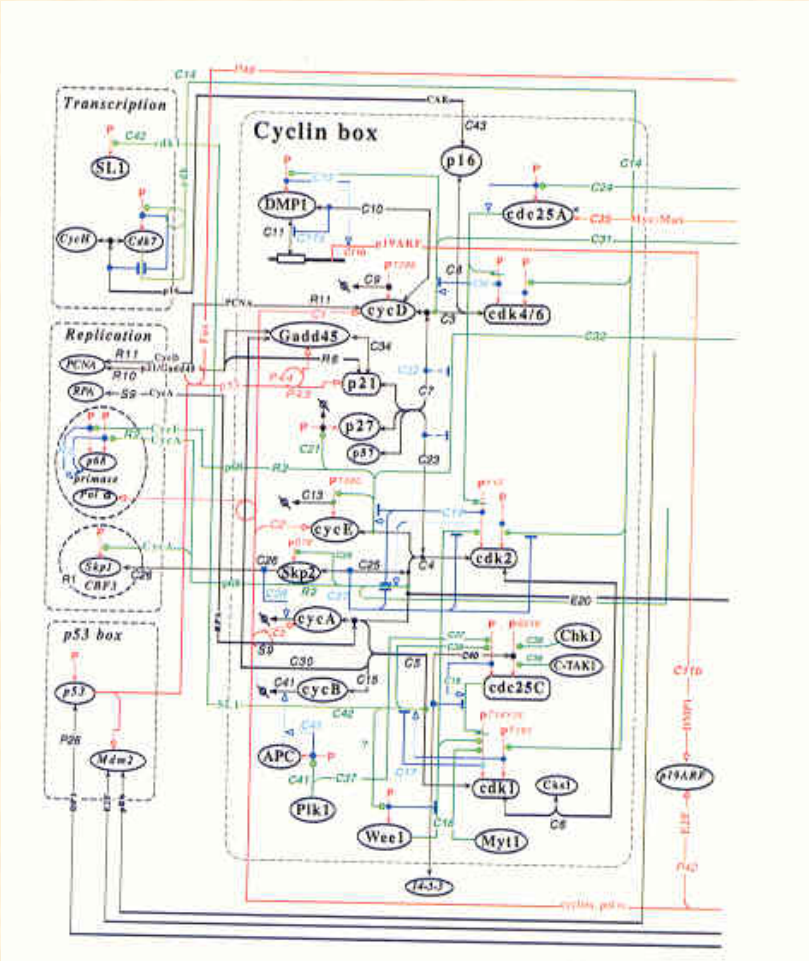
Complex Network Example: Routes of Airlines

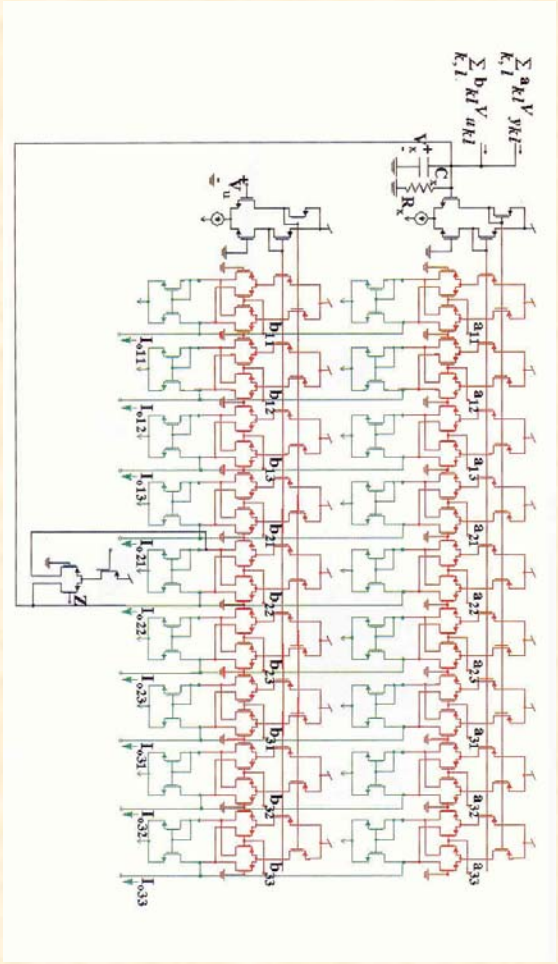
Complex Network Example: Usenet

(Naveen Jamal)

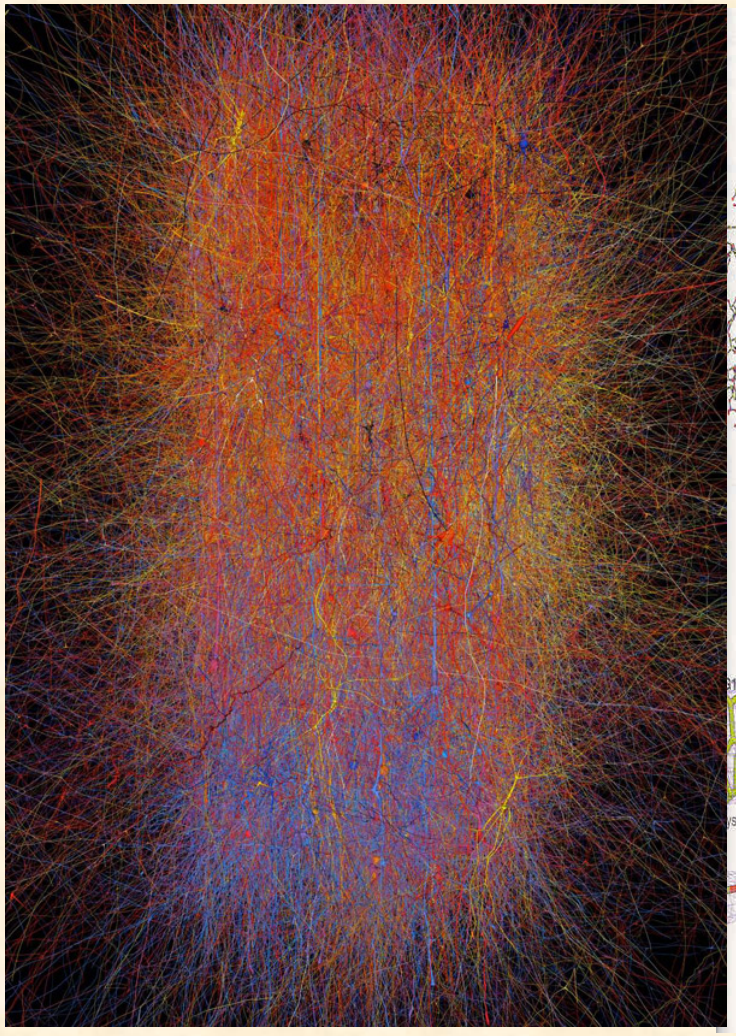


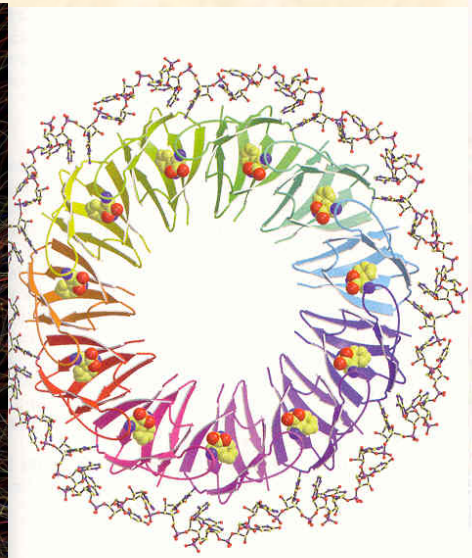
Complex Network Example: VLSI Circuits, CNN

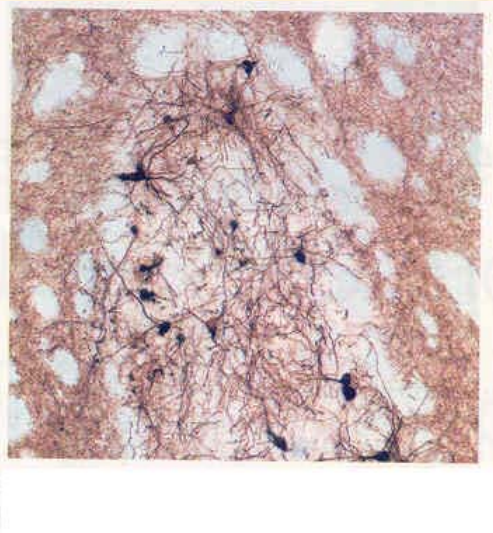


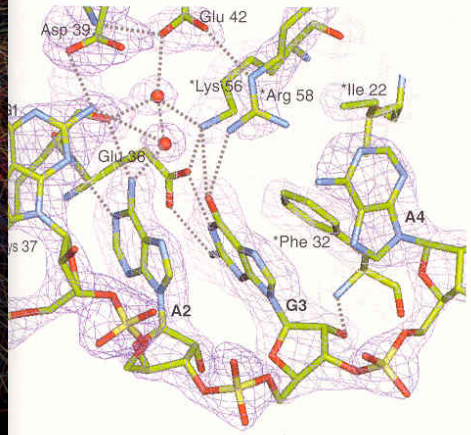


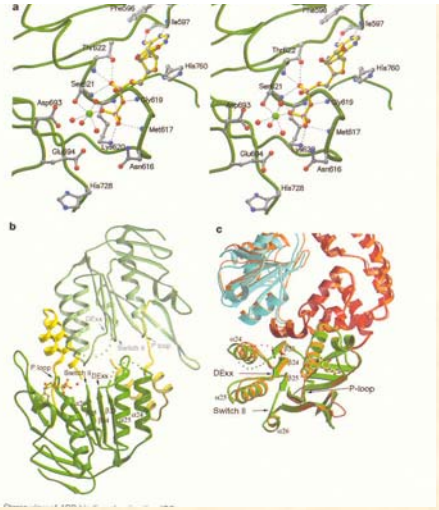
Complex Network Example: Biological Networks







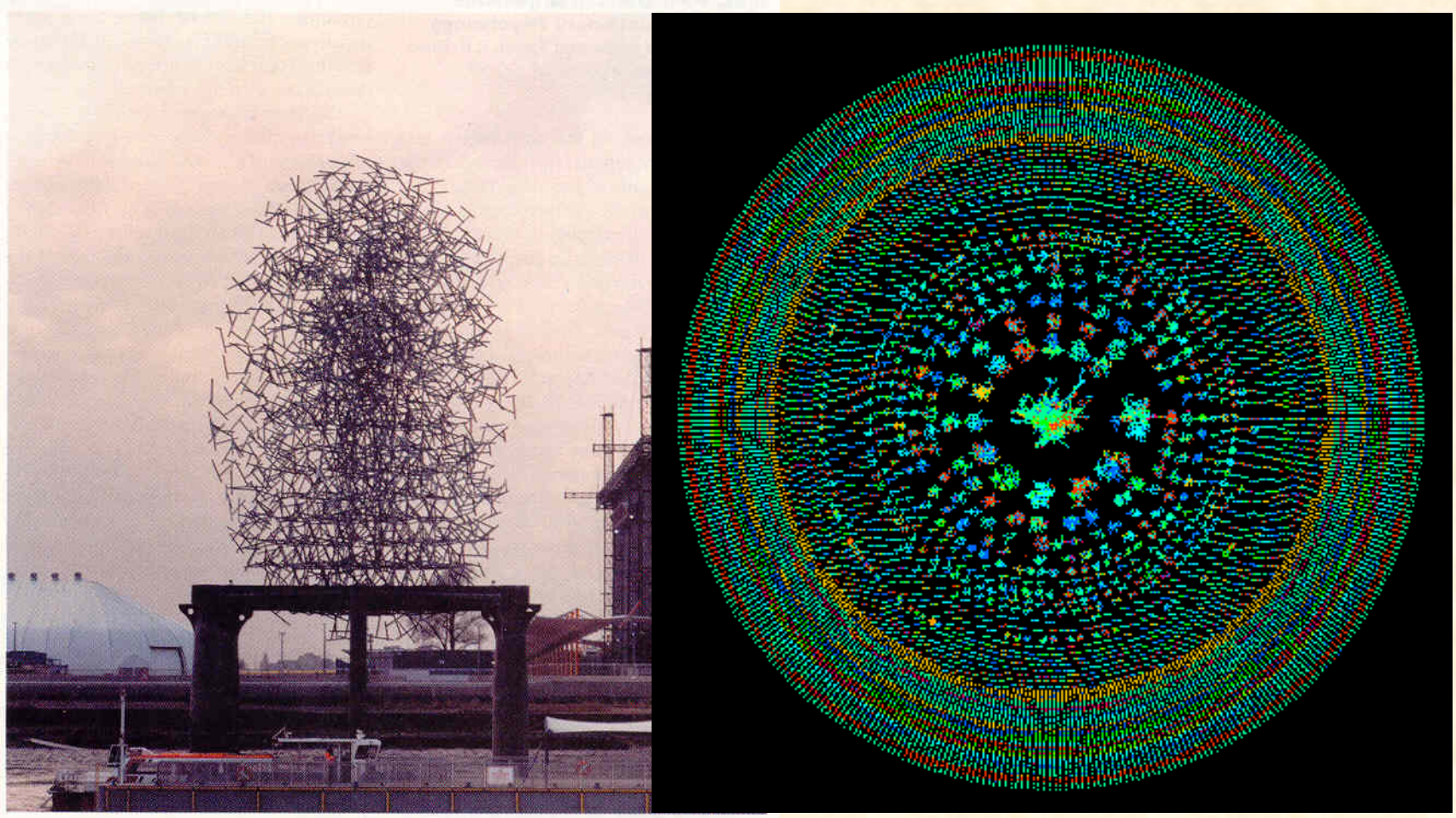




Complex Network Example: Swarms and Flocks

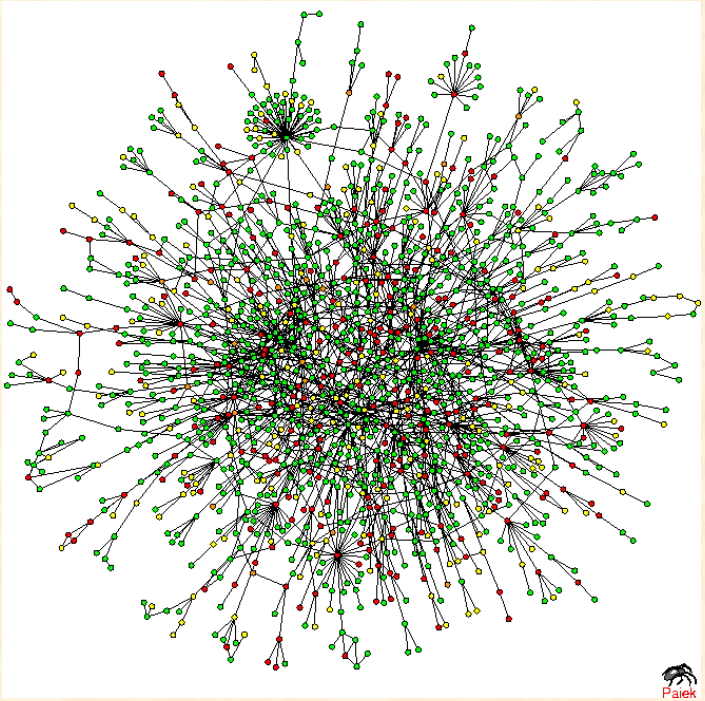
Complex Network Example: Human Relationships

Complex Network Example: Arts ☺



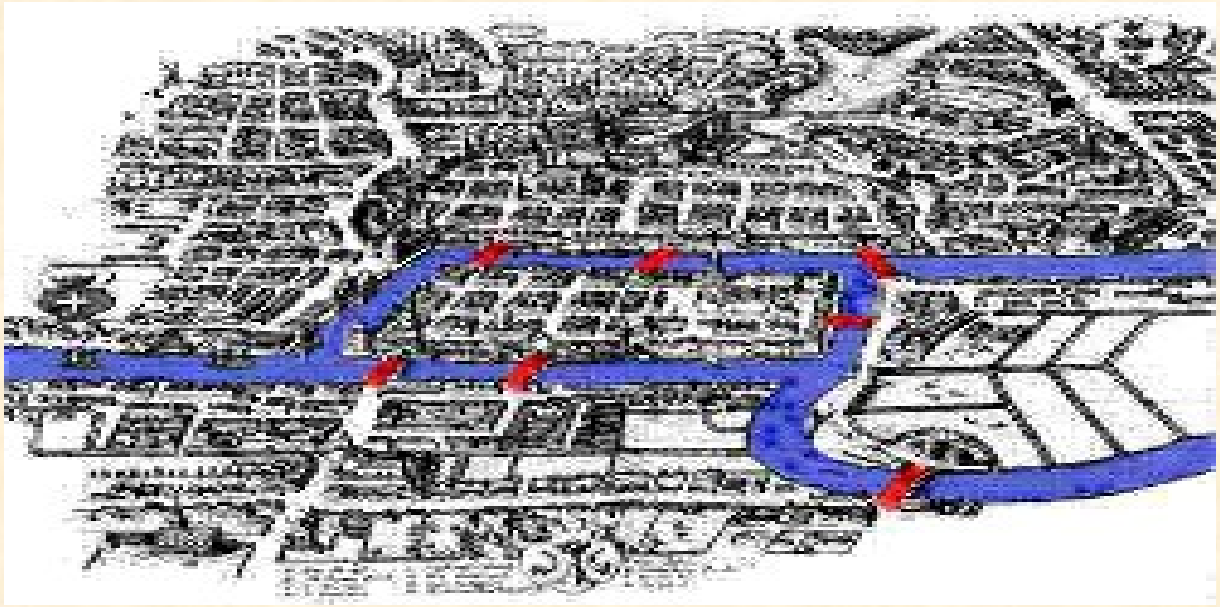
How To Model
All Such Complex Networks ?

Graph Theory !



Leonhard Eüler (1707-1783)

"Father of Graph Theory"

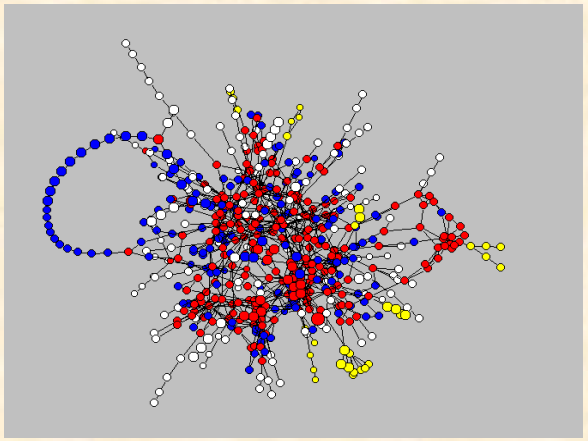


The town Königsburg and the seven bridges in year 1736

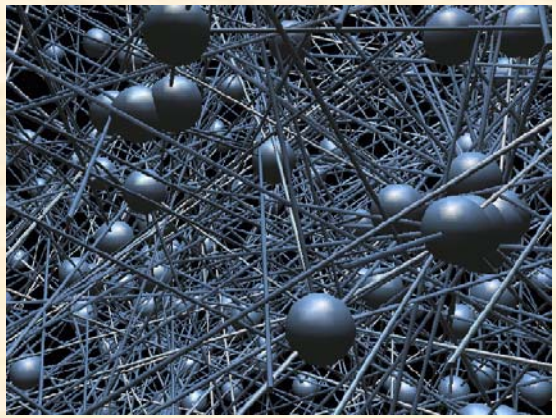
Q: Can one walk across all the seven bridges, once and once only, and then return to the starting point ?

Topics for Today

- **Mathematical Models of Networks**
  - **Random-Graph Network Model**
  - **Small-World Network Model**
  - **Scale-Free Network Model**
- **Some Real-World Examples**
- **References**

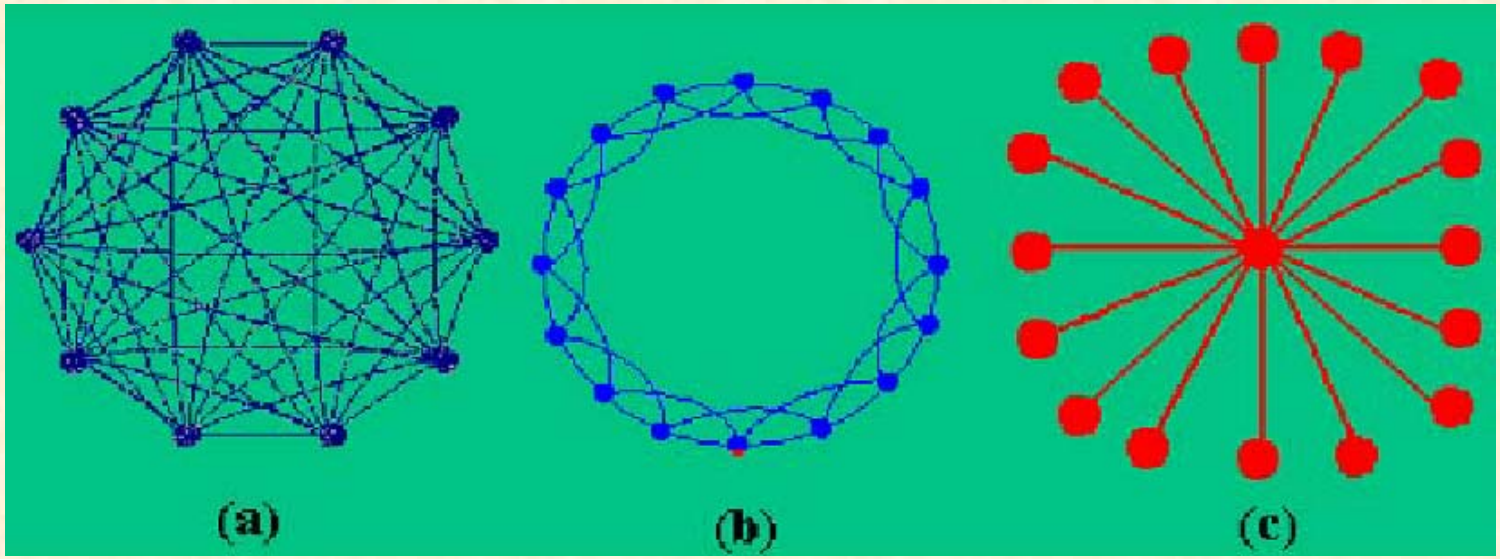


Network Topology



- A network is a graph,
  with a set of nodes
  interconnected via edges
  - Computer Networks: nodes – PCs edges – wires
  - Neural Networks: nodes – cells edges – nerves
  - Social Networks: nodes – individuals edges – relations
  - … …

Regular Networks



(a) globally coupled network
(b) ring-coupled network
(c) star-coupled network

degree, degree distribution, distance, clustering coefficient, …

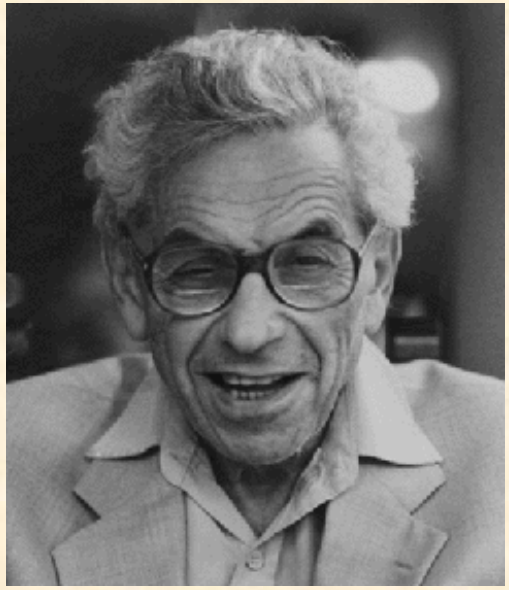
Basic Network Models

- **Random Graph Theory - Erdös and Rényi (1960)**
- **ER Random Graph model dominates for 50 years …… until recently**
- **Small-World effect (Watts and Strogatz, Nature, 1998)**
- **Scale-Free feature (Barabási and Albert, Science, 1999)**

Random Graph Theory

-- A **revolution** in the 1960s

Paul Erdös



Alfred Rényi

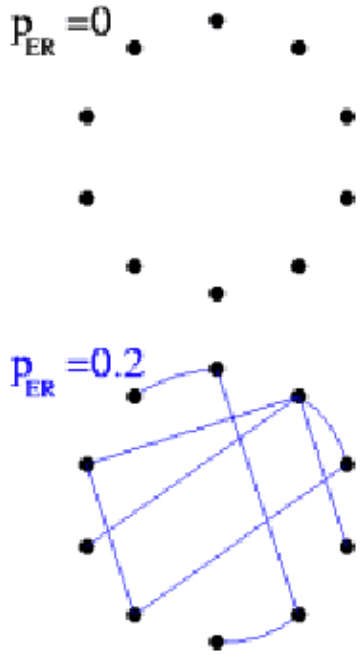
The simplest model for the most complex networks

ER Random Graph Models

Erdős-Rényi

(Publ. Math. Inst. Hung. Acd. Sci. **5,** 17 (1960))

N nodes, each pair of node is connected with probability p



Features:

- **Connectivity**
  node degree distribution - Poisson
- **Homogeneity**
  all nodes have about the same number of edges
- **Non-growing**

Random Graph and Poisson Degree Distribution

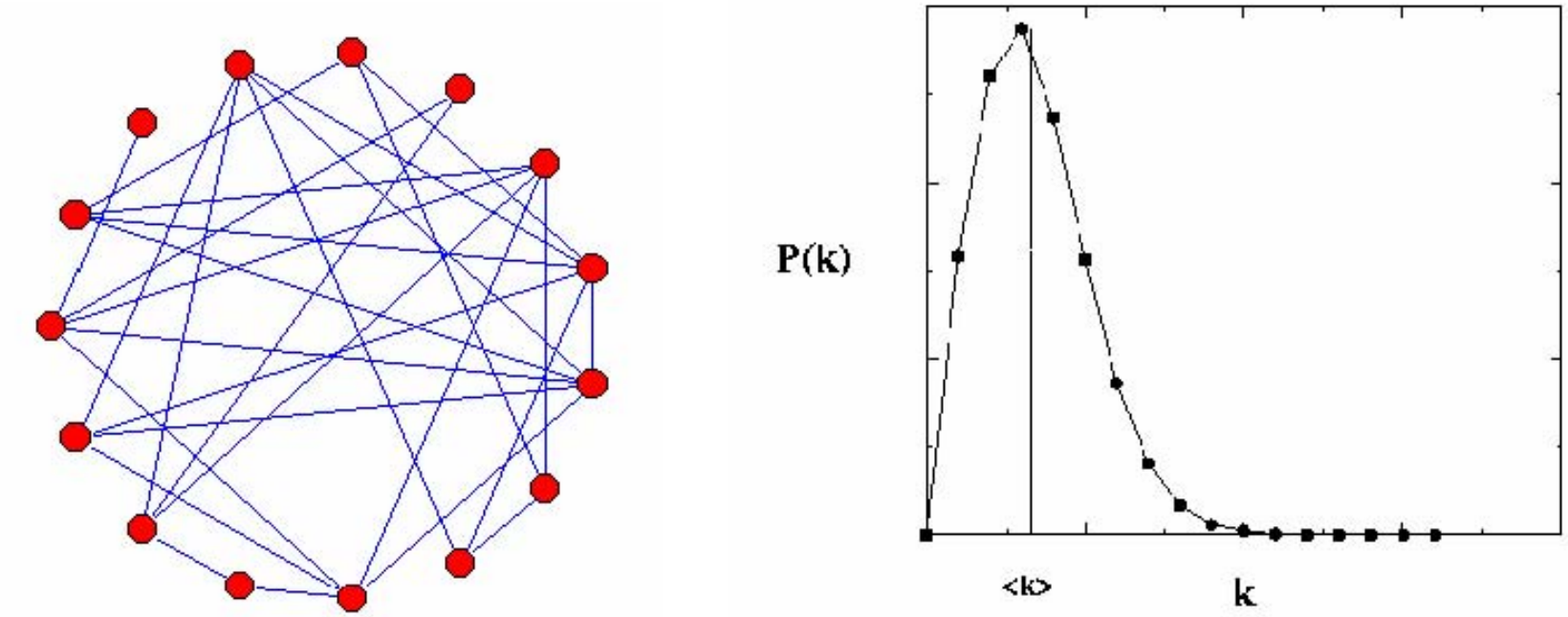


Illustration of Erdös-Rényi randon-graph network model

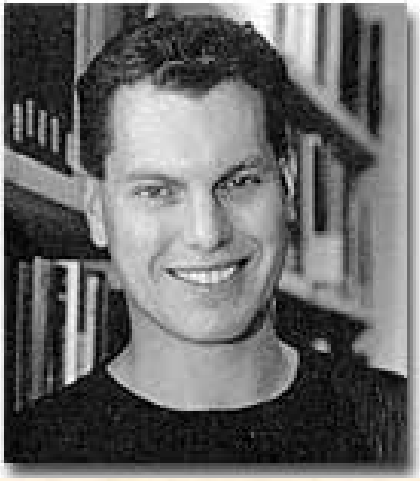
Small-World Networks

"Collective dynamics of

'small-world' networks"

--- Nature, 393: 440-442, 1998

D. J. Watts



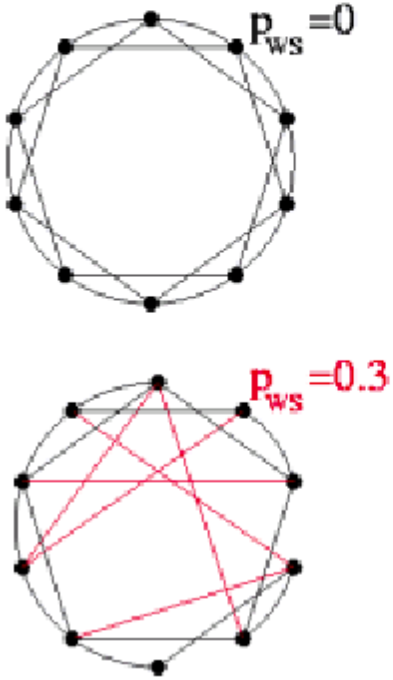
S. H. Strogatz

Cornell University

Small-World Networks

Watts-Strogatz

(Nature **393,** 440 (1998))



N nodes forms a regular lattice. With probability p, each edge is rewired randomly

Features:

(Similar to ER Random Graphs)

- **Connectivity**

  Poisson distribution

- **Homogeneity**

  all nodes have about the same number of edges

- **Non-growing**

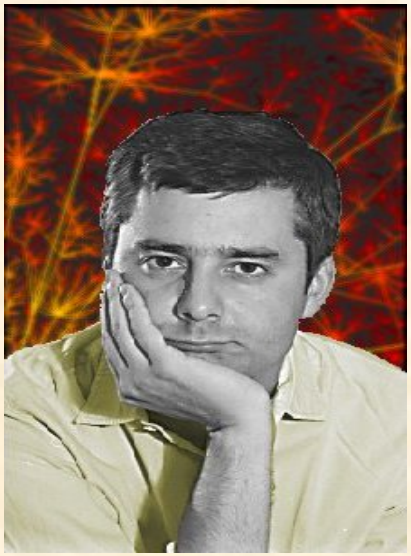
New: Small-World Property !

Scale-Free Networks

"Emergence of scaling in random networks"

Science, 286: 509 (1999)

A.-L. Barabási



R. Albert

Norte Dame University

Scale-Free Networks

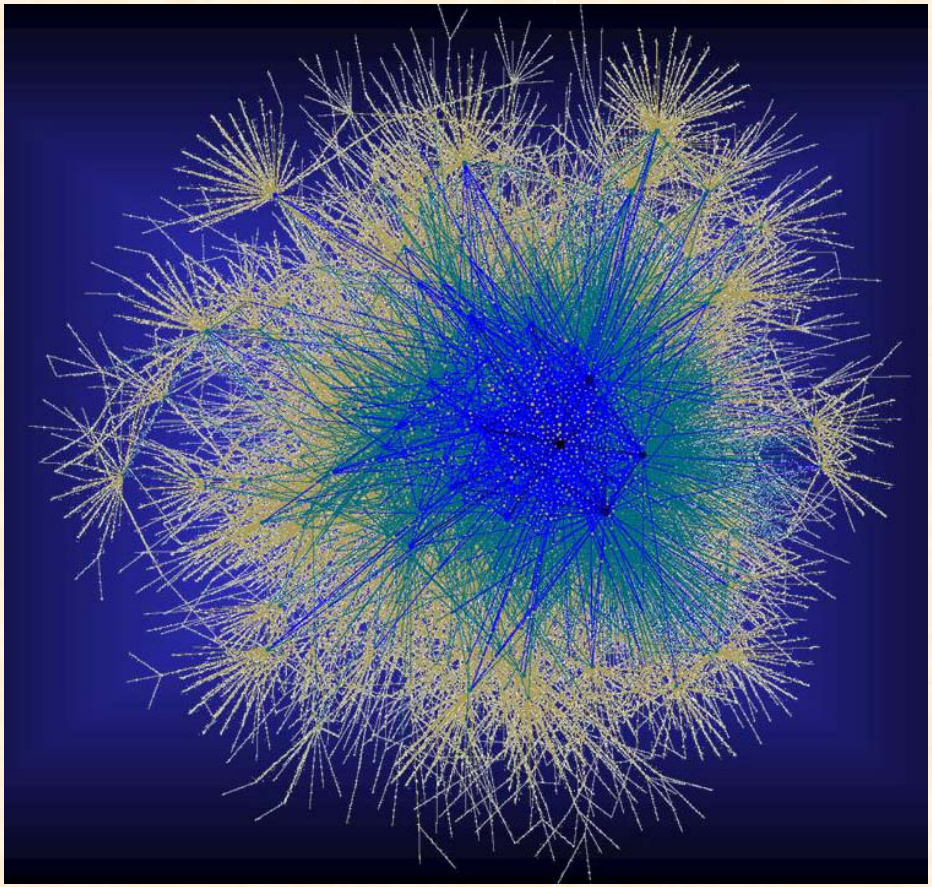
(**Barabasi-Albert**, **Science**, 1999)

(0) Start with a small connected network **(initialization)**

(i) Add new nodes (incremental growth):
With probability p, a new node is added into the network

(ii) Add new links (preferential attachment):
The probability q of the new node connect to an existing node is proportional to the degree of the existing node

Scale-Free Networks



Features:

- **Connectivity:**

  power-law form

  $$P(k) \sim k^{-\gamma}$$

- **Non-homogeneity:**

  very few nodes have many edges but most nodes have very few edges

- **Growing**

Complex Networks and Mathematics

International Congress of Mathematics (ICM)

22-28 August 2006, Madrid, Spain

Jon M Kleinberg (Comp. Sci.) received the Nevanlinna Prize for Applied Mathematics

He gave a 45-minute talk - "Complex Networks and Decentralized Search Algorithms"

J M Kleinberg, "Navigation in a small world," Nature, 2000

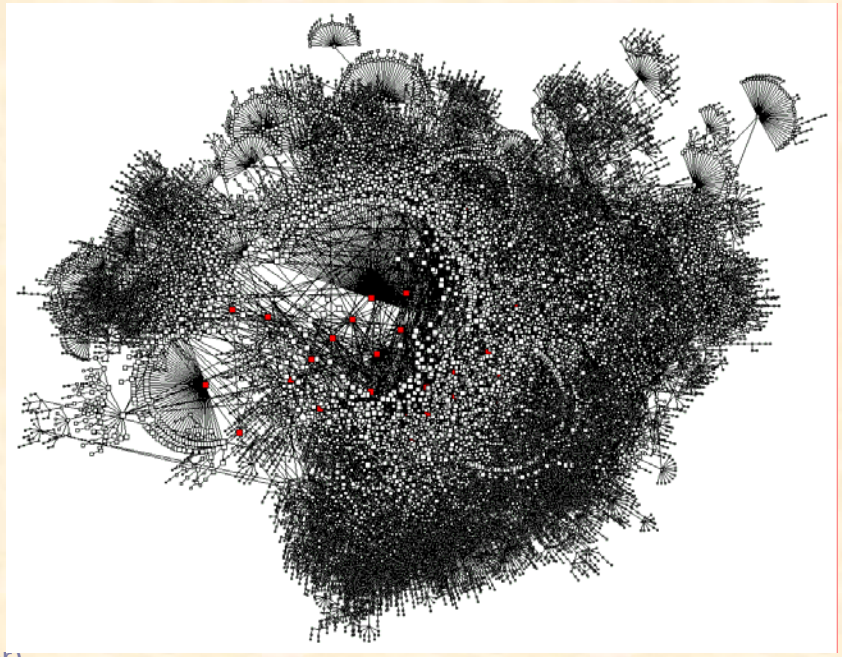
Cornell University

Comparison

	Degree Distribution	Average Distance	Clustering Coefficient	Homogeneity
Random Networks	Poisson	(Relatively) Large	Small	Homogeneous
Small-World Networks	Poisson	Small	Large	Homogeneous
Scale-Free Networks	Power-Law	Large	(Relatively) Small	Heterogeneous

Some Real Examples

- **Technology:**
  - World Wide Web
  - Internet
- **Social Science:**
  - 6 degree of separation
  - Movie actors network
  - Scientific cooperation



(Bradley Huffaker)

World Wide Web

R. Albert, H. Jeong, A.-L. Barabási, Nature, **401** 130 (1999)

Nodes: WWW documents
Links: URL links

800 million documents
(S. Lawrence, 1999)

ROBOT: collects all URL's found in a document and follows them recursively

World Wide Web

- **Average distance**
  - **Computed average distance *L* = 14**
  - **Diameter *L* = 19 → at most 19 clicks to any webpage**
- **Degree distribution**
  - **Outgoing edges:** $P(k) \sim k^{-\gamma}$ γ = 2.38~2.72
  - **Incoming edges:** $P(k) \sim k^{-\gamma}$ γ = 2.1

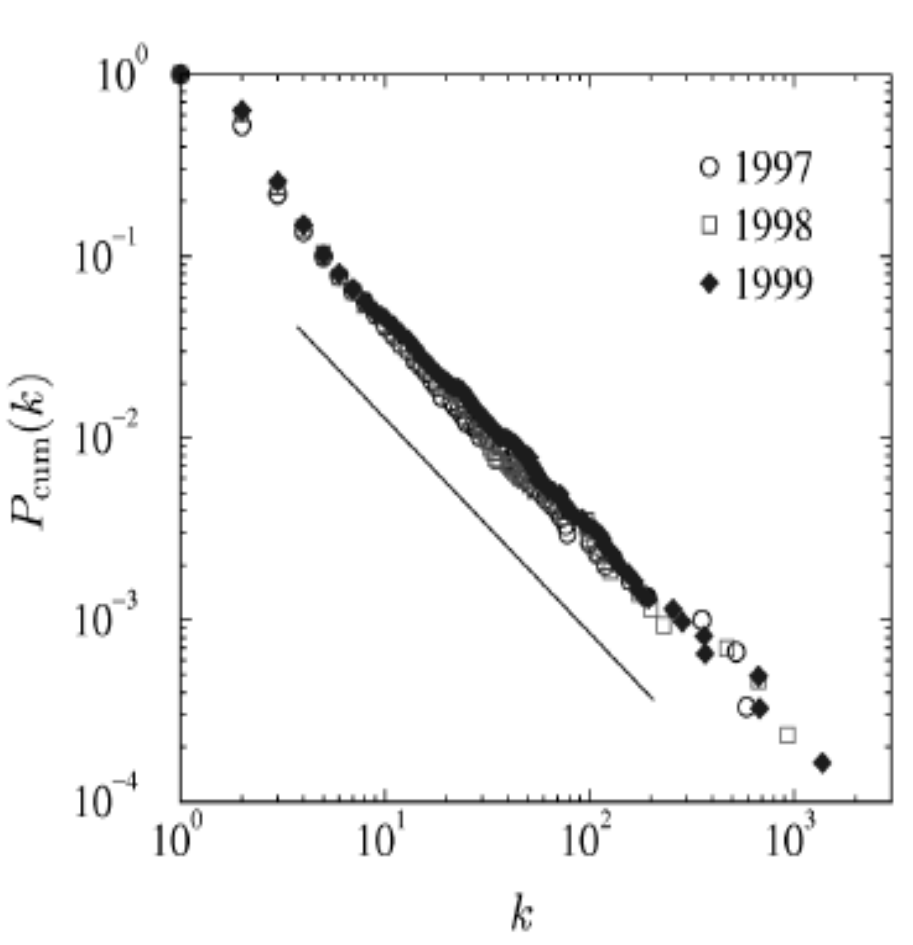
Internet

(Computed in 1995-1999, at both domain level and router level)

- **Average distance**
  - ***L*** = 4.0 (small)
  - **So, Internet is a small-world network**
- **Degree distribution**
  - **Obey power law:** $P(k) \sim k^{-\gamma} \quad \gamma = 2.2$
  - **So, Internet is a scale-free network**

→ **Small-world network is a good model for the Internet**

The Real Internet



$$P(k) \sim k^{-\gamma} \qquad \gamma = 2.2$$

(at the AS level)

Complex Networks:

More Examples …

Social Science

Small-World Experiment (1967)

Stanley Milgram, Harvard University

Question: How many acquaintances would it take to connect two randomly selected individuals in the USA ?

Answer: 6

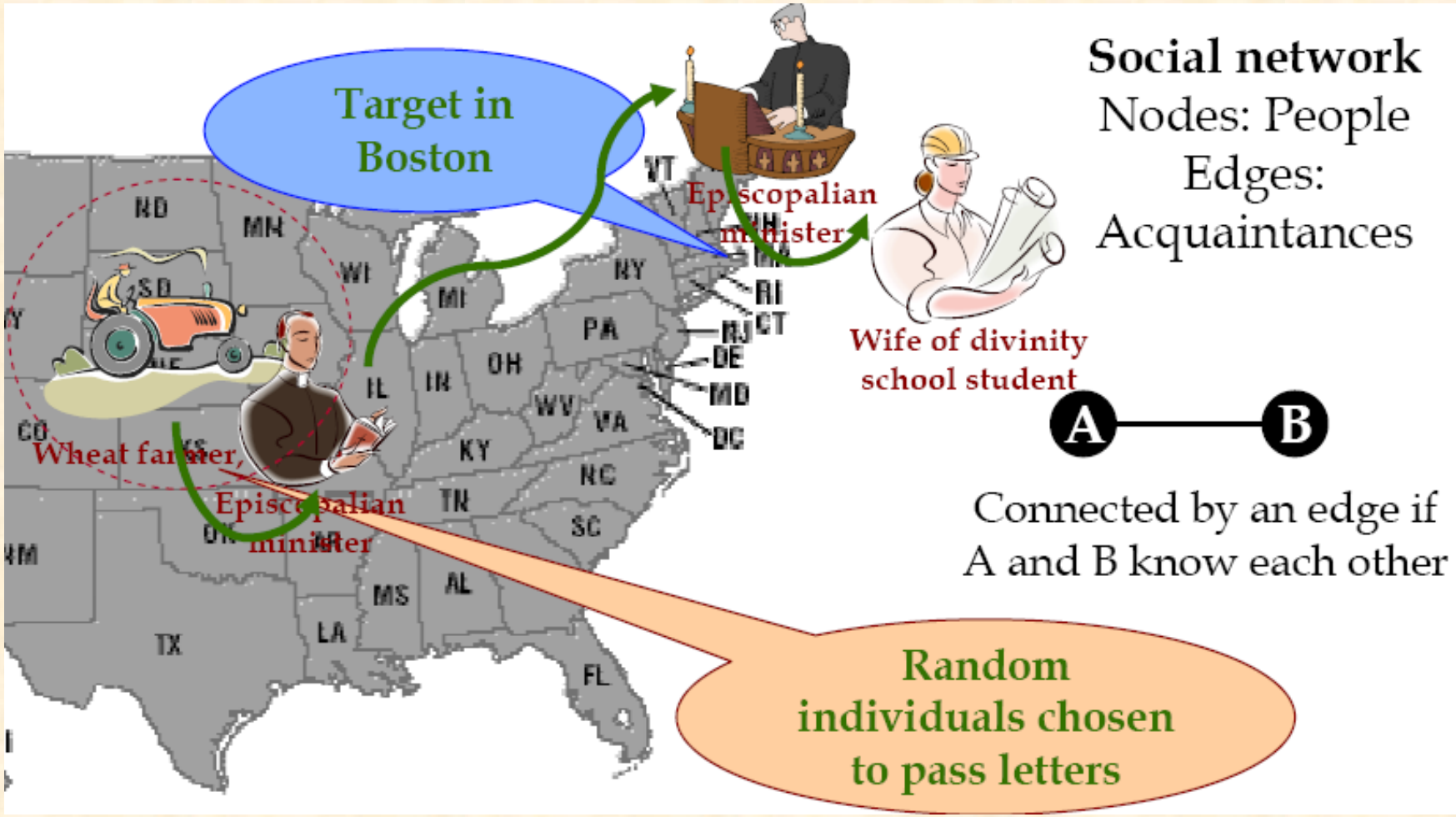
Alice → B → C → D → E → F → George

S. Milgram, The small world problem, *Psychology Today*, May 1967, 60-67.

Small World Experiment

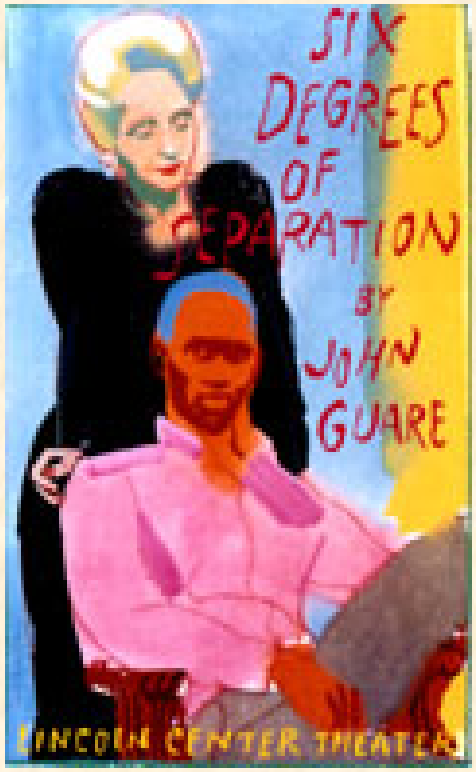
- A single “target” in Boston
- 300 initial “senders” in Boston and Omaha (Nebraska)
- Each sender forwarded a letter to a friend who was “closer” to the target
- In average, how many forwarding steps for a packet to arrive the target ?
  → **6 !**

The celebrated discovery of
“six degree of separation”



Six Degrees of Separation play

John Guare (1991)



(**Broadway, New York**)

In the play, Ousa tells her daughter:

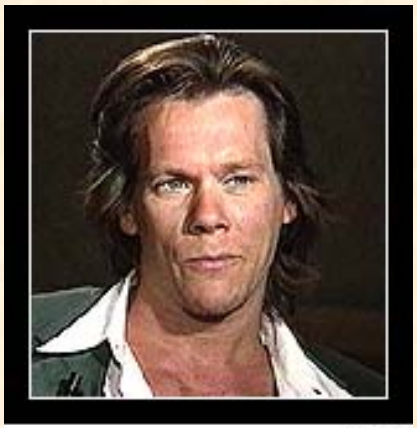
"Everybody on this planet is separated by only six other people. **Six degrees of separation …**"

Alice → **B** → **C** → **D** → **E** → **F** → **G**eorge

The Kevin Bacon Game

The average distance between **Kevin Bacon** and all other actors **= ?**

The braintrust behind this craze (CNN)



(CNN)

- In January 1994, Bacon's movie ***The Air Up There*** was airing on TV
- Then, three men were invited to appear on the CNN TV Show ***Stewart Show***, with Bacon; they tried to connect Bacon to any randomly-picked actor or actress in the Hollywood
- "*We are three men on a mission. Our mission is to prove…… that Bacon is God.*"
- **Result:** Every actor in the Hollywood could be connected to Kevin Bacon, with typically 2 to 3 connections

Six Degree Separation of the Kevin Bacon Game

Austin Powers:
The spy who
shagged me

Let's make
it legal

Robert Wagner

Mike Myers

Wild Things

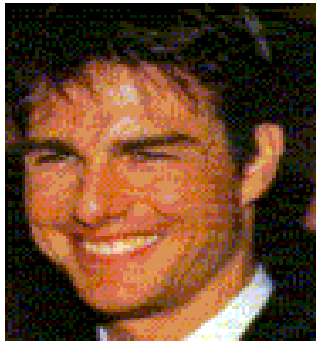
What Price Glory

Barry Norton

Tom Cruise

A Few
Good Man

Monsieur
Verdoux



L = 3.65 (small) and *C* = 0.79 (large) → **small-world !**

Bacon Number

Bacon numbers of actors and actresses (as of 1-1-2009)

http://oracleofbacon.org/cgi-bin/center-cgi?who=Kevin+Bacon

Kevin Bacon Number	# of People
0	1
1	2147
2	208553
3	634484
4	147122
5	9176
6	803
7	129
8	15

Total number of linkable actors: ***N*** = **1002430**

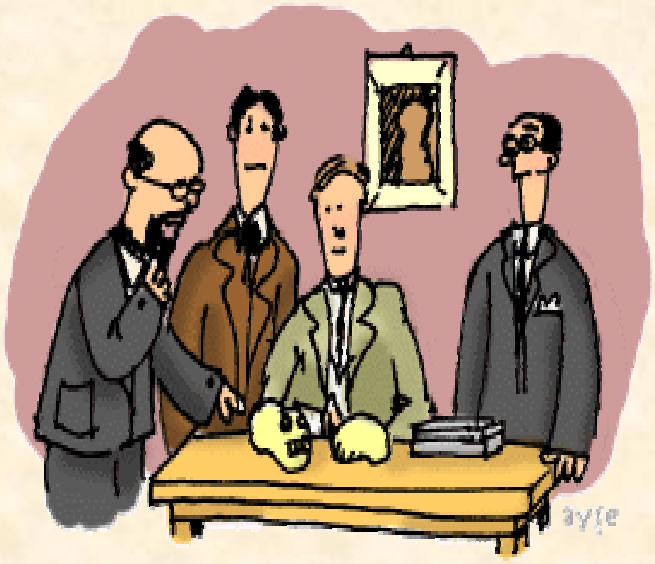
Average Kevin Bacon number: ***L*** = **2.956**

Scientific Collaboration

Nodes: authors (scientists)
Edges: writing joint papers

Web of Scientists

"Let's write a paper together"



$L = 4 \sim 9$

M.E.J. Newman (2001) and A.L. Barabási *et al.* (2001)

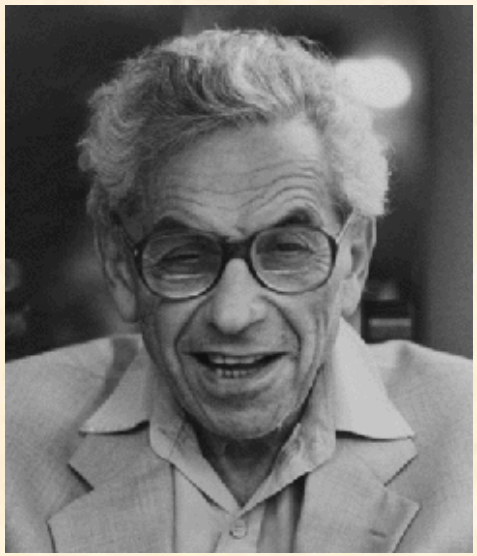
Pál Erdös (1913-1996)

- **"A mathematical genius of the first order, Paul Erdös was totally obsessed with his subject -- he thought and wrote mathematics for nineteen hours a day until the day he died. He traveled constantly, living out of a plastic bag, and had no interest in food, sex, companionship, art -- all that is usually indispensable to a human life."**
- Book: The Man Who Loved Only Numbers

  (Paul Hoffman, 1998)

- *"A mathematician is a machine that turns coffee into theorems"* -- *Erdös*

Pál Erdös (1913-1996)



- Erdös published > 1600 papers
  with > 500 coauthors
- Published 2 papers per month in 63 years

- Main contributions in modern mathematics: Ramsey theory, graph theory, Diophantine analysis, additive number theory, prime number theory, …

My Erdös Number is 2

P. Erdös

C. K. Chui

G. R. Chen

1. I. Borosh, C. K. Chui, and P. Erdos: ``On changes of signs in infinite series,'' Anal. Math., 4(1), 3-12, 1978.

2. C. K. Chui and G. R. Chen: Kalman Filtering with Real-Time Applications, Springer-Verlag (1st ed., 1987; 2nd ed., 1991; 3rd ed., 1999; 4th ed., 2009)

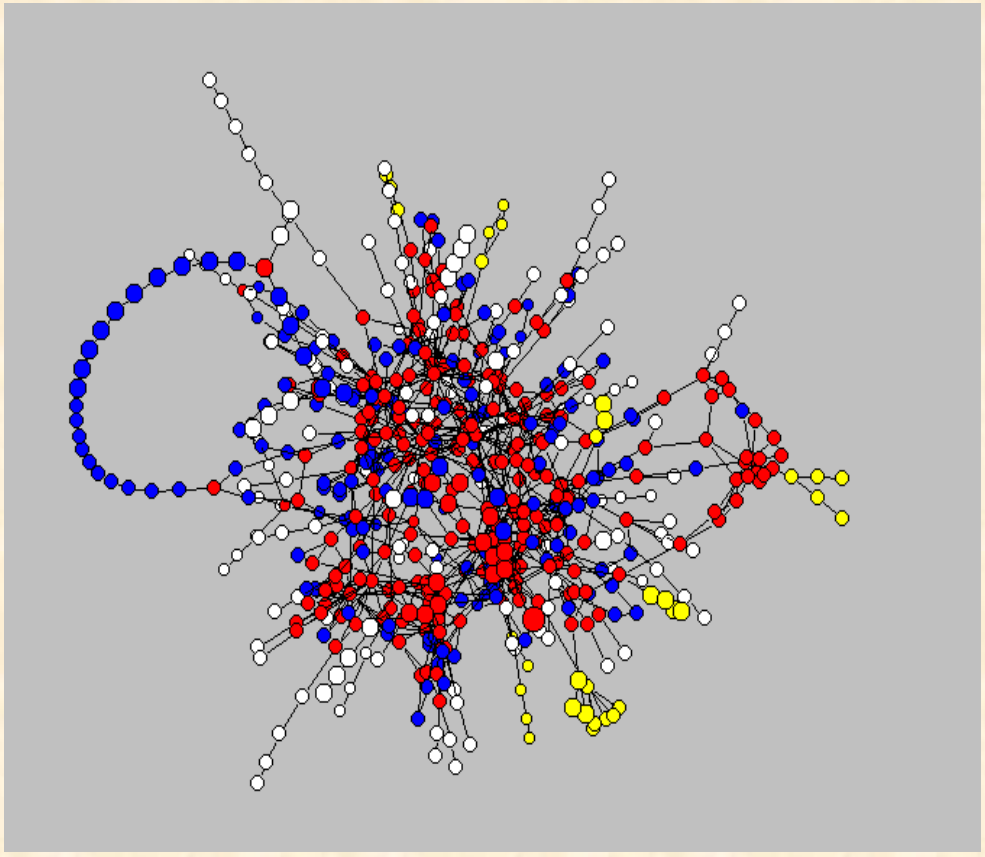
The Erdös number network is a small-world (and scale-free) network !!

Complex Networks:

Even More Examples …

Metabolic Networks

-- The metabolic network of 43 organisms is ***scale-free***



Nodes: chemicals (substrates)

Edges: bio-chemical reactions

*And, is also **small-world** with L = 3*

H. Jeong *et al.*,
Nature, 407: 651-654, 2000

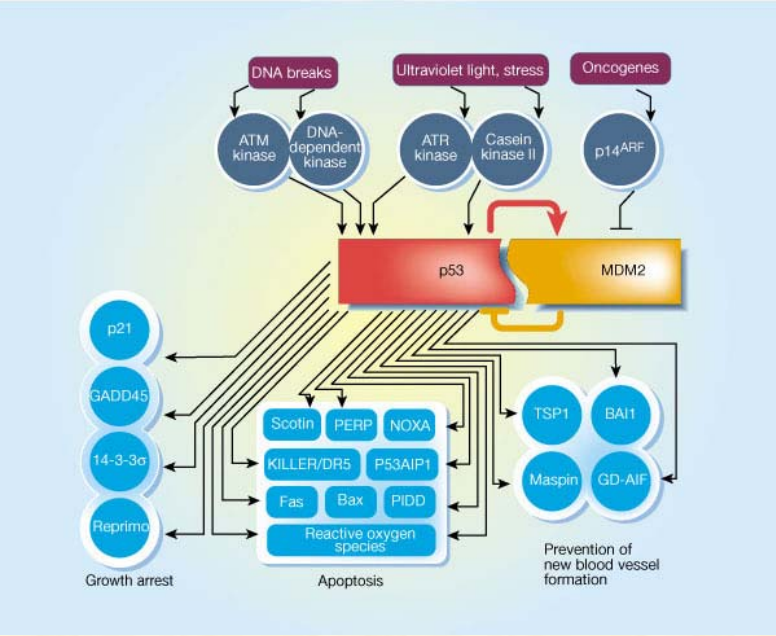
Nature **408** 307 (2000)

news and views feature

Surfing the p53 network

Bert Vogelstein, David Lane and Arnold J. Levine

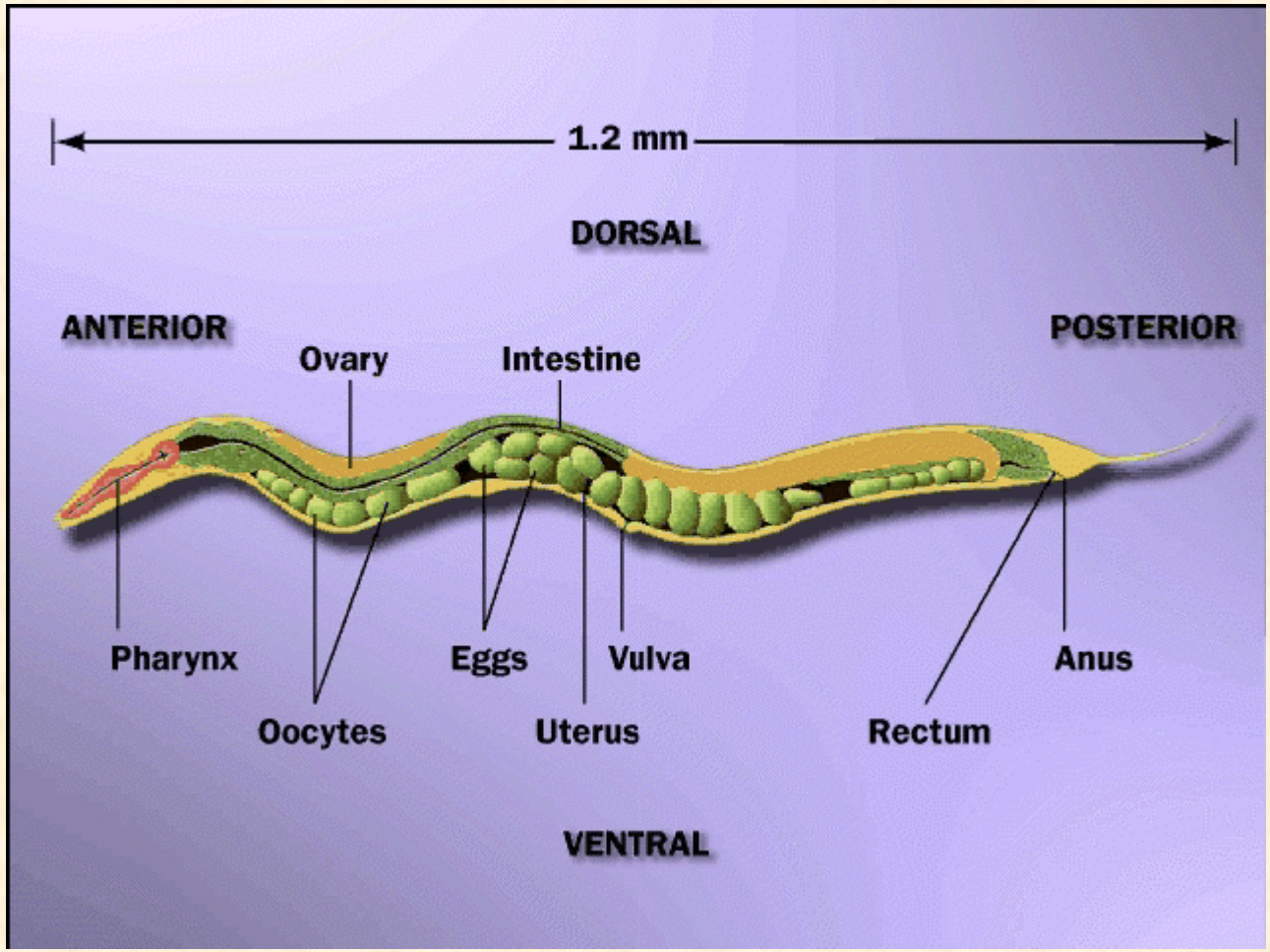
The p53 tumour-suppressor gene integrates numerous signals that control cell life and death. As when a highly connected node in the Internet breaks down, the disruption of p53 has severe consequences.



p53 gene is perhaps the most important discovery in cancer research

"One way to understand the p53 network is to compare it to the Internet. The cell, like the Internet, appears to be a 'scale-free network'."

C. Elegans Neural Network: Small-World Network

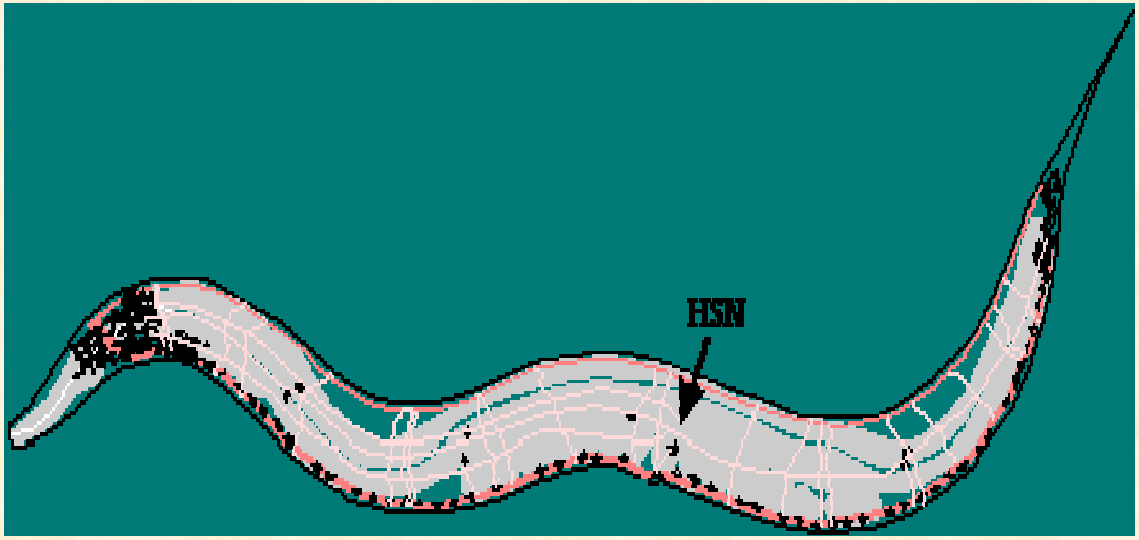


C. Elegans anatomy

http://www.imsc.res.in/~sitabhra/research/neural/celegans/index.html

C. Elegans Neural Network

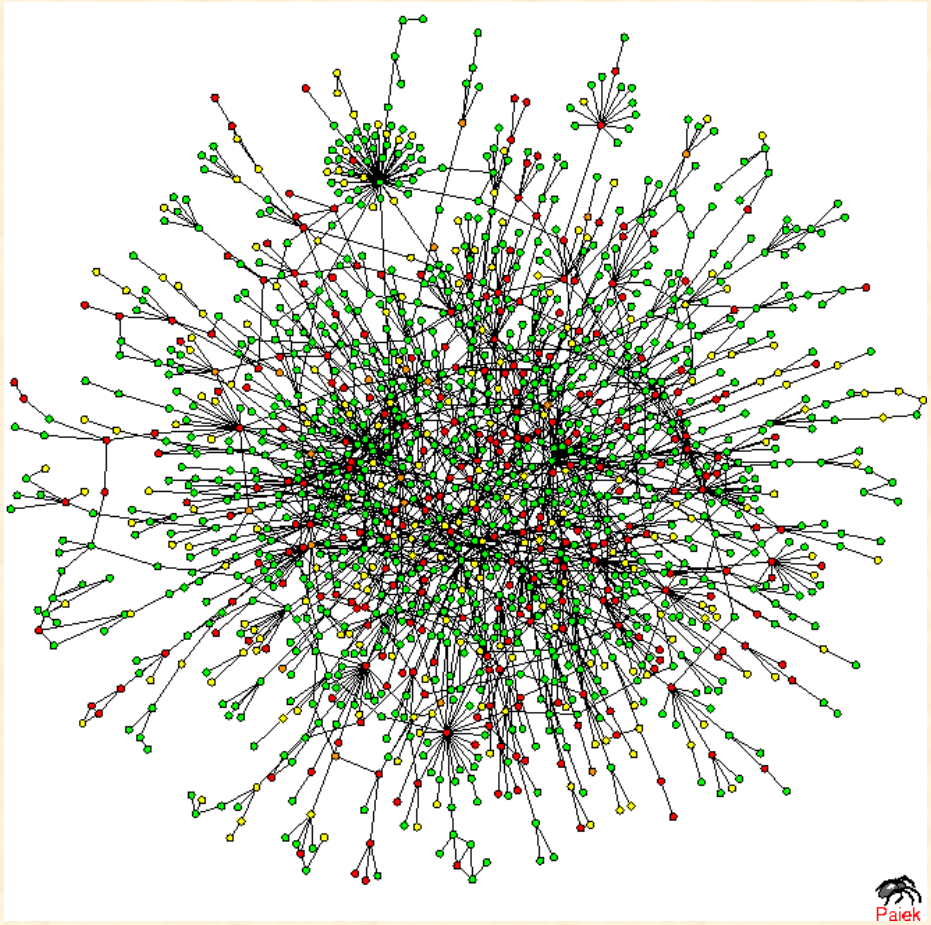
Nodes: **neurons** **Edges**: **synapses**



The 302 neurons of C. Elegans worm (black circles) make ~7000 synapses. The positions of each neuron and synapse are known.

L = **2.65** and C = **0.28**

Yeast Protein Interaction Network: Scale-Free Network

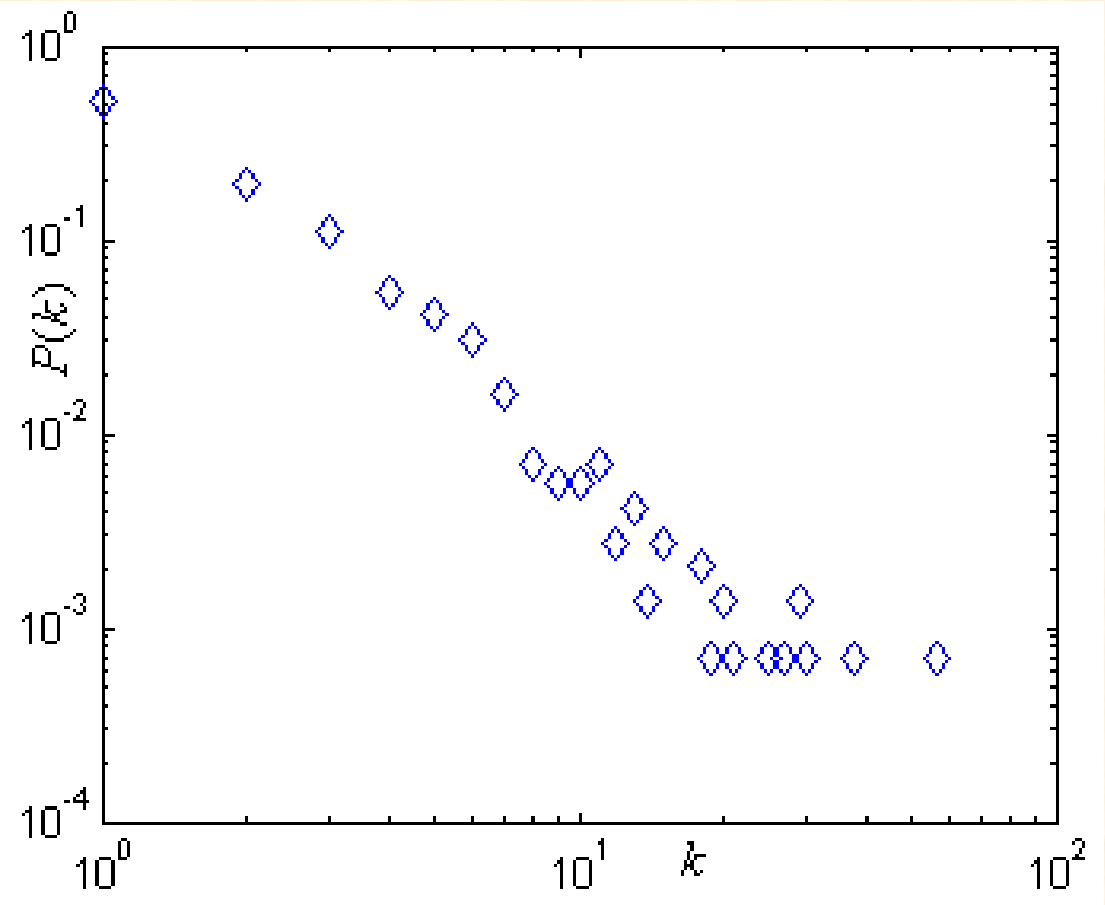


Red: Lethal
Green: non-lethal
Orange: slow growth
Yellow: unknown

Nodes: proteins
Edges: physical interactions

H. Jeong *et al.*, ***Nature***, 411: 41-42, 2001

Yeast Protein Interaction Network: Scale-Free Network



$$P(k) \sim k^{-\gamma}$$

$$\gamma = 2.5$$

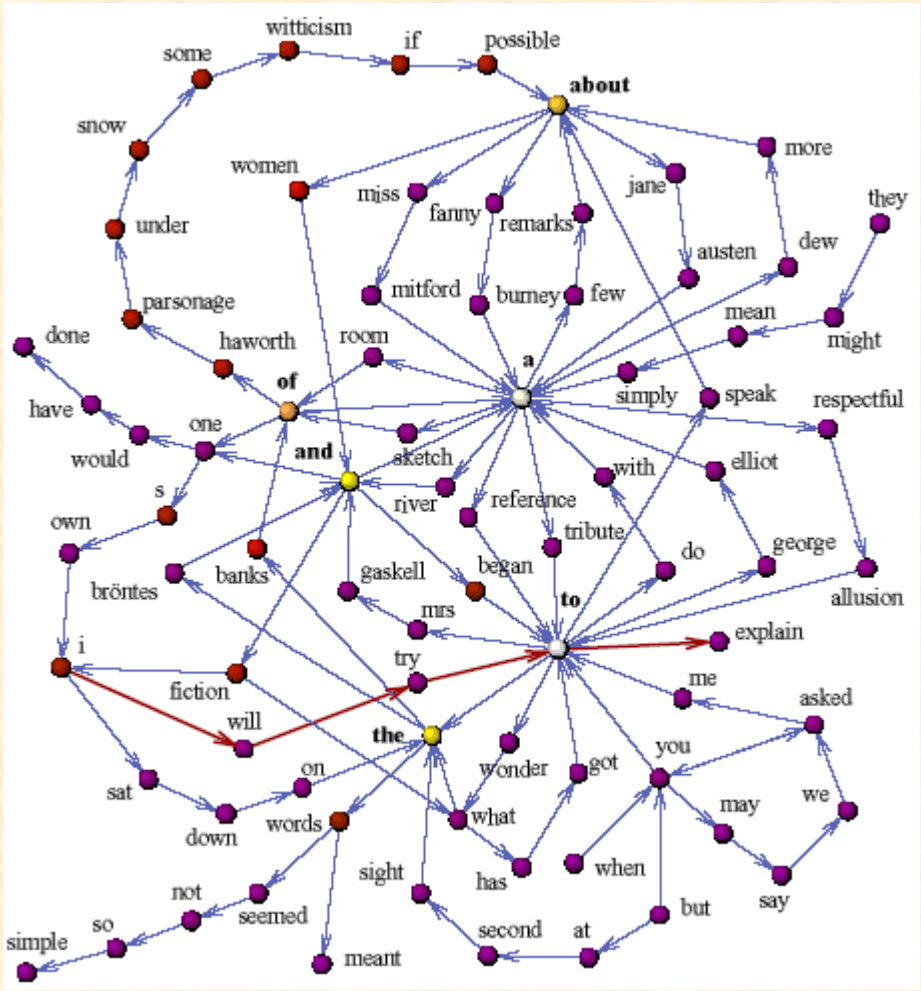
← Degree distribution

Language

- Words in human language interact like a **small-world network**
- Human brain can memorize 10^{4}~10^{5} words (Romaine, 1992)
- Average distance between two words

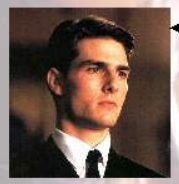
  d = 2~3 (small world)
- Degree distribution obeys a scale-free power-law:

  $P(k) = k^{-\gamma}$, $\gamma = 3$



ACTOR CONNECTIVITIES

nodes: actors
edges: casted jointly

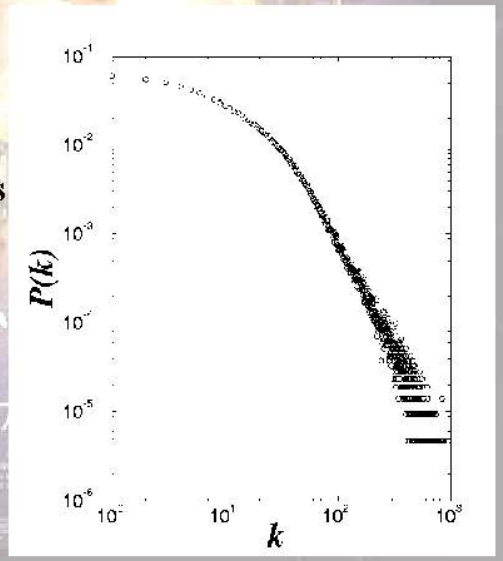


Days of Thunder (1990)
Far and Away (1992)
Eyes Wide Shut (1999)

N = 212,250 actors
⟨k⟩ = 28.78

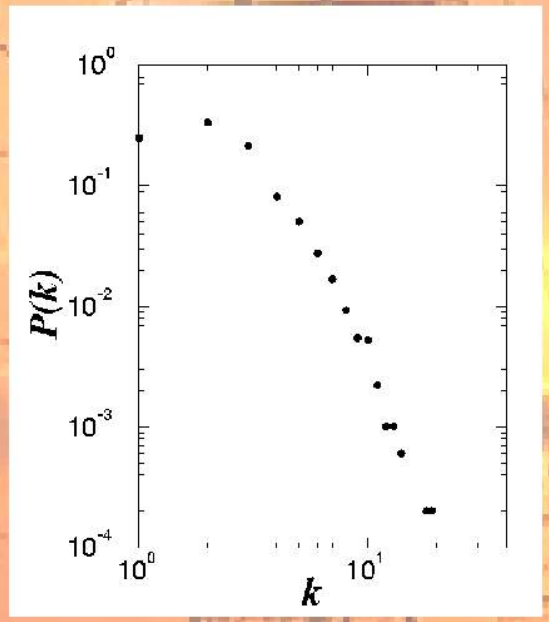
$P(k) \sim k^{-\gamma}$

$\gamma = 2.3$



ELECTRICAL POWERGRID

nodes: generators, transformers
edges: transmission lines



The electrical power grid of the western United States

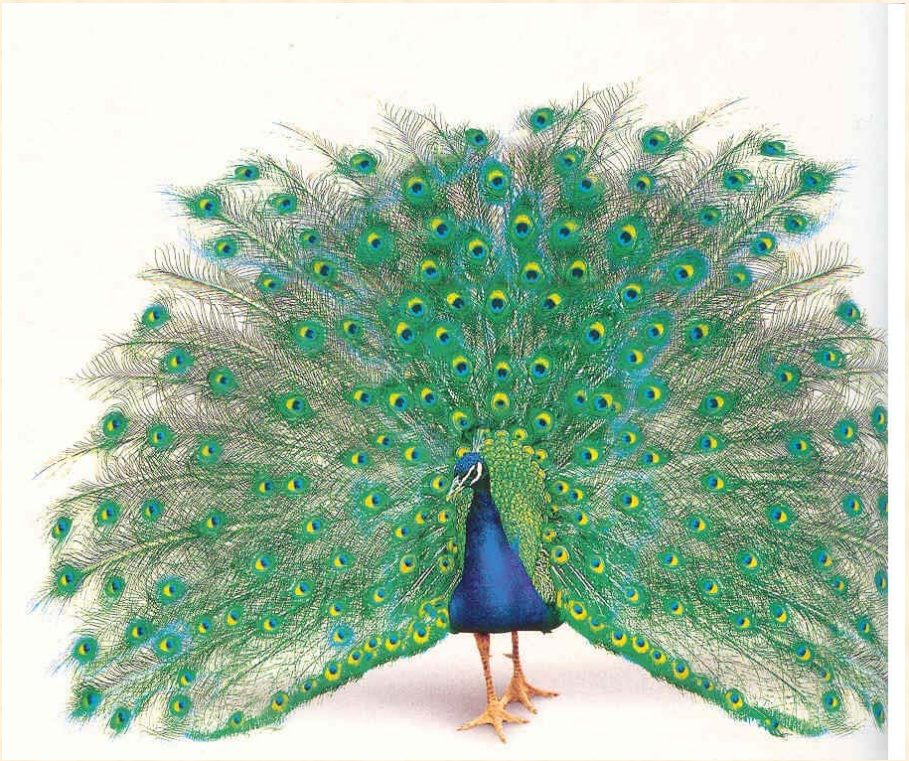
$\gamma \cong 4$

A random-graph model for scale-free network generation

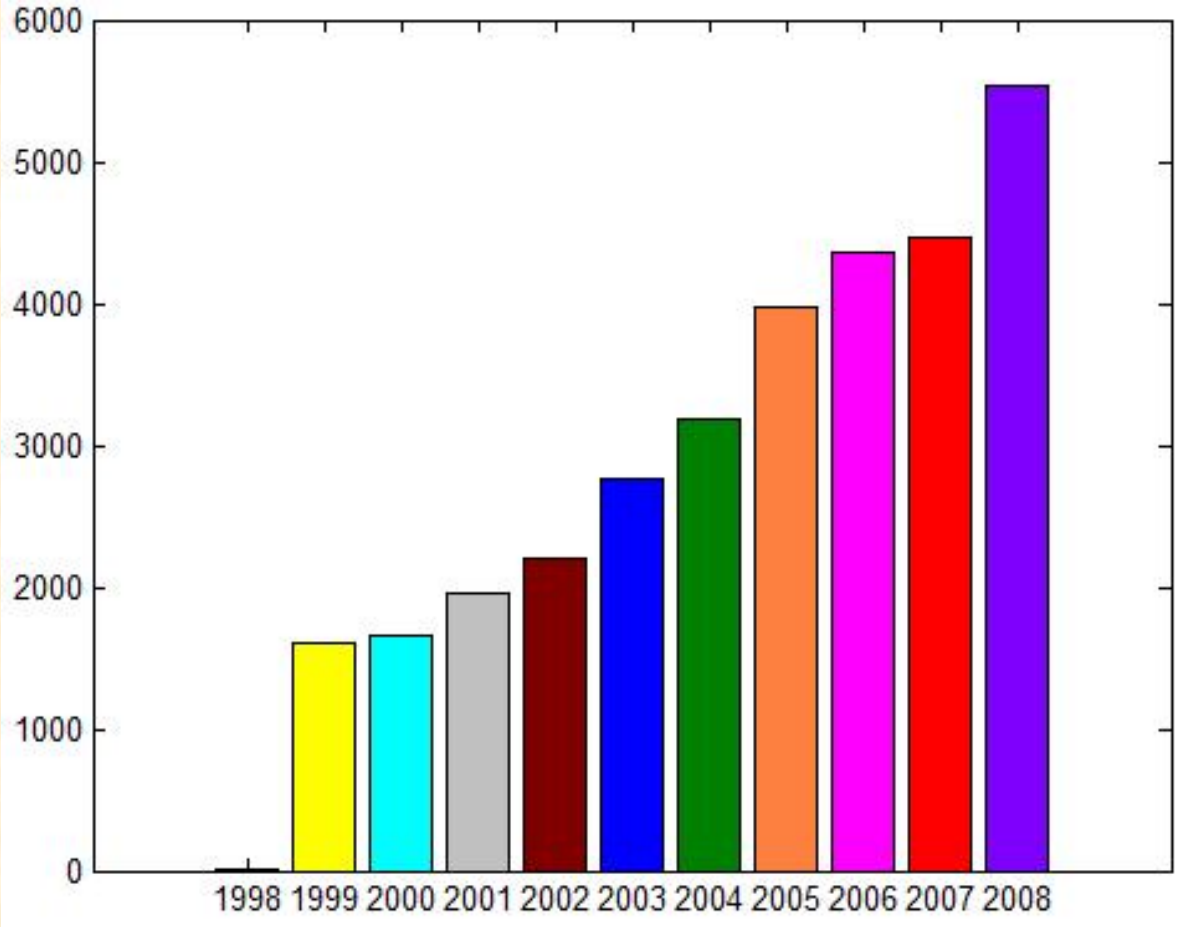
W. Aiello, F. Chung and L. Y. Lu (2001)

- Start with **no** nodes and **no** edges
- At each time, a new node is added with probability p
- With probability q, a random edge is added to the existing nodes
- Here, $p + q = 1$
- Theorem: The degree distribution of the network so generated satisfies a power law with $\gamma = 1 + 1/q$
- If $\frac{1}{2} < q < 1$ then $2 < \gamma < 3$

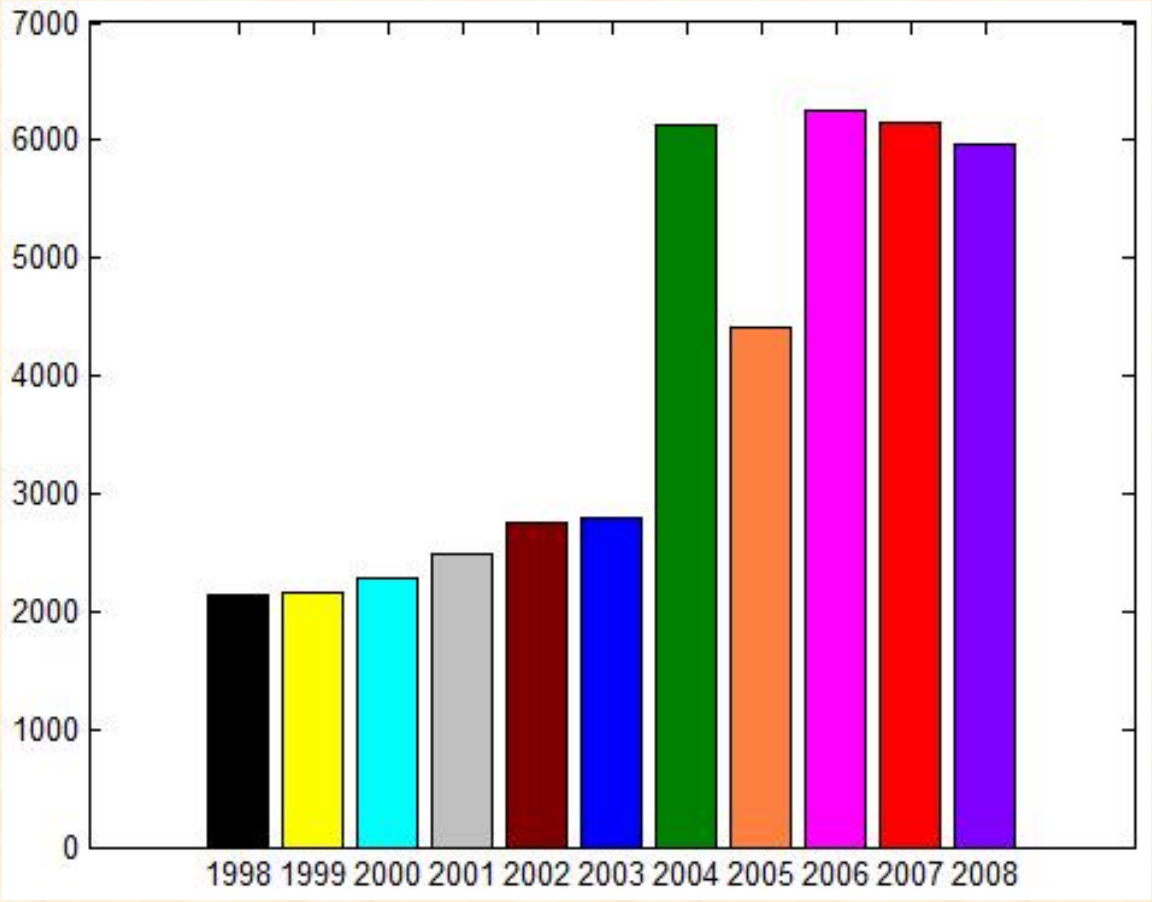
So much for today …



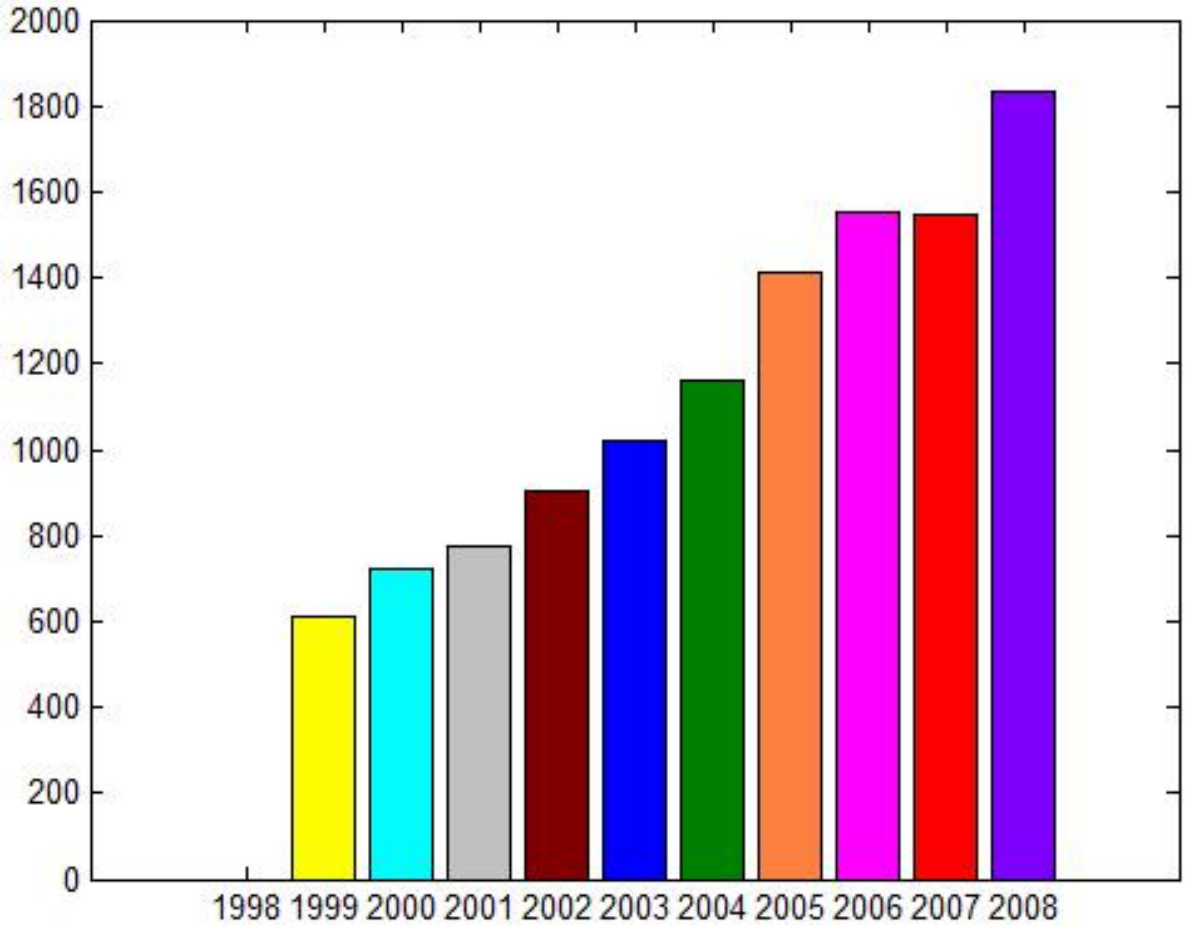
SCI papers: Complex Networks



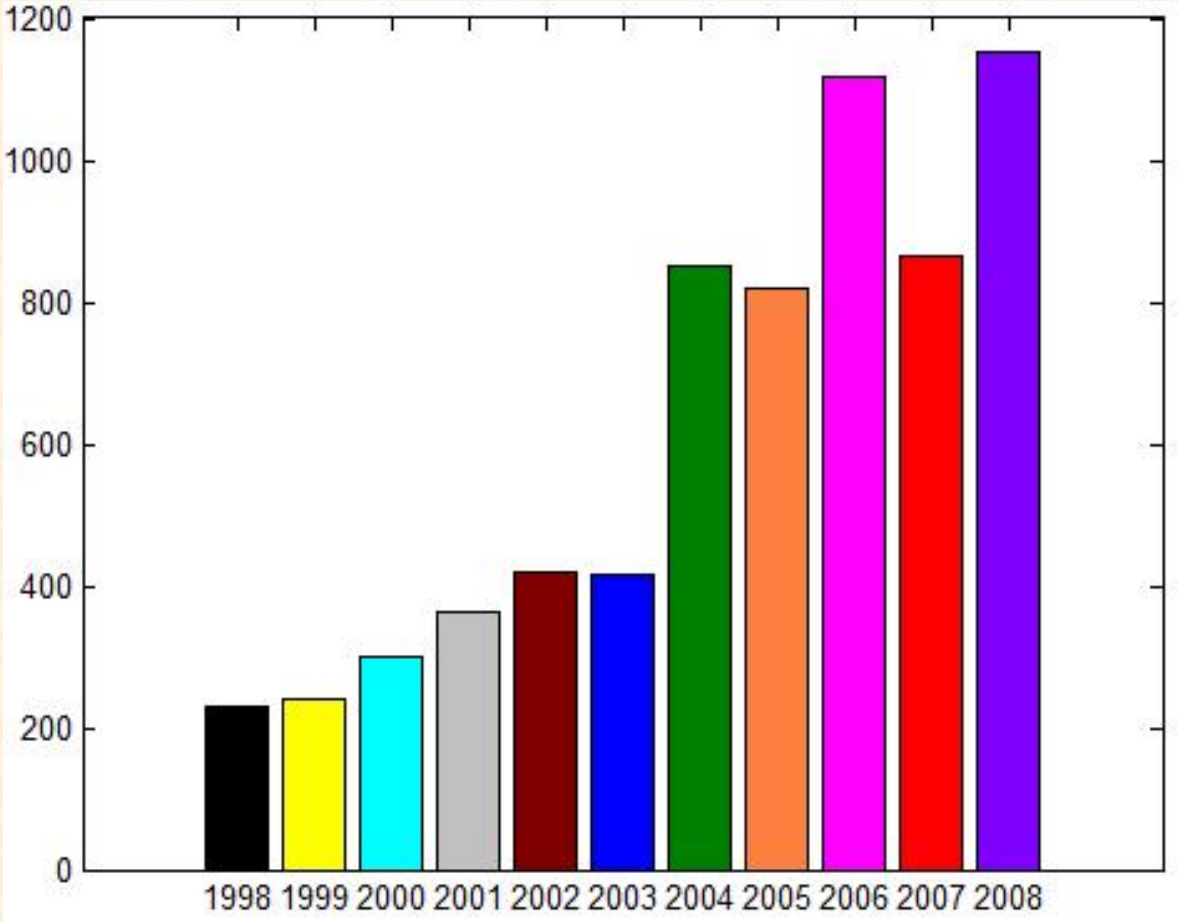
EI papers：Complex Networks



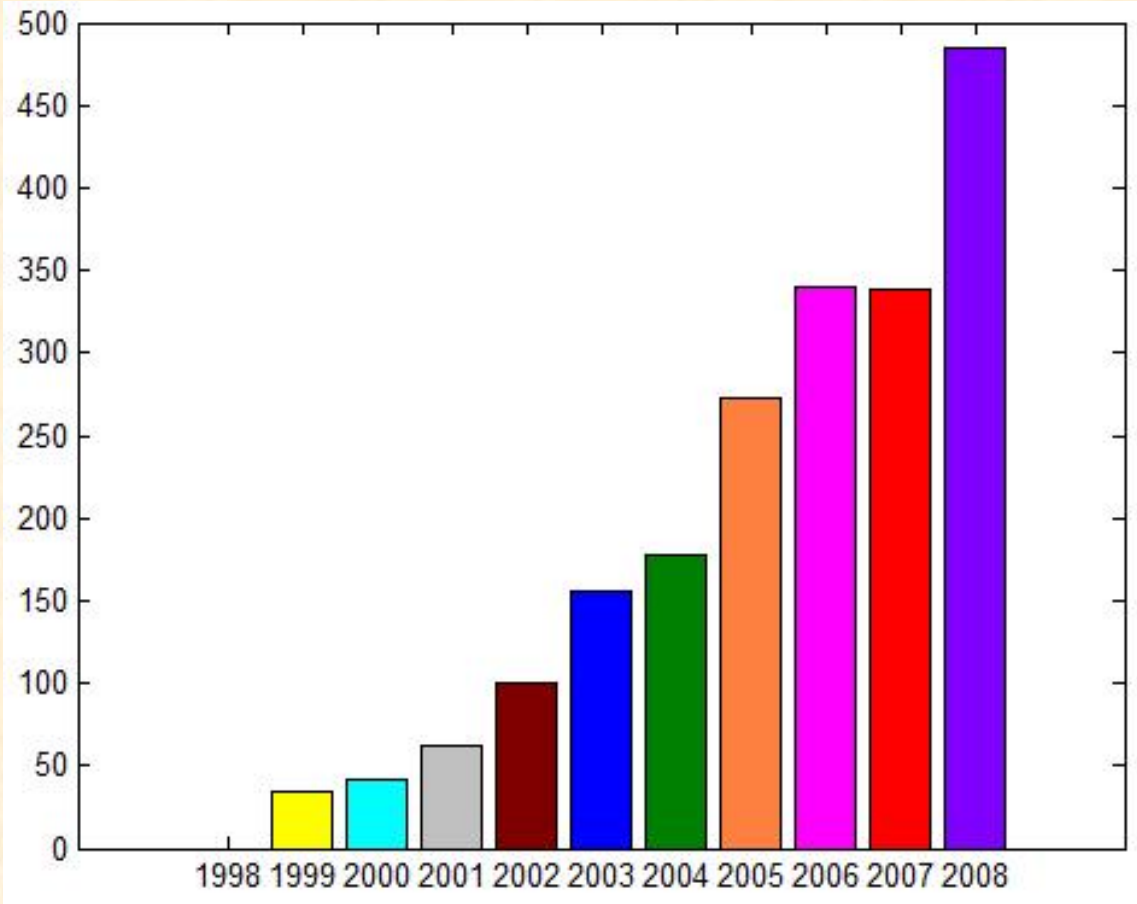
SCI papers: Small-World Networks



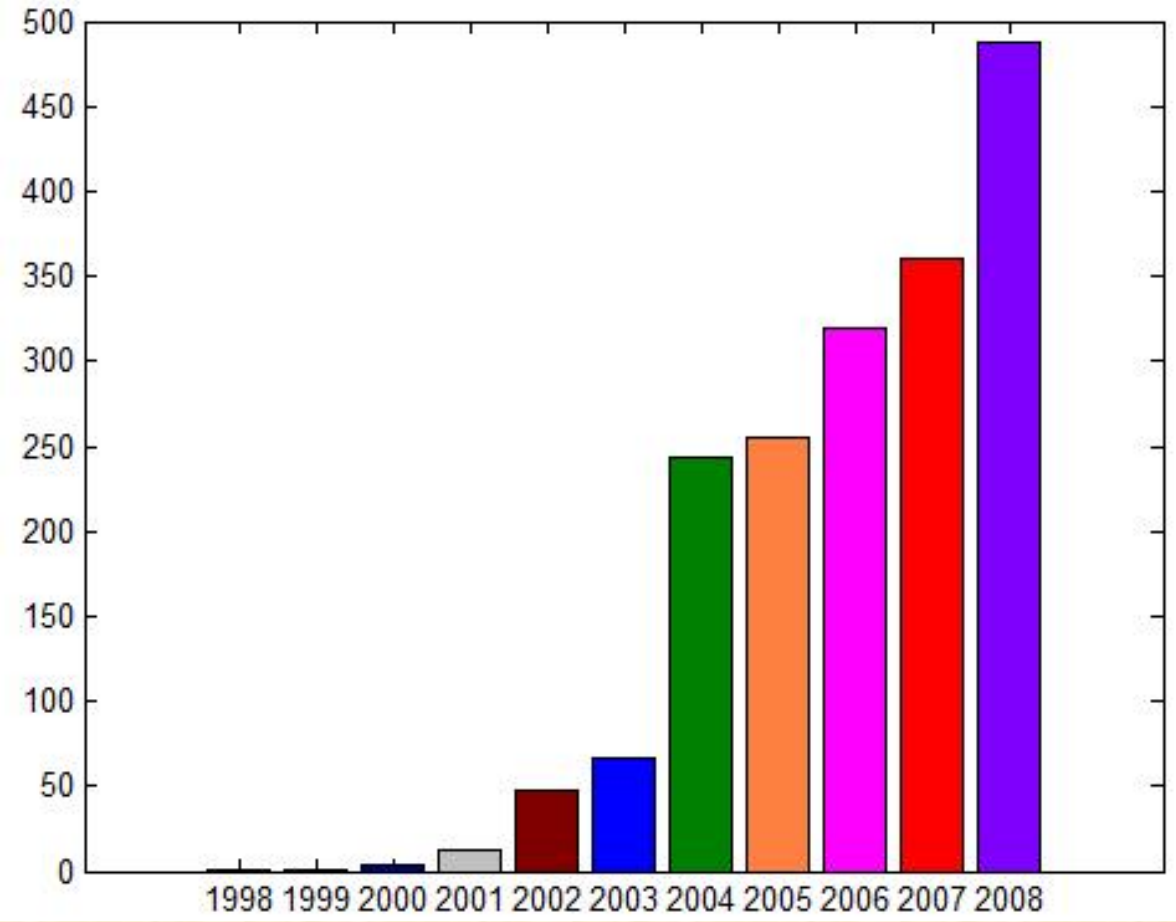
EI papers： Small-World Networks



SCI papers: Scale-Free Networks



EI papers： Scale-Free Networks



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Thank You !