College Admissions: Need-blind versus Need-aware, Financial Aid and Student Loan Programs

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Need-blind versus Need-aware College Admissions, Financial Aid and Student Loan Programs*

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Abstract: *We develop a stylized college admission model to analyze the effects of need-blind admission policy which requires admission decisions to be independent of applicants' financial needs. We show that the need-blind admission policy enhances needy students' enrollment if financial aid offers are made contingently on the student's need level, but reduces it if they are also made need-blindly. We also analyze student loan programs and investigate an optimal form of loan program in increasing the enrollment of needy students: the initial endowment gap of the colleges should be reduced with the support of the student loan program. (JEL Codes: C72, C78, I23)*

Keywords: college admission, need-aware admission policy, need-blind admission policy

1 Introduction

A majority of US colleges adopt need-blind admission policies, whereby they disregard applicant's financial status in admission decisions. According to the survey by the National Association for College Admission Counseling, 92% of public colleges and 81% of private colleges reported so in 2007, albeit down from 98% and 88% in 1994, respectively (Heller, 2008). Under growing attention on equity and diversity in higher education, this policy is widely perceived as colleges' endeavor to construct a more diverse and inclusive student body, encouraging applications from students in middle and low income families. However, achieving diversity also relies on these students finding financial means to support themselves through the college education, which has been an unrelenting issue given the spiralling tuition fees of selective colleges.

Many colleges announce that they meet the demonstrated needs of admitted students fully through their need-based financial aid programs. Yet, few colleges are able to do so with their own scholarships or grants (that need not be repaid) and students often seek other

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funding sources, such as federal student loans or work-study programs.[1] Colleges adopting need-blind admission policies may be in particularly stretched situations with their scholarship budget, as they extend admission offers to all qualified applicants regardless of their financial needs. Consequently, these colleges cannot offer sufficient scholarships to all admitted students and some students end up turning down the admission offers when they have no external funding or have to borrow excessively. Such cases are called *admit/deny*, as students are *admitted* but in effect *denied* due to inadequate financial aid. Despite well-intentioned policies, admit/deny cases may undermine the colleges' effort and create inefficiency.

As such, the impact of need-blind policies may hinge on how the college's financial aid programs are run in connection with other available funding sources. Insofar as the colleges are fundamentally interested in their academic standing, they may use any discretion allowed in making financial aid offers strategically both relative to rival colleges and across applicants, influencing the demography of enrolling students in directions tricky to predict. Taking into account how colleges will respond, federal agencies may also want to design student loan programs to promote their goal. To better understand what need-blind admission policies may achieve, therefore, all these aspects need to be studied in tandem.

We develop and analyze a stylized economic model equipped for comparing admission outcomes under alternative policies, with a view to deriving useful economic insights and policy implications. Specifically, two competing colleges make admission and financial aid offers to maximize the average academic ability of incoming students given fixed capacity and budget constraints. They observe (unbiased estimates of) academic abilities and demonstrated financial needs of all applicants but not their preferences over the two colleges.

We consider different regimes by varying restrictions on which characteristics of applicants the colleges may base their decisions on. By a *need-blind regime* we refer to environments in which colleges may base their admission decisions on the applicant's academic ability but not on their financial need level (hence, the term "need-blind"). We divide this regime into two categories depending on whether their financial aid decisions should also be need-blind or not. We compare these cases with the benchmark case where the colleges make their decisions without any restriction, which we call the *need-aware regime* (as the colleges base their decisions on the applicant's need level as well as their academic ability), to study if and when need-blind admission policies enhance needy students' enrollment. We also asso-

[1] The amount of loan that a student can bear varies with his/her current economic status, because it should be repaid and an excessive loan might also lower the graduation rate. Surprisingly, there is no rule in the US that mandates colleges to abide by to claim that they meet the full demonstrated need. For instance, colleges calculate the cost of attendance (COA) using their own measures, and this cost can be an underestimation of the actual cost. The composition of the financial aid package also matters even when the demonstrated need is fully met, as explained above. Currently, there are only a handful of colleges that commit *no-loan* policy to all admitted students.

ciate this analysis to the federal student loan programs operated by the U.S. Department of Education (DoED, henceforth). We analyze the effect of the current student loan program and provide a policy implication in designing optimal student loan programs.

To this end, we start with the baseline model where there are no external funding sources. A fraction of the applicant pool are students who need no financial support, hence only need an admission offer to enroll in a college. We call them *non-needy* students. All other students are in need of positive levels of financial support and should receive a full scholarship from the college in order to enroll (as there is no external funding), in addition to an admission offer. We call them *needy* students. Each student enrolls in a college if he/she can, in the preferred one if possible. Each college tries to fill its capacity with students of a highest possible average ability subject to its scholarship budget constraint and relevant restrictions of the regime under consideration.

As the intuition suggests, we show that colleges adopt *cutoff admission rules* in equilibrium, whereby they extend admission offers to all students above a certain cutoff ability level. The cutoff level may differ depending on the student's financial need level in the need-aware regime but must be the same regardless in the need-blind regime. All non-needy students above the cutoff can enroll without receiving any scholarship, but colleges have to expend their scholarship budgets to accommodate any needy student in the amount equal to the student's financial need.

In the need-aware regime, the equilibrium admission cutoff turns out to be higher for needy students than for non-needy students, increasing linearly in their need level as they become more "costly" to accommodate. This implies that the average amount of scholarships offered to all needy students above the cutoff schedule is lower than the *ex ante* mean need level of the initial pool of all needy students. The exact equilibrium cutoff schedule is determined by binding capacity and budget constraints.

In the need-blind regime, on the other hand, the colleges have to apply the same admission cutoff to all students. If they have to make financial aid offers need-blindly as well, there is no other option but to ration scholarships randomly with the same probability across all need levels. Then the average amount of scholarships offered to needy students is equal to the *ex ante* mean need level of all needy students, thus higher than that in the need-aware regime. Given a fixed budget, therefore, the total number/measure of needy students accommodated is smaller than that in the need-aware regime. This in turn means that the colleges lower the admission standard to admit more non-needy students to fill the capacity, by setting the common admission cutoff below the level set for non-needy students in the need-aware regime.

Consider the alternative case that colleges may make scholarship offers conditioning on the applicant's need level. Then they still have to ration scholarships among admitted needy students but they may do so cost-effectively by offering scholarships to students with lowest need levels above the common cutoff until the budget is depleted, insofar as the average ability above the cutoff remains the same across need levels. We show that they indeed do this in equilibrium, that is, each college offers scholarships to all needy students up to a certain threshold need level so long as they meet its admission cutoff. The college with a larger scholarship budget can reach out farther to needy students, hence sets a higher admission cutoff and a higher threshold need level. We show that the total needy students accommodated in this regime is larger than that in the need-aware regime. This is because scholarships are allocated in effect to maximizes the enrollment of needy students in this regime treating the common cutoff level as the minimal admittable ability, whereas in the need-aware regime they are allocated to equalize the marginal contribution (to average ability of enrolling students) of scholarship money spent across all need levels.

As such, admit/deny incidents are unavoidable under need-blind admission regimes, be they random or concentrated on highly needy students depending on whether scholarship offers are also need-blind or not. The *match inefficiency* is another issue that may affect needy students, particularly when scholarship offers are also need-blind and made randomly: some students admitted by both colleges may receive financial support only from their less preferred college and enroll there. Then, swapping the colleges for such students would bring about a Pareto improvement. Such misaligned outcomes may also arise in other regimes when the colleges have different budgets, but those incidents are inherent to the asymmetry of the colleges leading to different admission cutoffs between colleges and Pareto-improving swaps are not possible.

Next, we extend our analysis to environments where external funding is available, e.g., in the form of federal student loans. Students may now enroll in a college without receiving a full scholarship provided that they can take out a loan. Knowing this, the colleges adjust their scholarship offers accordingly. We study how such external funding affects the enrollment of needy students and thereby, how such funding may be designed to best promote it. We focus on the need-blind regime where financial aid may be need-dependent because needy students' enrollment is greatest in this environment as explained above.

Federal student loans are not limitless and their availability is a policy variable because the federal student loan programs are largely managed by the DoED in the U.S. We examine and characterize the DoED's optimal design of student loan programs, assuming that it aims to maximize the total enrollment of needy students given a fixed budget of federal loan.[2] Formally, we introduce a function representing the availability of student loans,

[2]The fixed total amount of loan is interpreted as the annual federal budget allotted to the federal student

which specifies the probability with which a loan application gets approved depending on the amount sought. Taking this function as given, the colleges choose their admission cutoffs and scholarship allocations and the applicants take out a loan when needed and decide where to enroll.

We first analyze the case in which the availability function is a single *step function*, so that loan applications are approved for sure up to a certain threshold amount but no higher amount is approved. In equilibrium, the colleges offer partial scholarships, inducing the applicants to make up the rest with a loan, and use the saved budget to recruit more qualified applicants with greater financial needs. Specifically, the colleges behave as if the students' need levels are reduced by the threshold loan amount, and the result above without external funding applies with need levels adjusted as such.

When the two colleges have identical scholarship budgets, they set the same admission cutoff and threshold need level as if one half of the federal budget was their own. This equilibrium outcome maximizes the enrollment of needy students among all loan availability functions. For colleges with similar budgets, therefore, this observation justifies the student loan program currently adopted by the DoED: there is a maximum credit limit for each applicant, depending on the student's school year and dependency status, and a student loan up to this limit is guaranteed.[3]

If they have different budgets, however, a single step function no longer maximizes the enrollment of needy students. In fact, we find that single-step loan programs exacerbate the imbalance of the two colleges originating from their asymmetric budgets. As explained above, in this case the college with a larger budget sets both a higher admission cutoff and a higher threshold need level than the other, thus admitting less non-needy students and reaching out farther to more needy students in equilibrium. Since the amount of loan that needy students take out increases with their need level up to the loan threshold, this means that the total loan taken out by the students enrolling in the better-endowed college is larger than that of the other college. As student loans are some sort of "subsidy" to the colleges that relaxes their budget constraints without imposing any burden on them, the better-endowed college receives more subsidy from the DoED under single-step loan programs. This aggravates the imbalance of the two colleges, which in turn hampers the total enrollment of needy students as will be elaborated in our analysis.

This suggests that federal loan programs need be designed to abate the initial endowment gap between colleges, in order to enhance the enrollment of needy students. We discuss how

loan programs. In 2021, for instance, $94 billion is requested for the student loans (excluding consolidation loans), according to the DoED's *Fiscal Year 2021 Budget Request*. For more details and various plans of federal student loan programs, see the "Student Loan Policies" section of the report.

[3] These limits are specified in the guideline of the *Federal Student Aid*, available at *https://studentaid.gov/understand-aid/types/loans.*

the federal loan programs can be modified in this direction. In particular, we fexplain how the current program may be augmented with a suitably designed "matching loan" scheme to help achieve the goal by reducing the endowment gap.

College admission has been the subject of both theoretical and empirical studies. However, to our knowledge, comparison of need-blind and need-aware admission policies and the role of financial aid systems have not been studied hitherto. For instance, Epple *et al.* (2006) present a general equilibrium model of college admissions where colleges determine optimal price-discrimination strategies as well as their educational expenditures, utilizing all attributes of students including their economic status. Hence, they do not study need-blind admission regimes. The comprehensive nature of their model renders it difficult to characterize equilibrium analytically; instead, they provide its structural estimation.

Another line of research on the topic focuses on *college admission games* where applicants are sorted and matched to the colleges under various specifications, abstracting from endogenous educational investment/expenditures. Kim (2010), for example, analyzes the effects of early decision admissions as a device to screen student's income for colleges that officially adopt need-blind admission policy. Without financial aid considerations, Avery and Levin (2010) analyze early decision/action, Chade *et al.* (2014) consider frictions with non-negligible application fees, and Che and Koh (2016) study the impact of enrollment uncertainties on admission strategies. Our analysis also stands on this line but we focus on the impact of need-blind policy in relation to internal and external financial aid systems.

On the other hand, *centralized mechanisms* for college admissions have been widely studied in the market design literature, which stems from the seminal work by Gale and Shapley (1962). In recent studies, financial aid has been incorporated to these matching mechanisms, and relevant stability concepts have been tested (Abizada (2016), Afacan (2020), Biró *et al.* (2020)). Nevertheless, applicants' economic backgrounds and different admission standards remain to be addressed.

The rest of the paper is organized as follows. In Section 2, we define the model and introduce admission and financial aid rules. In Section 3, we present our results under need-aware and need-blind policies, assuming that scholarships are the only source of financial aid. We extend this analysis in Section 4 by introducing federal student loans, followed by a discussion of an optimal design of student loan programs. Section 5 includes a few concluding remarks. Appendix contains deferred proofs.

2 Model

There are two selective colleges A and B, each with an enrollment capacity of measure $1/2$ and a respective budget of $M_A, M_B > 0$ for financial aid. The two selective colleges can accommodate one half of all students in a student pool S of measure 2.[4]

Each student $i \in S$ is characterized by his preference/taste $\tau_i \in \{A, B\}$ for colleges; his academic ability $v_i \in V \equiv [0, 1]$; and financial status $y_i \in Y \equiv [\underline{y}, \overline{y}]$ where $\underline{y} < 0$ and $\overline{y} = 1$ by normalization. The three characteristics are distributed independently of one another, as well as across students, according to a commonly known distribution described below.

Each student i is equally likely to prefer college A to college B (i.e., $\tau_i = A$) and the other way around ($\tau_i = B$). We divide S into two groups of students S_A and S_B of measure 1 each, where students in S_A prefer college A and students in S_B prefer college B.

In each group S_c where $c = \{A, B\}$, student i's academic ability, v_i, and financial status, y_i, are distributed, respectively, according to atomless (cumulative) distribution functions G_V and G_Y with full support. A student with a higher v_i is more able academically. A student's financial status y_i, if positive, specifies this student's financial need, namely, the amount of financial aid that he must secure to be able to enroll in either college, should he be admitted. A student with $y_i \leq 0$ does not need financial aid, thus only needs an admission offer to enroll in a college. We say a student i is *needy* if $y_i > 0$ and *non-needy* otherwise. Negative financial status levels $y_i \in [\underline{y}, 0]$ are used solely for expositional ease of representing a positive mass of non-needy students by a continuous distribution G_Y, thus are equivalent strategically. We use $G(v, y) = G_V(v) \cdot G_Y(y)$ to denote the cdf of (v_i, y_i) when convenient.

At the application stage, each student knows his characteristics (v_i, y_i) and the colleges observe them correctly.[5] The student's taste τ_i is his private information and cannot be credibly communicated. Application is costless, hence all students apply to both colleges.

Based on the commonly known distribution G, each college $c \in \{A, B\}$ decides on an admission policy that dictates whom to offer a place and how much financial aid to offer based on the student's characteristics (v_i, y_i). An admission policy consists of (i) an *admission rule*, $\alpha : V \times Y \to \{0, 1\}$, that specifies whether to offer admission ($\alpha(v, y) = 1$) or not ($\alpha(v, y) = 0$)[6] and (ii) an *aid rule*, $\beta : Y \to \Delta(\Re_+)$, that specifies a probability distribution $\beta(y)$

[4] We may assume that a third, non-selective college (least preferred by all students) absorbs all the remaining students, but this third college would play no role in our analysis.

[5] The academic ability v_i can be interpreted as a weighted sum of GPA, SAT and other relevant activities, which is perceived as unbiased estimate of the applicant's academic ability. The financial status y_i is submitted to the *Free Application for Federal Student Aid (FAFSA)* and the accuracy of these data is verified through income tax returns and W-2 statements and so on.

[6] We adopt a technical convention of treating admissions rules equivalently if the sets of student characteristics to which they extend admissions to, $\alpha^{-1}(1)$, differ only by a measure 0 set. This is innocuous because such admission rules are strategically equivalent regardless of the other college's strategy.

over amounts of financial aid to offer. Colleges are not allowed to offer a financial aid that exceeds each student i's financial need, $\max\{0, y_i\}$, and therefore, $\beta(y) \in \Delta([0, \max\{0, y\}])$. Thus, when we say that a needy student receives a *full scholarship*, the amount equals to his financial need.

An admission policy is *need-blind* if α depends only on v and not y, and *need-aware* if it depends on both v and y. Note that the financial aid rule, β, may depend on the student's financial status but not on his academic ability. This is because we focus on financial aid schemes that go through the *Free Application for Federal Student Aid* (FAFSA) and a student's financial status is basically the only relevant information verified in this process.

A student i is said to have *secured a place* in college $c \in \{A, B\}$ if he has received an admission offer and obtained a financial aid equal to his need y_i if $y_i > 0$ from college c and/or external sources to be detailed later. If he secures a place in both colleges, he enrolls in his preferred college; if he secures a place in just one college, he enrolls in that college. In line with Avery and Levin (2010) and Kim (2010), we assume that each college must fill up their capacity exactly with these students. The objective of each college is to maximize the average academic ability of the incoming students subject to its scholarship budget constraint under any admission/aid regime specified.

We formalize the situation above as the following game between the two colleges:

1) Each college $c \in \{A, B\}$ chooses an admission policy (subject to the restrictions of the admission regime under consideration) and extends admission and financial aid offers to all students according to it.
2) Each student enrolls in the (preferred) college he secured a place in, if any.
3) The payoff of college c is the average academic ability of the students who enroll in college c if such students fill up its capacity, $1/2$, exactly.

A Nash equilibrium is admission policies of the two colleges that are mutual best responses in the game described above. We characterize and compare Nash equilibria of this game under several different admission regimes which place different restrictions on admission policies the colleges may adopt.

The average academic ability of all incoming students of the two colleges is maximized when they (together) recruit the top one half of students in academic ability in the entire student pool S (of measure 2) so that they fill up the capacities of both colleges. These are all students with $v_i \geq v^*$ regardless of their financial status and taste, where v^* is the ability level such that $G_V(v^*) = G(v^*, \overline{y}) = 1/2$. By denoting the average need level of all needy students (i.e., with $y_i > 0$) in the entire student pool S by

$$\tilde{n} \equiv \int_0^{\overline{y}} \frac{y}{1 - G_Y(0)} \, dG_Y,$$

the total budget needed to support all needy students with ability above v^* in S_A (or in S_B) is $(1 - G_Y(0))(1 - G_V(v^*)) \cdot \tilde{n}$.

If the budget of each college $c \in \{A, B\}$ is sufficiently large to fund all such students in S_c, the college can fill up its capacity with students of ability above v^*, offering full scholarships to all those who need financial aid.[7] As will become clear in the sequel, this is the unique equilibrium outcome in every regime we will consider. To avoid such triviality, we assume that M_A and M_B are not sufficiently large to warrant such an ideal outcome:

Assumption 1. $M_A, M_B \ < \ (1 - G_Y(0))(1 - G_V(v^*)) \cdot \tilde{n}$.

As their budgets are insufficient to fill the capacity by supporting all needy students with abilities above the ideal cutoff v^*, the colleges need to admit more non-needy students (i.e., with $y_i \leq 0$) by lowering the academic standard for them. We aim to characterize and compare the extent to which they have to "favor" non-needy students as such (at the detriment of needy students) in different admission regimes, with a view to providing policy implications.

For an equilibrium to exist, each college must be able to fill its capacity with its scholarship budget. We ensure that this is possible for both colleges with any scholarship budget by assuming that there are enough non-needy students to fill both colleges, i.e., $G_Y(0) \geq 1/2$. Also, to warrant that equilibrium is unique in all regimes we consider (so that unambiguous comparison of outcomes is possible between them), we impose an additional assumption on G_Y below:

Assumption 2. $G_Y(0) \geq 1/2 \quad \text{and} \quad G_Y(y)/y$ is non-increasing in $y \in (0, 1)$,

The latter assumption means that financial needs are not highly concentrated at high levels. It is a relatively mild condition because, for $G_Y(y)/y$ to increases, $G_Y(y)$ must increase at a rate exceeding both $G_Y(y)/y > \frac{1}{2y}$ and 1 at some y, thus a sufficient condition is $g_Y(y) \leq \max\{\frac{1}{2y}, 1\}$ for $y \in (0, 1)$. This is satisfied unless $g_Y(\cdot)$ swings sharply at some levels of y, since a uniform distribution on $(0, 1)$ would mean $g_Y(y) \leq 1/2$ given $G_Y(0) \geq 1/2$. Assumption 2 warrants unique equilibrium for every possible scholarship budgets the two colleges may have, thus can be relaxed significantly once their budgets are fixed.

We close this section with a useful lemma. We say that an admission rule α is a *cutoff (admission) rule* if there is a "cutoff schedule" $x : Y \to V$ such that $\alpha(v, y) = 1$ if and only if $v > x(y)$. That is, an admission offer is made to everyone with academic ability above a certain cutoff $x(y)$ that may depend on their financial status. By the next lemma, we consider only cutoff rules as equilibrium admission rules, which facilitates presentation greatly.

[7] To be precise, in equilibrium college A gets the top measure $1/2$ of students in academic ability in S_A and college B gets the same students in S_B with ability above v^*.

Lemma 1 *In every equilibrium of the admission regimes that we consider, each college adopts a cutoff admission rule.*

The proof is provided in Appendix, but the intuition is clear: each college can enhance the average ability of incoming students by admitting students at the top end of academic ability subject to its capacity and budget.

3 College Admissions with No External Funding

In this section, we assume that there are no external sources of financial aid other than scholarship from the colleges' budget. Hence, a needy student i may enroll in a college only if he receives a full scholarship as well as an admission offer from the college. Section 4 extends the analysis to the cases where external funding sources exist to which needy students may resort.

3.1 Need-aware regime

In the *need-aware* regime, colleges adopt admission rules that may depend on both a student's ability v_i and need y_i. By Lemma 1, college c sets different ability cutoff levels for different y_i's, which we depict by a measurable function $x_c^a : Y \to V$, where the superscript a refers to the need-$a$ware admission regime. To facilitate the exposition we explain the equilibrium heuristically in the main text, deferring a formal proof to Appendix.

We first characterize an equilibrium for the symmetric budget case whereby the colleges have identical budgets; then we explain how the equilibrium changes for the asymmetric budget case. Since there is no other funding source, an admission offer to any needy student is pointless unless accompanied by a full scholarship, because without it, the student cannot enroll even with an admission offer. Hence, we assume without loss in the current regime that colleges offer full scholarships to all needy students whom they extend admission offers to.

Thus, an equilibrium is a pair of cutoff schedules (x_A^a, x_B^a) that are mutual best responses subject to filling the college's capacity without breaking the budget constraint. An equilibrium is symmetric if $x_A^a = x_B^a$, i.e., the two colleges adopt the same cutoff schedule and thus, make offers to the same set of students, i.e., those above the common cutoff schedule. In such equilibria, all such students enroll in their favorite college and each college c fills its capacity only with students in S_c.

As will be shown below, the equilibrium is unique and symmetric when the colleges have the same budget $M_A = M_B = M$. In identifying this (and subsequent) equilibrium, the following "solo (optimization) problem" proves useful.

Solo problem. A single college (without a rival college), say A, chooses an admission policy over a student pool, S_A in the current case, to maximize the average ability of incoming students subject to its capacity and budget constraints:

$$\max_{x:Y\to V} \int_{\substack{x(y)<v \\ \underline{y}<y<\overline{y}}} v\, dG \quad \text{subject to} \quad \int_{\substack{x(y)<v \\ \underline{y}<y<\overline{y}}} 1\, dG = \frac{1}{2} \quad \text{and} \quad \int_{\substack{x(y)<v \\ 0<y<\overline{y}}} y\, dG \le M. \tag{1}$$

Due to insufficient budget (Assumption 1), it is straightforward to verify that the budget constraint binds at the solution, which we take for granted in the sequel. Note that taste parameter τ_i is moot in this problem because every student will enroll if offered a place.

Let x^a be a cutoff schedule that solves the optimization problem (1), which we call a "solo-optimal policy". To characterize this, we solve the Lagrangian problem of (1) written as

$$\max_{\substack{x(y)\in[0,1],y\in Y \\ \mu,\lambda}} \int_{\substack{x(y)<v \\ \underline{y}<y<\overline{y}}} v\, dG + \mu\frac{1}{2} + \lambda M - \int_{\substack{x(y)<v \\ \underline{y}<y<\overline{y}}} (\mu + \lambda \max\{0, y\})\, dG$$

where μ and λ are multipliers for the capacity and budget constraints, respectively. The first order conditions (FOC) are:

$$\left.\begin{array}{lll} [\mu - x(y)]g(x(y), y) = 0 & \Leftrightarrow \quad \mu = x(y) & \text{if } y \le 0, \\ [\mu + \lambda y - x(y)]g(x(y), y) = 0 & \Leftrightarrow \quad \mu + \lambda y = x(y) & \text{if } y > 0 \text{ and } x(y) < 1, \end{array}\right\} \tag{2}$$

$$\int_{\substack{x(y)<v \\ \underline{y}<y<\overline{y}}} 1\, dG = \frac{1}{2}, \qquad \text{and} \qquad \int_{\substack{x(y)<v \\ 0<y<\overline{y}}} y\, dG = M \tag{3}$$

where $g(v, y)$ denote the density functions of $G(v, y) = G_V(v) \cdot G_Y(y)$.

By the conditions in (2), to maximize the average ability of incoming students, college A admits all non-needy students above a constant cutoff, say $x^a(y) = \mu^a > 0$ for all $y \le 0$; and all needy students above the cutoff that increases linearly at a rate, say $\lambda^a > 0$, in their financial need $y > 0$, i.e., $x^a(y) = \mu^a + \lambda^a y$, until $x^a(y)$ reaches the maximal ability 1.

It is clear that the solo-optimal cutoff level should be the same for all non-needy students because they are equivalent strategically; moreover, this cutoff is positive ($\mu^a > 0$) because the total enrolling students would exceed the capacity if $\mu^a = 0$ given $G_Y(0) \ge 1/2$.

It is also intuitive that the cutoff increases in $y > 0$, because otherwise the college may swap students of higher ability and lower needs with those of lower ability and higher needs without breaking the budget. To see, in addition, that $x^a(y)$ must increase *linearly* in $y > 0$ heuristically, suppose to the contrary that there are $y', y'' > 0$ such that $y'' = 2y'$ and $x^a(y'') - \mu^a \ne 2(x^a(y') - \mu^a)$, say $x^a(y'') - \mu^a > 2(x^a(y') - \mu^a)$. Then, the college could admit another student with financial need $y_i = y''$ (whose ability is $x^a(y'')$ at the margin) by releasing two students with $y_i = y'$ (whose ability is $x^a(y')$) to satisfy the budget constraint, and admit another non-needy student (whose ability is $x^a(0) = \mu^a$) to fill capacity, thus enhancing the average ability because $x^a(y'') + x^a(0) > 2x^a(y')$. As such, linearity of $x^a(y)$ captures that the marginal value of substituting scholarship spending across need levels is exactly balanced out by its implication on the non-needy student intake to meet the capacity.

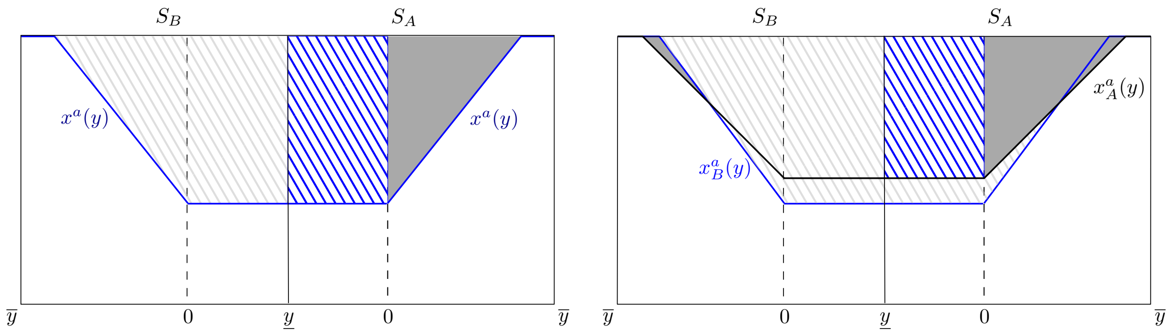
We are now ready to pin down the solo-optimal policy. For $\mu \le v^*$, let $\lambda(\mu) > 0$ be the solution to the latter equation of (3) when $x(y) = \mu$ for $y \le 0$ and $x(y) = \min\{\mu + \lambda(\mu)y, 1\}$ for $y > 0$, that is, the budget constraint binds when the said cutoff rule $x(y)$ is adopted. This strategy under-fills capacity at $\mu = v^*$ due to Assumption 1, and as μ decreases from v^*, binding budget constraint implies that $\lambda(\mu)$ increases and the enrollment of needy students increases continuously as well as that of non-needy students, eventually over-filling the capacity before μ hits 0 because $G_Y(0) \ge 1/2$. Therefore, there is a unique solution to the set of FOC's listed above, denoted by (μ^a, λ^a) where $\mu^a \in (0, v^*)$. Consequently, the solo-optimal policy (which exists as shown in Appendix) must be the cutoff strategy

$$x^a(y) = \begin{cases} \mu^a \in (0, v^*) & \text{for } y \le 0 \\ \min\{\mu^a + \lambda^a y, 1\} & \text{for } y \in [0, \bar{y}] \quad \text{where} \quad \lambda^a > 0. \end{cases} \tag{4}$$

Lemma 2 *There exists a unique solo-optimal policy that solves the optimization problem (1), which is $x^a(\cdot)$ in (4) where μ^a and λ^a solve (3).*

Remark 1: The solo problem (1) is written for the need-aware regime with S_A as the student pool, but an analogous solo problem can be defined in other regimes (to be specified later) and also for student distributions G that are not independent between the two traits v and y. In particular, if the student pool gets larger due to more students (of certain distribution of characteristics) being added, it is straightforward to see that the solo-optimal policy for the need-aware regime continues to be the unique cutoff strategy of the form (4) as described in Lemma 2. This observation will be used in the subsequent sections.

Having characterized the solo-optimal policy as above, we now show that both colleges adopting it constitutes a unique equilibrium when the colleges have identical budgets. If both adopt the solo-optimal policy, (4), each college $c \in \{A, B\}$ gets all students above the cutoff $x^a(y)$ who favor the college (i.e., in group S_c), as illustrated in Figure 1(a). To verify optimality, consider a deviation from x^a, say by college A. Due to the capacity constraint, this means that college A lowers its cutoff from x^a over some range in Y and raises the cutoff from x^a over some other range in Y: that is, college A newly admits some students from both S_A and S_B over the former range and denies some students from S_A over the latter range. If this deviation were beneficial, the college could have done even better in the solo problem (1) by newly admitting the students only from S_A over the former range and raising the cutoff only half way over the latter range so that both the capacity and budget



(a) When $M_A = M_B$ (b) When $M_A > M_B$

Figure 1: **An equilibrium under the need-aware regime**: (a) The right (left) half of the diagram depicts S_A (S_B) where the horizontal axis measures $y \in [\underline{y}, \bar{y}]$ increasing in the right (left) direction and the vertical axis measures $v \in [0,1]$. When $M_A = M_B$, both colleges set an identical admission cutoff line $x^a(\cdot)$ that varies over $[0, \bar{y}]$. Each student above these cutoff lines receives admission offers from both colleges and enrolls in college A (B) if he is in S_A (S_B). The solid-gray area depicts needy students enrolling in A and the blue-dashed area non-needy students enrolling in A. A symmetric description applies to college B and all students enrolling in college B are populated in the gray-dashed area. (b) When $M_A > M_B$, college B has a lower cutoff for non-needy students than college A; and the cutoff lines of the colleges intersect at a point. The solid-gray area depicts needy students enrolling in college A and the blue-dashed area non-needy students enrolling in A; all students enrolling in college B are populated in the gray-dashed area.

constraints are satisfied; but this would be in contradiction to x^a being a solo-optimal policy that solves the problem (1).

The analysis extends straightforwardly to the case of asymmetric budgets, that is, $M_A > M_B$ without loss of generality. In any equilibrium of this case, say (x^a_A, x^a_B), college A optimally chooses x^a_A in the knowledge that the students in S_B who will get offers from college B according to x^a_B will enroll in college B, but any other student will enroll in college A if given admission (with full scholarship). By definition, therefore, x^a_A is the solo-optimal policy that solves (1) when $M = M_A$ and G is redefined[8] to accommodate all students in S_A and those in S_B who do not get admission from the rival college B according to x^a_B; and x^a_B is the solo-optimal policy in a symmetrical situation. Since the earlier characterization of the solo-optimal policy is valid for this situation as well (cf. Remark 1), the Lagrangian problem for college c is (1) with M replaced by M_c and G modified as described above. Consequently, the solution is in the same form as (4) but college-specific (i.e., not symmetric), which we write as

$$x^a_c(y) \;=\; \begin{cases} \mu^a_c \in (0, v^*) & \text{for } y \le 0 \\ \min\{\mu^a_c + \lambda^a_c y, 1\} & \text{for } y \in [0, \bar{y}]. \end{cases} \tag{5}$$

Therefore, a strategy profile (x^a_A, x^a_B) is an equilibrium if each x^a_c is the solo-optimal policy

[8] To be precise, the redefined G is a "measure" (rather than a distribution) because the full measure exceeds 1; the redefined density function is $g(v, y) = g_V(v) g_Y(y)$ if $v \ge x^a_B(y)$ and $g(v, y) = 2g_V(v) g_Y(y)$ otherwise.

in this sense.

Lastly, we characterize how the equilibrium admission policies, $x^a_A(y)$ and $x^a_B(y)$, differ due to different scholarship budgets as follows:

$$x^a_A(0) > x^a_B(0), \quad \lambda^a_A < \lambda^a_B, \quad \text{and} \quad x^a_A(\hat{y}) = x^a_B(\hat{y}) < 1 \quad \text{at some} \quad \hat{y} \in (0, \bar{y}) \tag{6}$$

as illustrated in Figure 1(b). The first inequality reflects that the college with a larger budget, A, is more selective with non-needy students because it can afford to take in more needy students by providing financial support. Both colleges gradually toughen their admission standards for students with larger financial needs (owing to their limited scholarship budgets), but the "better-endowed" college allocates its budget relatively more evenly across different need levels by increasing the admission cutoff level more slowly (the second inequality); as a result, it reaches out to and takes in more severely financially constrained students than its less well-endowed rival (the third condition).

We summarize the findings on the need-aware regime in the next lemma, including the uniqueness of equilibrium (proved in Appendix). For comparison with other regimes later, we note that the average need level of incoming needy students, which determines the size of needy students enrollment, is lower than the ex-ante average $\tilde{n}$ as stated in part (b) below.

Lemma 3 *In the need-aware regime where $M_A > M_B$, there is a unique equilibrium (x^a_A, x^a_B). In this equilibrium, each college $c \in \{A, B\}$ adopts a cutoff strategy $x^a_c(y)$ where*

(a) *$x^a_c(y)$ is constant for $y \in [\underline{y}, 0]$ and increases at a constant rate $\lambda_c > 0$ for $y \geq 0$ until it reaches 1, in such a way that $x^a_A(y)$ crosses $x^a_B(y)$ from above at some $\hat{y} \in (0, \bar{y})$;*

(b) *the average need level of all needy students who enroll in either college is*

$$\tilde{n}^a := \frac{\int_{\substack{v > \min\{x^a_A(y), x^a_B(y)\} \\ 0<y<\overline{y}}} y \, dG}{\int_{\substack{v > \min\{x^a_A(y), x^a_B(y)\} \\ 0<y<\overline{y}}} 1 \, dG} < \tilde{n};$$

(c) *the total measure of needy students enrolling in either college is $\frac{M_A + M_B}{\tilde{n}^a}$;*

(d) *in the limit case when $M_A = M_B$, the equilibrium is symmetric, i.e., $x^a_A = x^a_B = x^a$, where x^a is the solo-optimal policy (4).*

3.2 Need-blind regime

Under the need-blind admission regime, each college must apply the same admission standard to all students regardless of their financial status. Thus, each college $c \in \{A, B\}$ sets the same admission cutoff level for all $y \in Y$, which we denote by $x_c \in [0, 1]$, that is, $x(y) \equiv x_c$.

Each college c also decides how to allocate scholarships to the admitted needy students by choosing an aid rule β_c. As mentioned already, any scholarship short of their need is useless since there is no other source of financial aid; thus, it is innocuous to assume that the colleges either offer a *full* scholarship equal to the student's need y, or no scholarship. Hence, abusing notation slightly, we use $\beta_c(y)$ to denote the probability with which college c offers a full scholarship to students with need level $y > 0$.

Since the colleges cannot fill their capacity with the ideal admission cutoff level v^* due to insufficient budget (Assumption 1), they set an admission cutoff x_c below v^* to admit more non-needy students. However, all needy students above the same cutoff receive admission offers as well (due to need-blind admission), pushing up the measure of students holding admission offers above the college's capacity. The colleges then have to ration the scholarships to only some of these needy students, leaving the remaining needy students with no option but to turn down the admission offers.

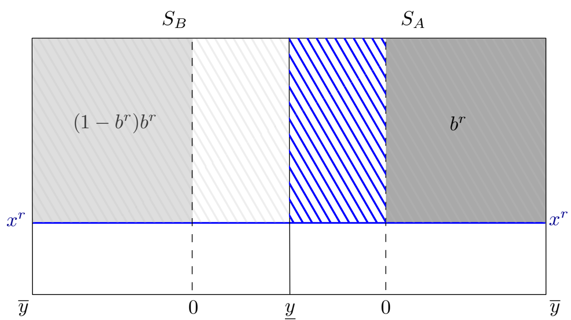
Below we first consider a *need-blind* aid scheme that does not discriminate students on the basis of their need level. That is, $\beta_c(y)$ remains the same for all $y > 0$. As both admission and financial aid decisions are made irrespective of the applicants' financial need, we call such environment the *fully need-blind regime*. We analyze this regime as a preliminary one that ideally complies with the non-discrimination principle on all accounts of the admission process. In the subsequent sections, we also explain why this regime is hardly seen in practice. Alternatively, an aid scheme can be *need-dependent*, whereby $\beta_c(y)$ may vary with y. We refer to such environments as the *need-blind admission with need-dependent aid regime*. We characterize the unique equilibrium that prevails in each regime.

(I) Fully need-blind regime: In this regime, there is no way to satisfy the budget constraint but to randomize awarding full scholarships across all needy students above the admission cutoff. Hence, each college c chooses a single admission cutoff $x_c \in [0,1]$ for all students and a fixed aid probability $\beta_c(y) \equiv b_c \in [0,1]$ for all needy students, that is, the college offers each needy student who clears the cutoff x_c a full scholarship with probability b_c and no scholarship with probability $(1 - b_c)$. We use the superscript r to denote variables under this regime, as scholarship offers are $r$andomized.

As in the previous regime, the equilibrium is symmetric if the two colleges have identical scholarship budgets but asymmetric otherwise. We describe and analyze the general case of asymmetric budgets below, which includes the symmetric case in the limit.

Let $\{(x_c^r, b_c^r)\}_{c=A,B}$ be an equilibrium in this regime where the scholarship budgets are $M_A > M_B$. Each college selects (x_c^r, b_c^r) to maximize the average ability of incoming students, while meeting the capacity and budget constraints, in the knowledge that the students who prefer the rival college will go there if given admission offers from that rival college (along with full scholarship for needy students).

All non-needy students who meet the higher cutoff enroll in their preferred college, while



(a) When $M_A = M_B$ (b) When $M_A > M_B$

Figure 2: **An equilibrium under the fully need-blind regime**: (a) The needy students enrolling in college A are populated over two shaded areas: those in S_A above the common cutoff x^r who receive a full scholarship from college A (with probability b^r; in the dark-solid area) and those in S_B above the cutoff with a full scholarship from college A only (with probability $b^r(1-b^r)$; in the light-solid area). All non-needy students enrolling in colleges A and B, respectively, are populated in the blue-dashed area and in the gray-dashed area. (b) The areas are analogous with $b^r_A > b^r_B$.

those who meet only the lower cutoff enroll in that college. Hence, the college that sets a higher admission cutoff takes in less non-needy students than the other college and thus, should take in a larger measure of needy students to fill the capacity. This requires a larger budget because the average scholarship bill for each needy student enrolled is the same at the unconditional mean need level, $\tilde{n}$, due to random allocation.

This means, as expected, that the better-endowed college, A, sets a higher admission standard in equilibrium, i.e., $x^r_A > x^r_B$. The less well-endowed college, by setting a lower admission cutoff, issues admission offers to a larger set of needy students and thus, should offer scholarships more sparingly, i.e., $b^r_A > b^r_B$. In the limit when the colleges have identical budgets, the differences disappear and we have $x^r_A = x^r_B$ and $b^r_A = b^r_B$. Figure 2(a) illustrates the equilibrium for the symmetric case and Figure 2(b) for the asymmetric case.

To pin down the equilibrium values precisely, we list the equilibrium conditions below:

$$G_Y(0)(1-G_V(x^r_A)) + \tfrac{M_A}{\tilde{n}} = \tfrac{1}{2} \quad (7)$$

$$2G_Y(0)(1-G_V(x^r_B)) - G_Y(0)(1-G_V(x^r_A)) + \tfrac{M_B}{\tilde{n}} = \tfrac{1}{2} \quad (8)$$

$$(1-G_Y(0))(1-G_V(x^r_A))[b^r_A + (1-b^r_B)b^r_A] = \tfrac{M_A}{\tilde{n}} \quad (9)$$

$$(1-G_Y(0))(1-G_V(x^r_A))[b^r_B + (1-b^r_A)b^r_B] + (1-G_Y(0))(G_V(x^r_A) - G_V(x^r_B))b^r_B = \tfrac{M_B}{\tilde{n}}. \quad (10)$$

The condition (7) is the capacity constraint for college A: $G_Y(0)(1-G_V(x^r_A))$ is the measure of non-needy students in S_A above the cutoff and $M_A/\tilde{n}$ is the measure of needy students it supports. The condition (8) is the corresponding constraint for college B who takes in all non-needy students above the cutoff x^r_B except those who enroll in college A. Conditions (9) and (10) are the budget constraints for colleges A and B, respectively. Note that all needy students above x_A^r get admission offers from both colleges, and enroll in their preferred college if offered scholarships from them, but enroll in the less preferred college if offered scholarships from that college only. All needy students with ability above x_B^r but below x_A^r get admission offers only from college B and enroll there if also offered scholarships.

The condition (7) determines x_A^r uniquely, then condition (8) determines x_B^r uniquely. With x_A^r and x_B^r determined as such, conditions (9) and (10) constitute a simultaneous equation system that determines b_A^r and b_B^r uniquely. The findings in the fully need-blind regime are summarized in the next lemma.

Lemma 4 *In the unique equilibrium under the fully need-blind regime where $M_A > M_B$,*

(a) *the colleges set admission cutoffs x_A^r and x_B^r characterized by (7) and (8) and offer a full scholarship to each needy student with probabilities b_A^r and b_B^r characterized by (9) and (10), where $x_A^r > x_B^r$ and $b_A^r > b_B^r$;*

(b) *the average need level of all needy students who enroll in either college is $\tilde{n}^r = \tilde{n}$;*

(c) *the measure of all needy students enrolling in either college is $\frac{M_A+M_B}{\tilde{n}}$;*

(d) *in the limit case when $M_A = M_B$, the equilibrium is symmetric, i.e., $x_A^r = x_B^r$ that solves (7) and $b_A^r = b_B^r$ that solves (9).*

(II) Need-blind admission with need-dependent aid regime: We now consider the other case in which each college c may make scholarship offers based on the student's financial need (although it chooses a single admission cutoff x_c that applies to all). We use the superscript d to denote variables under this regime, as scholarship offers are need-*d*ependent.

Once again, we describe and analyze the general case of asymmetric budgets with $M_A > M_B$ below, which includes the symmetric equilibrium of the symmetric budget case in the limit. Let $\{(x_c^d, \beta_c^d)\}_{c=A,B}$ be an equilibrium in this regime where $\beta_c^d(\cdot)$ is an aid rule that specifies the probability of offering full scholarship contingently on the need level $y > 0$.

As before, each college c selects (x_c^d, β_c^d) optimally in the knowledge that it can attract any student except for those who prefer the rival college and will secure a place there according to the equilibrium strategy. With $\tilde{G}_c$ denoting the distribution of such students,[9] therefore, (x_c^d, β_c^d) is the solution to the solo problem with $\tilde{G}_c$ replacing G under the restrictions of the current regime.

As will be verified later, the college with a larger budget sets a higher admission cutoff here as well, that is, $x_A^d \geq x_B^d$. Thus, all non-needy students in S_A above the cutoff x_A^d enroll in college A, and all other non-needy students above the cutoff x_B^d enroll in college B. All needy students above the cutoff x_A^d get admission offers from both colleges. College A

[9]Formally, $\tilde{G}_c$ is defined by a density function $\tilde{g}_c(v,y) = (2 - \beta_{c'}^d(v,y))g(v,y)$ if $v \geq x_{c'}^d$ where c' is the rival college of c and $\beta_{c'}^d(v,y) = 1$ for $y \leq 0$, and $\tilde{g}_c(v,y) = 2g(v,y)$ if $v < x_{c'}^d$.

can recruit them by offering scholarships unless they prefer college B and get a scholarship from B which happens with probability $\beta_B^d(y)$ where y is their need level. Since $\beta_B^d(y)$ is independent of the student's ability, the ability distribution of students that college A can recruit by offering scholarships is the same as the initial distribution conditional on $x_i > x_A^d$, irrespectively of the need level y. Therefore, college A must allocate scholarships most cost-effectively, i.e., to the least needy students (above the cutoff x_A^d) until its budget M_A is depleted, say up to a threshold need level denoted by $n_A^d > 0$.

Given this, if college B offers scholarships to students who meet its admission cutoff x_B^d, they will accept and enroll unless they prefer college A and secure a place there, i.e., unless their ability is above the higher cutoff x_A^d and need level below the threshold n_A^d. Thus, as before, the ability distribution of needy students that college B can recruit is the same for every need level below n_A^d, although it is worse than the initial distribution conditional on $x_i > x_B^d$. Hence, college B must allocate scholarships most cost-effectively among needy students with need levels up to n_A^d.

However, if college B offered scholarships to students with need levels marginally above n_A^d (who don't get scholarship from college A), they all would enroll and thus their average ability would be higher. Hence, college B might find it optimal to offer scholarships up to a certain need level strictly below n_A^d, then jump discontinuously and offer to those marginally above n_A^d. We show that this is not the case in equilibrium so long as the financial needs are not too concentrated in the sense of Assumption 2,[10] because then the jump is too large to be beneficial (as shown in Appendix). Consequently, college B also allocates scholarships most cost-effectively until its budget M_B is depleted, say up to a threshold n_B^d.

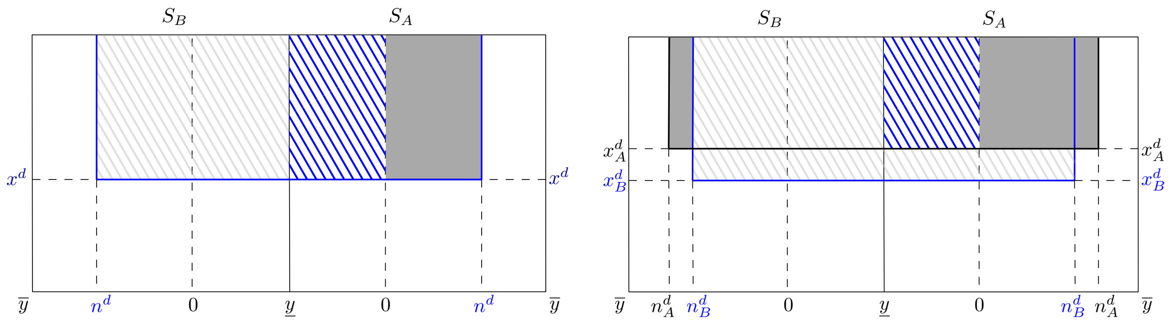
In equilibrium, therefore, each college $c \in \{A, B\}$ sets an admission cutoff x_c^d and offer full scholarships to all needy students above their cutoff up to a threshold need level denoted by $n_c^d > 0$, where x_c^d and n_c^d solve

$$\int_{\substack{x_c^d \leq v \leq 1 \\ y \leq n_c^d}} 1 \, d\tilde{G}_c = \frac{1}{2} \quad \text{and} \quad \int_{\substack{x_c^d \leq v \leq 1 \\ 0 < y \leq n_c^d}} y \, d\tilde{G}_c = M_c, \tag{11}$$

i.e., the binding capacity constraint and budget constraint, respectively. We represent an equilibrium in this regime by $\{(x_c^d, n_c^d)\}_{c=A,B}$, i.e., in terms of the upper threshold need level n_c^d for scholarship offers (in lieu of aid rule β_c^d).

To verify which college sets a higher admission cutoff, recall that the college with a higher cutoff takes in less non-needy students, hence should recruit a larger measure of needy students. This requires a higher threshold need level for scholarship offers and thus, a larger scholarship spend by the college. Therefore, the college with a larger budget, A, sets a higher ability cutoff as asserted earlier, and also a higher threshold need level.

[10]This is the only place where the second part of Assumption 2 is used.



(a) When $M_A = M_B$ (b) When $M_A > M_B$

Figure 3: **An equilibrium under the need-blind admission policy with need-dependent aid regime**: (a) All students enrolling in college A are populated in the gray-solid area (needy students) and the blue-dashed area (non-needy students). A symmetric description applies to college B and all students enrolling in college B are populated in the gray-dashed area. (b) The gray-solid area depicts all needy students enrolling in A; the blue-dashed area depicts all non-needy students enrolling in A; and all students enrolling in B are populated in the gray-dashed area.

The equilibrium is illustrated in Figure 3(b) for the case that $M_A > M_B$, where $x^d_A > x^d_B$ and $n^d_A > n^d_B$ as deduced above. The differences in equilibrium cutoffs and threshold levels arise because a larger budget bestows an advantage on college A in that it can impose a higher admission standard and support students with higher needs. Unsurprisingly, this advantage weakens as the difference in budget gets smaller:

$$x^d_A \ (x^d_B) \text{ increases (decreases) as } M_A \text{ increases while keeping } M_A + M_B \text{ constant.} \tag{12}$$

Note also that as x^d_B decreases, the total enrollment of non-needy students increases, hence that of needy students decreases. In the limit when the colleges have identical budgets, the equilibrium is symmetric, i.e., $(x^d_A, n^d_A) = (x^d_B, n^d_B)$, as illustrated in Figure 3(a). The next lemma summarizes the findings in the need-blind admission with need-dependent aid regime, including the uniqueness of equilibrium (proved in Appendix).

Lemma 5 *In the unique equilibrium under the need-blind admission with need-dependent aid regime where $M_A > M_B$,*
(a) *each college $c \in \{A, B\}$ sets ability cutoff at x^d_c and offers full scholarships to students with need levels $y_i \in (0, n^d_c]$, where x^d_A, x^d_B, n^d_A and n^d_B jointly solve the four equations in (11) for $c = A, B$, and satisfy $x^d_A > x^d_B$, $n^d_A > n^d_B$, and (12);*
(b) *the average need level of all needy students who enroll in either college is*

$$\tilde{n}^d \;=\; \frac{\int_{x^d_A}^{1}\int_0^{n^d_A} y\, dG_Y dG_V + \int_{x^d_B}^{x^d_A}\int_0^{n^d_B} y\, dG_Y dG_V}{\int_{x^d_A}^{1}\int_0^{n^d_A} 1\, dG_Y dG_V + \int_{x^d_B}^{x^d_A}\int_0^{n^d_B} 1\, dG_Y dG_V} \;<\; \tilde{n} \;=\; \tilde{n}^r;$$

(c) the measure of all needy students enrolling in either college is $\frac{M_A+M_B}{\tilde{n}^d}$;
(d) in the limit case when $M_A = M_B$*, the equilibrium is symmetric, i.e.,* $(x_A^d, n_A^d) = (x_B^d, n_B^d)$ *that solves (11) when* $\tilde{G}_c$ *is replaced by* G.

3.3 Does need-blind admission benefit needy students?

We come to the question that initially motivated our analysis: *Does need-blindness help needy students in college admissions?* Based on the findings in Sections 3.1–3.2, we compare the equilibrium outcomes under three regimes, namely, need-aware regime (NA), fully need-blind regime (NBr), need-blind admission with need-dependent aid regime (NBd). The result is clear as below regarding the total enrollment of needy students.

Proposition 1 *In equilibrium the total enrollment of needy students is highest in NBd, then in NA, and lowest in NBr.*

In the fully need-blind regime, the colleges have to allocate scholarships completely randomly among all needy students above the admission cutoff. In the other two regimes, they may use discretion in allocating scholarships for a more efficient use of budget to the extent allowed in each regime, resulting in a larger total enrollment of needy students than in NBr (which is evident from parts (b) and (c) of Lemmas 3–5).

The comparison is more complex between the regimes NA and NBd, and we explain it for the case of symmetric budgets (where the equilibrium is also symmetric). In NBd regime, as illustrated in Figure 2(a), each college sets an admission cutoff $x^d\,(=x_A^d=x_B^d)$ and fully supports all needy students up to a threshold need level $n^d\,(=n_A^d=n_B^d)$ as a result of maximizing x^d subject to filling the capacity by supporting needy students above the cutoff most efficiently for the given budget. In NA regime, each college sets cutoff levels that start from $x^a(0)$ for non-needy students and increase linearly in the student's need level $y > 0$ as illustrated in Figure 1(a). If the starting cutoff level $x^a(0)$ was equal to (or higher than) x^d, the colleges would take in the same (or smaller) set of non-needy students in NA than in NBd, thus they should take in a larger measure of needy students in NA than in NBd to fill the capacity. However, this is impossible given the fixed total budget because, as the cutoff level increases with need level in NA, the colleges admit students of higher need levels in place of students of lower need levels compared with what they do in NBd. This leads us to conclude that $x^a(0) < x^d$ must hold, implying a smaller total enrollment of non-needy students in NBd than in NA, hence a larger total enrollment of needy students in NBd. This result extends to the case of asymmetric budgets as well (as shown in Appendix).

Therefore, need-blind admission policies are effective in promoting the enrollment of needy students only when financial aid schemes are not need-blind. Need-blind aid schemes force the limited budget be allocated evenly across all levels of financial need, keeping the average amount of support for a needy student large at the ex-ante average need level, $\tilde{n}$. To meet the capacity given such a heavy burden on the college's budget, the colleges have to admit more non-needy students by lowering the admission standard. Due to need-blindness, however, the lower standard also applies to needy students and a larger number of needy students get admission offers. The colleges then have to ration scholarships more severely to these students to meet the budget constraint, which exacerbates admit/deny incidents.

Such random rationing of financial support also creates Pareto inefficient matches between students and colleges: students above the ability cutoff levels of both colleges may end up receiving aid offers only from their less preferred college and enrolling there. Since the distributions of characteristics (v_i, y_i) of such students are identical between the two colleges, swapping the colleges for these students would increase the welfare of students without affecting either college's utility or constraints. When the colleges have asymmetric budgets, such misaligned matches may also arise in the other regimes: some students in S_A end up enrolling in college B and *vice versa* in the need-aware regime and the need-blind admission with need-dependent aid regime. However, unlike the fully need-blind regime, no swap is possible among those qualified at the respective colleges: those in S_A enrolling in college B are of lower abilities than those in S_B enrolling in A as illustrated in Figures 1(b) and 3(b). Therefore, no eligible Pareto improvement can be made in these regimes.

Given that admit/deny incidents are unavoidable under need-blind admission policies as discussed above, we may also compare the "distributions" of admit/deny incidents. These incidents are spread out evenly among all needy students if the financial aid scheme is also need-blind, while concentrated on the students with large financial needs (i.e., those with needs above the threshold n^d) when the aid scheme is need-dependent, and the distribution of needy students accommodated in the need-aware regime in between. Therefore, more diverse accommodation across different need levels comes at the cost of reduced total enrollment of all needy students (cf. Proposition 1). Certainly, fairer and more diverse representation is desirable. However, it is unclear how to measure and compare the desirability of different degrees of diversification (which may also depend on the initial distribution G_Y in our context). Therefore, we abstract from it in the current paper and focus on the level of total enrollment of all needy students achievable in different regimes.

Summarizing, the fully need-blind regime performs poorly in all respects, except that it gives some highly needy students more chance to enroll, and it is also clear that the average ability of incoming students is lowest in the fully need-blind regime. This could explain why we hardly find this regime in practice, even if it is a system that ideally complies with the non-discrimination principle all the way in the admission process. In contrast, the need-blind admission with need-dependent aid regime is most effective in promoting the enrollment of needy students among the considered environments. In the next section, we focus on this regime and extend the analysis by introducing availability of external fund.

4 Need-blind Admission with External Fund

In the previous section, we analyzed equilibria under different regimes when a needy student can enroll in a college only if he receives a full scholarship offer. In practice, however, financial packages take more complicated forms and primarily combine (i) scholarship (or grant) funded by the college's budget and (ii) federal student loan. A student may enroll in a college as long as the combined amount of (i) and (ii) covers his financial need. Scholarships impose a financial burden on the college, but student loans fall on students' shoulders to repay. We refer to such student loans as *external funding* to distinguish them from scholarships paid out of the college's budget. In this section, we examine the need-blind admission with need-dependent scholarship regime when such external funding is available. We first introduce a function that formalizes the availability of external funding.

- **Availability of external funding**: A function $f : (0, \overline{y}] \to [0, 1]$ specifies the probability $f(z)$ of obtaining an amount z of external funding when sought.

Here, z is the amount of external funding sought, not the student's financial need. The availability function f is common to all needy students subject to the constraint that a student cannot take out a loan in excess of his financial need y_i minus any scholarship offered.[11]

Although no restriction is imposed on f a priori, in practice f may take a specific functional form. Broadly speaking, the current federal student loan program adopts f in the form of a *step function* with a *single* threshold: there exists a threshold $\bar{z} \in [0, \overline{y}]$ such that $f(z) = 1$ for each $z \leq \bar{z}$ and $f(z) = 0$ for each $z > \bar{z}$. That is, federal students loans are guaranteed up to a fixed credit line for each applicant, but no higher amount than this threshold is possible.[12] We start by analyzing the admission problem with such a step function f.

[11]This constraint is based on a general rule in the *packaging* of need-based financial aid: the total financial aid, including scholarship and student loans, must not exceed a student's financial need (Chapter 7 "Packaging Aid" of the 2020-2021 Federal Student Aid Handbook, available at *https://fsapartners.ed.gov/knowledge-center/library*).

[12]The U.S. Office of the DoED explicitly indicates that "the maximum amount you can borrow each year in Direct Subsidized Loans and Direct Unsubsidized Loans ranges from $5,500 to $12,500 per year, depending on what year you are in school and your dependency status" (*studentaid.gov/understand-aid/types/loans*). No other apparent criteria is indicated for approval in the guideline.

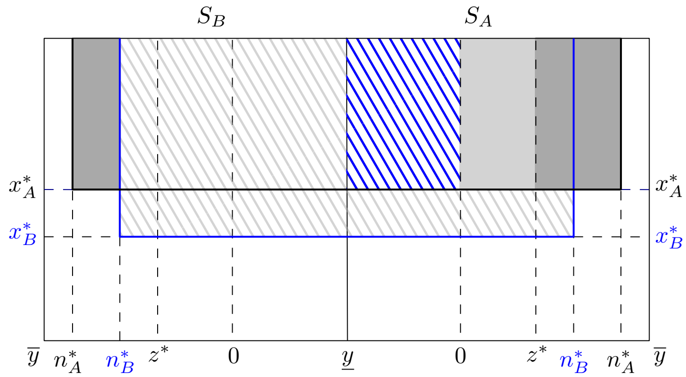


Figure 4: **An equilibrium under the need-blind admission policy with partial scholarship in the asymmetric budget cases**: The light-solid area depicts needy students enrolling in A receiving no scholarship; the dark-solid area depicts needy students enrolling in A receiving partial scholarship in the amount of $(y_i - z^*)$ from A; the blue-dashed area depicts non-needy students enrolling in college A. The gray-dashed area depicts those enrolling in B with analogous scholarships from B.

4.1 Partial scholarship scheme

With student loans available, colleges use their limited budget more sparingly to recruit more qualified, needy students: they may offer partial scholarships to admitted students, inducing them to take out loans for the remaining amounts. Specifically, we consider the need-blind admission with need-dependent aid regime when external funding is available according to a single-step function f. Taking this function as given, the colleges choose their admission cutoffs and scholarship allocations and the applicants seek loans when needed and decide where to enroll. We assume $M_A > M_B$ in what follows, setting aside the symmetric budget case as a limit.

Given a step function f with a threshold $\bar{z}$, any student with need $y_i > 0$ is assured of external funding up to the amount $\min\{y_i, \bar{z}\}$. Therefore, each college minimizes its scholarship spending on a needy admitted student by offering no scholarship if his financial need, y_i, is less than $\bar{z}$ and a partial scholarship in the amount of $(y_i - \bar{z})$ if it exceeds $\bar{z}$. The colleges then may allocate the saved scholarship budget to students whose needs are higher, so as to increase the admission cutoff within the constraints.

That is, due to the external funding available up to a threshold amount $\bar{z}$, the colleges behave as if every student's financial need is reduced by $\bar{z}$. Therefore, the equilibrium with the step function f is equivalent to the equilibrium obtained earlier in Section 3.2 (B) for the need-blind admission with need-dependent financial aid regime where every student i's financial status is redefined as $y_i' = y_i - \bar{z}$.[13]

[13]As before, if the budget of each college M_c is enough to support all needy students above the ideal cutoff v^* in this manner, the ideal outcome prevails where all students with ability above v^* in S_c enroll in college c.

Let $\{(x_c^*, n_c^*)\}_{c=A,B}$ denote an equilibrium with this federal loan program. By the result of the Section 3.2 (B), $x_A^* > x_B^*$ and $n_A^* > n_B^*$ where $n_B^* > \bar{z}$ because the colleges can admit students without scholarship offer up to the need level $y_i = \bar{z}$, as illustrated in Figure 4. More non-needy students enroll in college B than A because the admission cutoff level is lower for B. In terms of needy students, those in S_c with $(v_i, y_i) \in (x_A^*, 1) \times (0, n_B^*)$ enroll in each college c; those in $S_A \cup S_B$ with $(v_i, y_i) \in (x_A^*, 1) \times (n_B^*, n_A^*)$, denoted by $\tilde{S}^A$, enroll in college A and those in $S_A \cup S_B$ with $(v_i, y_i) \in (x_B^*, x_A^*) \times (0, n_B^*)$, denoted by $\tilde{S}^B$, enroll in college B.

Note that the students in $\tilde{S}^A$ are not only of a larger measure but receive a larger federal loan on average than those in $\tilde{S}^B$. Therefore, a larger fraction of the total federal loan budget is used to help college A—the better-endowed college—recruit needy students. This means that the current federal student loan program exacerbates the disadvantage of the less well-endowed college as college B is in effect receiving a smaller amount of federal "subsidy" in the form of student loans. When the colleges have an identical budget so that the disadvantage is irrelevant, however, a federal loan program with a single threshold turns out to be optimal in maximizing the total enrollment of needy students as discussed below.

4.2 Optimal availability of external funding

The availability of student loans is an important policy variable of the DoED and other relevant public sectors in higher education. Each year, a fixed fiscal budget is assigned to student loan programs so that students are offered low interest rates and various repayment plans. For the fiscal year 2021, for instance, \$94 billion has been requested by the DoED for student loan programs. In addition to the total budget, the manner in which student loans are made available is an important policy instrument.

In this section, we discuss how to design availability of student loans when the DoED, given a fixed budget F, aims to maximize the total enrollment of needy students in selective colleges. Recall that we represent availability of student loans by a function $f : (0, \overline{y}] \to [0, 1]$ where $f(z)$ is the probability of securing a student loan in the amount of z. If the budget F is large enough, it is easy to see that a step function f exists that implements the ideal outcome in which precisely those students with abilities above v^* enroll in one of the two colleges, which clearly maximizes the total enrollment of needy students. For this outcome, the grand total of scholarship and student loan budgets, $M_A + M_B + F$, should be sufficient to meet the financial needs of all students in S with ability above v^*, or equivalently,

$$(1 - G_Y(0))(1 - G_V(v^*))\tilde{n} \ \leq \ \frac{M_A + M_B + F}{2}.$$

Below we consider the cases where the colleges' budgets are not large enough for this ideal outcome to prevail.

We assume below that this is not the case, so that the ideal outcome cannot be induced in equilibrium. We first identify an upper bound of possible enrollment of needy students in this case. Recall from (12) that, in the current regime without external funding, the total equilibrium enrollment of needy students is largest when the two colleges have the same budget (i.e., when $M_A = M_B$) conditional on the combined scholarship budget, $M_A + M_B$, being a constant. Moreover, if the two colleges coordinate to maximize the enrollment of needy students over $S_A \cup S_B$ with their combined budget, each of them adopts the solo-optimal policy as if each had one half of the combined budget.

Consequently, in a hypothetical situation where the two colleges have at their disposal the grand total of financial budgets across the two colleges and the DoED, $M_A + M_B + F$ (i.e., without being constrained by a loan availability function f), the maximal possible enrollment of needy students is achieved when the two colleges set the solo-optimal policy for a scholarship budget of $\frac{M_A+M_B+F}{2}$ each: that is, they set the same need-blind cutoff level x^* and fund all needy students above x^* with needs below a threshold n^* such that they fill their capacity and exhaust the entire budget, or equivalently, x^* and n^* solve (11) when M_c and $\tilde{G}_c$ are replaced by $\frac{M_A+M_B+F}{2}$ and G, respectively: that is,

$$G_Y(n^*)(1 - G_V(x^*)) = \frac{1}{2} \quad \text{and} \quad (1 - G_V(x^*)) \int_0^{n^*} y \, dG_Y = \frac{M_A + M_B + F}{2}. \tag{13}$$

The enrollment of needy students in this outcome provides an upper bound of those achievable when the DoED budget F is available through any availability function f.

We now verify that, if the colleges have symmetric budgets ($M_A = M_B$), this outcome can be induced as the unique equilibrium under the need-blind admission with need-dependent aid regime if a federal loan program of a total budget F is available according to a carefully-chosen step function f. With x^* and n^* identified as above, we find $\bar{z}^*$ ($< n^*$) that satisfies

$$(1 - G_V(x^*)) \int_{\bar{z}^*}^{n^*} (y - \bar{z}^*) dG_Y \;=\; M_A = M_B \tag{14}$$

and let f be a step function with a threshold $\bar{z}^*$. Then, by the analysis in Section 4.1 and Lemma 5(d), both colleges set their admission cutoffs at x^* and offer scholarships in the amount of $(y_i - \bar{z}^*)$ to all admitted students with $y_i \in [\bar{z}^*, n^*]$. Thus, all needy students above the cutoff whose needs are lower than n^* enroll, funded either by a combination of college scholarship and federal loan if their needs exceed the threshold $\bar{z}^*$ or solely by a federal loan otherwise. Therefore, among all f's, the maximum enrollment of needy students is obtained by using the step function f with a threshold $\bar{z}^*$.

As such, when the colleges have symmetric budgets, the current federal loan program imposing a maximum credit limit for each applicant works well for the DoED. However, this observation does not extend to the case of asymmetric budgets because then a single-threshold loan program exacerbates the disadvantage of the less well-endowed college as explained earlier. Yet, the total equilibrium enrollment of needy students should be close to the upper bound identified above so long as the scholarship budget disparity is small.

As the scholarship budget disparity (i.e., $M_A - M_B$) gets larger, not only the enrollment of needy students falls further from the upper bound, but also the disadvantage of the less well-endowed college gets exacerbated more severely. What kind of alternative loan programs may help enhance the enrollment of needy students?

Recall from (12) that, absent any external funding, the total enrollment of needy students is maximized when the two colleges have balanced scholarship budgets. This is because, since the degree of "favoring" non-needy students is determined by the lower admission cutoff, the total enrollment of needy students is bound by how far the college with a smaller budget can reach out to needy students. When federal loans are available, the extent to which a college can reach out is augmented by the total amount of loans taken out by the students enrolling in that college. Therefore, federal loan schemes may be more effective in promoting the enrollment of needy students if they are designed in such a way that the less well-endowed college gets a larger share of the total loan, abating the endowment gap between the colleges.

If the DoED is free to allocate the total loan budget F in any manner between the two colleges (so that they can use the allocated fund as if it is their own budget), it will split F to minimize the endowment gap: the entire budget F will be given to B if F is less than the initial gap, $(M_A - M_B)$; otherwise, it will be allocated so as to equalize the financial aid budgets between the two colleges at $\frac{M_A+M_B+F}{2}$ each. The same result can also be obtained by introducing college-specific single-step availability functions, f_A and f_B, with suitably chosen threshold levels. Albeit simple, these schemes may be deemed draconian and discriminatory across colleges.

Matching financial aid offers

We discuss another, non-discriminatory federal loan scheme that may reduce the budget gap between the colleges. Drawing on the practice that some colleges "match" financial aid packages offered to their prospective students by competing colleges,[14] below we consider a special federal loan budget reserved for such matching financial aid offers, to be run in conjunction with a standard loan program with a single-step availability function.

Recall that under single-threshold loan programs, the less well-endowed college, B, ends up recruiting more of less needy students who take out smaller amounts of loans, receiving a smaller fraction of the federal loan budget. On the flip side, helped by a larger fraction of

[14] For example, Cornell University announced in 2010 to match the need-based financial aid for admitted students if they are also accepted to other competing schools (*https://news.cornell.edu/stories/2010/12/cornell-matches-financial-aid-offered-peer-schools*).

it, the other college gets to recruit the students of higher ability with higher levels of need regardless of their tastes over colleges (cf. Figure 4). The idea behind a special loan program is to alleviate this imbalance by helping college B tap into the subset of these students who prefer college B through matching federal loans.

Formally, a limited budget of $F_m\,(\leq F)$ is set aside for a matching loan program, which students may apply for if they are admitted to both colleges but with a full financial package from only one (presumably, their less preferred) college, while the standard loan program with a single threshold is run with the remaining budget $(F-F_m)$ as explained in Section 4.1. A matching loan, if approved, tops up the insufficient scholarship offer made by one of the colleges to fully meet the applicant's financial need. A condition for approval is that the amount of the initial (insufficient) scholarship be no lower than any other scholarship offered by the same college (which we assume is verifiable). This condition is needed to prevent abusive use of the matching loan program, as explained below. If the total amount of applications exceeds F_m, matching loans are allocated randomly.

Consider the case that $F \geq M_A - M_B$.[15] Recall the ability cutoff x^* and the threshold need level n^* of the solo-optimal policy when each college has $\frac{M_A+M_B+F}{2}$ as its budget, determined by (13), which gives the maximal possible enrollment of needy students subject to M_A+M_B+F being the grand total of budgets. We show that this outcome is achievable with a matching loan program of a specific budget F_m, together with a standard loan program of a suitably chosen threshold z^*.

In equilibrium (cf. Figure 5), the two colleges set their admission cutoff levels identically at $x_A^* = x_B^* = x^*$, and college A offers financial aid in the same way as before up to the threshold need level $n_A^* = n^*$, i.e., fully in federal loans for students with $y_i \leq z^*$ and by combining a scholarship of $(y_i - z^*)$ and a maximal federal loan of z^* for students with $y_i \in (z^*, n^*]$. College B also offers full financial packages analogously but only up to a lower need level $n_B^* \in (z^*, n^*]$, and makes insufficient aid offers to students with $y_i \in (n_B^*, n^*]$ composed of z^* amount of loan and $(n_B^* - z^*)$ amount of scholarship. The students in S_B with insufficient aid offers from college B apply for matching loans successfully, taking out $(y_i - n_B^*)$ amount of additional loan. Consequently, all students with $v_i \geq x^*$ and $y_i \leq n^*$ enroll in their preferred college.

Note that x^* and n^* are chosen to satisfy the aggregate capacity and budget constraints, hence each college's capacity constraint is also satisfied given symmetry. Additionally, each college's budget constraint needs to be satisfied in equilibrium:

[15] We defer the case of $F < M_A - M_B$ to Appendix A2, where the DoED allocates the entire budget to a matching loan program ($F_m = F$), inducing college B to receive all of it through matching loans to the students enrolling there. This minimizes the budgetary gap between the two colleges.

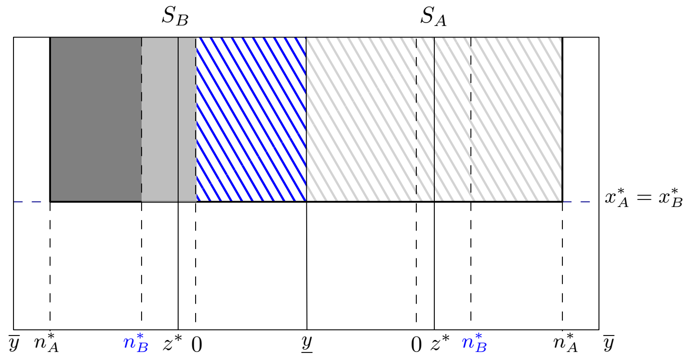


Figure 5: **Matching financial aid offers when $F > M_A - M_B$:** The dark-gray area depicts needy students who enroll in B with matching aid offers; the light-gray area depicts needy students enrolling in college B with partial scholarship in the amount of $\max(y_i - z^*, 0)$ from B; the blue-dashed area depicts non-needy students enrolling in B; those enrolling in A are populated in the gray-dashed area.

$$\int_{\substack{x^* \le v \le 1 \\ z^* \le y \le n^*}} (y - z^*)\, dG = M_A \quad \text{and} \quad \int_{\substack{x^* \le v \le 1 \\ z^* \le y \le n^*}} (\min\{y, n_B^*\} - z^*)\, dG = M_B.$$

The equilibrium value of z^* is determined by the first equation, then n_B^* is determined by the second. Finally, the matching loan budget is determined as $F_m = \int_{\substack{x^* \le v \le 1 \\ n_B^* < y < n^*}} (y - n_B^*)\, dG$.

Now we verify that the strategies of the two colleges described above are mutual best responses. Note that, no matter how B may deviate, all students in S_A with $y_i < n^*$ enroll in college A, each with a federal loan in the amount of $\min\{y_i, z^*\}$. Hence, college B's share of the total federal loan in the presumed equilibrium is the maximal it can get by any deviation. Since x^* and n^* constitute the solo-optimal policy when college B has the sum of this share of federal loan and M_B as its own budget, there is no beneficial deviation for B.

Turning to college A, for beneficial deviation it has to set a higher admission cutoff, say $x' > x^*$, and fill its capacity by recruiting students with $y_i > n^*$ or those in S_B with $y_i < n^*$. It cannot recruit students in S_B with $y_i < n^*$ because either they secure a place in college B with initial admission offers (if $y_i < n_B^*$) or they would get matching loans (if $y_i \in (n_B^*, n^*]$). Hence, it should recruit students with higher needs $y_i > n^*$ by offering a scholarship of $(y_i - z^*)$ (because they cannot use the matching loan program). This is more costly than recruiting any student in the presumed equilibrium, hence it must save its own scholarship budget in recruiting some students with lower need levels $y_i < n^*$. The only potential way to do so is offering insufficient scholarships to students with $y_i \in (z^*, n_B^*]$ so that they apply for matching loans, but for this it must not offer scholarships of any higher amount to any student with $y_i > n_B^*$, either.[16] As this implies that college A would be unable to recruit students with $y_i > n^*$ (defeating its purpose), there is no beneficial deviation for A, either. This verifies that the two colleges' strategies constitute an equilibrium.

5 Concluding Remarks

Need-blind admission policy prevails in the US college admissions and many expect it to enhance economic diversity of campus demographics. It is true under this policy that colleges do not screen applicants by "ability to pay" in their admission decisions, but we have seen that they can still use financial aid packaging in the selection process. In this paper, we thereby analyze admission policies in combination of feasible financial aid systems and find that for the need-blind admission policy to increase the enrollment of needy students, the accompanying financial aid offers need be made contingently on the student's need level. This equilibrium characterization extends with suitable modifications to the cases with external funding, such as the federal student loan programs. An interesting policy implication here is that, the current federal student loan programs may exacerbate the disadvantage of less well-endowed colleges, reducing the total enrollment of needy students.

We have analyzed the strategic interactions of the colleges assuming that they take the admission regime as given rather than as a part of their choices. We believe that their decisions on which regime to adopt are made at a different level than their normal operational decisions such as admission and financial aid offers, for instance, to promote their social responsibility or to conform to changing social norms. Insofar as need-blind admission policies have been adopted to promote diversity in college education, our analysis provides a theoretical investigation on whether and how such a change in landscape may indeed generate the intended outcome.

Lastly, to get a sense of how need-aware/blind policy has changed the student body in practice, we take a quick look at a few colleges that made a shift from one admission policy to the other in recent years. Those colleges include Wesleyan University and Macalester College, which switched from need-blind system to need-aware system in 2012 and 2006, respectively, and Vassar College switched the other way in 2007. A large number of colleges, such as Amherst College and Williams College, have kept their need-blind policy for the years covered, while Colby College has been adopting the need-aware policy.

A widely used proxy for economic diversity at the moment is the share of federal Pell Grant recipients in the undergraduate program.[17] In Table 1, we calculate this measure

[16] This is where the condition is needed that for a matching loan to be approved, the initial (insufficient) scholarship amount needs to be largest among all scholarships offered by the relevant college.

[17] Typically, students qualify for the Pell Grant when they are from households with annual income below

Table 1: **Share of Pell Grant recipients**

	Non-profit 4-year Private	Wesleyan University (NB→NA)	Macalester College (NB→NA)	Vassar College (NA→ NB)	Colby College (NA)	Amherst College (NB)	Williams College (NB)
2003-2004	27.9%	13.7%	13.1%	11.4%	8.5%	15.3%	10.6%
2007-2008	26.8%	12.1%	14.4%	10.4%	7.6%	17.9%	14.6%
2011-2012	35.8%	18.9%	15.4%	24.3%	10.6%	21.7%	20.2%
2015-2016	36.4%	18.8%	15.6%	24.1%	10.6%	25.2%	20.0%

Source: NCES 2019-487 (the second column), the Common Data Set of each college and the IPEDS (undergraduate enrollment) and the Pell Grant distribution data by institution (the number of Pell Grant recipients of each college) available at *https://www2.ed.gov/finaid/prof/resources/data/pell-institution.html*

every four years for the aforementioned selective colleges. The table also presents the overall share of recipients in the non-profit 4-year private college group, which subsumes these five colleges.[18]

As can be seen, non-profit 4-year private colleges exhibit an overall increase of 8.5% during 2003-2016. For Wesleyan, Macalester and Colby, the measure increases rather slowly, falling behind the overall trend. In contrast, Vassar had a sharp increase of 12.7%, while the rest of the need-blind institutions, Amherst and Williams, made an increase comparable to the overall trend, 9.9% and 9.4%, respectively. This observation is broadly consistent with our finding that the enrollment of needy students is higher when need-blind admissions are combined with appropriate aid schemes, than in the need-aware regime. Obviously, more careful empirical investigation should follow to test our theoretical findings, which we leave open for future research.

$50,000 US. The Pell Grant is *not* institutional, so it is a sort of monetary subsidy to these applicants. All of our arguments carry over by modifying a student i's need to be y_i net of any federal grant of this kind.

[18]The share of Pell Grant recipients is not a perfect measure to represent characteristics of the needy student group, but rich data sets are available on this measure, and the DoED, policymakers, and the media regard it as a proxy of enrollment of low-income students. For instance, the DoED publishes a series of annual reports on the distribution of Pell Grant funds by institutions since 1999; the *US news* also publishes rankings of colleges in regards to "economic diversity" by using this measure.

Appendix

Appendix A1. Deferred proofs

Proof of Lemma 1.

We prove the lemma for the need-aware regime here. The proof can be adapted to other regimes straightforwardly. Since an admission offer to a needy student is useless without a full scholarship, we represent each college's strategy by an admission rule and take it for granted that a full scholarship is also offered to every needy student who gets an admission offer.

Consider an equilibrium in which the strategy of one of the colleges, say college A, is α. Let $Q = \alpha^{-1}(1) = \{(v, y) \in V \times Y | \alpha(v, y) = 1\}$ be the set of student characteristics to which college A extends admissions to. Consider the following process: Divide Y into k subintervals of equal length as $[\underline{y}, \underline{y} + \frac{(\bar{y}-\underline{y})}{k}], (\underline{y} + \frac{(\bar{y}-\underline{y})}{k}, \underline{y} + \frac{2(\bar{y}-\underline{y})}{k}], \cdots, (\underline{y} + \frac{(k-1)(\bar{y}-\underline{y})}{k}, \underline{y} + \frac{k(\bar{y}-\underline{y})}{k}]$. For each subinterval $J_j^k = (\underline{y} + \frac{j(\bar{y}-\underline{y})}{k}, \underline{y} + \frac{(j+1)(\bar{y}-\underline{y})}{k}]$, let

$$v_j^k \;=\; \sup\{v \in V \,|\, G(Q \cap (V \times J_j^k)) = G(Q \cap ([v, 1] \times J_j^k))\}$$

where $G(Z)$ is the G-measure of $Z \subset V \times Y$; and define $x^k(y) = v_j^k$ for all $y \in J_j^k$. Then, using the sequence $k = 2, 2^2, \cdots$, define

$$x(y) \;=\; \lim_{n\to\infty} x^{2^n}(y), \qquad \forall y \in Y$$

which is well-defined (because $x^{2^n}(y)$ is non-decreasing in n for each y) and straightforwardly verified to be a measurable function. Let $X = \{(v, y) \in V \times Y | v \geq x(y)\}$.

By construction, $G(X) \geq G(Q)$. If $G(X) = G(Q)$, the admission rule α and the cutoff rule x are strategically equivalent (cf. footnote 6). Below we show $G(X) = G(Q)$.

Suppose to the contrary that $G(X) > G(Q)$. Then, there is a rectangle $L = [v', v''] \times [y', y''] \subset V \times Y$, such that $G(L \cap X) - G(L \cap Q) > 0$. Note that it is impossible that for each and every (v, y) in the interior of L there is $\epsilon > 0$ such that $G(([v, v+\epsilon] \times [y-\epsilon, y]) \cap X) = G(([v, v+\epsilon] \times [y-\epsilon, y]) \cap Q)$, because if so we would have $G(L \cap X) = G(L \cap Q)$. Hence, by taking an appropriate point in the interior of L as the vertex replacing (v', y'') if necessary, we may assume that

(A1) $G(([v', v'+\epsilon] \times [y''-\epsilon, y'']) \cap X) - G(([v', v'+\epsilon] \times [y''-\epsilon, y'']) \cap Q) > 0, \quad \forall \epsilon > 0.$

Divide $I = [v', v'']$ and $J = [y', y'']$ into $k \in \mathbb{N}$ equidistant grids so that L is divided into k^2 identical rectangles called "cells". Let I_k denote the lowest grid of I and J^k denote the highest grid of J. As $k \to \infty$, $G(I_k \times J) + G(I \times J^k) \to 0$. For all large enough k, therefore, there is a cell composed of neither the grid I_k nor J^k, denoted by ℓ^k, such that

(A2) $G(\ell^k \cap X) - G(\ell^k \cap Q) = m_k > 0$ and $\ell^k \cap (I_k \times J) = \ell^k \cap (I \times J^k) = \emptyset.$

Fix a sufficiently large k for which (A2) holds. By (A1), we also have

(A1') $G(\ell_* \cap X) - G(\ell_* \cap Q) > 0$ where $\ell_* = I_k \times J^k = [v', v' + \frac{v''-v'}{k}] \times [y'' - \frac{y''-y'}{k}, y'']$.

If $G(\ell_* \cap Q) = 0$, there is an interval $J' \subset J^k$ such that $G(([0, v'] \times J') \cap Q) > 0$ because otherwise, $G(\ell_* \cap X)$ would have been 0 according to the construction of $x(\cdot)$ and X described above. By redefining as $\ell_* = [0, v'] \times J'$ in this case (with some abuse of notation), therefore, there is a rectangle $\ell_* = I' \times J'$ such that

(A3) $G(\ell_* \cap Q) > 0$ where $\max_y\{(v, y) \in \ell^k\} \leq \min J'$ and $\max I' \leq \min_v\{(v, y) \in \ell^k\}$.

Finally, we modify α in a way to increase the payoff of college A. Denoting $\ell^k = I'' \times [y'_k, y''_k]$, modify α so that $\alpha_\epsilon(v, y) = 1$ for all $(v, y) \in I'' \times [y'_k, y'_k + \epsilon]$ for $\epsilon \geq 0$. As ϵ increases from 0 to $(y''_k - y'_k)$, the measure of new students with characteristics in ℓ^k in S who enroll in college A (relative to the initial equilibrium) continuously increases from 0 to some positive number no lower than $\frac{m_k}{2}$.

On the other hand, denoting $\ell_* = I' \times [y'_*, y''_*]$, modify α so that $\alpha_\eta(v, y) = 0$ for all $(v, y) \in I' \times [y'_*, y'_* + \eta]$ for $\eta \geq 0$. As η increases from 0 to $(y''_* - y'_*)$, the measure of students with characteristics in ℓ_* in S who cease to enroll in college A (relative to the initial equilibrium) continuously increases from 0 to some positive number no lower than $\frac{1}{2}G(\ell_* \cap Q)$.

Therefore, we can find $\epsilon > 0$ and $\eta > 0$ such that the measure of students who enroll in college A are the same after α has been modified in the two ways described above. Since those who newly enroll are more able and less needy than those who cease to enroll by (A3), it follows that the modified admission rule is feasible and increases the payoff of college A, i.e., it is a beneficial deviation, contradicting the presumed equilibrium. Therefore, we conclude that $G(X) = G(Q)$ and thus, the equilibrium strategy of college A is strategically equivalent to the cutoff rule x described above. The same holds for college B analogously. $\square$

Proof of Lemma 2.

It remains to prove that the solution to the optimization problem (1) exists. We say that an admission rule x is *feasible* if it satisfies the two constraints of (1); and is *effectively increasing* if it is weakly increasing in y on a subset $\tilde{Y} \subset Y$ with $m(\tilde{Y}) = m(Y)$, where m is the Lebesgue measure. Any effectively increasing x can be transformed to a (weakly) increasing rule by redefining $x(y) = \inf\{x(y') | y' \in \tilde{Y} \cap (y, \bar{y}]\}$ for all $y \notin \tilde{Y}$. Clearly, the two rules are strategically equivalent. Hence, below we identify any effectively increasing rule with the increasing (and right-continuous) rule obtained as such. We start by showing that

(A4) if a feasible x is not increasing, there is another feasible rule that "dominates" x in the sense that the average ability of enrolling students is strictly higher under the latter rule than under x.

If a feasible x is not increasing, then $x(y') > x(y'')$ if $y' \in Y'$ and $y'' \in Y''$ for some $\hat{y} \in (\underline{y}, \bar{y})$ and $Y' \subset [\underline{y}, \hat{y})$ and $Y'' \subset (\hat{y}, \bar{y}]$ with $m(Y') > 0$ and $m(Y'') > 0$. One can reduce $x(y)$ slightly for $y \in Y'$ and raise $x(y)$ for $y \in Y''$ slightly while keeping intact the G-measure of $\{(y, v) | y \in Y' \cup Y'', v \geq x(y)\}$, so that the students who would enroll given the modified x fills the capacity exactly. Since the newly enrolling students are more able and less needy than those who cease to enroll, the modified x dominates the initial x, establishing (A4).

Let v^a denote the supremum of the value of the objective function in (1) among all feasible increasing admission rules. Any feasible increasing admission rule that achieves v^a, if exists, is a solution to (1). We show below that such a rule exists.

Consider a sequence of feasible increasing admission rules $\{x^k\}_k$ such that the corresponding sequence of values converges to v^a. For each $\ell = 2, 3, \cdots$, let $\Gamma_\ell = \{\underline{y} + j(\bar{y} - \underline{y})/\ell \mid j = 0, 1, \cdots, \ell\}$ be the grid points that divide Y into ℓ subintervals of identical lengths. Since V is compact, for every $\ell = 2, 3, \cdots$, one can recursively construct a subsequence of $\{x^k\}_k$ such that $x^k(y)$ converges to, say $x^a(y)$, at every $y \in \cup_{2 \leq \ell' \leq \ell} \Gamma_{\ell'}$. For any $y \in \Gamma_\ell$ for some $\ell < \infty$, the limit value $x^a(y)$ is well-defined. For every other $y \in Y$, one can construct an infinite sequence $\{y_l\}$ that converges to y from above where each $y_l \in \Gamma_\ell$ for some ℓ. Since $\{x^a(y_l)\}_l$ is a non-increasing sequence bounded below, it has a limit value which we denote by $x^a(y)$.

Note that $x^a : Y \to V$ defined above is an increasing admission rule and there is a subsequence of $\{x^k\}_k$ that converges to x^a in the following metric: the distance between two admission rules x and x' is $\int_Y |x(y) - x'(y)| dy$. Since the value of the objective function in (1) is continuous in this metric, x^a achieves the supremum v^a and thus is a solution to (1). $\square$

Proof of Lemma 3.

A large part of the proof concerns the existence and uniqueness of equilibrium, and the properties (a)–(d) will be proved in the process. We start with some preliminaries. Consider the solo problem (1) of college A where G is replaced by $\tilde{G}_A$ that comprises all students in S_A and those in S_B who do not get admission offer from college B according to x^a_B in the form of (5).[19] The solo problem (1) of college B is defined symmetrically. We refer to this problem as the solo problem with $\tilde{G}_c$ for $c = A, B$.

It was proved in the proof of Lemma 2 that a solution to the solo problem (1) exists. This proof clearly extends to the case that the distribution G on (v, y) need not be independent between v and y (because this condition is needed nowhere in the proof). Hence, a solution exists to the solo problem with $\tilde{G}_c$ and this solution is in the form of (5) as verified in the main text. In the need-aware regime where $M_A \geq M_B$, by definition, a strategy profile (x^a_A, x^a_B) is an equilibrium if each x^a_c solves the solo problem with $\tilde{G}_c$ appropriately specified.

[19] $\tilde{G}_A$ is defined by the density function $\tilde{g}_A(v, y) = g(v, y)$ if $v \geq x^a_B(y)$ and $\tilde{g}_A(v, y) = 2g(v, y)$ if $v < x^a_B(y)$. $\tilde{g}_B(v, y)$ is defined symmetrically.

Existence of equilibrium.

In light of the above, we define a strategy as $(\mu, \lambda) \in [0,1] \times \Re_+$ where μ is the cutoff level for non-needy students and λ is the rate at which it increases in y. Let $\bar{\lambda} > 0$ be such that college B exhausts its scholarship budget M_B when it admits and fully supports every needy students in $S_A \cup S_B$ with $v \geq \bar{\lambda} y$. Since each college's budget binds in every equilibrium, we restrict strategies to $(\mu, \lambda) \in \Sigma := [0,1] \times [0, \bar{\lambda}]$.

Given a strategy $(\mu, \lambda) \in \Sigma$ of the rival college, let $BR_c(\mu, \lambda) \in \Sigma$ denote the optimal/best response of college $c \in \{A, B\}$ which is unique by Lemma 2. It is straightforward to see that $BR_c(\cdot)$ is continuous.[20] Therefore, $BR : \Sigma^2 \to \Sigma^2$ defined as $BR((\mu_A, \lambda_A), (\mu_B, \lambda_B)) := (BR_A(\mu_B, \lambda_B), BR_B(\mu_A, \lambda_A))$ is a continuous function on a convex and compact domain. Consequently, Brouwer Fixed Point Theorem dictates that BR has a fixed point, which constitutes an equilibrium by definition. This establishes existence of equilibrium.

Properties of equilibrium

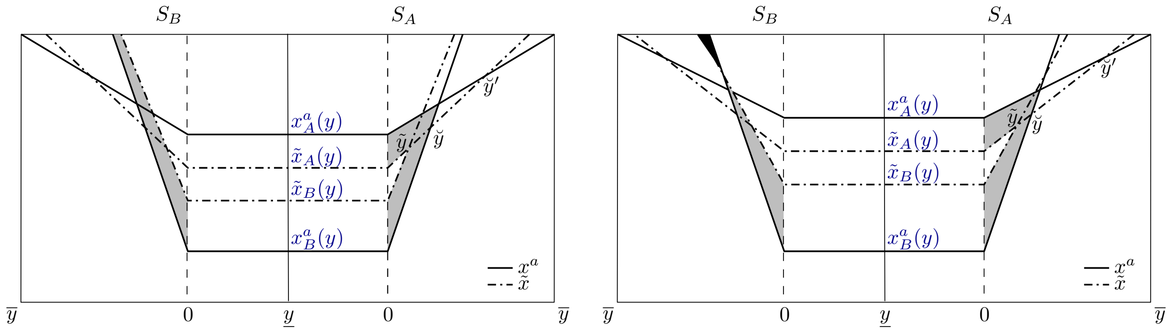
First consider an asymmetric case that $M_A > M_B$ and an equilibrium thereof, denoted by (x_A^a, x_B^a). Each cutoff strategy $x_c^a(y)$ is in the form of (5).

It is not possible that $x_A^a(y) \geq x_B^a(y)$ for all $y \in Y$ owing to binding capacity constraint, unless $x_A^a(y) = x_B^a(y)$ for all $y \in Y$; nor is $x_A^a(y) \leq x_B^a(y)$ for all $y \in Y$ by the same token. Also impossible is $x_A^a(y) = x_B^a(y)$ for all $y \in Y$, because it would imply that the scholarship bill would be same for the two colleges, contradicting the binding budget constraint in equilibrium.

Hence, $x_A^a(\cdot)$ and $x_B^a(\cdot)$ must cross at some need level $\hat{y} \in (0, \bar{y})$ where $x_A^a(\hat{y}) = x_B^a(\hat{y}) < 1$. If $x_A^a(0) < x_B^a(0)$, then college A would take in more non-needy students than college B, hence less of needy students than college B (due to capacity constraint). Moreover, the average scholarship amount offered for them is lower for college A because the admission cutoff x_A^a crosses x_B^a from below at $\hat{y}$. But, this would imply a larger total scholarship bill for college B than A, contradicting $M_A > M_B$. Hence, $x_A^a(0) > x_B^a(0)$ must hold. Together with the earlier observation that $x_A^a(\cdot)$ and $x_B^a(\cdot)$ cross at some $\hat{y} \in (0, \bar{y})$, this verifies (6) and establishes part (a) of the Lemma 3. Parts (b) and (c) follow straightforwardly.

Next, consider an equilibrium of the symmetric case that $M_A = M_B = M$, again denoted by (x_A^a, x_B^a). Unless the equilibrium is symmetric, $x_A^a(\cdot)$ and $x_B^a(\cdot)$ must cross at some $\hat{y} \in (0, \bar{y})$ for binding capacity constraint for both colleges, but this would imply unequal scholarship bills for the two colleges, contradicting binding budget constraints in equilibrium.

[20] As the rival college's strategy (μ^n, λ^n) converges to (μ, λ), the underlying distribution G^n for the other college $c \in \{A, B\}$ in solving for its solo-optimal policy converges to the limit distribution, say G^∞, in the sense that $G^n(X) - G^\infty(X) \to 0$ for any $X \subset V \times Y$ with a positive Lebesgue measure. Therefore, the solo-optimal policy $BR_c(\mu^n, \lambda^n)$ must also converge to that when the rival college adopts (μ, λ), $BR_c(\mu, \lambda)$, because otherwise either the capacity constraint or the budget constraint would fail for large enough n.



(a) When $\tilde{x}_B$ and x_B^a do not intersect

(b) When $\tilde{x}_B$ and x_B^a intersect

Figure 6: **Uniqueness of equilibrium when $\tilde{y} < \breve{y} < \breve{y}'$**

Hence, the equilibrium must be symmetric, i.e., $x_A^a = x_B^a = x^a$. Thus, x^a must solve the solo problem with $\tilde{G}$, but this solution only admits students with characteristics (v, y) in a subset over which the two measures G and $\tilde{G}$ coincide. Therefore, x^a must also solve the solo problem when $\tilde{G}$ is replaced by G, i.e., the problem (1), which is unique in the form of (4) by Lemma 2. This proves part (d) and the uniqueness of equilibrium when $M_A = M_B$.

Uniqueness of equilibrium when $M_A > M_B$

Let (x_A^a, x_B^a) be an equilibrium for the case that $M_A > M_B$ as per (5), so that x_A^a crosses x_B^a from above at $\hat{y} \in (0, \bar{y})$. Suppose there is another equilibrium, denoted by $(\tilde{x}_A, \tilde{x}_B)$, which should also conform to (5) and thus, $\tilde{x}_A$ crosses $\tilde{x}_B$ from above at some $\tilde{y} \in (0, \bar{y})$. We assume $\tilde{x}_B(0) \geq x_B^a(0)$ without loss, and present the proof presuming the inequality is strict because the logic extends straightforwardly to the case of equality. It proves useful to denote $\underline{\tilde{x}}(y) := \min\{\tilde{x}_A(y), \tilde{x}_B(y)\}$ and $\underline{x}^a(y) := \min\{x_A^a(y), x_B^a(y)\}$.

It is impossible that $\underline{\tilde{x}}(y) \geq \underline{x}^a(y)$ for all y, because that would imply the total enrollment of students (two colleges combined) being strictly higher under (x_A^a, x_B^a) than under $(\tilde{x}_A, \tilde{x}_B)$, a contradiction. In addition, $\underline{\tilde{x}}(y)$ crossing $\underline{x}^a(y)$ only once (from above) is impossible, because if it did, given a higher total enrollment of needy students under $(\tilde{x}_A, \tilde{x}_B)$ due to $\tilde{x}_B(0) > x_B^a(0)$, the total scholarship bill would also be higher under $(\tilde{x}_A, \tilde{x}_B)$, violating the binding budget constraint in equilibrium. Therefore, $\underline{\tilde{x}}(y)$ must cross $\underline{x}^a(y)$ at least twice, first from above and second from below. Then, it is clear that $\underline{\tilde{x}}(y)$ crosses $\underline{x}^a(y)$ first at some $\breve{y} \in (0, \hat{y})$ from above, and second at some $\breve{y}' \in (\hat{y}, \bar{y})$.

First, suppose that $\tilde{y} < \breve{y} < \breve{y}'$, as illustrated in Figure 6. Given $x_B^a(0) < \tilde{x}_B(0)$, $x_A^a(0) > \tilde{x}_A(0)$ should hold for $\tilde{y} < \breve{y} < \breve{y}'$. We figure out the enrollment change of needy students in B under (x_A^a, x_B^a), relative to $(\tilde{x}_A, \tilde{x}_B)$. College B newly takes in the students in S_B above x_B^a and below $\tilde{x}_B$ as well as the students in S_A below x_A^a and above x_B^a excluding those in S_A below $\tilde{x}_A$ and above $\tilde{x}_B$ (the gray area in Figure 6(a)(b)). College B releases the students in S_B above $\tilde{x}_B$ and below x_B^a (the black area in Figure 6(b)). Note that the demonstrated need of the released students is larger than that of a student who are newly taken in. The measure of needy students who are newly taken in should be larger than that of the released as college B exhausts its budget in both equilibria. However, the college would take in more non-needy students because $x_B^a(0) < \tilde{x}_B(0)$, violating B's capacity constraint.

Next, suppose that $\breve{y} < \tilde{y} < \breve{y}'$, as illustrated in Figure 7. Then, $\tilde{x}_A(\tilde{y}) = \tilde{x}_B(\tilde{y}) < \min\{x_A^a(\tilde{y}), x_B^a(\tilde{y})\}$ and $\tilde{x}_A(y) > x_A^a(y)$ for $y > \breve{y}'$. Thus, relative to (x_A^a, x_B^a), college A releases the students with (v, y) in the area above the graph of x_A^a and below $\tilde{x}_A$, denoted by r_1, and those in S_B above x_A^a and $\tilde{x}_B$ but below x_B^a, denoted by r_2; and newly takes in the students in S_A above $\tilde{x}_A$ and x_B^a but below x_A^a, denoted by t_1, and the union of those in S_A above $\tilde{x}_A$ but below x_A^a and x_B^a and those in S_B above $\tilde{x}_A$ and below x_A^a and $\tilde{x}_B$, denoted by t_2. The scholarships released from r_1 and r_2 are used to newly support more of needy students since they support students in t_1 and t_2 who have lower levels of needs. If the measure of r_2 is not larger than that of t_1, the money released from r_2 also support more of needy students (as it can support students in t_1), hence college A would take in more needy students in $(\tilde{x}_A, \tilde{x}_B)$. But this would violate capacity constraint because college A would take in more non-needy students as well by $\tilde{x}_A(0) < x_A^a(0)$.

If the measure of r_2 is larger than that of t_1, on the other hand, it can be shown analogously that college B newly supports students with higher levels of need in $(\tilde{x}_A, \tilde{x}_B)$ with money released from students with lower levels of need, hence take in less of needy students than in (x_A^a, x_B^a). But this would violate B's capacity constraint because college B would take in less non-needy students as well by $\tilde{x}_B(0) > x_B^a(0)$ and $\tilde{x}_A(0) < x_A^a(0)$.

Lastly, consider the remaining case that $\breve{y}' < \tilde{y}$. First, suppose that $\underline{\tilde{x}}(y)$ crosses $\underline{x}^a(y)$ three times, as illustrated in Figure 8(a). Relative to (x_A^a, x_B^a), in $(\tilde{x}_A, \tilde{x}_B)$ the two colleges release students with (v, y) in the area above the graph of x_B^a and below $\tilde{x}_B$, denoted by r_1, and those above x_A^a but below $\tilde{x}_A$ and $\tilde{x}_B$, denoted by r_2; and newly take in the students

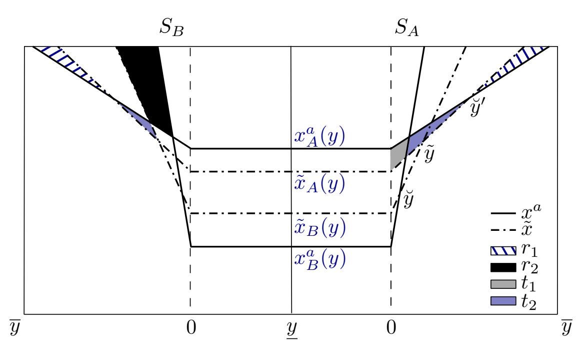
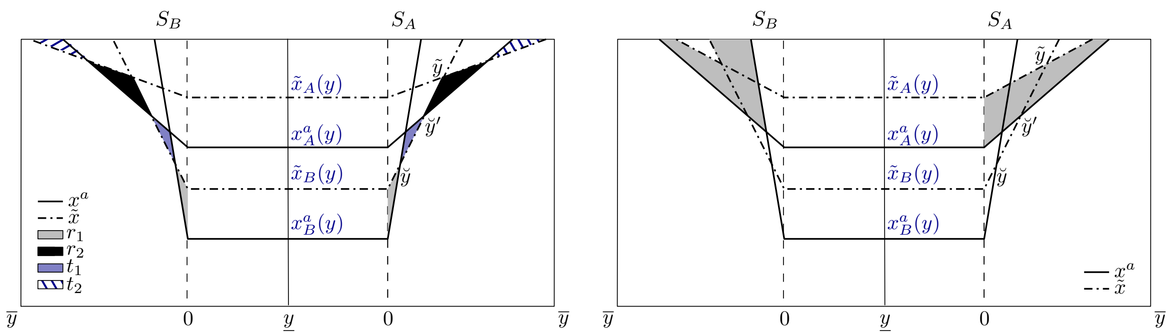


Figure 7: **Uniqueness of equilibrium when $\breve{y} < \tilde{y} < \breve{y}'$**



(a) When $\underline{\tilde{x}}(y)$ crosses $\underline{x}^a(y)$ three times (b) $\underline{\tilde{x}}(y)$ crosses $\underline{x}^a(y)$ two times

Figure 8: **Uniqueness of equilibrium when $\breve{y}' < \tilde{y}$**

above $\tilde{x}_B$ but below x^a_B and x^a_A, denoted by t_1, and those above $\tilde{x}_A$ but below x^a_A, denoted by t_2.

If the measure of t_2 is larger than that of r_2, since the total enrollment of needy students is larger under $(\tilde{x}_A, \tilde{x}_B)$ by $x^a_B(0) < \tilde{x}_B(0)$, it would follow that the total scholarship bill is strictly higher under $(\tilde{x}_A, \tilde{x}_B)$, a contradiction. If the measure of t_2 is no larger than that of r_2, on the other hand, it would follow that for college A the intakes of both needy students and non-needy students are smaller under $(\tilde{x}_A, \tilde{x}_B)$ than under (x^a_A, x^a_B), a contradiction to the capacity constraint.

Next, if $\underline{\tilde{x}}(y)$ crosses $\underline{x}^a(y)$ two times, as illustrated in Figure 8(b). In $(\tilde{x}_A, \tilde{x}_B)$, relative to (x^a_A, x^a_B), college A releases needy students in S_A above x^a_A and below $\tilde{x}_A$ as well as the students in S_B above x^a_A and below x^a_B excluding those in S_B above $\tilde{x}_A$ and below $\tilde{x}_B$ (the gray area in Figure 8(b)). College A takes in less non-needy students by $x^a_A(0) < \tilde{x}_A(0)$, violating its capacity constraint. This completes the proof of the uniqueness of equilibrium. $\square$

Proof of Lemma 5.

Without presuming which college has a larger budget, let A be the college that sets a higher admission cutoff, i.e, $x^d_A \geq x^d_B$, in equilibrium. We showed in the main text that this college offers scholarships to all needy students up to a threshold level which we denote by $n_A \in (0, \bar{y})$; then the other college, B, should offer scholarships most cost-effectively to needy students so long as their need levels are below n_A. (We drop the superscript from n^d_c for notational ease.)

First, we show it impossible in equilibrium that college B offer scholarships up to a certain need level strictly below the threshold level of college A, say $n_B < n_A$, then jumps and offers to those marginally above n_A. Suppose to the contrary that college B did so in an equilibrium. Then, binding capacity constraints imply that

$$[G_V(x^d_A) - G_V(x^d_B)]G_Y(n_B) < [1 - G_V(x^d_A)][G_Y(n_A) - G_Y(n_B)]. \tag{15}$$

We show that the following deviation of college B is beneficial: offer scholarships to those with needs marginally above n_B, instead of those marginally above n_A. Let the amount transferred as such is ϵn_B. Then, no longer recruited is measure $\epsilon\frac{n_B}{n_A}$ of students with need level n_A, whose average ability is $E(v|v > x_B^d)$. Recruited instead is measure ϵ of students with need level n_B. Of these students, a fraction $\frac{1-G_V(x_B^d)}{1+G_V(x_A^d)-2G_V(x_B^d)}$ is from S_B and their average ability is $E(v|v > x_B^d)$, and the remaining fraction $\frac{G_V(x_A^d)-G_V(x_B^d)}{1+G_V(x_A^d)-2G_V(x_B^d)}$ is from S_A and their average ability is $E(v|x_B^d < v < x_A^d) > x_B^d$. Since newly recruited measure of students is larger by $\epsilon(1-\frac{n_B}{n_A})$, college B can raise x_B^d to fill the capacity exactly, and by doing so it releases measure $\epsilon(1-\frac{n_B}{n_A})$ of students with ability marginally above x_B^d. Then, the expected gain in enrolling student ability is first-order approximated by

$$\begin{aligned}
&\epsilon\frac{[1-G_V(x_B^d)]E(v|v > x_B^d) + [G_V(x_A^d) - G_V(x_B^d)]E(v|x_B^d < v < x_A^d)}{1+G_V(x_A^d)-2G_V(x_B^d)} - \epsilon\frac{n_B}{n_A}E(v|v > x_B^d) - \epsilon\Big(1-\frac{n_B}{n_A}\Big)x_B^d \\
\geq\ & \frac{\epsilon}{1+G_V(x_A^d)-2G_V(x_B^d)}\Bigg([1-G_V(x_B^d)]E(v|v > x_B^d) + [G_V(x_A^d) - G_V(x_B^d)]x_B^d \\
&\qquad -[1+G_V(x_A^d)-2G_V(x_B^d)]\frac{n_B}{n_A}E(v|v > x_B^d) - [1+G_V(x_A^d)-2G_V(x_B^d)]\Big(1-\frac{n_B}{n_A}\Big)x_B^d\Bigg) \\
=\ & \frac{\epsilon}{1+G_V(x_A^d)-2G_V(x_B^d)}\Bigg([1-G_V(x_B^d)]\Big(1-\frac{n_B}{n_A}\Big) - [G_V(x_A^d) - G_V(x_B^d)]\frac{n_B}{n_A}\Bigg)[E(v|v > x_B^d) - x_B^d] \\
\geq\ & \frac{\epsilon}{1+G_V(x_A^d)-2G_V(x_B^d)}[1-G_V(x_B^d)]\Big(1-\frac{n_B}{n_A}-\frac{[G_V(x_A^d) - G_V(x_B^d)]}{1-G_V(x_A^d)}\frac{n_B}{n_A}\Big)[E(v|v > x_B^d) - x_B^d] \\
>\ & \frac{\epsilon}{1+G_V(x_A^d)-2G_V(x_B^d)}[1-G_V(x_B^d)]\Big(1-\frac{G_Y(n_A)}{G_Y(n_B)}\frac{n_B}{n_A}\Big)[E(v|v > x_B^d) - x_B^d] \\
>\ & 0
\end{aligned}$$

where the first inequality is due to $E(v|x_B^d < v < x_A^d) > x_B^d$, the second inequality is by $G_V(x_A^d) \geq G_V(x_B^d)$, the third inequality is by (15), and the last inequality is because $\frac{G_Y(n_A)}{n_A} < \frac{G_Y(n_B)}{n_B}$ by Assumption 2. Therefore, the presumed equilibrium scholarship allocation of college B is not optimal.

Therefore, the college that sets a lower admission cutoff also offers scholarships to all needy students up to a threshold level denoted by n^B. We showed in the main text that the college with a larger budget sets both a higher admission cutoff and threshold need level.

Thus, assume $M_A > M_B$ without loss and consider an equilibrium in which college c sets (x_c^d, n_c^d) where $x_A^d \geq x_B^d$ and $n_A^d \geq n_B^d$. Below we prove uniqueness of equilibrium and property (12). Then, parts (a), (b) and (c) follow straightforwardly from the discussion in the main text leading to Lemma 5. For part (d), observe that if $x_A^d \neq x_B^d$, say $x_A^d > x_B^d$, then $n_A^d > n_B^d$ must hold for both colleges to fill their capacity, but this would mean college B takes in more non-needy students and less of needy students than college A, which is impossible because the average scholarship amount is lower for college B given the same budget.

Uniqueness of equilibrium. We prove for the case that $M_A > M_B$; the proof is simpler if

$M_A = M_B$ and is omitted. Suppose there are two equilibria, $\{(x_c^d, n_c^d)\}_{c=A,B}$ and $\{(x'_c, n'_c)\}_{c=A,B}$, where we assume $x_A^d \geq x'_A$. If $x_A^d = x'_A$ then $n_A^d \neq n'_A$. If $n_A^d > n'_A$ ($n_A^d < n'_A$, resp.), the capacity constraint for college A would imply $n_B^d > n'_B$ ($n_B^d < n'_B$, resp.), but this would violate binding budget because the total scholarship bill for college A is larger (smaller, resp.) in $\{(x_c^d, n_c^d)\}_{c=A,B}$ than in $\{(x'_c, n'_c)\}_{c=A,B}$.

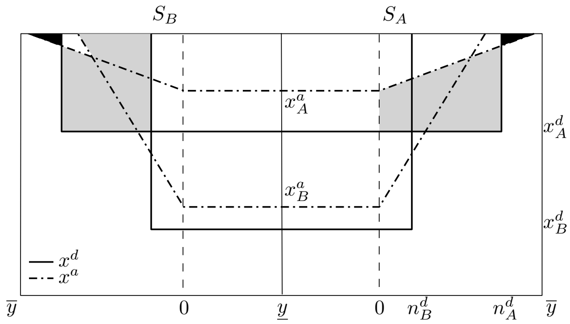
Hence, assume $x_A^d > x'_A$. First, if $n'_B \leq n_B^d$ then the average amount of scholarship is lower in $\{(x'_c, n'_c)\}_{c=A,B}$ for college B who thus takes in less of non-needy students, which in turn implies $x_B^d < x'_B$ due to capacity constraint. Then, relative to $\{(x'_c, n'_c)\}_{c=A,B}$, college A increased cutoff and college B reduced cutoff in $\{(x_c^d, n_c^d)\}_{c=A,B}$; moreover, college B's need threshold is higher in $\{(x_c^d, n_c^d)\}_{c=A,B}$. Therefore, the total intake of college B is strictly higher in $\{(x_c^d, n_c^d)\}_{c=A,B}$, a contradiction to capacity constraint.

Next, suppose $n'_B > n_B^d$ (and $x_A^d > x'_A$). (i) If $x'_B \leq x_B^d$ then $n'_A < n_A^d$ (because $n'_A \geq n_A^d$ would imply a greater total enrollment across the two colleges in $\{(x'_c, n'_c)\}_{c=A,B}$ than in $\{(x_c^d, n_c^d)\}_{c=A,B}$, a contradiction); this implies that total needy enrollment is larger in $\{(x_c^d, n_c^d)\}_{c=A,B}$, but less (more, resp.) of needy students are accommodated for $y < n'_A$ ($y \in (n'_A, n_A^d)$, resp.), which in turn means that the total scholarship spend is higher in $\{(x_c^d, n_c^d)\}_{c=A,B}$, a contradiction. (ii) If $x'_B > x_B^d$, again $n'_A < n_A^d$ should hold because otherwise total scholarship spend would be higher in $\{(x'_c, n'_c)\}_{c=A,B}$ by an analogous reasoning to just above. In $\{(x'_c, n'_c)\}_{c=A,B}$, relative to $\{(x_c^d, n_c^d)\}_{c=A,B}$, college B releases a group of needy students with $y > n_B^d$ and takes in another group of needy students with $y > n_B^d$. Since $x'_B > x_B^d$ and $x_A^d > x'_A$, the measure of the former group is smaller than that of the latter group to meet the capacity constraint. However, this would imply that B's scholarship spend is higher in $\{(x'_c, n'_c)\}_{c=A,B}$ than in $\{(x_c^d, n_c^d)\}_{c=A,B}$, a contradiction.

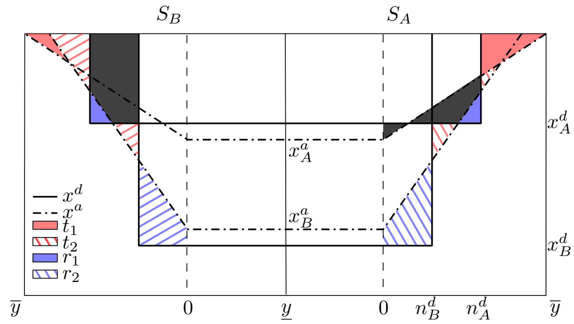
Property (12). To see this, suppose M_A increased to M'_A and M_B decreased to M'_B by the same amount, accompanied by the changes in equilibrium from $\{(x_c^d, n_c^d)\}_{c=A,B}$ to $\{(x'_c, n'_c)\}_{c=A,B}$. With a view to reaching a contradiction, suppose that $x_B^d \leq x'_B$. If $x'_A \leq x_A^d$ then college B has less non-needy students with M'_B and cannot have more needy students than before because $M'_B < M_B$. Hence, we should have $x'_A > x_A^d$.

If $n'_B \leq n_B^d$, then we should have $n'_A > n_A^d$ to fill capacity for both colleges. Then, relative to the initial equilibrium, the enrollment of needy students reduced for $y < n_A^d$ and increased for $y > n_A^d$. This implies that the total enrollment of needy students across the two colleges decreased (because the total budget across two colleges remain the same), which is impossible because the total enrollment of non-needy students also decreased (because $x_B^d < x'_B$).

Next, consider the case that $n'_B > n_B^d$. Then, we should have $n'_A > n_A^d$ to fill college A's capacity. Switching from $\{(x_c^d, n_c^d)\}_{c=A,B}$ to $\{(x'_c, n'_c)\}_{c=A,B}$, college B saved scholarship budget from the students in $S_A \cup S_B$ with $(v_i, y_i) \in (x_B^d, x'_B) \times (0, n_B^d)$ and spent a part of it on



(a) When $x^a_A(0) \geq x^d_A$



(b) When $x^a_A(0) < x^d_A$ and $x^a_A(n^d_B) > x^d_A$

Figure 9: **Proof of Proposition 1**

the students in $S_A \cup S_B$ with $(v_i, y_i) \in (x'_B, x^d_A) \times (n^d_B, n'_B)$. The remainder of the saved budget is "transferred" to college A for the students in S_A with $(v_i, y_i) \in (x^d_A, x'_A) \times (0, n'_B)$ and those in S_B with $(v_i, y_i) \in (x^d_A, 1) \times (n^d_B, n'_B)$ (these students enrolled in A in $\{(x^d_c, n^d_c)\}_{c=A,B}$ but switched to B in $\{(x'_c, n'_c)\}_{c=A,B}$). College A received the transfer from college B as described above and also saved its budget on the students in $S_A \cup S_B$ with $(v_i, y_i) \in (x^d_A, x'_A) \times (n'_B, n^d_A)$. College A spent all of them to take in the students in $S_A \cup S_B$ with $(v_i, y_i) \in (x'_A, 1) \times (n^d_A, n'_A)$ (where $n^d_A < n'_A$). Combining how the scholarship budgets of the two colleges have been moved from the initial equilibrium to the new equilibrium, we deduce that in net the money saved from less needy students' scholarships has been spent to support more needy students. This implies that the total enrollment of needy students across the two colleges decreased, which is again impossible because the total enrollment of non-needy students also decreased (because $x^d_B < x'_B$).

This establishes (12), completing the proof. □

Proof of Proposition 1

It remains to compare between NA and NBd when $M_A > M_B$. Let (x^a_A, x^a_B) and $\{(x^d_c, n^d_c)\}_{c=A,B}$ be the equilibrium in NA and in NBd, respectively. Recall that $x^a_A(0) > x^a_B(0)$ as well as $x^d_A > x^d_B$ and $n^d_A > n^d_B$. It suffices to show that $x^a_B(0) < x^d_B$ for then the total enrollment of needy students is higher in NBd than in NA.

Suppose, to the contrary, that $x^a_B(0) \geq x^d_B$. First, consider the case that $x^a_A(0) \geq x^d_A$, as illustrated in Figure 9(a). As the equilibrium moves from $\{(x^d_c, n^d_c)\}_{c=A,B}$ to (x^a_A, x^a_B), we deduce the following:

(i) The total enrollment of needy students is weakly larger in (x^a_A, x^a_B) because that of non-needy enrollment is weakly lower due to $x^a_B(0) \geq x^d_B$.

(ii) College A newly takes in some students with need levels above n^d_A (indicated by the two black areas in Figure 9(a)) because otherwise it would have a lower total enrollment than in $\{(x_c^d, n_c^d)\}_{c=A,B}$, contradicting capacity constraints.

(iii) For there to be a larger total enrollment of needy students in (x_A^a, x_B^a) as per (i) above, other needy students with lower need levels must also be newly taken in, which means that $x_B^a(n_B^d) < x_A^d$ as in Figure 9(a).

(iv) This implies that college A releases some needy students with need levels below n_B^d as indicated by gray areas in Figure 9(a);

(v) Since college A newly takes in students with higher need levels as explained in (ii), college A has a strictly lower enrollment of needy students in (x_A^a, x_B^a) than in $\{(x_c^d, n_c^d)\}_{c=A,B}$.

(vi) This is a contradiction because A's non-needy student enrollment is also lower.

Next, consider the case that $x_A^a(0) < x_A^d$. If $x_A^a(n_B^d) \leq x_A^d$, given the assumption $x_B^a(0) \geq x_B^d$, college B takes in less non-needy students as well as less needy students with $y_i < n_B^d$ in (x_A^a, x_B^a) than in $\{(x_c^d, n_c^d)\}_{c=A,B}$; then B cannot fill the capacity in (x_A^a, x_B^a) because the money released from needy students with $y_i < n_B^d$ is not enough to fill the capacity by supporting students with higher need levels $y_i \geq n_B^d$, a contradiction.

Suppose $x_A^a(n_B^d) > x_A^d$. Then, $x_B^a(n_B^d) < x_A^d$ must hold, as illustrated in Figure 9(b), because otherwise the total scholarship bill of the two colleges combined would be larger than that in the need-blind equilibrium, leading to a contradiction. When the colleges switch from $\{(x_c^d, n_c^d)\}_{c=A,B}$ to (x_A^a, x_B^a), some needy students are released by one college but are taken in by the other college (as shown in the dark-gray area in Figure 9(b)). Let q_{AB} be the set of needy students who are supported by A in $\{(x_c^d, n_c^d)\}_{c=A,B}$ but by B in (x_A^a, x_B^a), and q_{BA} be the set of needy students who are supported by B in $\{(x_c^d, n_c^d)\}_{c=A,B}$ but by A in (x_A^a, x_B^a). Let m_{AB} and m_{BA} be the budgets to support q_{AB} and q_{BA}, respectively.

Next, in (x_A^a, x_B^a), college A newly takes in needy students in S_A with $y_i \geq n_A^d$ and $v_i \geq x_A^a(y_i)$ and those in S_B with $y_i \geq n_A^d$ and $v_i \in (x_A^a(y_i), x_B^a(y_i))$ (denoted by t_1 in Figure 9(b)); college A may release needy students with $y_i \leq n_A^d$ and $v_i \in (x_A^d, x_B^a(y_i))$ (denoted by r_1 if exists). Note that students in t_1 have larger need levels than those in r_1. Similarly, we find that the needy students who are newly taken in to college B (denoted by t_2) have larger need levels than those released by the same college (denoted by r_2).

(i) Suppose $m_{AB} > m_{BA}$. From $\{(x_c^d, n_c^d)\}_{c=A,B}$ to (x_A^a, x_B^a), of the money m_{AB} released from q_{AB}, A transfers the amount m_{BA} to support q_{BA} and the remaining $(m_{AB} - m_{BA})$ to support new students in t_1. At the same time, B transfers the full amount m_{BA} released from q_{BA} to support a part of q_{AB}; and of the money released from r_2, B transfers $(m_{AB} - m_{BA})$ to support the rest of q_{AB} and the remaining money to support new students in t_2. Note that B's transfer of m_{BA} is simply replacing the same amount of money A transfers back to support q_{BA}; and B's transfer of $(m_{AB} - m_{BA})$ is simply replacing the same amount of money A transfers out to support new students in t_1. Therefore, from the perspective

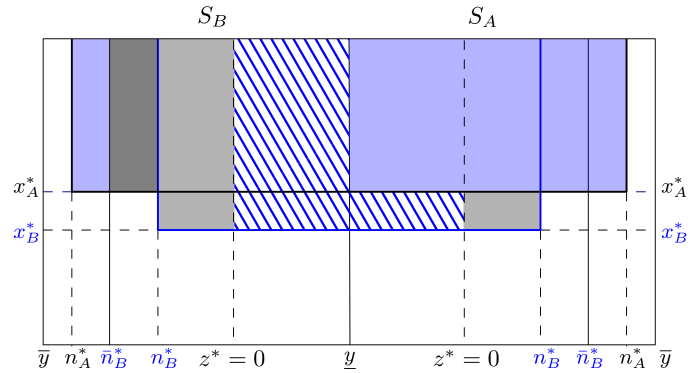


Figure 10: **Matching financial aid offers when $F < M_A - M_B$:** $z^* = 0$ and A's admission cutoff is higher than that of B. The needy students in the dark-gray area request matching aid offers; they either enroll in B with probability p^* or enroll in A otherwise. The light-gray area in S_B depicts all needy students enrolling in B after receiving the scholarship y_i; all non-needy students enrolling in college B are populated in the blue-dashed area; and those enrolling in A are populated in the blue-solid area.

of A and B combined, from $\{(x_c^d, n_c^d)\}_{c=A,B}$ to (x_A^a, x_B^a), of the money released from r_2, $(m_{AB} - m_{BA})$ is used to support new students in t_1 and the rest is used to support new students in t_2; and the money released from r_1, if exists, is used to support new students in t_1. Note that the money released is used to support new students of greater need levels, hence the total enrollment of needy students must be lower in (x_A^a, x_B^a). This is impossible because non-needy enrollment is lower in (x_A^a, x_B^a) as well.

(ii) If $m_{AB} < m_{BA}$, then B uses m_{BA} released from q_{BA} to support the whole of q_{AB} and some of t_2, and uses the money from r_2 to support the rest of t_2. Thus, the released money is used to support more expensive students in (x_A^a, x_B^a). Since non-needy enrollment is lower for B in (x_A^a, x_B^a) as well, this contradicts B's capacity constraint. □

Appendix A2: Matching financial aid offers when $F < M_A - M_B$

For the cases with $F < M_A - M_B$, it is optimal for the DoED to set $z^* = 0$ so that college B receives the entire loan subsidy in the form of matching loans. Due to the asymmetric budget, still, college A cannot set a smaller need threshold n_A^* than college B's threshold, n_B^*. Then, the students in S_B populated between these need thresholds and above A's cutoff apply for matching aid offers. It follows from $F < M_A - M_B$ that the total amount of applications exceeds $F(= F_m)$ and therefore, the DoED randomizes the approval with probability $p^*(< 1)$, in which case, the applicants get approved with probability p^* and enroll in B, whereas they get denied with probability $(1 - p^*)$ and end up enrolling in A. Altogether, A's admission cutoff should be higher than B's cutoff in order to satisfy the budget and capacity constraints, as illustrated in Figure 10.

Formally, we have $(z^* = 0, p^*)$, (x_A^*, n_A^*), and (x_A^*, n_B^*) should satisfy

$$\int_{\substack{x_A^* \leq v \leq 1 \\ \underline{y} \leq y \leq n_A^*}} 1\, dG + (1 - p^*) \int_{\substack{x_A^* \leq v \leq 1 \\ n_B^* \leq y \leq n_A^*}} 1\, dG = \frac{1}{2};$$

$$\int_{\substack{x_A^* \leq v \leq 1 \\ \underline{y} \leq y \leq n_B^*}} 1\, dG + 2 \int_{\substack{x_B^* \leq v \leq x_A^* \\ \underline{y} \leq y \leq n_B^*}} 1\, dG + p^* \int_{\substack{x_A^* \leq v \leq 1 \\ n_B^* \leq y \leq n_A^*}} 1\, dG = \frac{1}{2};$$

$$\int_{\substack{x_A^* \leq v \leq 1 \\ 0 \leq y \leq n_A^*}} y\, dG + (1 - p^*) \int_{\substack{x_A^* \leq v \leq 1 \\ n_B^* \leq y \leq n_A^*}} y\, dG = M_A;$$

$$\int_{\substack{x_A^* \leq v \leq 1 \\ 0 \leq y \leq n_B^*}} y\, dG + 2 \int_{\substack{x_B^* \leq v \leq x_A^* \\ 0 \leq y \leq n_B^*}} y\, dG + p^* \int_{\substack{x_A^* \leq v \leq 1 \\ n_B^* \leq y \leq n_A^*}} n_B^*\, dG = M_B;$$

$$p^* \int_{\substack{x_A^* \leq v \leq 1 \\ n_B^* \leq y \leq n_A^*}} (y - n_B^*)\, dG = F,$$

where the first two equations are the capacity constraints of the colleges, the next two equations are the budget constraints of the colleges, and the last equation is the DoED's budget constraint.

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