Hyperinflation and Explosive Behaviour in the General Price Level *

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Abstract

The study uses an indirect statistical approach to detect whether prices diverted from the market fundamentals in the hyperinflation episodes that took place in Latin American economies during the 1980s and more recently in Venezuela in the 2010s. The statistical methodology is a recursive unit root test that seeks to distinguish between periods where the time series of interest are difference-stationary from periods in which they exhibit explosive behaviour. The right-tailed unit root tests are applied to the time series of inflation rates and money growth rates finding supporting evidence of explosive behaviour in the former and non-explosive behaviour in the latter in countries such as Argentina, Peru and Venezuela. The statistical approach successfully identifies historical periods of price-level bubbles and collapses over some of the hyperinflationary periods being studied.

Keywords: Mildly explosive time series; Right-tailed unit root tests; Hyperinflation.
JEL classification: C22, E31, E41

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"*Does it follow that self-generating inflations are somehow impossible? There seems to be no reason why they could not occur; so far, they have just not been observed*."

Phillip Cagan (1956)

I. Introduction

The epigraph in this article is from Phillip Cagan's (1956) seminal work on the monetary dynamics of hyperinflation in which reference is made to a situation where prices depart from the market fundamentals.[1] Economists have been interested in the study of this sort of phenomena at least from the early decades of the twentieth century when some scholars analysed the Great Tulip Bubble of the seventeenth-century Netherlands commonly known as the Tulip mania (see, for example, Posthumus, 1929).

The possibility that prices take on a divergent path (a price-level bubble) could emerge in situations where the expected rate of market price change is an important factor determining the current market price. These circumstances have been investigated in markets such as the stock market, the bond market, precious metals markets, the foreign exchange market as well as for the general price level, specifically during hyperinflation episodes.

The presumption that a hyperinflationary process could be an appropriate environment to observe positive price-level bubbles is based on two reasons. Firstly, bubbles are usually associated with self-fulfilling expectations, which are outcomes that could be more clearly observed during rapid inflationary processes where expectations play a prominent role in the determination of the price level. Secondly, positive price-level bubbles require inflation and its higher-order rates of change to be accelerating over time, a situation that can only been observed during hyperinflation episodes. (Flood and Garber, 1980).

The idea of a market launching itself onto a divergent path has not escaped controversy though, and it has sparked a significant number of academic works both on theoretical and empirical grounds. Price-level bubbles are a feature of rational expectations macroeconomic models. These models have a large number of equilibria, among them is the well-known neoclassical solution where prices grow at the rate of monetary expansion, but there are also

[1] An explicit specification of the market fundamentals depends on the structure of the theoretical framework being used.

other solutions in which the equilibrium price level exhibits either explosive or implosive paths, even under circumstances where the money supply is assumed to be constant. In most studies these divergent paths of the equilibrium price level are ruled out by imposing certain transversality conditions and other theoretical considerations. Also, several studies have used these assumptions to provide cross-equation restrictions to facilitate identification and econometric estimations. Nevertheless, the validity of the direct testing procedures has been criticised by Hamilton and Whiteman (1985), who have argued that the proposition that prices are driven by bubbles or extraneous factors is empirically untestable. Instead, they propose indirect statistical methods as alternative procedures to detect situations where prices are in a non-convergent path.[2]

In this study a statistical methodology proposed by Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a,b), which identifies situations when prices are in a divergent path, is applied to Latin American economies that have experienced hyperinflation episodes (i.e. Argentina, Bolivia, Brazil, Nicaragua, Peru, and Venezuela). The testing procedure is based on recursive right-tailed unit root tests, which are implemented to determine the time span when the examined variable exhibits explosive behaviour. Supporting evidence of the existence of a bubble is found in cases where explosive behaviour in the general price level and non-explosive behaviour in the monetary base (the market fundamental) is noticed. Based on this indirect statistical approach, price-level bubbles are identified during the hyperinflation processes in Argentina (1989:05-1990:03), Peru (1990:07-08) and Venezuela (2017:12-2020:12).

The remainer of the study is organised as follows. The second section provides a general background on hyperinflation episodes giving the context surrounding the research topic. The third section presents a review of the theoretical and empirical literature on the study of divergent paths in the general price level. The fourth section describes the econometric methodology employed in the paper, which is based on the recursive right-tailed unit root tests developed by Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, b). The fifth section shows the empirical results of the application of these statistical tests to the hyperinflationary processes in Latin America, and in the last section, the final remarks and conclusion of the study are presented.

[2] The difficulties found in the interpretation of real-world data has prompted some researchers to recommend placing greater weight on theoretical considerations rather than on empirical findings (see, for example, Obstfeld and Rogoff, 1986).

II. General Background

Hyperinflation is a situation in which the general price level of goods and services exhibits such an unusually high rate of growth that the monetary system of the country reaches a stage of virtual collapse. During hyperinflation episodes the domestic money tends to lose its normal functions as a medium of exchange, unit of account and store of value, forcing the public to recur to limited forms of barter and to adopt more stable substitutes such as foreign currencies and assets. A distinctive feature of hyperinflation processes is their relative short duration and explosive nature. The main reference in the study of hyperinflations is Phillip Cagan's classic work *The Monetary Dynamics of Hyperinflation* (1956). In this study Cagan (1956, pp. 25) defines hyperinflations in the following terms: "I shall define hyperinflations as beginning in the month the rise in prices exceeds 50 percent and ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year." Over the years, alternative definitions of hyperinflation have been suggested. For example, Dornbusch *et al*. (1990) define hyperinflation as annual inflation rates exceeding 1,000 percent or inflation rates greater than 15 to 20 percent per month. Similarly, Reinhart and Rogoff (2011) set a more modest threshold by defining hyperinflation as the rise in the general price level of at least 500 percent per year. But it has been Cagan's definition of hyperinflation that has become the most widely used criterion to identify these extreme episodes.

Applying Cagan's criterion, economists have constructed a table of hyperinflation episodes (see Hanke and Krus, 2013; Hanke and Bushnell, 2017).[3] The record registers only 57 hyperinflations since the late eighteenth century. All the hyperinflation episodes recorded in the Hanke-Krus World Hyperinflation Table were experienced during the twentieth century except for the *assignat* inflation of France at the end of the eighteenth century (1795:05-1796:11). Nevertheless, it should be mentioned that two additional cases of extreme inflation prior to the twentieth century are mentioned in the extant literature, which are the American War of Independence (1775-1783) and the Confederacy in the American Civil War (1861-1865).[4] The fact that hyperinflations were almost inexistent before the twentieth century reveals that the phenomenon is essentially a problem of modern economies. In this respect, Capie (1991) argues that the rare use of paper money before the twentieth century explains the low occurrence of hyperinflations in the previous centuries, and that it has been its use

[3] More precisely, the criteria used in the *Hanke-Krus World Hyperinflation Table* are the following: (i) An inflation rate of at least 50 percent per month; (ii) The inflation rate must persist for at least 30 consecutive days; and (iii) The episode must be fully documented, and the inflation rate should be replicable (Hanke and Bushnell, 2017, pp. 6).

[4] Eugene Lerner (1956) provides a study of the extreme inflation during the Civil War in the United States. Like Phillip Cagan, he was one of Milton Friedman's students who were encouraged to study the rare events of very rapid inflation.

that made the technology available to significantly expand the money supply, which generates the explosion in the rise of the general price level.

During the first half of the twentieth century waves of hyperinflation were observed during the aftermaths of the First and Second World Wars. In the 1920s hyperinflation struck in countries such as Germany, Austria, Russia, Poland, Hungary and the Free City of Danzig. An important characteristic of most of these episodes was their relatively short duration as they were ended with the implementation of radical fiscal and monetary reforms, which restored currency convertibility and gave central banks independence to conduct monetary policy (Reinhart and Savastano, 2003). Similarly, in the 1940s hyperinflation episodes were experienced in Greece, China, the Philippines, Hungary and Taiwan. The hyperinflation in Hungary during this period is of particular interest as the country went through the highest inflation ever recorded in the history of these events.[5]

The hyperinflations of the 1920s, and particularly in the Weimar Republic, are probably the most intensively researched inflationary processes in the literature. The most salient features found in these episodes can be summarised as follows: Firstly, a process of demonetisation and declining real money balances, and consequently a significant rise in the velocity of money in circulation; the contraction in real money balances indicating that prices were increasing faster than the money stock as price-setters, acting on their inflation expectations, were moving faster than money and credit creators (i.e., the monetary authority and other financial institutions). Secondly, a significant drop in bank deposits, therefore a reduction in the size of the financial sector, as the public did not have much incentive to hold savings using the national currency except for making payments and money transfers. Thirdly, an exchange rate depreciating faster than the inflation rate reflecting the actions of money holders looking for more stable currencies to secure the value of their assets. Finally, an accelerating frequency in the adjustment of nominal wages as a response to the continuously increasing cost of living. (He, 2018, pp. 53-60).

Since the 1950s extreme inflation episodes have been confined to the developing countries and some formerly Centrally Planned Economies in their transition to the market economy. A wave of hyperinflations was observed in Latin American countries such as Argentina, Bolivia, Brazil, Nicaragua and Peru during the period 1984-1991. In contrast to the episodes that took place at the beginning of the twentieth century, these hyperinflations were neither sudden nor

[5] After the Second World War, Hungary experienced for the second time a hyperinflation process, which started in August 1945 and ended in July 1946. During the peak month, July 1946, the price level increased by a factor of 4.19×10^{16} percent (or 41.9 quadrillion percent).

short lasting; they were led by years of chronic high inflation, and price stability was not reached swiftly -a stop-go pattern of inflation-price stability followed by major blowups occurred in their economies. In addition, the inflation rates experienced in these countries were far below those recorded during the 1920s and the 1940s (Dornbusch *et al*.,1990). Likewise, countries of the former Soviet Union and allied states such as Armenia, Azerbaijan, Belarus, Bulgaria, Estonia, Georgia, Poland, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Moldova, Russia, Tajikistan, Turkmenistan, Ukraine, Uzbekistan and Yugoslavia crossed the Cagan's threshold during the 1990s. However, many of these economies exhibited the exceeding 50 percent monthly inflation rate only at the start of the transition process and for a very short period. For this reason, some economists have excluded these episodes in their hyperinflation studies as the observed price hikes were more in the nature of a price level adjustment during the structural reforms rather than an ongoing inflation process (Fischer, Sahay and Vegh, 2002). Finally, hyperinflation episodes were experienced in a few African nations, such as Angola and The Democratic Republic of The Congo in the 1990s, and most recently in Zimbabwe, during civil war or serious social unrest conflicts.[6]

Common to all hyperinflation episodes is a major expansion in the money stock, which is driven by endemic fiscal imbalances. Therefore, it is the government's need to obtain seigniorage -the revenue from printing money- that constitutes the underlying cause of the process. Governments recur to the inflationary financing of the deficit when they are unable to rise revenues by using conventional taxation or borrowing from the domestic and the international capital markets.[7] Hence, under the Cagan-Chicago tradition inflation becomes a special form of tax levied on the public's holding of money. The seigniorage revenues fall as inflation rises because the tax-base -the real money balances- decreases proportionally more than the money growth as the public reduces their real demand for money.[8]

The notion of inflation as a tax has changed the emphasis from monetary to fiscal factors as the root cause of hyperinflations. It is also known that an unstable feedback process between the deficit and the growth of money is triggered during these episodes. An initially money-driven inflation increases the fiscal deficit as the real value of other tax revenues fall because

[6] Zimbabwe and Venezuela have been the last countries experiencing extreme inflation processes. The hyperinflation in Zimbabwe took place between March 2007 and November 2008. During the peak month of this inflation, the price level rose by a factor of 7.96×10^{10} to become the second highest inflation rate in history. The hyperinflation in Venezuela occurred between December 2017 and December 2020.

[7] Budget deficits that are financed by borrowing either domestically or internationally are not necessarily inflationary. Therefore, it is the inability to borrow and the adoption of printing money to finance the deficit that could lead to hyperinflation.

[8] Empirical evidence of an "inflation-tax Laffer Curve" in high-inflation countries and high-inflation episodes can be found in studies such as Fischer, Sahay and Vegh (2002).

of the lags between the imposition of the tax and its collection -this is known in the economic literature as the Olivera-Tanzi effect.[9] Hence, for countries in which the only means to finance the public deficit is printing money, the deficit becomes the main determinant of money growth that fuels inflation, which in turns determines the public deficit. Under these circumstances, the government can lose control over the growth of money as the deficit ends financing itself automatically (Dornbusch et al., 1990).

In an influential study, Sargent (1982) examined the process of stopping hyperinflations in four of the major episodes of the twentieth century: Austria, Hungary, Germany and Poland. The essence of his view is that hyperinflations can be ended at essentially no output cost when fiscal and monetary reforms are introduced to eliminate either the deficit or the government's capacity to use seigniorage to finance it, or both.[10] Sargent argues that monetary expansion by itself does not lead to inflation; the prevailing and anticipated fiscal position must be considered in conjunction with monetary policy. Although no panacea for stopping hyperinflations seems to exist, the necessary conditions for a successful stabilisation programme have been studied. Accordingly, the most important elements are the following: Firstly, gaining control over monetary growth and ending government's dependence on printing money to finance its deficit. Secondly, a reorganisation of government finances by increasing taxes and reducing public expenditures. Thirdly, to provide legal authority to the central bank to refuse printing money to lend to the government. Fourthly, the provision of foreign loans or financial aids to buildup foreign exchange reserves and to temporarily finance the deficit while public confidence in the stabilisation programme is consolidated. Finally, the introduction of full convertibility of the new currency into gold or any key foreign currency. (Cagan, 1987).

[9] High inflation could reduce the fiscal deficit if the reduction of the real value of debts -the so-called Patinkin effect- outstrips the fall in the real tax revenues.

[10] The fact that price stability can be brought at low cost during hyperinflation episodes is commonly explained in terms of the shortening of contracts that takes place during these events, which reduces inflationary inertia. The inflationary inertia found at low inflation rates explains the Phillip curve output cost of lowering inflation (Fischer, Sahay and Vegh, 2002).

III. Self-Generating Inflation

In his seminal work, Phillip Cagan (1956) studied several hyperinflation episodes that took place during the first half of the twentieth century.[11] The main purpose was to explain the erratic behaviour exhibited by real cash balances during these events -although real cash balances declined over the whole hyperinflation period, their month-to-month variations tend to fluctuate drastically. Cagan advanced the theory that such variations in the real cash balances depended on the changes in the expected rate of inflation, which were assumed to be formed adaptatively. His monetary framework can be represented as follows:

$$\frac{M_t}{P_t} = e^{-\alpha\pi_t^* - \gamma} \tag{1}$$

$$\dot{\pi}_t^* = \beta(\pi_t - \pi_t^*) \quad \beta > 0 \tag{2}$$

Equation (1) shows the demand for real cash balances, which is assumed to be a function of the expected rate of inflation, π^*. M_t is the demand for nominal balances at time t, which is assumed to be always equals to the supply, P_t is the price level at time t, and α (which is positive) and γ are constants -the former being the semi-elasticity of real money demand with respect to expected inflation. In the model, real variables, such as real output, that could affect the real money demand are assumed invariant, and the nominal money supply is exogenous.

Equation (2) shows how the expected rate of inflation depends on the actual inflation rate, π; the expected rate of inflation is revised every period in proportion to its deviation from the actual rate of inflation. The constant β, the "coefficient of expectation", measures the speed with which individuals adjust their expected rate of inflation. Given this specification, it is possible to show that expected inflation is a weighted average of past rates of inflation.

In this theoretical framework, inflation is determined in the money market and its pace is driven by the rate of growth of money and the change in expected inflation. Cagan (1956) assessed whether the process of expectation formation itself could have been the only cause of

[11] The extreme price events were those of Austria (October 1921 - August 1922), Russia (December 1921 - January 1924), Germany (August 1922 - November 1923), Poland (January 1923 - January 1924), Hungary I (March 1923 - February 1924), Greece (November 1943 - November 1944) and Hungary II (August 1945 - July 1946).

hyperinflation episodes and concluded that the rate of growth of money was instead responsible for it.

Given the rate of money growth, the model shows that prices evolve along a stable equilibrium path when the value of the product of the parameters $\alpha\beta$, the "reaction index", is less than unity. By contrast, if the reaction index is greater than unity, a small increase in prices makes people completely lose faith in the domestic currency to the point at which real cash balances fall by such a magnitude that prices continue to increase under the impetus of falling balances *ad infinitum* (Cagan, 1956, pp. 72). In other words, under this scenario once there has been an initial rise in the quantity of money, the course of inflation has nothing to do with the evolution of the money supply, inflation is *self-generating*.

Empirical evidence for estimates of the reaction index in the seven hyperinflation episodes were provided in his study. Although the estimated reaction indices for the hyperinflations in Austria, Greece and Hungary II were lower than unity, for the German and the Russian hyperinflations these were greater than one (1.09 and 1.07, respectively), while for the Polish and the Hungarian I episodes the confidence intervals of the parameters were too wide for the value of unity to be rejected. Besides these results, Cagan (1956) considered it unlikely that a case of self-generating inflation could have taken place during these hyperinflationary events, arguing that the results were unreliable mainly due to random errors and bias. However, he did not completely rule out the possibility of the occurrence of self-generating inflations but considered that they could only occur after a prolonged period of hyperinflation, and for just a brief period because under these circumstances a currency reform would become a political and economic necessity (Cagan, 1956, pp. 73).

A line of criticism of Cagan's analysis has been the expectation mechanism. For a continuously increasing inflation rate, the use of adaptive expectations as a weighted average of past inflation rates generates correlated expectational errors, which is not consistent with rational behaviour. An alternative approach is the adoption of fully rational expectations. Economists introduced rational expectations to this theoretical framework, which sparked a line of research known in the extant literature as *monetary growth models*. Early examples of these macroeconomic models can be found in Sidrauski (1967), Sargent and Wallace (1973 a,b), Black (1974), Brock (1974, 1975), Taylor (1977), among many others. An important feature of these models is the existence of multiple equilibria, where the source of the indeterminacy comes from the presence of expected future prices (or their expected rates of change) in the equilibrium equation of current prices.[12]

Burmeister, Flood and Garber (1983) have classified the solutions of the macroeconomic models with rational expectations into two types: the *market fundamentals* solution and the *market fundamentals plus bubble* solution. Accordingly, the origin of the two forms of solutions is the result of the well-known property of linear difference and differential equation systems in which the general solution is given by a homogeneous solution plus a particular solution. In their classification the market fundamental solution is one that excludes the homogeneous solution. While the remaining solutions combine a particular solution with the homogenous solution. Using a Cagan-type monetary market equilibrium condition

$$m_t - p_t = -\alpha[E_t p_{t+1} - p_t] + \varepsilon_t \tag{3}$$

where m_t is the logarithm of the stock of money balances at time t, p_t is the logarithm of the price level at time t, and ε_t is a serially independent, identically distributed disturbance with zero mean and finite variance. It is assumed that the constant parameter $\alpha \neq 0$ and $\alpha \neq -1$. Price expectations are rational in the sense that E_t is the mathematical conditional expectation operator based on all the information available at time t, I_t. The market fundamentals solution consists of current and expected values of the money supply and the disturbance term. Formally,

$$p_t = \sum_{i=0}^{\infty} \beta_i E_t m_{t+i} + \sum_{i=0}^{\infty} \gamma_i E_t \varepsilon_{t+i} \tag{4}$$

with parameters $\beta_0 = 1/(1+\alpha)$, $\beta_{i+1} = [\alpha/(1+\alpha)]\beta_i$ and $\gamma_0 = -1/(1+\alpha)$. On the other hand, the market fundamentals plus bubble solution is given by

$$p_t = \sum_{i=0}^{\infty} \beta_i E_t m_{t+i} + \sum_{i=0}^{\infty} \gamma_i E_t \varepsilon_{t+i} + A_t \tag{5}$$

[12] Blanchard (1979b, pp. 115) explains the problem in the following terms: "In each period both the current price and the expected future price clear the market. Over any number of periods, there is one more price (or expected price) than markets to clear."

where the above restrictions for the parameters continue to hold and A_t is an arbitrary term such that

$$E_t A_{t+1} = [(1+\alpha)/\alpha] A_t \quad (6)$$

which is the general condition of a bubble as described in Flood and Garber (1980).

The idea that expectations could lead to self-fulfilling speculative price paths (or that it could be completely based on extraneous variables, also known as sunspots) has been a subject of controversy among economists at least since John Maynard Keynes, and it remains an open question. On theoretical grounds, several approaches have been proposed to address explosive price-level paths equilibria that are unrelated to monetary growth. Brock (1974, 1975) provides a model of identical utility-maximising individuals with perfect foresight in an infinite-horizon setup where the demand for money emerges by assuming that individuals' utility depend not only on the level of consumption, but also on their real balances. In this pure fiat monetary system with rational expectations, hyperinflationary price-level paths cannot be excluded unless some (severe) restrictions are imposed on individuals' preferences. Hence, Brock (1974,1975) assumes that money has intrinsic value even in situations in which the price level is infinite so that along implosive real balance paths individuals would try to permanently raise their nominal money holdings.[13] An alternative approach can be found in Blanchard (1979), who analyses a case where explosive price-level paths arise from purely extrinsic uncertainty. In this stochastic framework there is a non-zero probability each period that the bubble bursts along the explosive price-level paths and the price level returns to its stationary steady state value.

Obstfeld and Rogoff (1983) have criticised Brock's restriction on individuals' preferences as it implies that individuals cannot function without money -or that there is no finite stock of consumption goods that could compensate them for their complete stock of money balances, which is considered economically unreasonable. Building upon Brock's work, they incorporate productive capital as well as money in a similar maximising model and rule out the possibility of hyperinflationary equilibria by introducing a fractional backing scheme to the currency in

[13] Formally, the utility function is given by $U = \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(m_t)]$ and $\beta < 1$, where c_t is consumption at time t and m_t denotes the real balances at time t. The functions $u(\cdot)$ and $v(\cdot)$ are increasing and strictly concave, with the conventional smoothness and Inada properties. In this model, explosive price-level paths can be ruled out as equilibria, if and only if it is assumed that $\lim_{m \to 0} m v'(m) > 0$. Obstfeld and Rogoff (1983) show that this condition implies $v(0) = -\infty$.

which the government guarantees to redeem money for a small amount of capital.[14] Accordingly, the government intervention does not need to be certain; even a possibility of such intervention will rule out speculative price-level paths. Obstfeld and Rogoff (1986) also claim that by fractional backing of the currency a government can exclude speculative hyperinflationary equilibria not only when the bubbles are deterministic, but also when these are stochastic like in Blanchard (1979). Cochrane (2011, 2023) has questioned fractional currency backing arguing that it is an insufficient measure to foreclose hyperinflationary equilibria, and that even if it would be a sufficient measure under commitment, such a mechanism is not realistic or credible.[15] Other arguments presented in the extant literature that emphasize the difficulty in reconciling self-fulfilling price paths with optimising behaviour in general equilibrium models are the inherently discrete nature of transactions (Farmer, 1984) and the theoretical difficulty in getting the bubble started originally (Diba and Grossman, 1988).

The empirical literature on this subject is mainly based on the study of the German hyperinflation (1922:08-1923:12). The first study was conducted by Flood and Garber (1980) using Cagan's money demand function under rational expectations, and assuming that the money supply is exogenous with respect to the inflation rate. After conducting a series of tests for a deterministic price-level bubble starting in three different months (1920:07, 1922:06, and 1923:01) and ending in 1923:06, the empirical evidence did not reject the hypothesis that market fundamentals were the only cause driving the process (i.e. $A_0 = 0$ in equation 5).

Burmeister and Wall (1982) conducted tests for convergence paths for the general price level using the same theoretical framework of Flood and Garber (1980) but employing a different statistical methodology (a Kalman filtering algorithm). They considered six different possible beginning periods for the bubble (1919:01, 1919:10, 1920:07, 1921:06, 1922:06, and 1923:01) all lasting until 1923:06. The tests rejected the null hypothesis of convergence paths for the general price level for all the different durations except those starting in 1922:06 and 1923:01, the results were interpreted as evidence in favour of a stochastic bubble.[16]

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[14] Using a non-stochastic overlapping generation model, Wallace (1981) describes a similar *hybrid monetary system* where the government back the currency with the perishable consumption good rather than with a durable asset. Alternative backing schemes can also be found in Brock (1982), Nicolini (1996) and Obstfeld and Rogoff (2021), among others.

[15] See Obstfeld and Rogoff (2021) for a discussion on Cochrane's (2011, 2023) criticisms.

[16] More precisely, a model using variables in first differences was employed after having estimation difficulties with the models for the variables in levels, which presented no convergence in the estimation interactive procedure.

Given the abundant historical and empirical evidence for a feedback process from inflation to the money supply in the German hyperinflation,[17] Burmeister and Wall (1987) built on their previous work introducing an alternative specification to their model by making the expected rate of inflation a determinant of the money supply. Their study corroborated their previous findings by rejecting the null that rationally formed expectations are always convergent.

Contrary to Burmeister and Wall's (1987) results, Casella (1989) was unable to reject the hypothesis of no stochastic bubble when the money supply is allowed to respond to prices using the period 1920:05 to 1923:06. Invoking the Wold decomposition theorem, she estimated a univariate representation for the money supply, which is a valid representation whether money is exogenous or endogenous, and Cagan's money demand function, allowing the error terms of these equations to be correlated. Using West's (1987) version of Hausmann specification test, she sought for inconsistencies in the estimate of α in a real balance equation using the market fundamentals (and no bubble) solution for the price level and the estimate in an instrumental variable estimation of the money demand equation (3), which -absent of other specification problems- is consistent whether a price-level bubble exists or not. If the former specification is incorrect, the absence of the bubble term will lead to inconsistent estimates. In her study the comparison of the two estimates of α is used as a test for bubbles. Although the study pointed out to the absence of a bubble during the German hyperinflation, her results were not robust to changes in the instrument sets employed in the estimation process.[18]

İmrohoroğlu (1992) corroborates the absence of a price-level bubble during the German hyperinflation. Using a version of Sargent and Wallace's (1987) rational expectation model with a government budget equation and a linearised Cagan's portfolio balance equation, the model generates multiple equilibrium paths, which could be driven by sunspots or market fundamentals, along a stochastic Laffer curve. In this theoretical framework hyperinflation is interpreted as the movement along the Laffer curve from an unstable low inflation stationary equilibrium to a stable high inflation stationary equilibrium. Applying the Kalman Filter algorithm, he conducted hypothesis tests for both deterministic bubbles and stochastic bubbles concluding that they were absent during the German hyperinflation. Furthermore, his results supported Burmeister and Wall's (1987) conclusion that rational expectations were not

[17] See, for example, Sargent and Wallace (1973a), Evans (1978), and Webb (1985).

[18] Casella (1989) uses two sets of instruments; a large set that includes the variables lagged rates of money growth and lagged inflation rates, and a small set with the lagged rates of money growth only. The conclusion that no bubble was present during the German hyperinflation when money is allowed to respond to prices is based on the estimates using the small set of instruments.

dynamically stable during the episode but explained the “divergent rational expectations” in terms of market fundamentals rather than the presence of a bubble.

Most of the empirical tests mentioned previously have been subject to criticism by Hamilton and Whiteman (1985). These researchers have argued that the proposition that prices are driven by bubbles (or extraneous factors) is empirically untestable.[19] Accordingly, it is always possible to relax the restrictions on the dynamics of the fundamental driving variables, which are commonly imposed in these tests, to interpret what appear to be a speculative price bubble as rational responses to economic fundamentals seen by agents but not observed by the econometrician.[20]

Time series techniques have also been used to analyse the dynamics of economic variables in hyperinflation episodes. A pioneer work in this area is Taylor (1991) in which it is argued that if real money balances ($m - p$) and inflation (Δp) are integrated of order one, then Cagan’s theoretical framework implies a cointegrating relationship between the variables. The work sparked a new interest on the study of hyperinflations using Cagan’s model applying time series approaches. Similarly, on the investigation of rational bubbles several indirect tests have been proposed to identify the possible presence of these phenomena during hyperinflation events.

Hamilton and Whiteman (1985) suggested the comparison of the order of integration of the general price level and the fundamentals to infer the presence of a bubble. The existence of a deterministic bubble introduces an explosive root into the time series behaviour of the price level, making it impossible to remove the stochastic trend by differencing the data once or several times. In other words, the series is not integrated of any order. For finite samples, this feature has been interpreted as the price level would exhibit a higher order of integration (or non-stationarity) than the fundamentals. Hence, a necessary condition for the absence of a bubble is met when the price level and the fundamentals have the same order of integration. A similar reasoning is used in the application of cointegration tests to determine the presence of a bubble where the difference between the market fundamentals solution (equation 4) and the market fundamentals plus bubble solution (equation 5) of the equilibrium price level is exploited. If the latter equilibrium specification is correct, the exclusion of the bubble term will lead to the rejection of a cointegration relationship between the price level and the fundamentals, in such situation the variables will tend to diverge infinitely over time.

[19] See also Hamilton (1986).

[20] A testing procedure to address the observational equivalence between expected future changes in the fundamentals and bubbles has been proposed by Blackburn and Sola (1996).

The application of integration and cointegration tests as means to reject the hypothesis of the existence of a bubble has been criticised, for example, by Evans (1991). He showed that stochastic bubbles that are “periodically collapsing” cannot be detected employing standard unit root tests to determine whether prices are more or less explosive than the fundamentals. His argument is that, even though collapsing bubbles register explosive conditional means, they will appear as integrated processes of order zero (stationary processes) when applying these tests. Accordingly, the only way the bubbles can be detected using integration and cointegration approaches is when they last for most of the period under investigation (Evans, 1991, pp. 925).

To address the challenges imposed by stochastic bubbles like those exposed by Evans (1991), several extensions of the Augmented Dickey-Fuller unit root tests have been proposed. A well-known approach, the Markov-Switching Augmented Dickey-Fuller (MSADF) tests, makes use of Hamilton’s (1989) Markov switching-regime models to identify periods in which the time series of interest might exhibit an explosive behaviour. The models assume that the path of the variable may be subject to occasional discrete shifts or regimes. The regime for each observation in the data is determined endogenously by realisation of a homogeneous first-order Markov process.[21] The null hypothesis in these tests is that of a unit root versus an alternative hypothesis that there is either a root greater (explosive process) or lower (stationary process) than one. Hence, evidence that one regime is non-stationary, maybe with an explosive root, while the other is stationary -the variable collapsing back towards the fundamental solution- indicates the presence of a bubble.

Different model specifications have been proposed for the MSADF tests. Funke, Hall and Sola (1994) employed a model in which changes in the autoregressive coefficients (due to the existence of a bubble) and changes in the error variance take place simultaneously as the variable goes through switches in regimes. The study examined the hyperinflation process in Poland in the late 1980s and early 1990s.[22] On the other hand, Hall, Psaradakis and Sola (1999) used a similar model specification but assumed that the error variances of the two regimes were constant.[23] The analysis was conducted for the hyperinflation episodes in

[21] van Norden and Vigfusson (1998, pp. 4) use different transition equations where the probability of observing the collapsing regime is an increasing function of the relative size of the bubble.
[22] The term hyperinflation was used loosely since the monthly inflation rates in Poland did not exceed Cagan’s 50% threshold during the period.
[23] Comparison of the size and the power for both statistical tests can be found in Shi (2013). The simulations conducted in his work show that the Markov switching approach is susceptible to false detection or spurious

Argentina during the years 1985, 1988 and 1989. These studies made use of the time series for the money base, the exchange rate, and the consumer price level. The money base was used as a proxy for the economic fundamentals. The inclusion of the exchange rate, a variable believed to be driven by the same fundamentals as the consumer price, was to distinguish rational bubbles from fundamentally determined bubble-like behaviours.[24] In other words, if it was observed that both prices and exchange rates change regimes simultaneously, this would suggest that the non-stationarity of prices was attributable to the market fundamentals. By contrast, if the changes in regimes were non-synchronous, then the likely presence of a bubble in one or both series could be concluded.

In the study of the Polish hyperinflation, Funke, Hall and Sola (1994) found evidence that the main determinant of the hyperinflation was a rational bubble in the exchange rate market -the three variables showed a switch to the unstable regime (a root greater than one), but the timing of the switching took place firstly in the exchange rate followed by prices and only later by the money supply. In the case of the hyperinflations in Argentina, Hall, Psaradakis and Sola's (1999) findings suggested a price bubble during the period 1988:06-08 and a bubble in the exchange rate in 1984-1985 -all variables seem to have switched simultaneously to the explosive regime in 1989:04 suggesting as the most likely explanation that the rapid expansion of money was responsible for the observed explosive behaviour in prices.

Recently, Morita, Psaradakis, Sola and Yanis (2024) proposed a generalisation of the Hall, Psaradakis and Sola's (1999) tests where changes in the intercept, the error variance and the autoregressive coefficients are governed by independent Markov processes. The study examines the hyperinflations in countries such as Argentina, Brazil, Germany, and Poland. In the case of the hyperinflations in Argentina and Germany they found no evidence in favour of a rational bubble during the events.[25] Conversely, for the rapid inflation processes in Brazil and Poland the empirical findings point towards the existence of a rational price bubble during the whole sample period.

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explosiveness. He also explored the properties of a model where the autoregressive coefficients and the error variance are governed by independent first-order Markov chains.

[24] A rational bubble reflects self-confirming belief that prices depend on a variable (or a combination of variables) that is intrinsically irrelevant (i.e. no part of the fundamentals), or on genuinely relevant variables in a way that includes parameters that are not part of market fundamentals (Diba and Grossman, 1988, pp. 35-36).

[25] In the case of the Argentinian hyperinflation, the different results of the two studies are attributed to a more flexible model specification, which allows for independent Markov changes in its parameters (Morita, *et al*. 2024, pp. 33).

An alternative statistical approach, the sup ADF test, has been proposed by Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015). The methodology is a recursive unit root test that, like the MSADF tests, seeks to distinguish between periods where the time series of interest are difference-stationary from periods in which they exhibit explosive behaviour. The statistical approach will be used in this study to examine the hyperinflations episodes in countries such as Argentina, Bolivia, Brazil, Nicaragua, Peru, and Venezuela. A brief description of the methodology is presented in the following section.

IV. Econometric Methodology

Phillips, Wu and Yu (2011) proposed the application of recursive regression techniques to identify periods of (mildly) explosive dynamics in the time series of an economic variable.[26] The recursive nature of the tests has the attractive feature that it enables the location of the origination, termination, and extent of the explosive behaviour in time series data. At the centre of the statistical approach is the right-tailed augmented Dickey and Fuller (1979; ADF) test for a unit root where the following autoregressive specification is estimated by least squares,

$$\Delta y_t = a_{r_1,r_2} + \gamma_{r_1,r_2} y_{t-1} + \sum_{j=1}^{k} \psi_{r_1,r_2}^{j} \Delta y_{t-j} + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \sigma_{r_1 r_2}^2\right) \tag{7}$$

where y_t denotes the variable of interest at time t, Δy_{t-j} are the lagged first differences of the series to address serial correlation, ε_t is the Gaussian disturbance term, and a_{r_1,r_2}, γ_{r_1,r_2}, and ψ_{r_1,r_2}^{j} are the parameters to be estimated.[27] The subscripts r_1 and r_2 are fractions of the total sample data (T) indicating the starting and ending dates of the subsample period. Hence, $r_2 = r_1 + r_w$ where $r_w > 0$ is the (fractional) window size of the regression.[28] The null hypothesis of a unit root is $H_0\colon \gamma_{r_1,r_2} = 0$ and the right-tailed alternative hypothesis of explosive dynamics is $H_1\colon \gamma_{r_1,r_2} > 0$. The ADF statistic corresponding to the null hypothesis is given by,

$$ADF_{r_1}^{r_2} = \frac{\hat{\gamma}_{r_1,r_2}}{se\left(\hat{\gamma}_{r_1,r_2}\right)} \tag{8}$$

In the forward recursive approach, model (7) is estimated repeatedly, using subsets of the sample data augmented by an additional observation at each step. In this setting, the starting

[26] In the context of economic and financial data this commonly implies a characteristic root in the region [1.002,1.05] (Phillips, Wu, Yu, 2011, pp. 208).

[27] Simulation evidence suggests that the test works well using a small number of lags (e.g. k equals 0 or 1).

[28] The standard ADF unit root test is conducted by setting $r_1 = 0$ and $r_2 = 1$.

of the sub-sample is kept constant at $r_1 = 0$, and the ending point, r_2, increases from r_0 (the smallest window size) to one (the full sample period).[29] The process generates a sequence of $ADF_0^{r_2}$ statistics where the *supremum* of the sequence, the SADF statistic, is defined as follows,

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \tag{9}$$

whose limit distribution is given by,

$$\sup_{r_2 \in [r_0, 1]} \frac{\int_0^{r_2} W dW}{\left(\int_0^{r_2} W^2\right)^{1/2}} \tag{10}$$

where W is a standard *Wiener* process. A rejection of the null hypothesis of a unit root in favour of the alternative of explosiveness in part(s) of the series requires that the SADF statistic exceeds the corresponding right-tailed critical value.

The proposed methodology to identify the origin and end of the explosive period is based on the comparison of the time series of the recursive *backward* ADF (BADF) test statistic against the right-tailed critical value of the distribution of the standard ADF statistic.[30] Denoting r_e as the fraction of the total sample for the origination of the explosive period, the estimated starting date $[\hat{r}_e T]$ is determined as the first chronological observation in which the ADF statistic exceeds the critical value. Similarly, indicating r_f as the fraction of the data when the explosive period ends, the estimated termination date $\left[T\hat{r}_f\right]$ of the bubble is the first chronological observation, after a minimum duration condition is satisfied, where the ADF statistic goes below the critical value.[31] The estimated starting and ending dates can then be calculated by the following crossing time equations,

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : ADF_0^{r_2} > cv_{r_2}^{\alpha} \right\} \tag{11}$$

[29] Phillips, *et al.* (2015a,b) recommend setting the minimum window size based on the rule of thumb: $r_0 = 0.01 + 1.8/\sqrt{T}$.

[30] The backward ADF test conducts a $sup\ ADF$ test on a backward expanding sample sequence in which the ending point of each sample is fixed at r_2 (the fraction related to the endpoint of the window), and the starting point changes from 0 to $r_2 - r_0$ (the sample fraction related to the origination of the window) (Phillips, *et al.* 2015, pp. 1051).

[31] To avoid short lived blips in the estimated autoregressive coefficients, Phillips, *et al.* (2015a,b) recommend focusing on episodes with a minimal duration of $\delta log\ (T)$ units of time, where δ is a frequency-dependent scaling parameter.

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e + \delta log(T)/T, 1]} \left\{ r_2 : ADF_0^{r_2} < cv_{r_2}^{\alpha} \right\} \tag{12}$$

where $cv_{r_2}^{\alpha}$ is the $100(1-\alpha)\%$ critical value of the ADF statistic corresponding to a significance level of α.

An important weakness of the SADF statistic is that it shows reduced power and inconsistency whenever the sample data presents more than one explosive period. To overcome this limitation, Phillips, Shi and Yu (2015a,b) proposed an extension to the statistical test, the generalised sup ADF (GSADF), which enables the identification of multiple bubbles. The GSADF test has the same null and alternative hypotheses as the SADF test, but the ending point, r_2, and the starting point, r_1, can change during the test. The greater flexibility of the approach allows the coverage of a larger number of subsamples. Formally, the GSADF statistic is given by,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \tag{13}$$

Under the null hypothesis of a unit root, the limit distribution of the GSADF is,

$$\sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} \left\{ \frac{\frac{1}{2} r_w [W(r_2)^2 - W(r_1)^2 - r_w] - \int_{r_1}^{r_2} W(r) dr [W(r_2) - W(r_1)]}{r_w^{\frac{1}{2}} \left\{ r_w \int_{r_1}^{r_2} W(r)^2 dr - \left[\int_{r_1}^{r_2} W(r) dr \right]^2 \right\}^{\frac{1}{2}}} \right\} \tag{14}$$

Similarly, a rejection of the null hypothesis against the alternative hypothesis of explosiveness in part(s) of the sample needs the GSADF statistic to be higher than the right-tailed critical value of its limiting distribution.

The date-stamping strategy for this case is like that suggested for the SADF methodology. The procedure is based on the sequence of recursive backward SADF (BSADF) statistics,

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} SADF_{r_1}^{r_2} \tag{15}$$

and estimates of the starting and ending points are constructed from the crossing time equations,

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) > scv_{r_2}^{\alpha} \right\} \tag{16}$$

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e + \delta log(T)/T, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^{\alpha} \right\} \tag{17}$$

where $scv_{r_2}^{\alpha}$ is the $100(1-\alpha)\%$ critical value of the SADF statistic for the $[r_2 T]$ observations.

V. Empirical Results

This section presents the statistical results of the right-sided augmented Dickey-Fuller unit root tests and its further developments (i.e. SADF and GSADF) on the time series of inflation and money growth in countries such as Argentina, Bolivia, Brazil, Nicaragua, Peru, and Venezuela. The presumption is that bubbles, if they exist, are more likely to occur during periods of hyperinflation episodes, a phenomenon that has been observed in all these economies. Since differences of an explosive process still manifest explosive characteristics, the tests are conducted on the rates of growth of the Consumer Price Index (CPI) and the monetary base. The empirical methodology in the study will be looking for explosive behaviour in inflation and non-explosive behaviour in money growth as supporting evidence of the existence of a price-level bubble in an economy during a particular time span.

Data

The data sources of the variables analysed in this section are presented in Table 1. Monthly observations for the inflation rates during the period 1970:01-2019:12 were collected for Argentina, Bolivia, Brazil and Peru. Similarly, monthly data of money growth rates during the same period were available for Argentina and Brazil. Monthly observations for the inflation rates during the years 1973:01-2022:12 and 1978:02-2024:03 were gathered for Venezuela and Nicaragua, respectively. Finally, monthly times series of the monetary base growth rates in Bolivia, Nicaragua, Peru and Venezuela were available during the time intervals 1980:02-2019:12,1980:02-2017:07, 1989:02-2019:12 and 1993:02-2022:12, respectively.

Table 1
Variables and Data Sources

Country	Prices	Monetary Base
Argentina	Datos Argentina (www.datos.gob.ar) IMF International Financial Statistics (www.imf.org/en/Data)	Banco Central de la República Argentina (www.bcra.gob.ar) IMF International Financial Statistics (www.imf.org/en/Data)
Bolivia	Instituto Nacional de Estadística (www.ine.gob.bo)	Banco Central de Bolivia (www.bcb.gob.bo) IMF International Financial Statistics (www.imf.org/en/Data)
Brazil	IPEADATA (www.ipeadata.gov.br) IMF International Financial Statistics (www.imf.org/en/Data)	IPEADATA (www.ipeadata.gov.br) IMF International Financial Statistics (www.imf.org/en/Data)
Nicaragua	Banco Central de Nicaragua (www.bcn.gob.ni) Lankes (1993) INIDE (https://www.inide.gob.ni/) IMF International Financial Statistics (www.imf.org/en/Data)	Banco Central de Nicaragua (www.bcn.gob.ni) Lankes (1993) IMF International Financial Statistics (www.imf.org/en/Data)
Peru	Banco Central de la Reserva del Perú (www.bcrp.gob.pe) IMF International Financial Statistics (www.imf.org/en/Data)	Banco Central de Reserva de Perú (www.bcrp.gob.pe) IMF International Financial Statistics (www.imf.org/en/Data)
Venezuela	Banco Central de Venezuela (www.bcv.org.ve) IMF International Financial Statistics (www.imf.org/en/Data)	Banco Central de Venezuela (www.bcv.org.ve) IMF International Financial Statistics (www.imf.org/en/Data)

Hyperinflation

This work considers all the hyperinflation processes experienced in Latin America except for the hyperinflation in Chile, an episode that lasted only a month when the inflation rate reached 50 percent in October 1973. Some important features of the different episodes are presented in Table 2.

Table 2
Hyperinflation Episodes

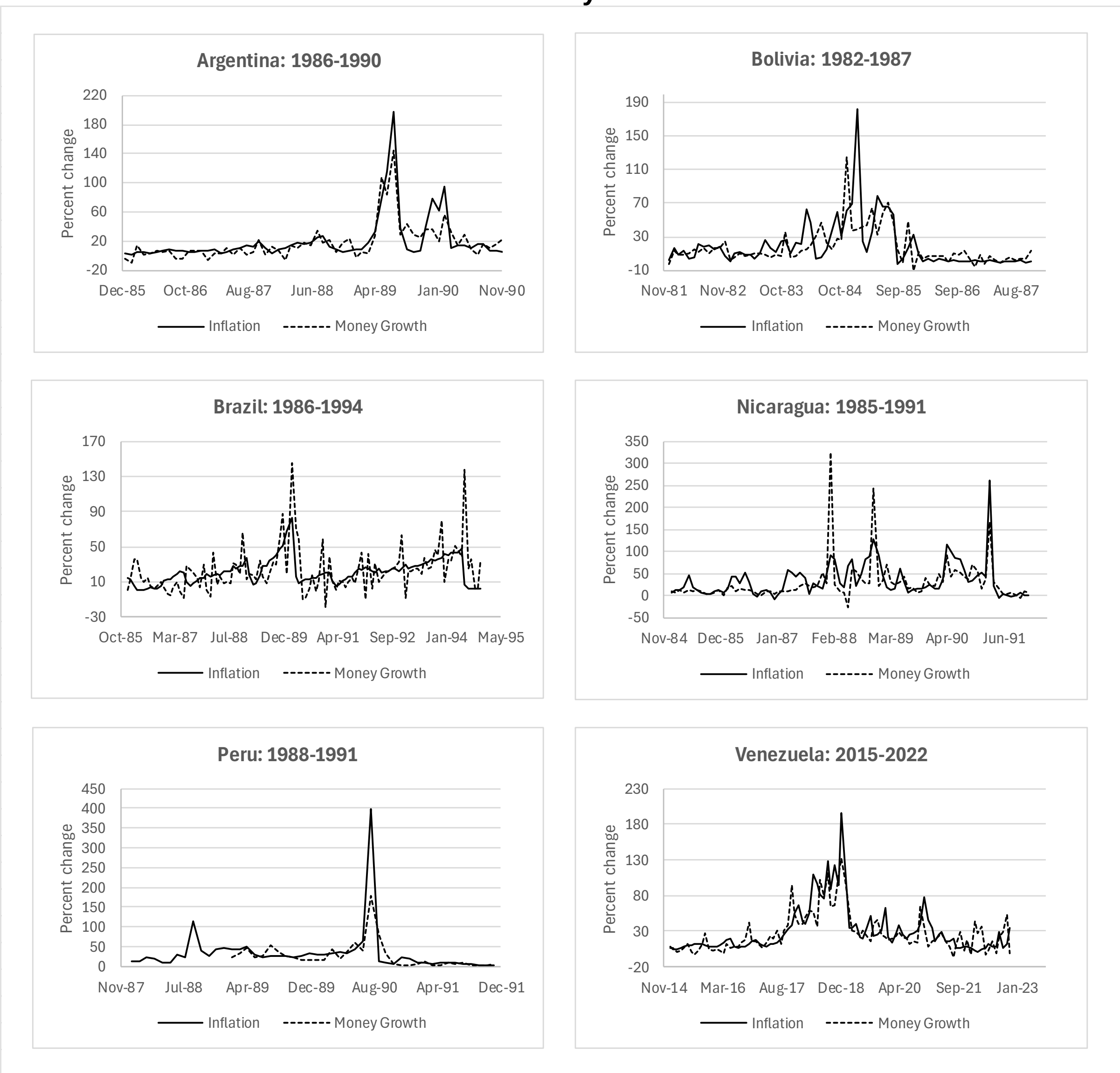
Country	Start Date	End Date	Average Monthly Inflation Rate	Month with Highest Inflation Rate	Highest Monthly Inflation Rate
Argentina	1989:05	1990:03	52 %	1989:07	197%
Bolivia	1984:04	1985:09	44 %	1985:02	183%
Brazil	1989:12	1990:03	52 %	1990:03	82%
Nicaragua	1986:06	1991:03	39 %	1991:03	261%
Peru	1988:09	1988:09	114%	1988:09	114%
	1990:07	1990:08	101%	1990:08	397%
Venezuela	2017:12	2020:12	51%	2019:01	197%

Most of the hyperinflations in Latin America took place in the 1980s, a period that is commonly known as the "lost decade" of economic stagnation in these countries. This decade was preceded by a period of financial liberalisation and integration with the international markets in which the Latin American governments contracted significant foreign debts at relatively low interest rates. In the 1980s these economies faced an adverse external environment characterised by a deterioration of the terms of trade, a hike in world interest rates and a lack of access to the international credit markets. Under these conditions the highly indebted governments resorted to printing money as the main source of financing public expending.

Latin American hyperinflations usually exhibited lower inflation rates than the European hyperinflation episodes of the interwar period. The highest inflation rate was recorded in Peru in 1990:08 at 397% per month, followed by Nicaragua's 1991:03 peak monthly hyperinflation rate of 261%. Hanke and Krus (2013) rank these episodes as the twelfth and eighteenth among fifty-six hyperinflations in the world, respectively.[32] It is worth noting that some of these economies experienced multiple hyperinflation processes (e.g. Peru experienced two periods of hyperinflation).[33] In terms of duration, the longest hyperinflation episode that has ever been documented in the world took place in Nicaragua (58 months) followed by Greece (56 months) in the 1940s and Venezuela (37 months). Figure 1 shows a graphical representation of these hyperinflation processes together with the rates of growth of the monetary base.[34]

[32] Hanke and Bushnell (2017) added to the World Hyperinflation Table a 57th episode, Venezuela's hyperinflation.
[33] In December 1956 the inflation rate in Bolivia reached 72% exceeding Cagan's hyperinflation criterion, however, this event is not recorded in the Hanke-Krus World Hyperinflation Table.
[34] The money growth rates in Bolivia during the second quarter of 1986 are monthly averages using the compound growth formula. A similar approach was used to compute the monthly growth rates in 1988.

Figure 1
Inflation and Money Growth Rates



A visual inspection of the inflation rates and money growth in Figure 1 reveals that hyperinflation processes tended to coincide with a large expansion of the monetary base, therefore, from this perspective hyperinflation seems to be a monetary phenomenon in Friedman's sense. Furthermore, in most cases inflation and money growth reached their peaks during the same period.[35] However, the increases in the monetary base were not of the same magnitude as those exhibited by the price level, apart from the hyperinflations in Brazil

[35] The exception was Bolivia, a country in which money growth reached its maximum value (124%) two months before the February 1985 peak monthly hyperinflation rate (183%).

and Nicaragua.[36] The explosive properties of these time series applying the right-tailed ADF tests (and their extensions) are analysed subsequently.

ADF_1, $SADF_r$ and $GSADF_r$

Table 3 shows the results of the right-tailed statistical tests ADF_1, $SADF_r$ and $GSADF_r$ for the time series of the inflation rates in Argentina, Bolivia, Brazil, Nicaragua, Peru and Venezuela. The tests have been conducted using 600 observations, except for the time series of Nicaragua that contains 554 observations. The number of (transient) lags order applied in the tests is $k = 1$, and the minimum window size is 50 observations (47 observations for Nicaragua). The 95% critical values of these statistics are -0.0146, 1.520 and 2.237, respectively (-0.069, 1.5242 and 2.2239 for the statistical tests on Nicaragua's inflation rates). The critical values were obtained from the R Core Team (2020) package `exuber`, developed by Vasilopoulos, Pavlis, and Martinez-Garcia (2022). Similarly, the statistical analysis was carried out using the STATA module `radf` of Baum and Otero (2021).

Table 3
Right-Tailed ADF Tests for Inflation

Country	Period	Test statistic		
		ADF_1	*$SADF_r$*	*$GSADF_r$*
Argentina	1970:01 - 2019:12	-9.332	4.589	5.773
Bolivia	1970:01 - 2019:12	-8.971	2.885	2.885
Brazil	1970:01 - 2019:12	-5.144	2.672	2.678
Nicaragua	1978:02 - 2024:03	-8.528	-2.249	0.891
Peru	1970:01 - 2019:12	-12.136	4.746	5.437
Venezuela	1973:01 - 2022:12	-5.119	8.138	8.138

Notes: t_{ADF1}, t_{SADFr}, and t_{GSADFr} 95% tabulated critical values are -0.0146, 1.520 and 2.2370, respectively. The minimum window size is 50 observations. The (transient) lag order $k = 1$. Nicaragua: t_{ADF1}, t_{SADFr}, and t_{GSADFr} 95% tabulated critical values are -0.069, 1.5242 and 2.2239, respectively. The minimum window size is 47 observations. The (transient) lag order $k = 1$.

A comparison of the ADF_1 statistics against the right-tailed critical value indicates that the tests cannot reject the null hypothesis $H_0: \gamma_{r_1 r_2} = 0$ in favour of the right-tailed alternative hypothesis $H_0: \gamma_{r_1 r_2} > 0$ at the 5% significance level. Hence, we could conclude that there was no significant evidence of explosiveness in the price data. Nevertheless, these tests have been criticised because of their low power in detecting temporary or short-lived explosiveness in economic and financial variables (see, for example, Evans, 1991). By contrast, both the $SADF_r$ and the $GSADF_r$ tests give strong evidence of explosive behaviour in the inflation rate in all

[36] In March 1990 the price level and the monetary base increased in Brazil by 82% and 145%, respectively. Similarly, higher increments in the money supply relative to the price level were present in Nicaragua during February 1988 (326% versus 91%) and December 1988 (244% versus 127%).

these economies except for the Nicaragua's inflation rate as their statistics are considerably higher than the 95% critical values.

It has been argued that the performance of the forward recursive approach developed by Phillips, Yu and Wu (2011), the $SADF_r$, can be affected by unconditional heteroskedasticity and important size distortions in their applications (see Harvey, Leybourne, Sollis and Taylor, 2016). In addition, the sequential nature of the methodology makes it prone to a problem known as the *multiplicity* or *family-wise size control* in which the probability of making false positive conclusions increases with the number of hypothesis tests that are conducted. The same problem is expected to be present in the flexible recursive approach of Phillips, Shi and Yu (2015a,b), the $GSADF_r$. To mitigate the problem, Phillips and Shi (2020) proposed a new bootstrap procedure, which is applied in this study to obtain the right-tailed critical values of the statistical tests using 499 replications. The results are presented in Table 4.

Table 4
Right-Tailed ADF Tests for Inflation: Wild Bootstrap (Phillips and Shi, 2020)

Country	Test Stat.	Finite Sample Critical Values		
		90%	95%	99%
Argentina:				
ADF_1	-9.332	-0.3511	0.1485	0.7524
$SADF_r$	4.589	1.8411	2.0632	2.5752
$GSADF_r$	5.773	0.7524	2.5752	4.7593
Bolivia:				
ADF_1	-8.971	-0.5326	-0.2282	0.3027
$SADF_r$	2.885	1.8314	2.4541	3.5401
$GSADF_r$	2.885	3.6988	4.3079	7.5351
Brazil:				
ADF_1	-5.144	-0.3511	0.1485	0.7524
$SADF_r$	2.672	1.8411	2.0632	2.5752
$GSADF_r$	2.678	2.9808	3.9946	4.7593
Nicaragua:				
ADF_1	-8.528	-0.427	0.035	0.990
$SADF_r$	-2.249	0.697	2.255	3.470
$GSAD_r$	0.891	3.653	4.313	5.206
Peru:				
ADF_1	-12.136	-0.3004	0.0525	0.5311
$SADF_r$	4.746	2.2112	2.9490	5.1872
$GSADF_r$	5.437	4.1156	4.8713	6.8900
Venezuela:				
ADF_1	-5.119	-0.4243	-0.0291	0.7116
$SADF_r$	8.138	1.6187	2.1059	3.4940
$GSADF_r$	8.138	3.4932	4.0856	5.7696

Notes: The right-tailed Monte Carlo critical values based on the wild bootstrap by Phillips and Shi (2020) are computed using 499 replications.

The results presented in Table 4 show that the simulated bootstrap critical values are much larger than the corresponding critical values tabulated by Vasilopoulos, Pavlis, and Martinez-Garcia (2022). Using the simulated 95% critical values and the $GSADF_r$ statistics, which are known to be superior to the $SADF_r$ statistics, evidence of explosive behaviour in the inflation rates can be found in countries such as Argentina, Peru and Venezuela. The results reached for Argentina and Brazil contrast with those found in Morita, Psaradakis, Sola and Yanis (2024) where evidence favouring a price-level bubble is found in the case of Brazil but not for Argentina using an alternative econometric methodology.

In Table 5 the results of conducting the right-sided unit root tests and their extensions to the money growth rates in Argentina, Peru and Venezuela are presented. Argentina's $tADF_1$, $tSADF_r$, and $tGSADF_r$ 95% tabulated critical values are -0.0146, 1.520 and 2.2370, respectively. The minimum window size is 50 observations. Peru's $tADF_1$, $tSADF_r$, and $tGSADF_r$ 95% tabulated critical values are -0.0718, 1.4444 and 2.1508, respectively. The minimum window size is 40 observations. Lastly, Venezuela's $tADF_1$, $tSADF_r$, and $tGSADF_r$ 95% tabulated critical values are -0.0880, 1.4136 and 2.1366, respectively. The minimum window size is 37 observations. The (transient) lag order in all the tests was set to $k = 1$.

Table 5
Right-Tailed ADF Tests for Money Growth

Country	Period	Test statistic		
		ADF_1	*$SADF_r$*	*$GSADF_r$*
Argentina	1970:01 - 2019:12	-14.018	-1.820	-0.263
Peru	1989:02 - 2022:12	-7.753	-2.600	-2.447
Venezuela	1993:02 - 2022:12	-5.903	-2.833	0.333

Notes: Argentina's t_{ADF1}, t_{SADFr}, and t_{GSADFr} 95% tabulated critical values are -0.0146, 1.520 and 2.2370, respectively. The minimum window size is 50 observations. Peru's t_{ADF1}, t_{SADFr}, and t_{GSADFr} 95% tabulated critical values are -0.0718, 1.4444 and 2.1508, respectively. The minimum window size is 40 observations. Venezuela's t_{ADF1}, t_{SADFr}, and t_{GSADFr} 95% tabulated critical values are -0.0880, 1.4136 and 2.1366, respectively. The minimum window size is 37 observations. The (transient) lag order is $k = 1$.

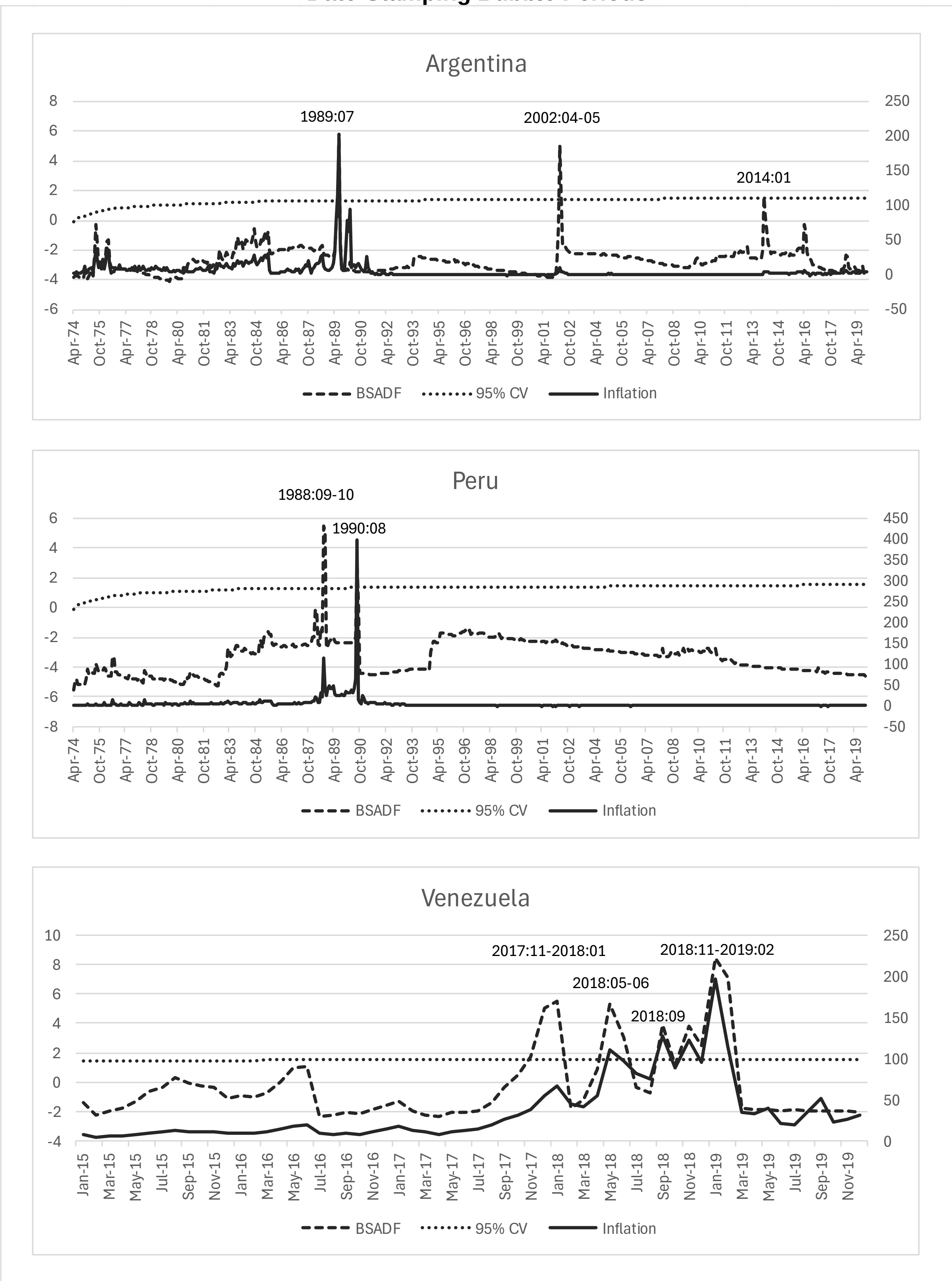
Based on the statistical results presented in Table 5, where the null hypothesis of a unit root cannot be rejected in favour of the right-tailed alternative hypothesis in the time series of money growth rates, we should conclude that there seems to be supporting evidence of the presence of price-level bubbles in countries such as Argentina, Peru and Venezuela during some time spans.

Date-stamping

In the last stage of the statistical approach, the BSADF tests proposed by Phillips, Wu and Yu (2011) is computed to identify the periods in which inflation rates exhibits explosive behaviour.

The results of the tests for the time series of inflation in Argentina, Peru and Venezuela are presented in Figure 2.

Figure 2
Date-Stamping Bubble Periods



The statistical tests applied to the inflation rate in Argentina show evidence of a price-level bubble in 1989:07 when the variable reached its highest rate of growth (197%) during the hyperinflation episode of 1989:05-1990:03. Similarly, there is evidence favouring explosiveness in the general price level in 2002:04-05, a period preceded by the 2002:01 collapse of the currency board that was adopted in 1991:04, and in 2014:01. It is worth mentioning Calvo's (2021, pp. 73) observations on the evolution of the general price level in Argentina during the periods analysed here. After drawing attention to a downward trend in the government deficit as a share of GDP (see Figure 3) and arguing against fiscal imbalances financed by seigniorage as a sufficient explanation of the evolution of prices in Argentina, he states the following: "Argentina's inflation explosions are worth exploring in great detail. I call them 'explosions' because they exhibit inflation rates that are orders of magnitude higher than those occurring shortly before and, as a general rule, are cases in which financial and other considerations dominate primary fiscal deficits. Figure 2 [average monthly inflation rate per year] very clearly shows the 1989 and 2001 explosions that seem to come from nowhere."

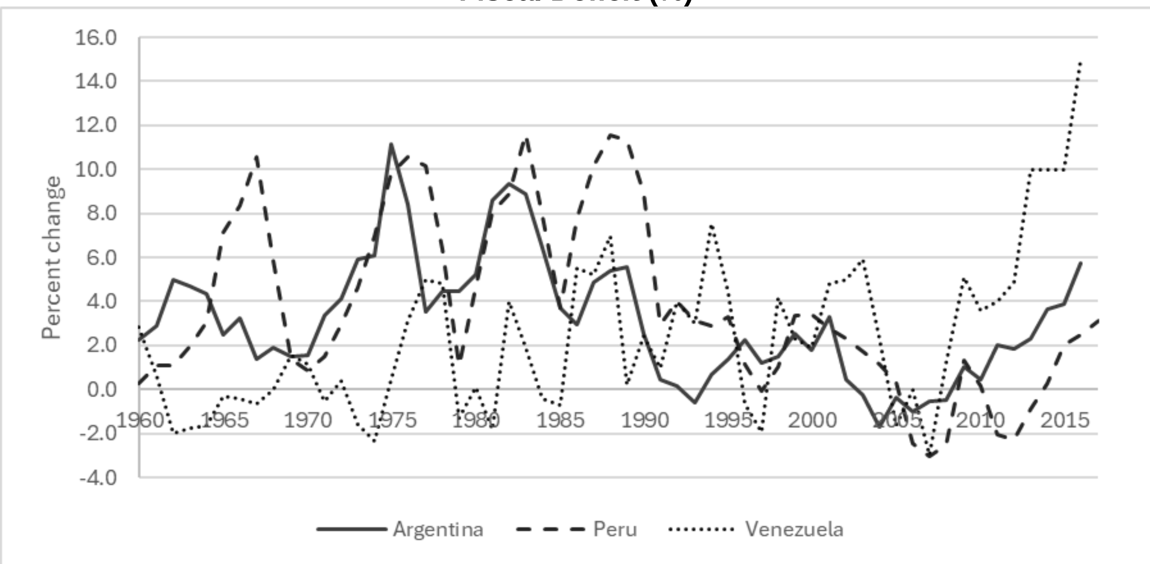
It could be argued that a similar picture emerges from analysing Peru's results. In this economy there is evidence of explosiveness in prices during 1988:09-10 and 1990:08.[37] These periods match the months in which inflation reached the highest rates during the hyperinflation processes in 1988:09 and 1990:07-08 -the inflation rates in 1988:09 and 1990:08 were 114% and 397%, respectively. In addition, these were months in which macroeconomic stabilisation attempts were introduced in the country.[38] A visual inspection of Figure 3 shows that the government deficit as a share of GDP reaches a maximum in 1988 and then describes a downward sloping trend for the following years. In this respect, Bigio (2021, pp. 446) makes the following remark: "If you inspect the path of revenues and expenditures, you see a clear picture: in the period between 1987 through 1989, the government was trying hard to cut its deficit by reducing expenditures…This was not a situation in which expenses were increasing; there was a reduction in both government income and expenses together."

[37] Given that the money growth time series does not cover the year 1988, we are unable to identify a price-level bubble during that period. Nevertheless, the study found no evidence of explosiveness in money growth in any country, which could indicate the presence of a price bubble in 1988:09-10.

[38] In September 1988 a drastic attempt to correct controlled prices, popularly known as the "Salinazo" after Abel Salinas the finance minister of Allan Garcia's first administration (1985-90), was implemented; the stabilisation programme led to a 757% devaluation of the exchange rate. Similarly, in August 1990 the administration of Alberto Fujimori (1990-2000) announced another large stabilisation programme and a structural reform agenda, popularly named the "Fuji shock", aiming to deregulate markets and reduce the size of the state in the economy.

Perhaps, the strongest evidence favouring divergent paths in the general price level, given their time span and recurrence, is found in Venezuela. Several periods of explosiveness in prices, mostly during the 2017:12-2020:12 hyperinflation episode, are identified. The price-level bubbles were present during the periods 2017:11-2018:01, 2018:05-06, 2018:09 and 2018:11-2019:02 -the highest inflation rate during the hyperinflation process took place in 2019:01 (197%). Unfortunately, there is no official data to determine whether the fiscal deficit increases or decreases during the hyperinflationary process. However, estimates of the deficit for the restricted public sector during the period 2017-2020 can be found in Iyer and Rodriguez (2021, Table 1, pp. 4).[39] Accordingly, the fiscal deficit as a percentage of GDP for the years from 2017 to 2020 were 26%, 11.9%, 8.9% and 3.9%, respectively. In other words, the public deficit and the usage of seigniorage to finance fiscal imbalances peaked in 2017, a year earlier to the detected explosiveness in prices.[40]

Figure 3
Fiscal Deficit (%)



Source: Kehoe and Nicolini (eds.) (2021).

Ultimately, the time span of the observed explosive behaviours in prices is a matter that requires some consideration. It was mentioned previously that Cagan's view was that even though a price's diversion from its fundamentals is feasible, this should last for only a short

[39] The restricted public sector sums fiscal accounts from the central government and state-owned enterprises such as the Venezuela's oil company, PDVSA. It should be noticed that although the Venezuela's legislation forbids to the monetary authority to monetise the deficit (i.e. the direct purchases of public bonds), the law was circumvented by financing PDVSA, which transferred the funds to the government via higher fiscal contributions (see Iyer and Rodriguez, 2021, pp. 10-11).

[40] Iyer and Rodriguez (2021) estimates of total seigniorage as a percentage of GDP for the period from 2017 to 2020 are 11.4%, 1.9%, 1.4% and 0.1%, respectively.

period because under these conditions a currency reform becomes a political and economic necessity. In addition, Phillips, *et at*. (2015a,b) recommend setting of a minimal duration, which is unavoidably discretionary, for the observed explosive results to be classified as bubbles. Several of the results obtained in this work indicate explosive periods in the price level for just a month, which could be associated with short lived blips in the estimated autoregressive coefficients. Therefore, the short length of these bubbles could put into question the validity of the reported statistical results of the study. Nevertheless, in several cases the explosiveness in prices has lasted longer than a single period such as the three- and four-months bubbles of 2017:11-2018:01 and 2018:11-2019:02 during the hyperinflation in Venezuela. These explosive behaviours have had a relative long duration given that hyperinflation processes tend to be short-lasting -for example, the median duration of the inflationary episodes in Latin America is eleven months.

VI. Final Remarks and Conclusion.

Despite a wealth of literature on the subject, the possibility that prices could take a divergent path from the market fundamentals remains characterised by a lack of consensus among economists. As a result of this controversy two distinct lines of economic research have emerged. Firstly, there is a theoretical approach based on fully specified general equilibrium models that investigates whether sunspots or price-level bubbles are consistent with rational agents' optimising behaviour. And secondly, an empirical approach that explores the implications of rational-expectations paths characterised by such phenomena. The present study has followed the latter tradition.

By drawing on improvements in econometric methodology in recent years, this work seeks to shed light on the important subject of currency deflation. By using the recursive testing procedure and dating algorithm developed by Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, b), which identifies the periods when a variable exhibits explosive characteristics, supporting evidence of divergent paths in the general price level were found in several Latin American economies that experienced hyperinflation episodes. Specifically, confirmatory evidence of mildly explosive behaviour in the inflation rates of Argentina (1989:07, 20002:04-05 and 2014:01), Peru (1988:09-10 and 1990:08) and Venezuela (2017:11-2018:01, 2018:05-06, 2018:09 and 2018:11-2019:02) were identified while indication of explosiveness in the market fundamentals (i.e. money growth rates) were not detected. Both the $SADF_r$ and the $GSADF_r$ tests give strong evidence of explosive behaviour in the inflation rate in these economies as the statistics are considerably higher than the 95% critical values. Furthermore, the results reached by the $GSADF_r$ tests are robust against problems such as unconditional heteroskedasticity and multiplicity or family-wise size control.

On a final note, it should be noticed that the study has not attempted to identify the explicit economic sources of the explosive price-level paths as this requires precise formulation of alternative models and suitable model determination methods to empirically distinguish between the theoretical frameworks. Consequently, the empirical results could be opened to various interpretations including the occurrence of price-level bubbles, herd behaviour or alternative explanations of the dynamics of prices like the proposed fiscal theory of the price level.

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