

# Multi-Stable Cylindrical Lattices

***A. Pirrera, X. Lachenal, S. Daynes,  
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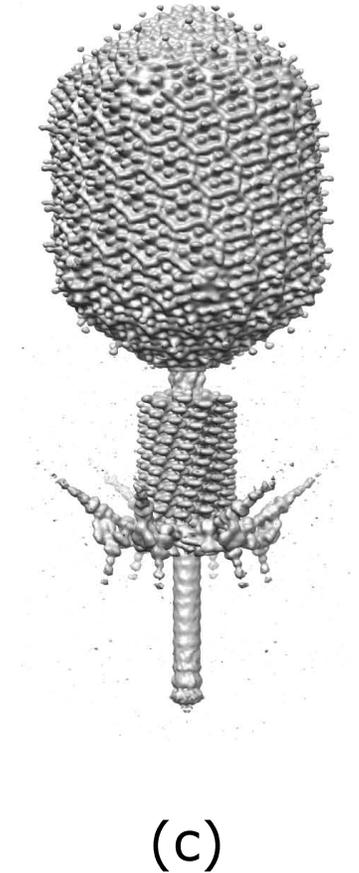
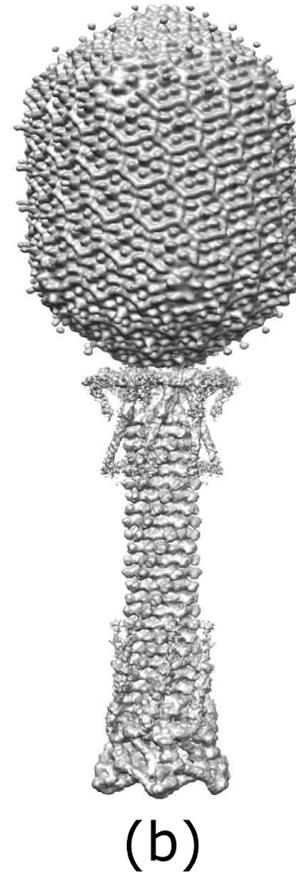
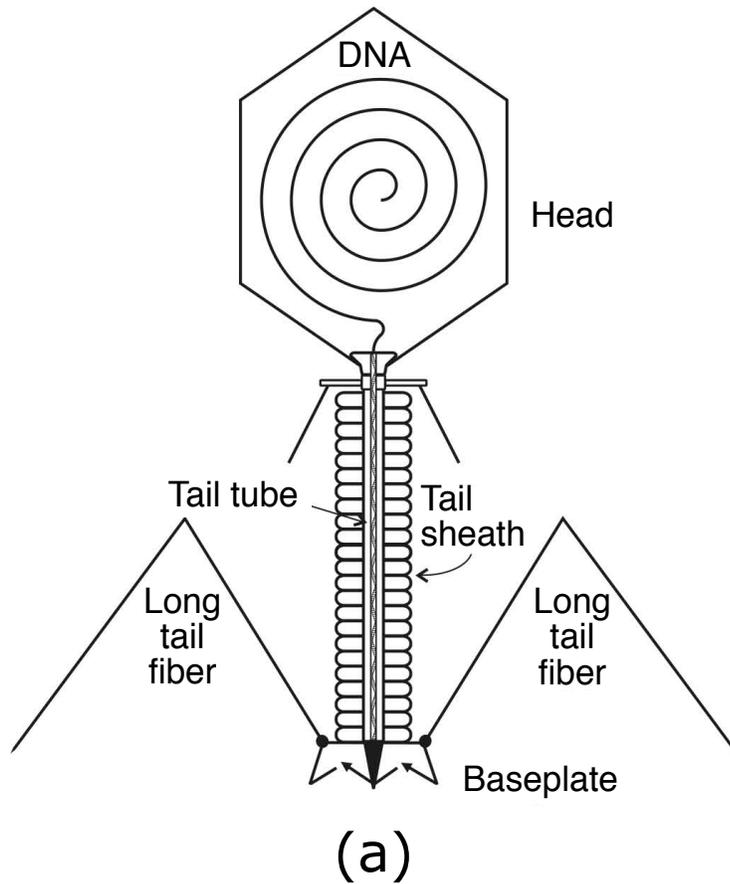
A. Pirrera, X. Lachenal, S. Daynes, P.M. Weaver, I.V. Chenchiah,  
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Physics of Solids*, Volume 61, Issue 11, 2013.

# Content

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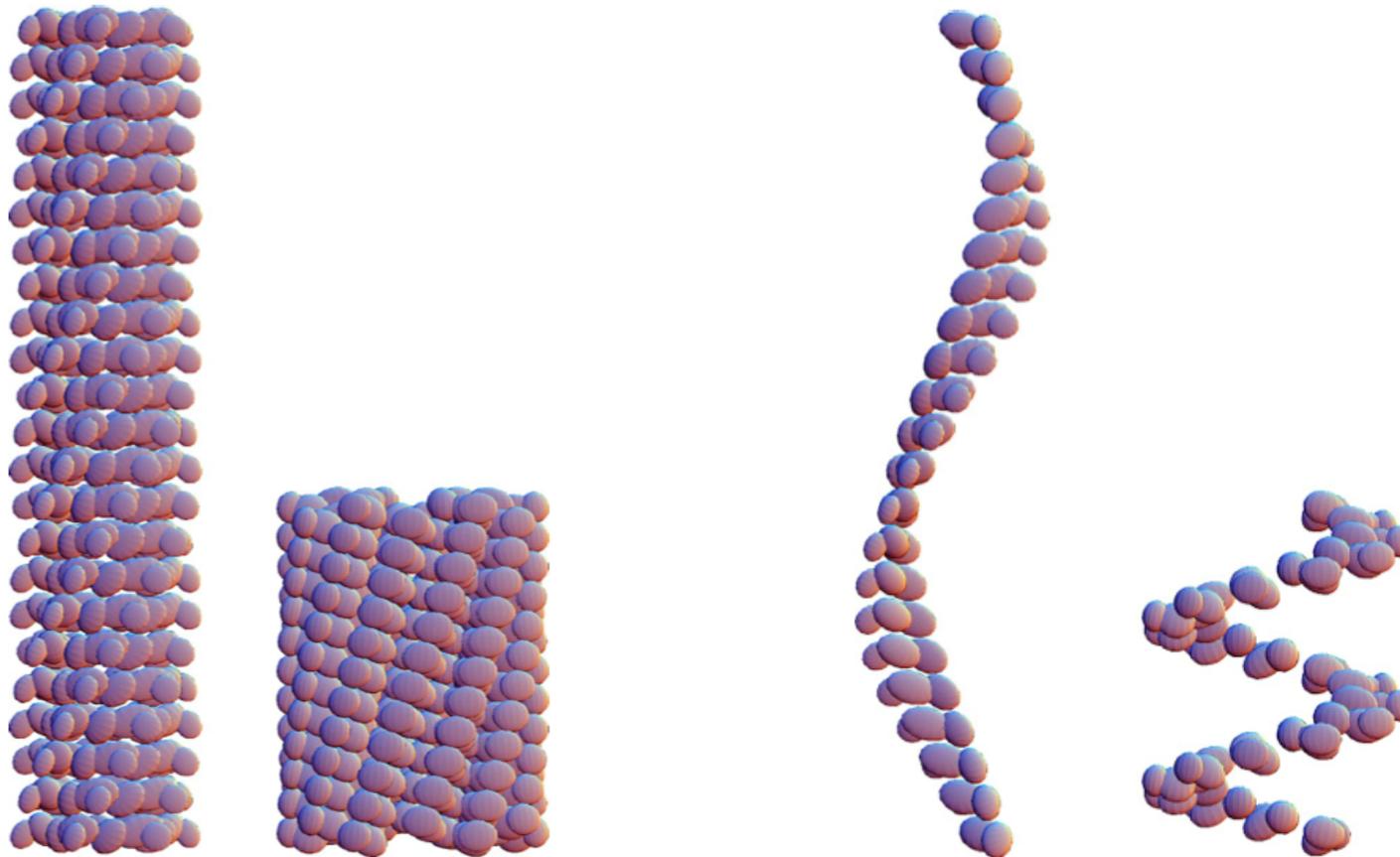
- Inspiration and Motivation
- The device
- Its kinematics and elasticity
- Results, conclusions and take-away message

# Inspiration and motivation: Bacteriophage T4



(a) Schematic representation and model of Bacteriophage T4 prior to (b) and upon (c) host cell attachment. [Leiman et al. \(2010\)](#)

# Inspiration and motivation: Bacteriophage T4



Visualisation of the ([left](#)) tail sheath and ([right](#)) one of the six main helices of Bacteriophage T4 in the extended and contracted states. [Falk and James \(2006\)](#).

# Inspiration and motivation

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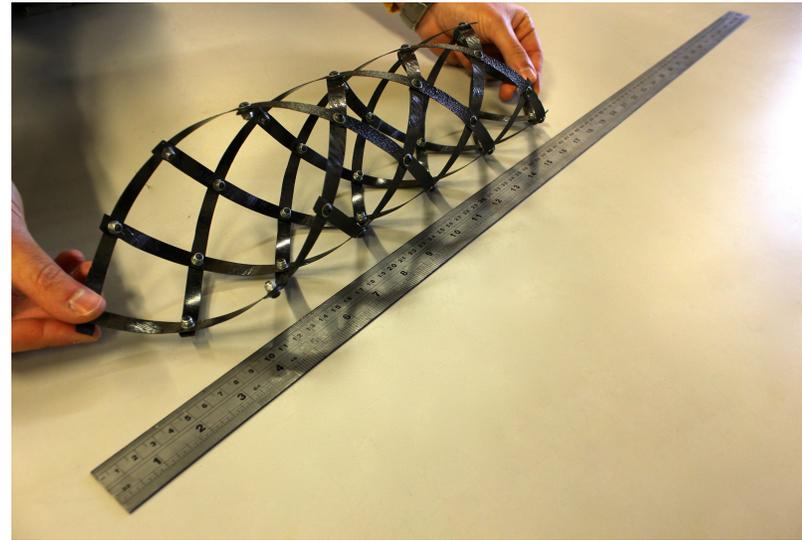
Why?

# Inspiration and motivation

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- A multi-stable cylindrical structure is in itself of engineering interest.
- We are motivated by a much broader vision, namely of translating molecular mechanisms into engineer-able mechanisms.

# The device



# The device

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- On the macroscale we have achieved this by using composite materials in a **lattice** arrangement to exploit interplay between **pre-stress**, **material properties** and **structural geometry**.
- Engineering applications of such multi-stable structures include **tailorable non-linear springs**, **non-linear dampers** and **shock-absorbers**, **exoskeletons**, and **deployable structures**, especially for space applications.

# Kinematics and elasticity

## Strain Energy

$$\begin{aligned} \Pi = & \frac{\ell_+}{2R^2} \begin{bmatrix} \cos^2 \vartheta_+ - \kappa_{11}^+ R \\ \sin \vartheta_+ \cos \vartheta_+ - \kappa_{12}^+ R \end{bmatrix}^T \begin{bmatrix} D^+ & D_{16}^{*+} \\ D_{16}^{*+} & D_{66}^{*+} \end{bmatrix} \begin{bmatrix} \cos^2 \vartheta_+ - \kappa_{11}^+ R \\ \sin \vartheta_+ \cos \vartheta_+ - \kappa_{12}^+ R \end{bmatrix} \\ & + \frac{\ell_-}{2R^2} \begin{bmatrix} \cos^2 \vartheta_- - \kappa_{11}^- R \\ -\sin \vartheta_- \cos \vartheta_- - \kappa_{12}^- R \end{bmatrix}^T \begin{bmatrix} D^- & D_{16}^{*-} \\ D_{16}^{*-} & D_{66}^{*-} \end{bmatrix} \begin{bmatrix} \cos^2 \vartheta_- - \kappa_{11}^- R \\ -\sin \vartheta_- \cos \vartheta_- - \kappa_{12}^- R \end{bmatrix} \end{aligned}$$

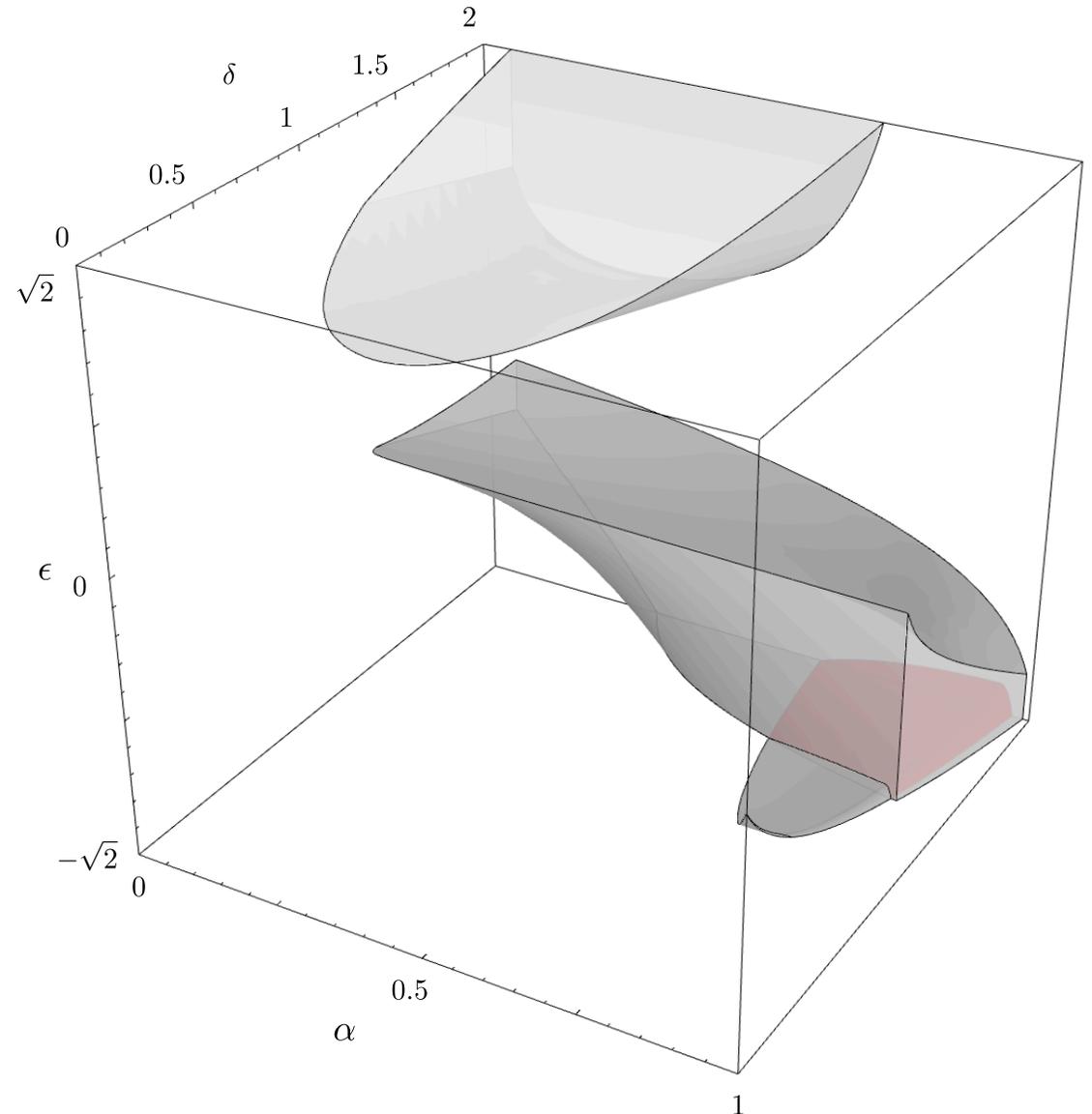
$$\cos \vartheta_- = \frac{1}{2\ell_-} R + \frac{\ell_-^2 - \ell_+^2}{2\ell_-} \frac{1}{R},$$

$$\cos \vartheta_+ = \frac{1}{2\ell_+} R + \frac{\ell_+^2 - \ell_-^2}{2\ell_+} \frac{1}{R}.$$

# Design space investigation & Conclusions

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- We demonstrate computationally that **multi-stability is a robust phenomenon**.
- We also show analytically that **it is possible to choose the design variables so that the energy is independent of the radius**, thus resulting in every state of the structure being stable.
- Exploitation is the next step...



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Questions?