Control, Cost, and Confidence: Perseverance and Procrastination in the Face of Failure

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Abstract

We study effort provision and the development of control beliefs over time: a student is uncertain whether she has control over success through her effort or whether it is determined by her innate ability, which she also does not know. In each period, what she can learn about her control and her ability depends on the level of effort she exerts. We characterize the student's optimal effort policy in this two-dimensional bandit problem, which may feature repeated switching of the effort level. Moreover, we analyze how control, cost, and confidence impact perseverance and procrastination in the face of failure. Finally, we relate our results to findings in educational psychology and discuss policies to foster perseverance and to lower procrastination.

JEL: D83, D81, D91, I21

Keywords: correlated bandit, optimal experimentation, perseverance, procrastination, locus of control, mindsets.

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1 Introduction

The willingness to work hard even after experiencing setbacks is documented as a key predictor of success: According to Duckworth, Peterson, Matthews and Kelly (2007) grit is an essential determinant of high, possibly even more important than talent.[1] Heckman, Stixrud and Urzua (2006) find that, for a given level of cognitive skills, non-cognitive skills such as perseverance strongly influence social and economic success.[2] Both in education and the workplace it is, therefore, essential to understand how students or workers can be motivated to exert high effort, even if they experience failures.

A central factor that seems to determine the willingness to work hard is the "control belief" – the belief that we have control over our own fate, or that we can achieve success if we work hard enough. The connection is simple: if we believe that we can achieve success eventually by exerting effort, we are less likely to give up after experiencing a failure.[3] The control belief is at the core of the psychological concept of *mindset*: Dweck (2006) distinguishes subjects with a "fixed mindset" – who believe that success is based on innate ability – from subjects with a "growth mindset" – who believe that success comes from hard work. Consequently, when facing a failure fixed types stop exerting effort, whereas growth types increase their effort level. Closely related are the concepts of *locus of control*[4] and *self-efficacy*[5], which are also centered around the control belief. All three concepts consider the control belief to be a personality trait, that is, essentially, an exogenously given constant. Therefore, interventions aiming at increasing effort often try to directly enhance the students' control beliefs.[6]

In this paper, we try to expand the set of possible policies to induce high effort by considering the control belief as a variable that evolves over time. In our analysis, we take the view of a student who decides whether to work hard or not. The key assumption is that the student is uncertain if high effort is decisive for success. As she experiences successes and failures, the student learns about how much her effort matters. We show how the

[1] Duckworth et al. (2007) develop a survey-based measure for grit. They find that it is a significant predictor for success, even when controlling for factors like IQ.

[2] Heckman et al. (2006) report that early childhood programs such as the Perry Preschool Program or Headstart had positive impact on life outcomes through non-cognitive skills, such as perseverance, while they did not have an effect on IQ.

[3] For empirical evidence in a school context, see, for example, Coleman and DeLeire (2003). Caliendo, Cobb-Clark and Uhlendorff (2015) analyze effort in job search.

[4] According to Rotter (1966), subjects with an "external locus of control" see success and failure beyond their reach, whereas subjects with an "internal locus of control" attribute outcomes more to their own behavior. Thus, after failure, an internal locus of control leads to perseverance of high effort, while an external locus results in despondence.

[5] Bandura (1997) holds that the extend to which people believe to have control over their lives determines their resilience towards difficulties.

[6] Dweck, in particular, has conducted several interventions in schools, to promote a high control belief in students (for a summary, see Dweck (2000)).

development of the control belief over time depends on the student's confidence in her own ability, as well as her effort cost, patience, and luck. Our analysis provides insights on possible policies to promote effort, both in the short and in the long run.

We design a simple theoretical model in which a student is uncertain about both the efficacy of effort and her own ability level: she does not know if success can be achieved through effort or if it is all down to innate ability. Moreover, if ability is decisive, she does not know if her ability is sufficient to succeed. This uncertainty translates to three possible states of the world: (α) success is down to ability and the student has high ability, (β) success is down to ability, but the student has low ability, or (ω) success can be achieved through high effort. The belief that the state is ω – i.e., that the student can affect the outcome through effort – is interpreted as her control belief. The student's belief that ability matters and that she has high ability (α) rather than low ability (β), is interpreted as her confidence.

The student faces a two-dimensional correlated bandit problem in which she simultaneously learns about her ability and control. Each period, the student decides whether to exert costly high effort or not and, subsequently, observes a success or a failure. Different levels of effort, combined with success or failure, result in different posterior beliefs about the state. For example, a student who succeeds with low effort would be certain that ability matters and she is a high type (α); by contrast, a student who observes a failure with low effort does not know whether she failed because high effort is necessary (ω), because she is a low type (β), or because she is a high type and had bad luck (α). It is particularly interesting to note that, for a given control belief, a student's confidence determines how she updates her control belief – even in a setting as ours in which ability is irrelevant when effort matters and *vice versa*. Thus, two students with identical prior control beliefs and identical experienced histories can draw very different conclusions, depending on their confidence. A student with low confidence tends to attribute failure with high effort to her lack of ability, while a student with high confidence tends to blame bad luck.

Interestingly, we find that a simple stopping rule is not optimal in general: the student might optimally change her effort level repeatedly, despite the fact that the state is fixed. Intuitively, after experiencing failure with low effort, the student becomes more convinced that high effort is necessary for success. Conversely, after experiencing a failure with high effort, the student becomes more convinced that effort was futile. Importantly, this implies that the value function is a complex expression which depends on the precise sequence of high and low effort. Thus, we cannot simply solve the Bellman equation to find the value function and derive the optimal behavior. Instead, we derive a linear belief cutoff rule that characterizes the student's effort choices in each period, i.e., her optimal policy. The cutoff rule is determined by comparing the pivotal plans that prescribe switching the effort level immediately after observing any failure. We show that any optimal plan prescribes the same effort level as the cutoff rule.

With uncertainty about the state, the optimal policy achieves the maximum expected welfare. Thus, the student's behavior is optimal, given her belief. Taking the perspective that the true state of the world is ω and that the student can be successful if and only if she exerts high effort, we discuss how to induce high effort to increase the student's welfare. We are interested in both the student's short-run and long-run welfare.

We define *procrastination* as the tally of failures with low effort that it takes before the student is willing to exert high effort.[7] Procrastination is a (negative) measure for short-run welfare. The longer it takes for the student to start exerting high effort, the longer she delays the possibility for success. We define *perseverance* as the tally of failures with high effort that it takes before the student gives up forever. Perseverance is a measure for long-run welfare. Intuitively, the longer the student tries to succeed with high effort, the more likely she eventually observes a success. Thus, to improve welfare we are interested in policies that decrease the student's procrastination and that increase her perseverance.

In accordance with the psychological theories mentioned above, our model predicts that a higher control belief encourages effort, both in the short and in the long run. Furthermore, we find that high effort costs lead to less perseverance and more procrastination. Equivalently, a student who assigns a lower payoff to success is less willing to work hard. We also predict that a student is more perseverant if she is more patient. While these results are intuitive, the role of confidence is more surprising. The effect of higher confidence on effort crucially depends on the time-frame considered: In the short run, higher confidence can increase the student's procrastination. In the long run, higher confidence can increase the student's perseverance.

From a theoretical perspective, the problem studied in our paper is a two-armed bandit problem: the student can choose between high and low effort which corresponds to choosing between two arms of a bandit. The probability of success from both high and low effort is uncertain. By choosing an effort level, the student receives both an immediate payoff and information about the payoff distribution which is valuable for her future actions. Consequently, the student faces the classical trade-off between exploitation of the option with the highest expected immediate payoff and exploration of the other option.

Rothschild (1974) shows that an agent choosing between two arms with unknown (and possibly correlated) payoff distributions will not necessarily learn the optimal action. This central insight on bandit problems also applies to our setting. A student may stop exerting

[7]We use procrastination as a neutral term, signifying the delay of effort. While procrastination is often understood as a dysfunctional behavior or trait, we model it as a rational reaction to learning about the efficacy of effort. For a discussion, see Kim and Seo (2015).

high effort for good – even though the state is ω. Gittins (1979) develops a simple index which characterizes the optimal policy for bandits with independent arms. The Gittins index is not applicable to our model, as the arms are correlated: the outcome with high effort also affects the student's belief about the success probability with low effort and *vice versa*. For bandit problems with correlated arms, there is no general analytical solution for the optimal policy, and the existing theoretical work is limited.

Camargo (2007) is able to show the optimality of cutoff belief strategies for a class of correlated two-armed bandits that satisfy some mild continuity assumption. By focusing on a specific information structure, we are able to give an explicit solution for the optimal cutoff belief strategy and thereby fully characterize the agent's optimal policy. This allows us to derive a formal notion of procrastination and perseverance and to analyze how these are affected by changes in the different parameters of our model.

Klein and Rady (2011) consider the case of negatively correlated arms that are operated by two different agents. While we consider a single-agent decision problem, they analyze the strategic interaction between the agents.[8] For perfectly negative correlation, they characterize all Markov perfect equilibrium strategies in closed form. The extension to imperfectly correlated arms is particularly related to our setting, even though the correlation between the arms is different. As in our case, Klein and Rady show existence of an equilibrium in cutoff strategies and that learning may remain incomplete. Pastorino (2019) provides an application of bandits with correlated arms. The paper analyzes a structural model of job and wage mobility in a firm and finds evidence for a novel mechanism through which learning about ability shapes wages.

Our paper connects to a literature featuring interdependent learning on two dimensions: the belief about one variable affects learning about another. Heidhues, Kőszegi and Strack (2018) consider misspecified beliefs: The agent's outcome depends on her action, her ability, and some external factor. The agent is uncertain about the external factor, while she is certain, but incorrect about her ability. The agent's overconfidence in her ability leads to incorrect inferences about the environment, and the agent's adjusted actions lead her further away from the truth. In contrast to model misspecification, our paper features model uncertainty: agents learn about two variables at the same time. The agent is both uncertain about her ability and about the efficacy of her effort. She might never learn the true production function even if her beliefs are not misspecified. Model uncertainty is also featured in Hestermann and Yaouanq (2020) and Piketty (1995).

In Hestermann and Yaouanq (2020), the agent may start with an incorrect prior belief about her ability. Similarly to our paper, the agent faces two-dimensional uncertainty and

[8] Starting with Bolton and Harris (1999) and Keller, Rady and Cripps (2005) there has been a rich economic literature on strategic experimentation in which arms are operated by multiple agents.

the ability belief influences learning in the other dimension. In a series of two-armed bandit problems, she can choose to take a new draw of the exogenous variable – i.e., sample a new environment – or stick to the current one. While an overconfident agent will eventually learn her true ability as she keeps sampling new environments, an underconfident individual may get stuck in a sub-optimal environment and never learn. This outcome corresponds to our finding that a student with low confidence in her ability may never learn that she can succeed with effort. There is no effort choice in Hestermann and Yaouanq (2020) and, thus, the agent's problem is different from ours. We provide a characterization of the optimal strategy with forward-looking agents.

In Piketty (1995), as in our model, the agents are uncertain about the efficacy of effort, and the level of effort they exert determines the information they can acquire. Agents with low control beliefs never strive and never learn that exerting high effort would be valuable, while others – with high control beliefs – work unnecessarily hard. The main difference to our approach is that in Piketty (1995) agents only live one period and, thus, behave myopically in their effort choice. Instead, our paper solves a bandit problem: the agent takes the information rent and the expected future payoffs into account, when deciding about the effort level.

Lemoine (2020) considers agents who are uncertain about their ability and how their efforts translate into output. Differently from our model, effort and ability are complementary. Surprisingly, agents become either persistently overconfident or persistently underconfident on average, depending on how greater effort affects the variance of an agent's payoffs.

Our paper also relates to a small literature in information acquisition and learning: Che and Mierendorff (2019), Nikandrova and Pancs (2018), and Mayskaya (2019) consider the choice between two actions that give different information about the state of the world. Differently from bandit problems, here, the agent does not receive any payoff until she takes a final decision.

We proceed as follows. We state our model in Section 2. Section 3 illustrates an example of two periods. In Section 4, we derive our main result and introduce a formal notion of perseverance and procrastination. Section 4.3 discusses the determinants of the student's short- and long-run behavior and possible implications for policy. A final section concludes. All proofs are in the Appendix.

2 Model

We consider a single agent decision problem over time. Time is discrete, and we discount the output of future periods with discount factor $\delta \in (0, 1)$. In each period, the student chooses a level of effort $e \in \{H, L\}$. Exerting high effort, $e = H$, has cost $c \in (0, 1 - \varepsilon)$ for some

$\varepsilon \in (0,1)$, exerting low effort is costless.[9] In each period, after choosing the level of effort, the student observes either a success S or a failure F. A success gives the student a payoff of 1, while a failure yields a payoff of 0.[10]

The state of the world takes a value in $\{\alpha, \beta, \omega\}$ (high ability, low ability, work) and is fixed for all periods. The student's choice of effort determines the probability of success for each state. In particular, if the state is ω (work), the student can only succeed with high effort, independent of her ability: the probability of success is $1-\varepsilon$ if she exerts high effort, and 0 if she exerts low effort. On the other hand, if the state is α or β (high or low ability), the probability of success is independent of the level of effort. In state α, the probability of success is $1-\varepsilon$, while in state β, the student cannot succeed. The parameter $\varepsilon \in (0,1)$ denotes the probability of having bad luck – i.e., with probability ε, the student observes a failure, even though she had all the prerequisites for success. The student is uncertain about the realization of the state, and the triple $\boldsymbol{p} = (p_\alpha, p_\beta, p_\omega) \in [0,1]^3$ with $p_\alpha + p_\beta + p_\omega = 1$ specifies the student's belief about the state. We interpret p_ω as the *control belief* and p_α as the *confidence* of the student in her own ability.

It proves convenient to specify the *outcome* of a period $h \in \{SH, FH, SL, FL\}$ as a combination of the choice of effort H, L and the observed success S or failure F in that period. The respective probabilities of these outcomes are given in Table 1. An outcome h, together

	α	β	ω
$Pr(SH)$	$1-\varepsilon$	0	$1-\varepsilon$
$Pr(FH)$	ε	1	ε
$Pr(SL)$	$1-\varepsilon$	0	0
$Pr(FL)$	ε	1	1

Table 1: Probabilities of outcomes for the states.

with a belief $\boldsymbol{p} = (p_\alpha, p_\beta, p_\omega)$, induces a posterior belief $\boldsymbol{p}(h) = (p_\alpha(h), p_\beta(h), p_\omega(h))$. The student's updating of her belief follows Bayes rule. Her posterior beliefs after the different outcomes are given in Table 2.

The student's updating is illustrated in Figure 1. The dashed (red) arrows depict updating after exerting high effort, while the solid (blue) arrows depicts updating after low effort. The student's updating is linear: After exerting high effort, the student's belief moves on a straight line between the point $p_\beta = 1$ and the edge between $p_\omega = 1$ and $p_\alpha = 1$. After exerting low effort, the student's belief moves on a straight line between the point $p_\alpha = 1$ and the edge between $p_\omega = 1$ and $p_\beta = 1$.

[9] The assumption that c is bounded by $1-\varepsilon$ ensures that exerting effort may be valuable for the student.

[10] We normalize the payoff to success to 1, but allow heterogeneity in effort cost c. Equivalently, we could normalize the cost and allow the payoff to vary, or both.

$\boldsymbol{p}$	p_α	p_β	p_ω
$\boldsymbol{p}(SH)$	$\frac{p_\alpha}{p_\alpha+p_\omega}$	0	$\frac{p_\omega}{p_\alpha+p_\omega}$
$\boldsymbol{p}(FH)$	$\frac{p_\alpha\varepsilon}{p_\alpha\varepsilon+p_\beta+p_\omega\varepsilon}$	$\frac{p_\beta}{p_\alpha\varepsilon+p_\beta+p_\omega\varepsilon}$	$\frac{p_\omega\varepsilon}{p_\alpha\varepsilon+p_\beta+p_\omega\varepsilon}$
$\boldsymbol{p}(SL)$	1	0	0
$\boldsymbol{p}(FL)$	$\frac{p_\alpha\varepsilon}{p_\alpha\varepsilon+p_\beta+p_\omega}$	$\frac{p_\beta}{p_\alpha\varepsilon+p_\beta+p_\omega}$	$\frac{p_\omega}{p_\alpha\varepsilon+p_\beta+p_\omega}$

Table 2: Posterior beliefs after observing outcomes.

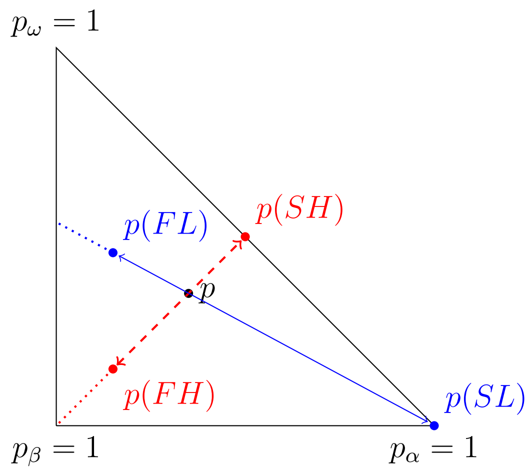

Figure 1: Updating beliefs after high and low effort.

Note that a student's confidence significantly affects how she updates her control belief for a given outcome. The effect is easiest to see for extreme beliefs. Assume a student is certain that, if ability matters, her ability is high – i.e., $p_\beta = 0$. In this case, the student learns nothing from high effort: her belief remains unchanged after observing SH or FH. Low effort, instead, moves her belief towards $p_\omega = 1$ for FL and towards $p_\alpha = 1$ for SL. Conversely, a student with $p_\alpha = 0$ learns nothing from exerting low effort.

Discussion of the assumptions. We assume binary effort and ability levels where ability does not matter if effort is decisive and *vice versa*. An alternative interpretation – which is equally consistent with the model – is that there are three different ability levels: at the lowest level (β), the student never succeeds, independently of effort; at the intermediate level of ability (ω), the student can succeed if and only if she exerts high effort; at the highest level (α), the student can succeed, independently of effort.

In our setup, the student's success probabilities in states α and β are independent of effort. However, results will be similar for any model in which the return to effort depends on ability non-monotonically – i.e., if effort and ability are not always complementary.[11]

[11]In practice, it seems reasonable to assume that the return to effort is not always strictly increasing

As is standard in the literature, we only model one type of noise – bad luck. As a result, the student learns immediately that the state is α upon observing SL and that the state cannot be β upon observing SH. We expect symmetrically similar result when replacing bad luck by good luck. In practice, it seems more reasonable to assume that a student may fail despite high effort and high ability, than that a student could succeed despite low effort and low ability. Finally, we assume that the probability of bad luck ε is the same in the states ω and α. Different probabilities would allow for an additional channel of learning: the student could learn about the relative probabilities of the state by keeping her effort constant. In particular, after successfully exerting high effort for sufficiently long, the student could infer from different values of ε whether the state is ω or α just from observing the failure likelihood.

3 Illustrative example

To illustrate the basic trade-offs at work, we consider simplified one- and two-period versions of our model without discounting ($\delta = 1$).

One period: The choice of effort only impacts the immediate payoff but has no informational externality. The student's expected payoff from exerting high effort is $EU(H) = (p_\omega + p_\alpha)(1-\varepsilon) - c$, while the expected payoff from low effort is $EU(L) = p_\alpha(1-\varepsilon)$. Thus, the student prefers to exert high effort if and only if $p_\omega(1-\varepsilon) \geq c$. The cost of high effort c is weighed against the expected benefit $p_\omega(1-\varepsilon)$, that results from the probability of effort being necessary to get a success, p_ω, and not having bad luck, $1-\varepsilon$.

Two periods: A plan $\boldsymbol{\pi} = (\pi_1(\emptyset); \pi_2(h))$ specifies an action for both periods; the action in period two depends on the observed outcome h in period one. We solve the student's decision problem by backward induction. In the last period, the student's problem is identical to the one-period case: she exerts high effort whenever her (posterior) belief $p_\omega(h)$ satisfies $p_\omega(h)(1-\varepsilon) \geq c$. The choice of effort in the first period is more complex, since it affects not only the immediate payoff but also the information available in the second period.

Lemma 1 *For two periods, one of the following plans is optimal, depending on the student's prior* $\boldsymbol{p}$.

1. *Persevering: plan* $(\pi_1(\emptyset) = H; \pi_2(SH) = H, \pi_2(FH) = H)$ *is optimal for* $p_\omega \geq \max\{c\frac{p_\alpha}{1-\varepsilon-c}, \frac{c}{c+\varepsilon}\left(\frac{1}{1-\varepsilon} - p_\alpha\right)\}$.

in ability – especially, for any non-continuous payoff scheme, e.g., for success/failure or discrete grading schemes. When a student crosses a payoff threshold, a small increase in effort would not increase her payoff any further.

2. *Despondent: plan* $(\pi_1(\emptyset) = H; \pi_2(SH) = H, \pi_2(FH) = L)$ *is optimal for* $\frac{c}{c+\varepsilon}\left(\frac{1}{1-\varepsilon} - p_\alpha\right) > p_\omega \geq \max\{\frac{2cp_\alpha}{1-\varepsilon-c}, \frac{c}{2-\varepsilon-c}\left(\frac{1}{1-\varepsilon} + p_\alpha\right)\}$.

3. *Motivated: plan* $(\pi_1(\emptyset) = L; \pi_2(SL) = L, \pi_2(FL) = H)$ *is optimal for* $\min\{c\left(\frac{1}{1-\varepsilon} + p_\alpha\right), \frac{2cp_\alpha}{1-\varepsilon-c}\} > p_\omega \geq c\left(\frac{1}{1-\varepsilon} - p_\alpha\right)$.

4. *Fatalist: plan* $(\pi_1(\emptyset) = L; \pi_2(SL) = L, \pi_2(FL) = L)$ *is optimal for* $p_\omega < c\left(\frac{1}{1-\varepsilon} - p_\alpha\right)$.

We find four optimal plans: For a high control belief p_ω, the student exerts high effort in both periods, regardless of the outcome of the first period (persevering). For a medium control belief p_ω and low confidence p_α, the student exerts high effort in $t = 1$. After observing a success, she continues with high effort in $t = 2$, while she switches to low effort after a failure (despondent). For a medium control belief p_ω and high confidence p_α, the student exerts low effort in $t = 1$. After observing a success, she continues with low effort in $t = 2$, while she switches to high effort after a failure (motivated). For a low control belief p_ω, the student exerts low effort in either period, regardless of the outcome of the first period (fatalist). We illustrate the different parameter regions described in the lemma in the left panel of Figure 2. The y-axis measures p_ω, the student's control belief that effort leads to success, while the x-axis measures p_α, her confidence in her ability when ability is key to success.

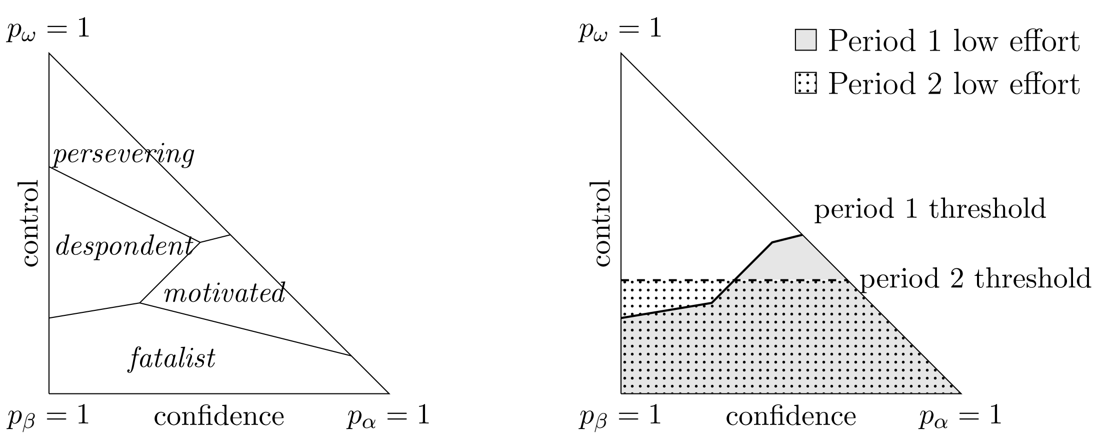

Figure 2: Left: optimal plans depending on initial belief.
Right: belief thresholds for effort in periods one and two. Both for $c = \frac{1}{4}$ and $\varepsilon = \frac{1}{4}$.

The effect of the informational externality that arises for two periods compared to one period is illustrated in the right panel of Figure 2. The gray area indicates beliefs where the student exerts low effort in period 1, while the dotted area indicates low effort in period

2.[12] Thus, in the white dotted area, the student exerts high effort in period one, but not in period 2. While the cost of high effort is higher than the immediate benefit, in this area, the information that can be gained makes high effort worthwhile: a success allows the student to exclude state β; this information is particularly valuable if the confidence p_α is relatively low. Meanwhile, in the gray area without dots, the student chooses low effort in period one and high effort in period 2. In this area, the cost of high effort is lower than the immediate benefit, but experimenting with low effort is worthwhile for the information gain: a success implies that the state is α; this information is particularly valuable if the confidence p_α is relatively high.

The optimal plans for the two period example illustrate some general features of our model. Whenever the student observes a success, she sticks to her choice of effort for the remaining period. After observing SL, she is fully informed that low effort is optimal. After observing SH, the student continues to exert high effort. Assume she preferred to exert low effort after SH – i.e., $p_\omega(SH)(1-\varepsilon) < c$. Then she would also exert low effort after FH, since $p_\omega(FH) < p_\omega(SH)$. However, if she does not condition her effort in period two on the outcome of period one h_1, she does not gain valuable information from high effort in period one. This can only be the case when she purely cares about her immediate payoff. Hence she prefers to exert low effort in period one, since $p_\omega(1-\varepsilon) < p_\omega(SH)(1-\varepsilon) < c$, but this precludes observing SH.

After a failure, the student may optimally change her effort level. A student who starts with high (low) effort may switch to low (high) effort after a failure, since it is now less (more) likely that the state is ω. A student with low control belief p_ω never exerts high effort, even after observing FL, and never learns if the state is actually ω and high effort would be worthwhile.

4 Infinite horizon

When the time horizon is infinite, the student's decision problem becomes stationary, and her effort choice is only determined by her current belief. The student's belief follows the rules outlined in Table 2, from any period to the next. A *history of outcomes* $\boldsymbol{h}$ is a vector with entries $h \in \{SH, FH, SL, FL\}$. We denote by $n_h(\boldsymbol{h}) = n_h$ the *tally of outcome* $h \in \{SH, FH, SL, FL\}$ in a history $\boldsymbol{h}$. Any history of outcomes $\boldsymbol{h}$ can thus be expressed as the vector $(n_{SH}, n_{SL}, n_{FH}, n_{FL})$. Notably, the student's posterior belief only depends on the tally of observed outcomes, but not on their order. For example, the student's posterior belief after observing FL and FH is identical to the student's posterior after observing FH

[12]Note that the beliefs are updated after observing the outcome of period 1. Thus, the beliefs change from period 1 to period 2 as illustrated in Figure 1.

and FL. The student's belief after observing history $\boldsymbol{h}$ is denoted by $\boldsymbol{p}(n_{SH}, n_{SL}, n_{FH}, n_{FL})$.

A *plan* $\boldsymbol{\pi}$ specifies a path of actions $e(\boldsymbol{h}) \in \{H, L\}$, for each possible history of outcomes $\boldsymbol{h}$. A plan $\boldsymbol{\pi}$ is called *optimal given belief* $\boldsymbol{p}$ if the student's expected payoff for $\boldsymbol{\pi}$ is higher than for any alternative plan $\boldsymbol{\pi}', EU[\boldsymbol{\pi}, \boldsymbol{p}] \geq EU[\boldsymbol{\pi}', \boldsymbol{p}]$. A *policy* $e(\boldsymbol{p})$ specifies an action $e \in \{H, L\}$ for any belief $\boldsymbol{p} \in [0, 1]^3$ with $p_\omega + p_\alpha + p_\beta = 1$. Note that an *optimal policy* specifies the same action at belief $\boldsymbol{p}$ which the optimal plan at $\boldsymbol{p}$ prescribes. Without loss of generality, we assume that the student chooses high effort H whenever she is indifferent between exerting high and low effort. As a result, the optimal policy is unique.

4.1 Optimal policy

To find the optimal policy, we have to compare infinitely many possible plans for each belief $\boldsymbol{p}$. The classical approach to this kind of problem involves analyzing the Bellman equation to derive the value function and the optimal policy. As mentioned before, since the payoffs from exerting low and high effort are correlated, in our setup, we need to use a different concept. Note, in particular, that the optimal policy typically does *not* invoke a simple stopping rule: as already seen in the example, the student's optimal effort can switch from high to low as well as from low to high. The value function depends on the exact sequence of high and low effort, and as the student may switch the effort level repeatedly, the value function becomes a much more complex object than in a setting which reduces to a stopping problem.

We, therefore, develop an alternative method to obtain the optimal policy. The key insight is that it is sufficient to compare two *pivotal plans*. These plans pin down the belief threshold that determines the optimal effort choice and, thus, the optimal policy. We apply the principle of optimality and show that there are no profitable one-shot deviations. This method of deriving the optimal policy without recourse to the Bellman equation is, to the best of our knowledge, new.

The two pivotal plans $\boldsymbol{\pi}_H$ and $\boldsymbol{\pi}_L$ prescribe immediate switching of the effort level after experiencing failure; after a success, the effort level is kept unchanged for both plans. After a failure with both high and low effort, both plans prescribe following an arbitrary fixed continuation plan $\overline{\boldsymbol{\pi}}$. For an illustration, see Figure 3.

$\boldsymbol{\pi}_H$: Choose H. After observing SH, play H forever. After observing FH, play L. After observing SL, play L forever. After observing FL, follow the plan $\overline{\boldsymbol{\pi}}$.

$\boldsymbol{\pi}_L$: Choose L. After observing SL, play L forever. After observing FL, play H. After observing SH, play H forever. After observing FH, follow the plan $\overline{\boldsymbol{\pi}}$.

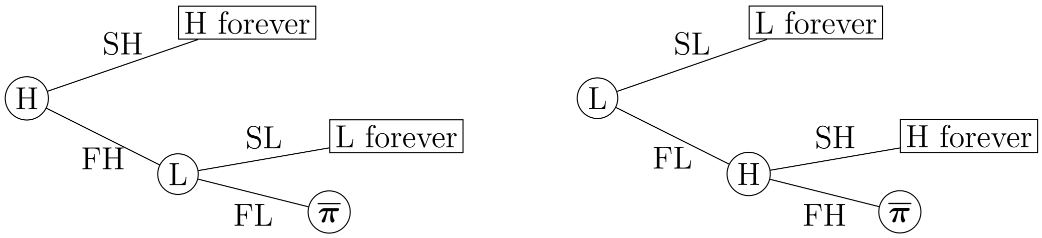

Figure 3: Pivotal plans $\boldsymbol{\pi}_H$ (left) and $\boldsymbol{\pi}_L$ (right).

Lemma 2 *The student is indifferent between following $\boldsymbol{\pi}_H$ and $\boldsymbol{\pi}_L$ if and only if her belief satisfies $p_\omega = \overline{p}_\omega(p_\alpha)$ with*

$$\overline{p}_\omega(p_\alpha) := \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon} + p_\alpha \frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right]. \tag{1}$$

The derivation of $\overline{p}_\omega(p_\alpha)$ follows from equating the expected payoffs of the pivotal plans $\boldsymbol{\pi}_H$ and $\boldsymbol{\pi}_L$ and solving for p_ω. Note that the choice of the continuation plan $\overline{\boldsymbol{\pi}}$ does not affect the comparison of the payoffs from $\boldsymbol{\pi}_H$ and $\boldsymbol{\pi}_L$. In both plans, the student reaches the decision node after observing FH and FL. The probability of this event and the student's posterior beliefs are independent of the order of observations. We show that the student optimally exerts high effort if and only if she prefers $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$, and, thus, if and only if her control belief p_ω is above the threshold $\overline{p}_\omega$:

Proposition 1 *The optimal policy has the form*

$$e^*(\boldsymbol{p}) = \begin{cases} H & \text{for } p_\omega \geq \overline{p}_\omega(p_\alpha) \\ L & \text{otherwise.} \end{cases}$$

In the Appendix, we formally show that the threshold $\overline{p}_\omega(p_\alpha)$ indeed prescribes the optimal action for any possible belief $\boldsymbol{p}$. Hence, $\overline{p}_\omega(p_\alpha)$ characterizes the optimal policy. The student optimally exerts high effort H if her belief is weakly above the threshold, and low effort L otherwise. To prove optimality, we consider all possible one-step deviations. We show that either the deviation is not optimal or the deviation is coherent with the optimal policy, i.e., it induces the same immediate effort level. The proof has two parts:

Part 1 *In any optimal plan, after observing a success SH (SL), the student continues to exert high (low) effort forever.*

First, in both plan $\boldsymbol{\pi}_H$ and $\boldsymbol{\pi}_L$, the student keeps the effort level unchanged after a success. To show that this must be the case for any optimal plan, we first consider a deviation to choosing H after observing SL. After SL the student knows with certainty that the state is α. In this case, there is no more learning and the student can save the effort cost by choosing L. Second, we consider a deviation to choosing L after observing SH. This is always dominated by either $\boldsymbol{\pi}_H$ or $\boldsymbol{\pi}_L$ and, thus, can never be optimal. Intuitively, if the student preferred to exert low effort after observing SH, she would have optimally chosen low effort already in the preceding period, precluding the observation of SH in the first place.[13]

Part 2 *When the optimal plan at belief* $\boldsymbol{p}$ *begins with high (low) effort, the student prefers plan* $\boldsymbol{\pi}_H$ *to* $\boldsymbol{\pi}_L$ *(*$\boldsymbol{\pi}_L$ *to* $\boldsymbol{\pi}_H$*).*

The plans $\boldsymbol{\pi}_H$ and $\boldsymbol{\pi}_L$ are the most responsive plans as they prescribe switching the effort level *immediately* after a failure. Therefore, they are decisive at the belief threshold $\overline{p}_\omega$: $\boldsymbol{\pi}_H$ is the optimal plan just above this threshold and $\boldsymbol{\pi}_L$ just below. We show that any optimal plan prescribes the *same* effort level as the threshold $\overline{p}_\omega$ indicates. For beliefs further away from the threshold, other plans may be optimal – in particular, the student might want to try the same level of effort again despite a failure. However, the immediate effort level of these plans is coherent with that of the threshold $e^*(\boldsymbol{p})$. To prove the second part, we need to address all plans in which – differently from $\boldsymbol{\pi}_H$ or $\boldsymbol{\pi}_L$ – the student does not immediately switch the effort level after a failure but only at a later period. We show that if a plan is optimal, it must induce the same immediate effort choice as indicated by $e^*(\boldsymbol{p})$. First, consider a plan such that the student chooses L directly after FL. If such a plan is optimal, $\boldsymbol{\pi}_L$ is preferred to $\boldsymbol{\pi}_H$ after observing FL – i.e., at belief $\boldsymbol{p}(FL)$. We show that this implies that $\boldsymbol{\pi}_L$ is also preferred to $\boldsymbol{\pi}_H$ at the initial belief $\boldsymbol{p}$. Hence, we have $p_\omega < \bar{p}_\omega$, and this optimal plan is consistent with the optimal policy $e^*(\boldsymbol{p})$ which prescribes L. Second, consider a plan such that the student chooses H directly after FH. If such a plan is optimal, $\boldsymbol{\pi}_H$ is preferred to $\boldsymbol{\pi}_L$ after observing FH – i.e., at belief $\boldsymbol{p}(FH)$. Again, we show that this implies that $\boldsymbol{\pi}_H$ is also preferred to $\boldsymbol{\pi}_L$ at the initial belief $\boldsymbol{p}$. Hence, we have $p_\omega \geq \bar{p}_\omega$, and this optimal plan is consistent with the optimal policy $e^*(\boldsymbol{p})$ which also prescribes H.

Since the analysis holds for any starting belief $\boldsymbol{p}$ and the problem is stationary, the reasoning applies to any decision node and, therefore, covers all possible multi-step deviations. In conclusion, the threshold $\bar{p}_\omega$ gives a necessary and sufficient condition for the optimal action.

[13]For an intuition, assume that the optimal plan prescribes high effort in period t and – independently of the outcome – low effort in period $t+1$. That is, even after observing SH the student exerts low effort to have a chance to observe SL and discover that the state is α. Note that the likelihood of eventually observing SL in this plan is the same as in an alternative plan prescribing low effort in period t and high effort in $t+1$, conditional on observing FL. However, if the student prefers low effort after SH, she strictly prefers the latter plan. This is because in the latter plan, the student has the possibility of saving the cost of high effort all together (if the state is in fact α) or delaying it (if the state is β). For a geometric intuition, note that the slope of updating after observing SH is steeper than that of the threshold $\overline{p}_\omega$.

4.2 Perseverance and procrastination

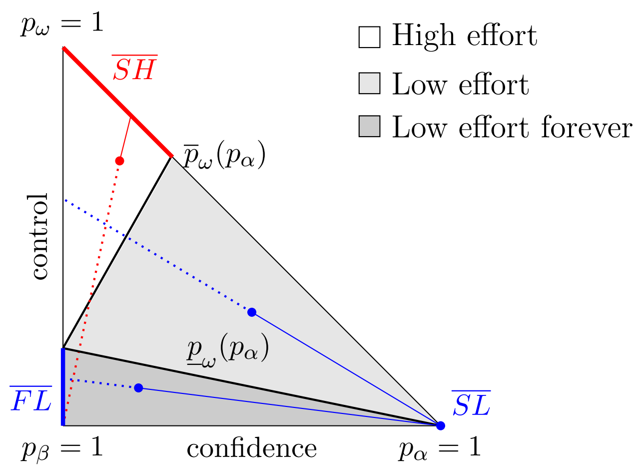

Figure 4: The thresholds $\overline{p}_\omega(p_\alpha)$ and $\underline{p}_\omega(p_\alpha)$ for $\delta = 0.7$, $\varepsilon = 0.1$, and $c = 0.4$.

The belief threshold $\overline{p}_\omega$, as illustrated in Figure 4, is decisive for the student's short-run behavior. The student exerts high effort when her current belief $\boldsymbol{p}$ is above the threshold and low effort when it is below. Her responses to success and failure depend on the level of effort she has exerted. After success with low effort, SL, the student is fully informed that the state is α, and that low effort is optimal. She continues to exert low effort, and her belief remains in $\overline{SL} := \{p_\alpha = 1\}$. After success with high effort, SH, the student knows that the state cannot be β, and her belief moves to $\boldsymbol{p}(SH) \in \overline{SH} := \{\boldsymbol{p}|p_\beta = 0 \text{ and } p_\omega \geq \overline{p}_\omega\}$ on the edge between $p_\omega = 1$ and $p_\alpha = 1$. She continues to exert high effort and, therefore, does not gain new information.

While the student maintains her effort level after a success, she may change it after a failure. After observing a failure with high effort, FH, her belief moves towards $p_\beta = 1$. If her posterior belief $\boldsymbol{p}(FH)$ crosses the threshold $\overline{p}_\omega$, exerting low effort becomes optimal.[14] On the other hand, after observing a failure with low effort, FL, her belief moves towards the edge between $p_\omega = 1$ and $p_\beta = 1$. In particular, her control belief $p_\omega = 1$ increases. As a consequence, her posterior belief $\boldsymbol{p}(FL)$ moves towards threshold $\overline{p}_\omega$. If her posterior belief crosses this threshold, exerting high effort becomes optimal. Updating her beliefs in this fashion, after observing failures with low and high effort, the student may cross the threshold $\overline{p}_\omega$ several times.

For a sufficiently low control belief, the student's belief can never cross the threshold $\overline{p}_\omega$ again, even when n_{FL} goes to infinity. The next lemma characterizes the set of these beliefs.

[14]Geometrically, FH, decreases the beliefs in ω and α on a steeper slope than that of $\overline{p}_\omega$.

Lemma 3 *Optimally, the student exerts low effort forever if her belief satisfies* $p_\omega < \underline{p}_\omega(p_\alpha)$ *with*

$$\underline{p}_\omega(p_\alpha) := \frac{(1-\delta)c}{(1-\varepsilon)(1-\delta\varepsilon-\delta c)}\,(1-p_\alpha)\,. \tag{2}$$

For beliefs below the threshold $\underline{p}_\omega$, even observing failure with low effort repeatedly can not increase the student's control belief sufficiently to cross the threshold $\overline{p}_\omega$. This is illustrated in the dark gray area in Figure 4: for $\boldsymbol{p} < \underline{p}_\omega$, the student's belief after a failure FL necessarily remains below $\overline{p}_\omega$. With an increasing number of observations FL, her posterior belief converges to $p_\alpha = 0$, to the set of absorbing beliefs $\overline{FL} := \{\boldsymbol{p}|p_\alpha = 0 \text{ and } p_\omega < \overline{p}_\omega\}$. Hence, once her belief drops below the threshold $\underline{p}_\omega$, she exerts low effort forever. As a consequence, she never learns whether the state is β or ω. If the state is ω and effort could lead to success, the student exerts an inefficiently low level of effort forever.

When the student observes a failure with high effort, FH, her belief moves towards $p_\beta = 1$ and, thus, closer to the threshold $\underline{p}_\omega$. We can determine how many failures with high effort, n_{FH}, a student is at most willing to observe before her belief drops below $\underline{p}_\omega$, and she never exerts high effort again. The belief after observing the tally n_{FH} of failures with high effort is given by $\boldsymbol{p}(0, 0, n_{FH}, 0)$. Inserting this posterior belief in condition (2), we obtain the following condition for low effort forever:

$$p_\omega(0, 0, n_{FH}, 0) < \underline{p}_\omega\left(p_\alpha(0, 0, n_{FH}, 0)\right). \tag{3}$$

From this inequality, we can derive the following result:

Proposition 2 *The maximal tally of failures with high effort that a student is optimally willing to observe is given by*

$$\overline{n}_{FH}(\boldsymbol{p}) := \begin{cases} \min\{n_{FH} \in \mathbb{N}_0 : \textit{inequality } (3) \textit{ holds}\} \\ \infty, \ \textit{if } \{n_{FH} \in \mathbb{N}_0 : \textit{inequality } (3) \textit{ holds}\} = \emptyset. \end{cases}$$

We define the tally $\overline{n}_{FH}$ as the student's *perseverance*. It specifies how many failures with high effort she is willing to tolerate before giving up for good. When the true state is ω, perseverance measures the probability that the student gives up exerting high effort due to a series of unlucky failures, given by $\varepsilon^{\overline{n}_{FH}}$. For an illustration, see Figure 4: A student with belief $p_\omega \geq \underline{p}_\omega$ (white and light gray areas) is willing to exert high effort immediately (white) or eventually (light gray), and we have $\overline{n}_{FH} > 0$. Note that for beliefs that satisfy $p_\alpha + p_\omega = 1$, we have $\overline{n}_{FH} = \infty$, since such beliefs are unchanged after observing FH. If the initial belief is already below the threshold $\underline{p}_\omega$ (dark gray area), the student exerts low effort forever without additional observations of FH and $\overline{n}_{FH} = 0$. The student's perseverance depends on the student's belief as well as cost, patience, and noise. We discuss the comparative statics regarding $\overline{n}_{FH}$ in Section 4.3.

We have seen that the student's control belief moves towards the threshold $\overline{p}_\omega$ when she observes a failure with low effort FL. We want to derive the tally n_{FL} of failures with low effort such that the student's belief $\boldsymbol{p}$ crosses the threshold and she exerts high effort. From Proposition 1, we know that the student exerts high effort if and only if $p_\omega \geq \overline{p}_\omega$. Inserting the posterior beliefs after observing a tally of n_{FL} failures with low effort in condition (1), we obtain the following condition for high effort:

$$p_\omega(0,0,0,n_{FL}) \geq \overline{p}_\omega\left(p_\alpha(0,0,0,n_{FL})\right). \tag{4}$$

From this inequality, we can derive the following result:

Proposition 3 *The tally of failures with low effort, n_{FL}, that a student needs to observe before exerting high effort is given by*

$$\underline{n}_{FL}(\boldsymbol{p}) := \begin{cases} \min\left\{n_{FL} \in \mathbb{N}_0 : \textit{inequality } (4) \textit{ holds}\right\} \\ \infty, \ \textit{if } \left\{n_{FL} \in \mathbb{N}_0 : \textit{inequality } (4) \textit{ holds}\right\} = \emptyset. \end{cases}$$

We define the tally $\underline{n}_{FL}$ as the student's *procrastination*: the number of failures with low effort after which the student is willing to exert high effort. For an illustration, see Figure 4: A student with belief $p_\omega \geq \overline{p}_\omega$ (white area) exerts high effort immediately, and we have $\underline{n}_{FL} = 0$. A student with belief $p_\omega < \overline{p}_\omega$ (gray areas) exerts low effort. If after a tally of $\underline{n}_{FL}$ failures her updated belief $p_\omega(0,0,0,\underline{n}_{FL})$ becomes sufficiently high, the student is willing to try exerting high effort (light gray area). In this case, we have $\underline{n}_{FL} \in \mathbb{N}$. However, a student with belief $p_\omega < \underline{p}_\omega$ (dark gray area) is never willing to exert high effort for any tally of failures with low effort; we have $\underline{n}_{FL} = \infty$. The higher $\underline{n}_{FL}$, the more failures with low effort are required to motivate the student to exert high effort. The student's procrastination depends on the student's belief as well as cost, patience, and noise. We discuss the comparative statics regarding $\underline{n}_{FL}$ in Section 4.3.

4.3 Welfare and policies

To analyze the optimal effort choices from the perspective of the student, we have assumed that she has an initial belief $\boldsymbol{p} \in (0,1)^3$, where all states have positive probability. The policy $e^*(\boldsymbol{p})$ derived in Proposition 1 achieves maximum welfare for any belief $\boldsymbol{p}$. Thus from the student's point of view her behavior is already optimal and there is no need for an intervention.

For this section, we take the perspective that the true state of the world is ω and that the student can be successful if and only if she exerts high effort. Since the student is uncertain about the state, there is potential to increase her welfare. We are interested in both the short-run and the long-run welfare. procrastination $\underline{n}_{FL}$ is a (negative) measure for short-run welfare. The longer it takes for the student to start exerting high effort, the longer she delays the possibility for success. Thus, policies that aim at *decreasing* the student's procrastination will improve her welfare. Perseverance $\overline{n}_{FH}$ is a measure for long-run welfare. A student with higher perseverance has a lower probability of giving up high effort for good due to a series of unlucky failures. Intuitively, the longer the student tries to succeed with high effort, the more likely she eventually observes a success. Thus, policies that aim at *increasing* the student's perseverance will improve her welfare.

Our model predicts that several parameters affect the student's perseverance and procrastination. The most direct channel is the control belief but also confidence plays an important role. In addition, the cost of effort, patience, and noise will affect the student's welfare.

Proposition 4 *Perseverance $\overline{n}_{FH}$ is decreasing in cost c and increasing in control p_ω, confidence p_α, and patience δ; it is in- or decreasing in noise ε depending on the belief $\boldsymbol{p}$. Procrastination $\underline{n}_{FL}$ is decreasing in control p_ω and increasing in confidence p_α, cost c and noise ε; it is in- or decreasing in patience δ depending on the belief $\boldsymbol{p}$.*

We discuss the implications of control, confidence, cost, patience, and noise on the student's perseverance and procrastination in the following paragraphs.

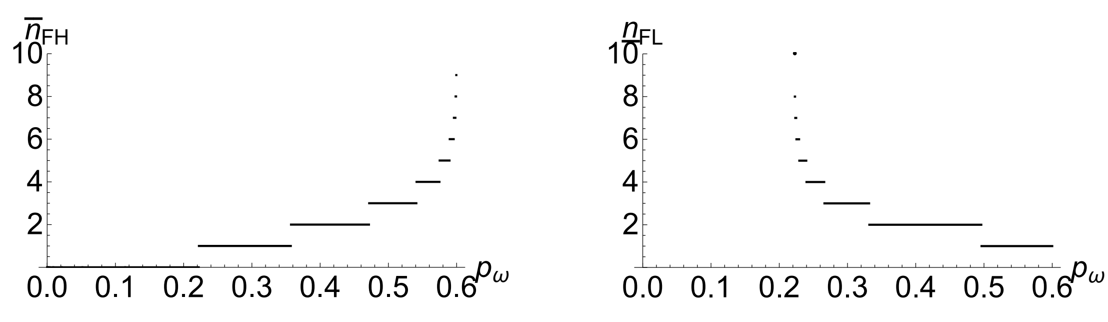

Figure 5: Perseverance $\overline{n}_{FH}$ (left) and procrastination $\underline{n}_{FL}$ (right) as functions of the control belief p_ω, for $p_\alpha = 0.4, \varepsilon = 0.4, \delta = 0.8$, and $c = 0.4$.

Control Clearly, a student with higher control belief is more likely to exert high effort, both in the short and in the long run. She thinks that it is more likely that high effort is necessary to succeed. In the short run, the student's procrastination $\underline{n}_{FL}$ is decreasing in

p_ω, implying that a student with higher control belief starts sooner to exert high effort when failing with low effort. From a long-term perspective, a student with higher control belief shows higher perseverance; $\overline{n}_{FH}$ is increasing in p_ω. The student tolerates a higher number of failures with high effort before she gives up exerting high effort forever. Thus, increasing the student's control belief is welfare enhancing from a short and from a long-run perspective. For an illustration see Figure 5.

Directly increasing the control belief through feedback that emphasizes the importance of effort is the main focus of the interventions proposed in Dweck (2006). Similarly, Bandura (1997) and Duckworth et al. (2007) state that a high control belief is key to foster perseverance. Our analysis shows that this type of feedback has positive short- and long-term consequences, the student not only starts earlier to exert high effort but also stops later in the face of failures. Examples for feedback that stress effort are "Good effort" or "Keep working hard".

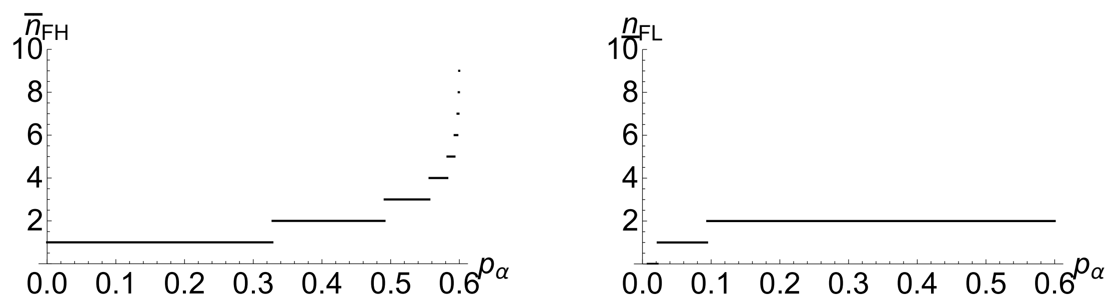

Figure 6: Perseverance $\overline{n}_{FH}$ (left) and procrastination $\underline{n}_{FL}$ (right) as functions of confidence p_α, for $p_\omega = 0.4, \varepsilon = 0.4, \delta = 0.8$, and $c = 0.4$.

Confidence Our model provides novel insights on the role of confidence. Mueller and Dweck (1998) find that praise for ability can decrease the students' motivation. We find that the effect of the student's ability belief on their effort level depends very much on the time frame considered; in the long term, a carefully designed message can, in fact, have a positive effect. For an illustration see Figure 6.

In the short run, the student's procrastination $\underline{n}_{FL}$ increases in confidence p_α. The student continues to exert low effort for longer before trying to succeed with high effort. This is coherent with the findings of Mueller and Dweck (1998) who consider a short time frame with a single instance of feedback. By contrast, in the long run the student's perseverance $\overline{n}_{FH}$ increases in confidence p_α. A student with high confidence is less easily discouraged as she tends to attribute failures to bad luck. This is coherent with the findings of Stinebrickner and Stinebrickner (2012). They elicit the ability beliefs of college students at the start of their course and find that, controlling for exam results, students with higher initial ability beliefs are less likely to drop out. In short, while higher confidence may lead to less effort in the short run, it increases effort in the long run. Thus, interventions increasing the student's confidence could increase perseverance.

Another key insight from our model is that such interventions must be designed with care. Mueller and Dweck (1998) warn that praising ability can carry the message that ability – instead of effort – is decisive for success. This message would decrease the student's control belief, which would both increase procrastination and reduce perseverance. Our model can give guidance on how to phrase messages about ability that are not detrimental to effort. Importantly, such message does not imply that only ability is decisive for success and effort is irrelevant (increasing both p_α and p_β). An example for a potentially harmful message would be: "You can do it – you are so smart!" Instead, a message that the student's ability is not too low to achieve success (decreasing p_β) is more beneficial for increasing perseverance. An example for a helpful message would be: "You may have to work hard, but eventually you will make it."

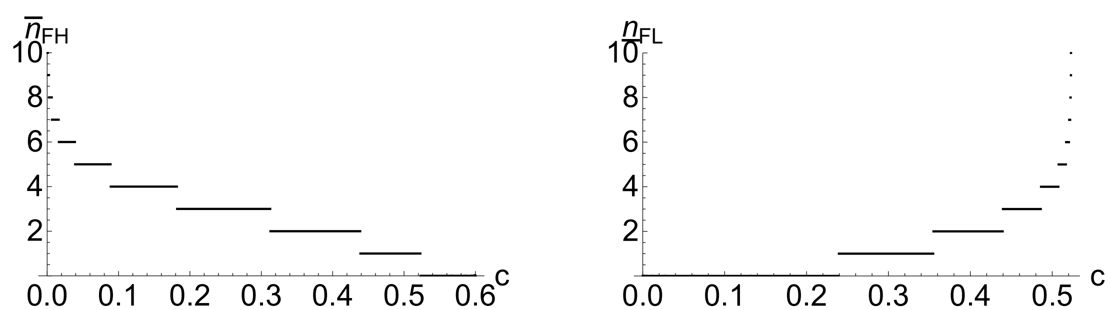

Figure 7: Perseverance $\overline{n}_{FH}$ (left) and procrastination $\underline{n}_{FL}$ (right) as functions of cost c, for $p_\omega = 0.4, p_\alpha = 0.4, \varepsilon = 0.4$, and $\delta = 0.8$.

Cost Unsurprisingly, an increase in the cost of effort c makes the student less willing to exert high effort, both in the short and in the long run. The student's procrastination $\underline{n}_{FL}$ is increasing in c; thus, it takes longer until the student switches to exerting high effort. The student's perseverance $\overline{n}_{FH}$ is decreasing in c; thus, she gives up high effort earlier. Hence, higher cost are detrimental for welfare in a long and in a short-run perspective. For an illustration see Figure 7.

This indicates that it is important to think about policies that decrease a student's pecuniary and non-pecuniary costs of studying. Individualizing learning materials and ensuring a positive learning environment are likely to reduce the student's costs, therefore, reducing the student to teacher ratio can be an important step. Increasing the salary of teachers can potentially increase the quality of learning. Giving students more choice on what to learn reduces their cost and increases their motivation. Both will have a positive effect on perseverance and will decrease procrastination.

In our model, a high cost can equivalently be interpreted as a low payoff from success, i.e., low returns to education. There is evidence that students – especially students from disadvantaged backgrounds – underestimate returns to education. As a consequence, they underinvest in education. Better information about the returns to education or the exposure to relevant role models could encourage students to increase their investment in education; see, e.g., Jensen (2010), Wiswall and Zafar (2015), and Delavande, Bono and Holford (2020). Our analysis shows that policies which increase the students' estimate of returns to education will also increase their perseverance and reduce their procrastination.

Patience In the short term, the impact of patience δ on the student's procrastination $\underline{n}_{FL}$ depends on her confidence p_α. If her confidence is high, the student's procrastination increases in her patience, if her confidence is low, it decreases. In other words, a confident student waits longer before trying to succeed with high effort, while a student with low confidence starts earlier. A patient student has a higher gain from future payoffs and is, therefore, more willing to sacrifice current payoff in order to learn the true state. In the long term, higher patience increases perseverance, $\overline{n}_{FH}$ increases in δ. Intuitively, future payoffs are relatively more valuable and the student is willing to tolerate a higher number of failures with high effort before giving up forever.

Indeed, Golsteyn, Grönqvist and Lindahl (2014) find that a lower discount factor is associated with lower educational attainment. Alan and Ertac (2018) provide evidence that patience can be enhanced through interventions teaching the imagination of future scenarios. Alternatively, one could overcome low patience by providing students with more prompt rewards. Levitt, List, Neckermann and Sadoff (2016) show that immediate monetary or non-monetary incentives can increase effort provision. In our setting, incentivizing high effort temporarily would give students the opportunity to discover that effort can bring about success; this would make the provision of incentives superfluous in the long run.

Noise From the perspective of the student, the connection between her effort and the outcome is noisy because of the possibility of bad luck. In the short run, procrastination $\underline{n}_{FL}$ increases in the probability of bad luck ε. The effect is similar to the effect of a higher cost of effort. An increase in ε decreases the expected return to effort: effort is less likely to lead to success, even if the state is ω. This, clearly, decreases the incentives to exert high effort and the student's procrastination gets larger; it takes longer until the student is willing to try high effort to succeed.

The effect of an increase in noise on perseverance is ambiguous; $\overline{n}_{FH}$ is non-monotonic in ε. Intuitively, on the one hand, the expected return to effort is reduced with more noise; on the other hand, a failure is less informative and, thus, less discouraging.

In a sports context, Foll, Rascle and Higgins (2006) find that, for a given control belief, subjects who attribute failure to unstable causes – such as luck – show greater perseverance than those who attribute failure to permanent causes. This indicates a positive effect of some uncertainty. Indeed, with no uncertainty the student can rule out being a high type α after a failure. However, a large increase in noise substantially increases the likelihood of never exerting high effort despite failures, indicating that some reliability of the outcome is important to encourage students to try hard.

5 Conclusion

In this paper, we aim at a better understanding of perseverance and procrastination in the face of failure. We develop a simple model of belief formation that addresses reasons for differing reactions to failure. In our model, a student who is uncertain about her own ability and the efficacy of effort tries to learn over time whether and how she can succeed. Depending on her control belief, cost of effort, patience, and confidence, she chooses whether to exert high or low effort in each period. We rationalize why some students react to failure by ceasing to exert effort, while others continue or increase their effort level. We characterize the optimal effort policy and show that it is a linear threshold rule. Moreover, we show that the optimal effort choices cannot be described by a simple stopping rule: depending on her beliefs, the student may optimally repeatedly switch between high and low effort, despite the fact that the state does not change over time.

Our characterization result allows us to derive expressions for perseverance and procrastination in the face of failure. Under the assumption that effort is necessary, procrastination is detrimental for the student's welfare in the short run, while perseverance is beneficial in the long run. We show that a student's procrastination is decreasing in control but increasing in confidence, cost, and noise. From a long-run perspective, we show that higher control, higher confidence, and higher patience are conducive to perseverance, whereas higher costs are detrimental. Our model confirms that increasing the control belief is an effective method to promote perseverance in the face of failure. In addition, our analysis reveals that interventions that aim at increasing patience and decrease the cost of exerting effort can also be used to foster perseverance. The effect of promoting confidence crucially depends on the time-frame considered: while higher confidence can decrease the effort level in the short run and lead to more procrastination, it increases perseverance in the long run.

Heckman and Rubinstein (2001) and Duckworth et al. (2007) explain success and failure by focusing on fixed personality traits (non-cognitive skills or grit). However, there has been criticism towards this approach: it can be tempting to blame a fault of character for failure while neglecting the inequalities in support and resources students face (Yeh; 2017). By endogenizing perseverance and procrastination we reconcile the two approaches. High effort can only prevail if the cost and the benefit are commensurate. If the reality is such that a student can not succeed by working hard, she reasonably stops trying eventually. It is important to invest in schools and teachers, so they can give sufficient support to students. Showing students goals they can aspire to and making sure that it is actually within their control to reach them are necessary preconditions for motivating them to work hard.

A Appendix

Proof of Lemma 1.

In the second period, the student exerts high effort if and only if $p_\omega(h_1)(1-\varepsilon)-c \geq 0$, for $h_1 \in \{SH, FH, FL, SL\}$. Using the posterior beliefs derived in Table 2, we get
$p_\omega(SH)(1-\varepsilon)-c \geq 0 \Leftrightarrow p_\omega \geq p_\alpha \frac{c}{1-\varepsilon-c} =: p_\omega^{SH}(p_\alpha)$;
$p_\omega(FH)(1-\varepsilon)-c \geq 0 \Leftrightarrow p_\omega \geq \frac{c}{c+\varepsilon}\left(\frac{1}{1-\varepsilon}-p_\alpha\right) =: p_\omega^{FH}(p_\alpha)$;
$p_\omega(FL)(1-\varepsilon)-c \geq 0 \Leftrightarrow p_\omega \geq c\left(\frac{1}{1-\varepsilon}-p_\alpha\right) =: p_\omega^{FL}(p_\alpha)$.
The student never exerts effort after observing SL, since this implies $p_\alpha = 1$. Note that $p_\omega^{FH}(p_\alpha) \geq p_\omega^{SH}(p_\alpha), p_\omega^{FL}(p_\alpha)$.

In the first period, the student anticipates her effort choice in the second period for each possible outcome of the first period. There are five different cases to consider, depending on the student's cost c, her control belief p_ω, and her confidence p_α.

1. For a prior $\boldsymbol{p}$ such that $p_\omega \geq p_\omega^{FH}(p_\alpha)$, the student chooses high effort in period 2, after any outcome, except for SL. In period 1, the student exerts high effort if

$$
\begin{aligned}
& EU\left((\pi_1(\emptyset)=H; \pi_2(SH)=H, \pi_2(FH)=H)\right) \geq EU\left((\pi_1(\emptyset)=L; \pi_2(SL)=L, \pi_2(FL)=H)\right) \\
\Leftrightarrow \quad & 2\left[p_\omega(1-\varepsilon)-c\right] \geq \left[1-p_\alpha(1-\varepsilon)\right]\left[\frac{p_\omega}{1-p_\alpha(1-\varepsilon)}(1-\varepsilon)-c\right] \\
\Leftrightarrow \quad & p_\omega \geq c\left(\frac{1}{1-\varepsilon}+p_\alpha\right) =: p_\omega^1(p_\alpha).
\end{aligned}
$$

Thus, plan $(\pi_1(\emptyset)=H; \pi_2(SH)=H, \pi_2(FH)=H)$ is optimal if $p_\omega \geq \max\{p_\omega^{FH}(p_\alpha), p_\omega^1(p_\alpha)\}$. Plan $(\pi_1(\emptyset)=L; \pi_2(SL)=L, \pi_2(FL)=H)$ is optimal if $p_\omega^1(p_\alpha) > p_\omega \geq p_\omega^{FH}(p_\alpha)$.

2. For a prior $\boldsymbol{p}$ such that $p_\omega^{FH}(p_\alpha) > p_\omega \geq \max\{p_\omega^{SH}(p_\alpha), p_\omega^{FL}(p_\alpha)\}$, the student chooses high effort in period 2 after both FL and SH, but not after SL or FH. In period 1, the student exerts high effort if

$$
\begin{aligned}
& EU((\pi_1(\emptyset)=H; \pi_2(SH)=H, \pi_2(FH)=L)) \geq EU((\pi_1(\emptyset)=L; \pi_2(SL)=L, \pi_2(FL)=H)) \\
\Leftrightarrow \quad & p_\omega(1-\varepsilon)-c+(p_\alpha+p_\omega)(1-\varepsilon)\left[\frac{p_\omega}{p_\alpha+p_\omega}(1-\varepsilon)-c\right] \\
& \geq \left[1-p_\alpha(1-\varepsilon)\right]\left[\frac{p_\omega}{1-p_\alpha(1-\varepsilon)}(1-\varepsilon)-c\right] \\
\Leftrightarrow \quad & p_\omega \geq \frac{2cp_\alpha}{1-\varepsilon-c} =: p_\omega^2(p_\alpha).
\end{aligned}
$$

Since $p_\omega^2(p_\alpha) > p_\omega^{SH}(p_\alpha)$, plan $(\pi_1(\emptyset)=H; \pi_2(SH)=H, \pi_2(FH)=L)$ is optimal if $p_\omega^{FH}(p_\alpha) > p_\omega \geq \max\{p_\omega^2, p_\omega^{FL}(p_\alpha)\}$. Plan $(\pi_1(\emptyset)=L; \pi_2(SL)=L, \pi_2(FL)=H)$ is optimal if $\min\{p_\omega^{FH}(p_\alpha), p_\omega^2(p_\alpha)\} > p_\omega \geq \max\{p_\omega^{SH}(p_\alpha), p_\omega^{FL}(p_\alpha)\}$.

3. For a prior $\boldsymbol{p}$ such that $p_\omega^{FL}(p_\alpha) > p_\omega \geq p_\omega^{SH}(p_\alpha)$, the student chooses high effort in period 2 after SH, but not after SL, FH, or FL. In period 1, the student exerts high effort if

$$\begin{aligned}
& EU((\pi_1(\emptyset) = H; \pi_2(SH) = H, \pi_2(FH) = L)) \geq EU((\pi_1(\emptyset) = L; \pi_2(SL) = L, \pi_2(FL) = L)) \\
\Leftrightarrow \quad & p_\omega(1-\varepsilon) - c + (p_\alpha + p_\omega)(1-\varepsilon)\left[\frac{p_\omega}{p_\alpha + p_\omega}(1-\varepsilon) - c\right] \geq 0 \\
\Leftrightarrow \quad & p_\omega \geq \frac{c}{2-\varepsilon-c}\left(\frac{1}{1-\varepsilon} + p_\alpha\right) =: p_\omega^3(p_\alpha).
\end{aligned}$$

Thus, plan $(\pi_1(\emptyset) = H; \pi_2(SH) = H, \pi_2(FH) = L)$ is optimal if $p_\omega^{FL}(p_\alpha) > p_\omega \geq p_\omega^3(p_\alpha)$. Plan $(\pi_1(\emptyset) = L; \pi_2(SL) = L, \pi_2(FL) = L)$ is optimal if $\min\{p_\omega^{FL}(p_\alpha), p_\omega^3(p_\alpha)\} > p_\omega \geq p_\omega^{SH}(p_\alpha)$

4. For a prior p such that $p_\omega^{SH}(p_\alpha) > p_\omega \geq p_\omega^{FL}(p_\alpha)$, the student chooses high effort in period 2 after FL, but not after SL, FH, or SH. In period 1, the student exerts high effort if

$$\begin{aligned}
& EU((\pi_1(\emptyset) = H; \pi_2(SH) = L, \pi_2(FH) = L)) \geq EU((\pi_1(\emptyset) = L; \pi_2(SL) = L, \pi_2(FL) = H)) \\
\Leftrightarrow \quad & p_\omega(1-\varepsilon) - c \geq [1 - p_\alpha(1-\varepsilon)]\left[\frac{p_\omega}{1-p_\alpha(1-\varepsilon)}(1-\varepsilon) - c\right] \\
\Leftrightarrow \quad & 0 \geq p_\alpha(1-\varepsilon)c.
\end{aligned}$$

Since this can never hold in this interval, plan $(\pi_1(\emptyset) = L; \pi_2(SL) = L, \pi_2(FL) = H)$ is optimal if $p_\omega^{SH}(p_\alpha) > p_\omega \geq p_\omega^{FL}(p_\alpha)$.

5. For a prior p such that $\min\{p_\omega^{FL}(p_\alpha), p_\omega^{SH}(p_\alpha)\} > p_\omega$, the student always chooses low effort in period 2. In period 1, the student exerts high effort if

$$\begin{aligned}
& EU((\pi_1(\emptyset) = H; \pi_2(SH) = L, \pi_2(FH) = L)) \geq EU((\pi_1(\emptyset) = L; \pi_2(SL) = L, \pi_2(FL) = L)) \\
\Leftrightarrow \quad & p_\omega(1-\varepsilon) - c \geq 0 \\
\Leftrightarrow \quad & p_\omega \geq \frac{c}{1-\varepsilon}.
\end{aligned}$$

Since $\frac{c}{1-\varepsilon} > p_\omega^{FL}(p_\alpha), p_\omega^{SH}(p_\alpha)$ plan $(\pi_1(\emptyset) = L; \pi_2(SL) = L, \pi_2(FL) = L)$ is optimal when $\min\{p_\omega^{FL}(p_\alpha), p_\omega^{SH}(p_\alpha)\} > p_\omega$.

□

Proof of Lemma 2.

To derive the threshold $\overline{p}_\omega$, note first that the student's beliefs do not depend on the order of observed outcomes. Therefore, we can define $\boldsymbol{p}(FL, FH)$ as the belief resulting from $\boldsymbol{p}$ after observing FH and FL in arbitrary order. Let $\overline{\boldsymbol{\pi}}$ be some arbitrary fixed plan. We denote by $EU(\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FH))$ the expected payoff that results from following an arbitrary plan $\overline{\boldsymbol{\pi}}$ at belief $\boldsymbol{p}(FL, FH)$. Comparing the expected payoffs from plans $\boldsymbol{\pi}_H$ and $\boldsymbol{\pi}_L$ – defined in the main text – at belief $\boldsymbol{p}$ yields

$$
\begin{aligned}
& EU\left(\boldsymbol{\pi}_H, \boldsymbol{p}\right) \geq EU\left(\boldsymbol{\pi}_L, \boldsymbol{p}\right) \\
\Leftrightarrow\quad & p_\omega(1-\varepsilon)\left[1+\frac{\delta(1-\varepsilon)}{1-\delta}\right]+p_\alpha\frac{1-\varepsilon}{1-\delta} \\
& -c\left[1+(p_\omega+p_\alpha)\frac{\delta(1-\varepsilon)}{1-\delta}\right]+\delta^2\left(p_\omega\varepsilon+p_\alpha\varepsilon^2+p_\beta\right)EU\left[\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FH)\right] \\
& \geq \delta p_\omega(1-\varepsilon)\left[1+\frac{\delta(1-\varepsilon)}{1-\delta}\right]+p_\alpha\frac{1-\varepsilon}{1-\delta} \\
& -\delta c\left[(p_\omega+p_\alpha\varepsilon+p_\beta)+(p_\omega+p_\alpha\varepsilon)\frac{\delta(1-\varepsilon)}{1-\delta}\right]+\delta^2\left(p_\omega\varepsilon+p_\alpha\varepsilon^2+p_\beta\right)EU\left[\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FH)\right] \\
\Leftrightarrow\quad & p_\omega(1-\varepsilon)(1-\delta)\left[1+\frac{\delta(1-\varepsilon)}{1-\delta}\right] \\
& \geq c\left\{(1-\delta)\left[1+(p_\omega+p_\alpha)\frac{\delta(1-\varepsilon)}{1-\delta}\right]+\delta p_\alpha(1-\varepsilon)\left[1+\frac{\delta(1-\varepsilon)}{1-\delta}\right]\right\} \\
\Leftrightarrow\quad & p_\omega \geq \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon}+p_\alpha\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right]. \qquad (5)
\end{aligned}
$$

□

Proof of Proposition 1.

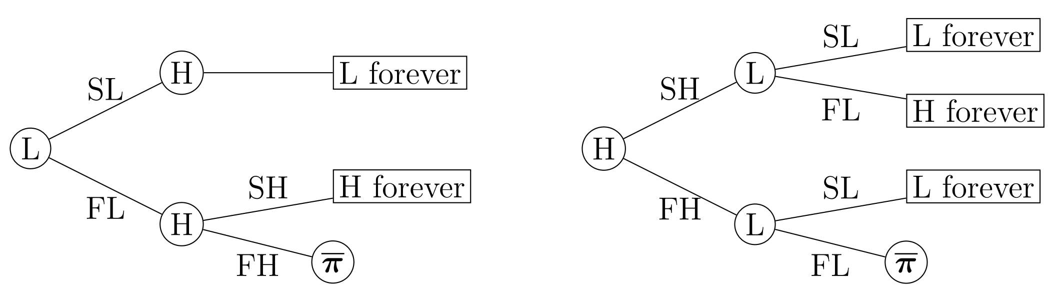

Figure 8: Plans $\boldsymbol{\pi}_H^{SL}$ (left) and $\boldsymbol{\pi}_L^{SH}$ (right).

Part 1: In any optimal plan, after a success, the student continues to exert the same effort level forever.

1) After a success with low effort (SL), it is always optimal to exert low effort forever. Consider a plan $\boldsymbol{\pi}_H^{SL}$ that prescribes H at any point after SL. See Figure 8 (left), for an illustration. Note that after observing SL once, the student knows with certainty that the state is α and thus effort is futile. At any subsequent history, the immediate payoff from L is $1-\varepsilon$ while the payoff from H is $1-\varepsilon-c$. Moreover, no more information can be gained for the future. Therefore, an optimal plan can never prescribe high effort after observing SL.

2) After a success with high effort (SH), it is always optimal to exert high effort forever. Assume for contradiction that a plan $\boldsymbol{\pi}_L^{SH}$ which prescribes L after SH is optimal. For an illustration, see Figure 8 (right). If plan $\boldsymbol{\pi}_L^{SH}$ is optimal, it must give a higher payoff than $\boldsymbol{\pi}_H$:

$$
\begin{aligned}
& EU\left(\boldsymbol{\pi}_L^{SH}, \boldsymbol{p}\right) > EU\left(\boldsymbol{\pi}_H, \boldsymbol{p}\right) \\
\Leftrightarrow\quad & p_\omega(1-\varepsilon)\left[1+\frac{\delta^2(1-\varepsilon)}{1-\delta}\right] + p_\alpha\frac{1-\varepsilon}{1-\delta} \\
& - c\left[1+(p_\omega+p_\alpha\varepsilon)\frac{\delta^2(1-\varepsilon)}{1-\delta}\right] + \delta^2\left(p_\omega\varepsilon + p_\alpha\varepsilon^2 + p_\beta\right) EU\left[\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FH)\right] \\
& > p_\omega(1-\varepsilon)\left[1+\frac{\delta(1-\varepsilon)}{1-\delta}\right] + p_\alpha\frac{1-\varepsilon}{1-\delta} \\
& - c\left[1+(p_\omega+p_\alpha)\frac{\delta(1-\varepsilon)}{1-\delta}\right] + \delta^2\left(p_\omega\varepsilon + p_\alpha\varepsilon^2 + p_\beta\right) EU\left[\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FH)\right] \\
\Leftrightarrow\quad & \frac{p_\omega}{p_\omega+p_\alpha} < c\left[\frac{1}{1-\varepsilon} + \frac{\delta}{1-\delta}\frac{p_\alpha}{p_\omega+p_\alpha}\right] \\
\Leftrightarrow\quad & p_\omega(SH) < c\left[\frac{1}{1-\varepsilon} + \frac{\delta}{1-\delta}p_\alpha(SH)\right] \\
\Leftrightarrow\quad & p_\omega < c\left[\frac{p_\omega+p_\alpha}{1-\varepsilon} + \frac{\delta}{1-\delta}p_\alpha\right].
\end{aligned}
\tag{6}
$$

If plan $\boldsymbol{\pi}_L^{SH}$ is optimal, it must also give a higher payoff than $\boldsymbol{\pi}_L$. Assume instead that $\boldsymbol{\pi}_L$ is preferred:

$$
\begin{aligned}
& EU\left(\boldsymbol{\pi}_L, \boldsymbol{p}\right) > EU\left(\boldsymbol{\pi}_L^{SH}, \boldsymbol{p}\right) \\
\Leftrightarrow \quad & \delta p_\omega(1-\varepsilon)\left[1+\frac{\delta(1-\varepsilon)}{1-\delta}\right] + p_\alpha \frac{1-\varepsilon}{1-\delta} \\
& - \delta c\left[\left(p_\omega + p_\alpha\varepsilon + p_\beta\right) + \left(p_\omega + p_\alpha\varepsilon\right)\frac{\delta(1-\varepsilon)}{1-\delta}\right] \\
& + \delta^2\left(p_\omega\varepsilon + p_\alpha\varepsilon^2 + p_\beta\right) EU\left[\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FH)\right] \\
& > p_\omega(1-\varepsilon)\left[1+\frac{\delta^2(1-\varepsilon)}{1-\delta}\right] + p_\alpha\frac{1-\varepsilon}{1-\delta} \\
& - c\left[1 + \left(p_\omega + p_\alpha\varepsilon\right)\frac{\delta^2(1-\varepsilon)}{1-\delta}\right] \\
& + \delta^2\left(p_\omega\varepsilon + p_\alpha\varepsilon^2 + p_\beta\right) EU\left[\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FH)\right] \\
\Leftrightarrow \quad & p_\omega < c\left[\frac{1}{1-\varepsilon} + \frac{\delta}{1-\delta}p_\alpha\right]. \qquad (7)
\end{aligned}
$$

We see that it is equivalent to say that the student prefers plan $\boldsymbol{\pi}_H^{SH}$ to $\boldsymbol{\pi}_H$ at belief $\boldsymbol{p}$ and that the student prefers plan $\boldsymbol{\pi}_L$ to $\boldsymbol{\pi}_H^{SH}$ at belief $\boldsymbol{p}(SH)$. However, inequality (7) is easier to fulfill than inequality (6). Therefore, whenever $\boldsymbol{\pi}_L^{SH}$ is preferred to $\boldsymbol{\pi}_H$, it is inferior to $\boldsymbol{\pi}_L$ and whenever $\boldsymbol{\pi}_L^{SH}$ is preferred to $\boldsymbol{\pi}_L$, it is inferior to $\boldsymbol{\pi}_H$. Thus $\boldsymbol{\pi}_L^{SH}$ is either dominated by $\boldsymbol{\pi}_L$, by $\boldsymbol{\pi}_H$, or by both.

Next, consider a plan $\boldsymbol{\pi}_L^{SHi}$ which prescribes starting with H and playing L repeatedly after observing SH. Specifically, it prescribes playing L after observing SH and continuing with L after observing FL i times, with $i \geq 1$. If there is no success, it prescribes playing H forever, after observing FL for the $(i+1)$-st time. If plan $\boldsymbol{\pi}_L^{SHi}$ is optimal, then it must yield a higher payoff than any other plan. We can construct an alternative plan $\boldsymbol{\pi}_H^{SHi}$, which is identical to $\boldsymbol{\pi}_L^{SHi}$ except that it prescribes L after observing FL $(i-1)$ times and H forever after observing FL for the i-th time. We then compare these plans:

$$
\begin{aligned}
& EU\left(\boldsymbol{\pi}_L^{SHi}, \boldsymbol{p}\right) > EU\left(\boldsymbol{\pi}_H^{SHi}, \boldsymbol{p}\right) \\
\Leftrightarrow\quad & p_\omega(1-\varepsilon)\left[1+\frac{\delta^{i+2}(1-\varepsilon)}{1-\delta}\right] - c\left[1+(p_\omega + p_\alpha\varepsilon^{i+1})\frac{\delta^{i+2}(1-\varepsilon)}{1-\delta}\right] \\
& > p_\omega(1-\varepsilon)\left[1+\frac{\delta^{i+1}(1-\varepsilon)}{1-\delta}\right] - c\left[1+(p_\omega + p_\alpha\varepsilon^{i})\frac{\delta^{i+1}(1-\varepsilon)}{1-\delta}\right] \\
\Leftrightarrow\quad & \frac{p_\omega}{p_\omega + p_\alpha\varepsilon^i} < c\left[\frac{1}{1-\varepsilon} + \frac{\delta}{1-\delta}\frac{p_\alpha\varepsilon^i}{p_\omega + p_\alpha\varepsilon^i}\right] \\
\Leftrightarrow\quad & p_\omega(SH, FL^i) < c\left[\frac{1}{1-\varepsilon} + \frac{\delta}{1-\delta} p_\alpha(SH, FL^i)\right] \\
\Leftrightarrow\quad & p_\omega < c\left[\frac{p_\omega + p_\alpha\varepsilon^i}{1-\varepsilon} + \frac{\delta}{1-\delta} p_\alpha\varepsilon^i\right]. \qquad (8)
\end{aligned}
$$

We see that inequality (8) is even harder to fulfill than inequality (6). Observing an additional FL makes low effort less attractive.

Next, consider a plan $\hat{\boldsymbol{\pi}}_L^{SHi}$ which is identical to $\boldsymbol{\pi}_L^{SHi}$, except that the first two periods are switched: $\hat{\boldsymbol{\pi}}_L^{SHi}$ starts with L and, after observing FL, H is played in period 2. Then, it prescribes playing L after observing SH and continuing with L after observing FL for $(i-1)$ times. If there is no success, it prescribes playing H forever, after observing FL for the $(i+1)$-st time overall. If plan $\boldsymbol{\pi}_L^{SHi}$ is optimal, it must also give a higher payoff than $\hat{\boldsymbol{\pi}}_L^{SHi}$. Assume instead that $\hat{\boldsymbol{\pi}}_L^{SHi}$ is preferred:

$$
\begin{aligned}
& EU\left(\hat{\boldsymbol{\pi}}_L^{SHi}, \boldsymbol{p}\right) > EU\left(\boldsymbol{\pi}_L^{SHi}, \boldsymbol{p}\right) \\
\Leftrightarrow\quad & \delta p_\omega(1-\varepsilon)\left[1+\frac{\delta^{i+1}(1-\varepsilon)}{1-\delta}\right] - \delta c\left[(p_\omega + p_\beta + p_\alpha\varepsilon) + (p_\omega + p_\alpha\varepsilon^{i+1})\frac{\delta^{i+1}(1-\varepsilon)}{1-\delta}\right] \\
& > p_\omega(1-\varepsilon)\left[1+\frac{\delta^{i+2}(1-\varepsilon)}{1-\delta}\right] - c\left[1+(p_\omega + p_\alpha\varepsilon^{i+1})\frac{\delta^{i+2}(1-\varepsilon)}{1-\delta}\right] \\
\Leftrightarrow\quad & p_\omega < c\left[\frac{1}{1-\varepsilon} + \frac{\delta}{1-\delta} p_\alpha\right]. \qquad (9)
\end{aligned}
$$

We see that it is equivalent to say that the student prefers plan $\boldsymbol{\pi}_L^{SHi}$ to $\boldsymbol{\pi}_H^{SHi}$ at belief $\boldsymbol{p}$ and that the student prefers plan $\hat{\boldsymbol{\pi}}_L^{SHi}$ to $\boldsymbol{\pi}_L^{SHi}$ at belief $\boldsymbol{p}(SH, FL^i)$. Inequality (9) is easier to fulfill than inequality (8). Therefore, whenever $\boldsymbol{\pi}_L^{SHi}$ is preferred to $\boldsymbol{\pi}_H^{SHi}$, it is inferior to $\hat{\boldsymbol{\pi}}_L^{SHi}$ and whenever $\boldsymbol{\pi}_L^{SHi}$ is preferred to $\hat{\boldsymbol{\pi}}_L^{SHi}$, it is inferior to $\boldsymbol{\pi}_H^{SHi}$. Thus $\boldsymbol{\pi}_L^{SHi}$ is either dominated by $\hat{\boldsymbol{\pi}}_L^{SHi}$, by $\boldsymbol{\pi}_H^{SHi}$, or by both.

The argument extends to plans where L is played – one or several times – not directly after the first observation SH, but j periods later. The reason is that the student's belief after observing SH adjust to $p_\beta = 0$ and does not change as long as the student chooses H.

Part 2: When the optimal plan at belief $\boldsymbol{p}$ begins with high (low) effort, the agent prefers plan $\boldsymbol{\pi_H}$ to $\boldsymbol{\pi_L}$ ($\boldsymbol{\pi_L}$ to $\boldsymbol{\pi_H}$).

While any optimal plan must prescribe continuing with the same effort choice after a success, after a failure different effort choices may be optimal for different beliefs. Thus, $\boldsymbol{\pi}_H$ or $\boldsymbol{\pi}_L$ will not always be optimal. We now show that if a plan different from $\boldsymbol{\pi}_H$ or $\boldsymbol{\pi}_L$ is optimal at a given belief, then it induces exactly the same "next" effort choice as the preferred plan of $\boldsymbol{\pi}_H$ and $\boldsymbol{\pi}_L$.

For this, it is useful to consider the effect of observing FL and FH on inequality (5) which signifies that plan $\boldsymbol{\pi}_H$ is weakly preferred to $\boldsymbol{\pi}_L$. After observing FL, resulting in posterior belief $\boldsymbol{p}(FL)$, $\boldsymbol{\pi}_H$ is weakly preferred to $\boldsymbol{\pi}_L$ if and only if

$$\begin{aligned}
& EU\left[\boldsymbol{\pi}_H, \boldsymbol{p}(FL)\right] \geq EU\left[\boldsymbol{\pi}_L, \boldsymbol{p}(FL)\right] \\
\Leftrightarrow \quad & \frac{p_\omega}{p_\omega + p_\beta + p_\alpha \varepsilon} \geq \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon} + \frac{p_\alpha \varepsilon}{p_\omega + p_\beta + p_\alpha\varepsilon}\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right] \\
\Leftrightarrow \quad & p_\omega \geq \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon}\left[1-p_\alpha(1-\varepsilon)\right] + p_\alpha\varepsilon\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right] \\
& < \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon} + p_\alpha\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right] = \overline{p}_\omega(p_\alpha).
\end{aligned}$$

The minimum value of belief p_ω such that the student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ after observing FL is *below* the threshold $\overline{p}_\omega$ – for which the student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ with the prior belief $\boldsymbol{p}$. This means that observing FL makes it easier to fulfill inequality (5). Therefore, whenever a student prefers plan $\boldsymbol{\pi}_L$ to $\boldsymbol{\pi}_H$ at the posterior belief $\boldsymbol{p}(FL)$, she must also prefer it at the prior belief $\boldsymbol{p}$.

After observing FH, resulting in posterior belief $\boldsymbol{p}(FH)$, $\boldsymbol{\pi}_H$ is weakly preferred to $\boldsymbol{\pi}_L$ if and only if

$$\begin{aligned}
& EU\left[\boldsymbol{\pi}_H, \boldsymbol{p}(FH)\right] \geq EU\left[\boldsymbol{\pi}_L, \boldsymbol{p}(FH)\right] \\
\Leftrightarrow \quad & \frac{p_\omega\varepsilon}{p_\omega\varepsilon + p_\alpha\varepsilon + p_\beta} \geq \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon} + \frac{p_\alpha\varepsilon}{p_\omega\varepsilon + p_\alpha\varepsilon + p_\beta}\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right] \\
\Leftrightarrow \quad & p_\omega \geq \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{(1-\delta)}{1-\varepsilon}\left[1 + \frac{1-\varepsilon}{\varepsilon}(1-p_\omega-p_\alpha)\right] + p_\alpha\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right] \\
& > \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon} + p_\alpha\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right] = \overline{p}_\omega(p_\alpha).
\end{aligned}$$

The minimum value of belief p_ω such that the student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ after observing

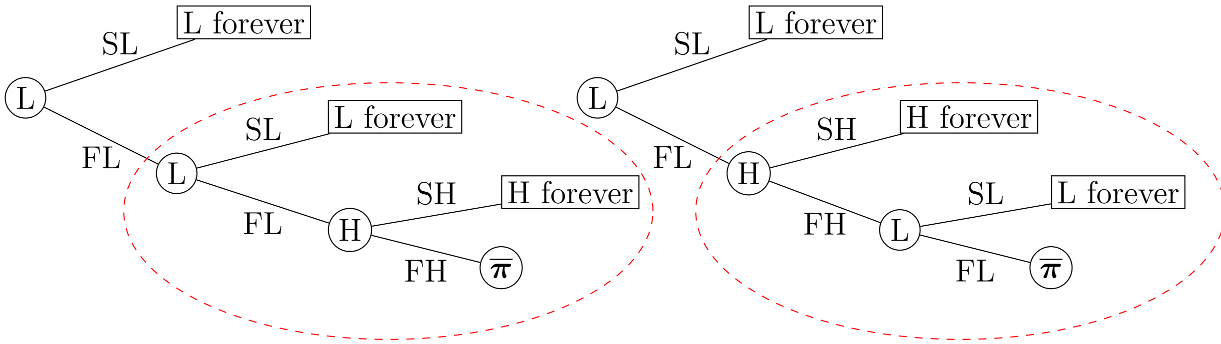

Figure 9: Plans $\boldsymbol{\pi}_L^{FL}$ (left) and $\boldsymbol{\pi}_H^{FL}$ (right).

FH is *above* the threshold $\overline{p}_\omega$ – for which the student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ with the prior belief $\boldsymbol{p}$. This means that observing FH makes it harder to fulfill inequality (5). Therefore, whenever a student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ at the posterior belief $\boldsymbol{p}(FH)$, she must also prefer it at the prior belief $\boldsymbol{p}$.

1) If the optimal plan starts with $\boldsymbol{L}$ and prescribes to continue with $\boldsymbol{L}$ after failure ($\boldsymbol{FL}$), then the optimal policy $\boldsymbol{e^*(p)}$ also prescribes $\boldsymbol{L}$. We show that for such a plan to be optimal, we must have that the student prefers plan $\boldsymbol{\pi}_L$ to $\boldsymbol{\pi}_H$ at the posterior belief $\boldsymbol{p}(FL)$. As we have seen, she must then also prefer $\boldsymbol{\pi}_L$ at the prior belief $\boldsymbol{p}$.

Consider a plan $\boldsymbol{\pi}_L^{FL}$ which starts with L. After SL it prescribes L forever. After FL, it prescribes following plan $\boldsymbol{\pi}_L$ – i.e., play L once more and continue with L in case of success, but switch to H in case of failure. If the plan $\boldsymbol{\pi}_L^{FL}$ is optimal, then it must yield a higher expected payoff than any other plan. We can construct an alternative plan $\boldsymbol{\pi}_H^{FL}$, which is identical to $\boldsymbol{\pi}_L^{FL}$, except that it prescribes following plan $\boldsymbol{\pi}_H$ after FL. For an illustration, see Figure 9.

We show that it is equivalent to say that the student prefers plan $\boldsymbol{\pi}_L^{FL}$ to $\boldsymbol{\pi}_H^{FL}$ at belief

$\boldsymbol{p}$ and that the student prefers plan $\boldsymbol{\pi}_L$ to $\boldsymbol{\pi}_H$ at belief $\boldsymbol{p}(FL)$:

$$
\begin{aligned}
& EU\left(\boldsymbol{\pi}_L^{FL}, \boldsymbol{p}\right) > EU\left(\boldsymbol{\pi}_H^{FL}, \boldsymbol{p}\right) \\
\Leftrightarrow\quad & \delta^2 p_\omega (1-\varepsilon)\left[1+\frac{\delta(1-\varepsilon)}{1-\delta}\right] + p_\alpha \frac{1-\varepsilon}{1-\delta} \\
& - \delta^2 c\left[\left(p_\omega + p_\beta + p_\alpha \varepsilon^2\right) + \left(p_\omega + p_\alpha \varepsilon^2\right)\frac{\delta(1-\varepsilon)}{1-\delta}\right] \\
& + \left(p_\omega \varepsilon + p_\beta + p_\alpha \varepsilon^2\right) EU\left(\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FL, FH)\right) \\
& > \delta p_\omega (1-\varepsilon)\left[1+\frac{\delta(1-\varepsilon)}{1-\delta}\right] + p_\alpha \frac{1-\varepsilon}{1-\delta} \\
& - \delta c\left[\left(p_\omega + p_\beta + p_\alpha \varepsilon\right) + \left(p_\omega + p_\alpha \varepsilon\right)\frac{\delta(1-\varepsilon)}{1-\delta}\right] \\
& + \left(p_\omega \varepsilon + p_\beta + p_\alpha \varepsilon^2\right) EU\left(\overline{\boldsymbol{\pi}}, p(FL, FL, FH)\right) \\
\Leftrightarrow\quad & \frac{p_\omega}{p_\omega + p_\beta + p_\alpha \varepsilon} < \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon} + \frac{p_\alpha \varepsilon}{p_\omega + p_\beta + p_\alpha \varepsilon}\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right] \\
\Leftrightarrow\quad & EU\left[\boldsymbol{\pi}_H, p(FL)\right] < EU\left[\boldsymbol{\pi}_L, p(FL)\right].
\end{aligned}
$$

The comparison of $\boldsymbol{\pi}_L^{FL}$ and $\boldsymbol{\pi}_H^{FL}$ at belief $\boldsymbol{p}$ reduces to the comparison of $\boldsymbol{\pi}_L$ and $\boldsymbol{\pi}_H$ at the posterior belief $\boldsymbol{p}(FL)$. As the plans $\boldsymbol{\pi}_L^{FL}$ and $\boldsymbol{\pi}_H^{FL}$ are identical after observing SL, the respective payoffs cancel out. We have already shown that, whenever a student prefers plan $\boldsymbol{\pi}_L$ to $\boldsymbol{\pi}_H$ at the posterior belief $\boldsymbol{p}(FL)$, she must also prefer it at the prior belief $\boldsymbol{p}$. Therefore, if plan $\boldsymbol{\pi}_L^{FL}$ is optimal then it prescribes the same immediate action as the policy $e^*(\boldsymbol{p})$: L.

Since the analysis holds for any possible starting belief $\boldsymbol{p}$ and the problem is stationary, the reasoning applies to any decision node. Thus, we can extend this analysis to plans prescribing playing L several times after observing FL: Consider a plan $\boldsymbol{\pi}_L^{FLi}$ which prescribes playing L i times after observing FL, with $i \geq 1$. Specifically, it prescribes to play L after observing FL $(i-1)$ times and, after observing FL for the i-th time, to follow plan $\boldsymbol{\pi}_L$. We compare this plan to an alternative plan $\boldsymbol{\pi}_H^{FLi}$ which is identical except that it prescribes plan $\boldsymbol{\pi}_H$ after observing FL for the i-th time. The comparison of the expected payoffs is identical to the one above, but with new starting belief $\boldsymbol{p}(FL^i)$, which results from observing FL i times. Recall that, whenever a student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ at the posterior belief $\boldsymbol{p}(FL)$, she also prefers $\boldsymbol{\pi}_H$ at the prior belief $\boldsymbol{p}$. Since the problem is stationary, this implies that whenever a student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ at the posterior belief $\boldsymbol{p}(FL^i)$, she must also prefer $\boldsymbol{\pi}_H$ at the prior belief $\boldsymbol{p}(FL^{i-1})$, and thus, also at the prior belief $\boldsymbol{p}$. Thus, if a plan $\boldsymbol{\pi}_L^{FLi}$, which prescribes L i times after observing FL, is optimal, then the first action is coherent with $e^*(\boldsymbol{p}) = L$.

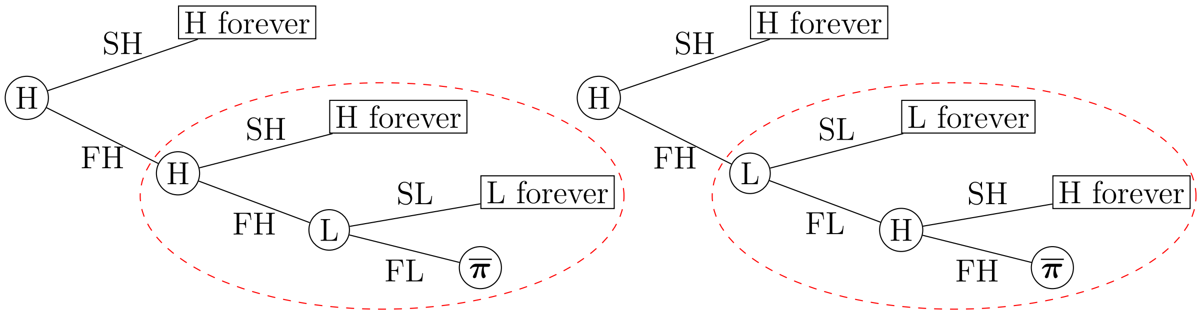

Figure 10: Plans $\boldsymbol{\pi}_H^{FH}$ (left) and $\boldsymbol{\pi}_L^{FH}$ (right).

2) If the optimal plan starts with high effort and prescribes to continue high effort even after failure ($\boldsymbol{FH}$), then the optimal policy $\boldsymbol{e^*(p)}$ also prescribes $\boldsymbol{H}$. We show that for such a plan to be optimal, we must have that the student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ at the posterior belief $\boldsymbol{p}(FH)$. As we have seen, she then also prefers $\boldsymbol{\pi}_H$ at the prior belief $\boldsymbol{p}$.

Consider a plan $\boldsymbol{\pi}_H^{FH}$ which starts with H. After SH it prescribes H forever. After FH it prescribes following plan $\boldsymbol{\pi}_H$ – i.e., play H once more and continue with H in case of success, but switch to L in case of failure. If the plan $\boldsymbol{\pi}_H^{FH}$ is optimal, then it must yield a higher expected payoff than any other plan. We can construct an alternative plan $\boldsymbol{\pi}_L^{FH}$, which is identical to $\boldsymbol{\pi}_H^{FH}$, except that it prescribes following plan $\boldsymbol{\pi}_L$ after FH. For an illustration, see Figure 10.

We now show that it is equivalent to say that the student prefers plan $\boldsymbol{\pi}_H^{FH}$ to $\boldsymbol{\pi}_L^{FH}$ at belief $\boldsymbol{p}$ and that the student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ at belief $\boldsymbol{p}(FH)$:

$$\begin{aligned}
& EU\left(\boldsymbol{\pi}_H^{FH}, \boldsymbol{p}\right) \geq EU\left(\boldsymbol{\pi}_L^{FH}, \boldsymbol{p}\right) \\
\Leftrightarrow \quad & p_\omega(1-\varepsilon)\left[1+\delta+\frac{\delta^2(1-\varepsilon^2)}{1-\delta}\right]+p_\alpha\frac{1-\varepsilon}{1-\delta} \\
& -c\left[1+\delta+(p_\omega+p_\alpha)\frac{\delta^2(1-\varepsilon^2)}{1-\delta}\right]+\left(p_\omega\varepsilon^2+p_\alpha\varepsilon^2+p_\beta\right)EU\left(\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FH, FH)\right) \\
& \geq p_\omega(1-\varepsilon)\left[1+\frac{\delta(1-\varepsilon)}{1-\delta}\right]+p_\alpha\frac{1-\varepsilon}{1-\delta} \\
& -c\left[1+(p_\omega+p_\alpha)\frac{\delta(1-\varepsilon)}{1-\delta}\right]+\left(p_\omega\varepsilon^2+p_\alpha\varepsilon^2+p_\beta\right)EU\left(\overline{\boldsymbol{\pi}}, \boldsymbol{p}(FL, FH, FH)\right) \\
\Leftrightarrow \quad & \frac{p_\omega\varepsilon}{p_\omega\varepsilon+p_\alpha\varepsilon+p_\beta} \geq \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon}+\frac{p_\alpha\varepsilon}{p_\omega\varepsilon+p_\alpha\varepsilon+p_\beta}\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right] \\
\Leftrightarrow \quad & EU\left[\boldsymbol{\pi}_H, \boldsymbol{p}(FH)\right] \geq EU\left[\boldsymbol{\pi}_L, \boldsymbol{p}(FH)\right].
\end{aligned}$$

The comparison of $\boldsymbol{\pi}_H^{FH}$ and $\boldsymbol{\pi}_L^{FH}$ at belief $\boldsymbol{p}$ reduces to comparing the payoff from $\boldsymbol{\pi}_H$ and $\boldsymbol{\pi}_L$ at the posterior belief $\boldsymbol{p}(FH)$. As the plans $\boldsymbol{\pi}_H^{FH}$ and $\boldsymbol{\pi}_L^{FH}$ are identical for all other histories, the respective payoffs cancel out. We have already shown that, whenever a student prefers plan $\boldsymbol{\pi}_H$ to $\boldsymbol{\pi}_L$ at the posterior belief $\boldsymbol{p}(FH)$, she must also prefer it at the prior belief $\boldsymbol{p}$. Therefore, if plan $\boldsymbol{\pi}_H^{FH}$ is optimal then it prescribes the same immediate action as the policy $e^*(\boldsymbol{p})$: H. Analogously to the case above, this argument directly extends to plans which prescribe playing H several times after FH.

Since the plans after observing both FL and FH are unspecified, any possible action following these outcomes is covered in the analysis. In conclusion, the threshold $\bar{p}_\omega$ gives a necessary and sufficient condition for the optimal action. □

Proof of Lemma 3. Consider a student with prior belief $\boldsymbol{p} = (p_\omega, p_\alpha, p_\beta)$. The posterior beliefs, after observing history $(0, 0, 0, n_{FL})$ are given by:

$$p_\omega((0,0,0,n_{FL})) = \frac{p_\omega}{p_\omega + p_\alpha \varepsilon^{n_{FL}} + 1 - p_\omega - p_\alpha},$$
$$p_\alpha((0,0,0,n_{FL})) = \frac{p_\alpha \varepsilon^{n_{FL}}}{p_\omega + p_\alpha \varepsilon^{n_{FL}} + 1 - p_\omega - p_\alpha}.$$

Taking the limits, we obtain

$$\lim_{n_{FL}\to\infty} p_\omega((0,0,0,n_{FL})) = \frac{p_\omega}{1-p_\alpha},$$
$$\lim_{n_{FL}\to\infty} p_\alpha((0,0,0,n_{FL})) = 0.$$

From Proposition 1, we know that the student exerts high effort if and only if $p_\omega \geq \overline{p}_\omega$. This inequality becomes easier to fulfil as n_{FL} increases. In order to find beliefs for which the threshold $\overline{p}_\omega$ cannot be reached, no matter how often the student observes the outcome FL, we consider the posterior beliefs as $n_{FL} \to \infty$. Using the posterior beliefs derived above, we obtain the following condition for low effort forever:

$$\lim_{n_{FL}\to\infty} p_\omega((0,0,0,n_{FL})) < \lim_{n_{FL}\to\infty} \overline{p}_\omega\left(p_\alpha((0,0,0,n_{FL}))\right)$$
$$\Leftrightarrow \qquad \frac{p_\omega}{1-p_\alpha} < \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon} + 0\cdot\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right]$$
$$\Leftrightarrow \qquad p_\omega < \frac{c(1-\delta)}{(1-\varepsilon)(1-\delta\varepsilon-\delta c)}(1-p_\alpha) =: \underline{p}_\omega(p_\alpha).$$

The threshold $\underline{p}_\omega$ defines all beliefs such that inequality (5) is violated in the limit for $n_{FL} \to \infty$. □

Proof of Proposition 2.

Consider a student with belief $\boldsymbol{p} = (p_\omega, p_\alpha, p_\beta)$. The posterior beliefs after observing the history $(0,0,n_{FH},0)$ are given by:

$$p_\omega((0,0,n_{FH},0)) = \frac{p_\omega \varepsilon^{n_{FH}}}{p_\omega \varepsilon^{n_{FH}} + p_\alpha \varepsilon^{n_{FH}} + 1 - p_\omega - p_\alpha},$$
$$p_\alpha((0,0,n_{FH},0)) = \frac{p_\alpha \varepsilon^{n_{FH}}}{p_\omega \varepsilon^{n_{FH}} + p_\alpha \varepsilon^{n_{FH}} + 1 - p_\omega - p_\alpha}.$$

From Lemma 3, we know that the student exerts low effort forever if and only if $p_\omega < \underline{p}_\omega$. This inequality becomes easier to fulfil as n_{FH} increases. We want to determine $\overline{n}_{FH}$, the maximal tally of failures with high effort that a student is optimally willing to observe before giving up high effort forever. This is equivalent to calculating the minimal tally n_{FH} such that:

$$p_\omega((0,0,n_{FH},0)) < \underline{p}_\omega\left(p_\alpha((0,0,n_{FH},0))\right).$$

For prior beliefs $\boldsymbol{p} < \underline{p}_\omega$ this inequality is fulfilled for $n_{FH} = 0$. Beliefs that satisfy $p_\alpha + p_\omega = 1$ are unchanged after observing FH. Thus, the belief can never fall below $\underline{p}_\omega$, and we have $\overline{n}_{FH} = \infty$. □

Proof of Proposition 3.

Consider a student with belief $\boldsymbol{p} = (p_\omega, p_\alpha, p_\beta)$. The posterior beliefs after observing the history $(0,0,0,n_{FL})$ are given in the proof of Lemma 3.

From Proposition 1, we know that the student exerts high effort if and only if $p_\omega \geq \overline{p}_\omega$. This inequality becomes easier to fulfil as n_{FL} increases. We are looking for the smallest number of n_{FL} such that:

$$p_\omega((0,0,0,n_{FL})) \geq \overline{p}_\omega\left(p_\alpha((0,0,0,n_{FL}))\right).$$

For beliefs $\boldsymbol{p} \geq \overline{p}_\omega$ this inequality is fulfilled for $n_{FL} = 0$. For beliefs $\boldsymbol{p} < \underline{p}_\omega$ this inequality is never fulfilled as the student exerts low effort forever. For this case, we define $\underline{n}_{FL} := \infty$. □

Proof of Proposition 4. First, we rewrite the definition of perseverance and calculate the comparative statics. Perseverance is derived from inequality (3). For $p_\omega = 0$, the student exerts low effort forever, and $\overline{n}_{FH} = 0$. Thus, we assume $p_\omega > 0$ for the remainder of the proof. We rewrite (3) as follows:

$$\begin{aligned}
& p_\omega(0,0,n_{FH},0) < \underline{p}_\omega\left(p_\alpha(0,0,n_{FH},0)\right) \\
\Leftrightarrow \quad & \frac{p_\omega \varepsilon^{n_{FH}}}{p_\omega \varepsilon^{n_{FH}} + p_\alpha \varepsilon^{n_{FH}} + 1 - p_\omega - p_\alpha} \\
& < \frac{c(1-\delta)}{(1-\varepsilon)(1-\delta\varepsilon-\delta c)}\left(1 - \frac{p_\alpha \varepsilon^{n_{FH}}}{p_\omega \varepsilon^{n_{FH}} + p_\alpha \varepsilon^{n_{FH}} + 1 - p_\omega - p_\alpha}\right) \\
\Leftrightarrow \quad & \varepsilon^{n_{FH}} < \frac{1-p_\alpha-p_\omega}{p_\omega}\frac{c(1-\delta)}{(1-\varepsilon-c)(1-\delta\varepsilon)} \\
\Leftrightarrow \quad & n_{FH} > \frac{1}{\ln(\varepsilon)} \ln\left(\frac{1-p_\alpha-p_\omega}{p_\omega}\frac{c(1-\delta)}{(1-\varepsilon-c)(1-\delta\varepsilon)}\right) =: \tilde{n}_{FH}. \qquad (10)
\end{aligned}$$

Note that $\overline{n}_{FH}$ is the minimal step function satisfying condition (10). We derive monotonicity properties of $\overline{n}_{FH}$ by taking the derivatives of the right side of inequality (10). The relevant range is $\boldsymbol{p} \geq \underline{p}_\omega$, since outside of this range $\overline{n}_{FH}$ is constant. Consequently, if the derivative with respect to a variable is positive (negative) in this range, $\overline{n}_{FH}$ weakly increases (decreases) in this variable. Taking the derivatives of the right side of inequality (10), we obtain

$$\begin{aligned}
\frac{\partial}{\partial p_\omega}\tilde{n}_{FH} &= -\frac{1-p_\alpha}{p_\omega(1-p_\alpha-p_\omega)\ln(\varepsilon)} > 0, \\
\frac{\partial}{\partial p_\alpha}\tilde{n}_{FH} &= -\frac{1}{(1-p_\alpha-p_\omega)\ln(\varepsilon)} > 0, \\
\frac{\partial}{\partial c}\tilde{n}_{FH} &= \frac{1-\varepsilon}{c(1-c-\varepsilon)\ln(\varepsilon)} < 0, \\
\frac{\partial}{\partial \delta}\tilde{n}_{FH} &= -\frac{1-\varepsilon}{(1-\delta)(1-\delta\varepsilon)\ln(\varepsilon)} > 0, \\
\frac{\partial}{\partial \varepsilon}\tilde{n}_{FH} &= \frac{1+\delta(1-c-2\varepsilon)}{(1-c-\varepsilon)(1-\delta\varepsilon)\ln(\varepsilon)} - \frac{1}{\varepsilon\ln(\varepsilon)^2}\ \ln\left(\frac{c(1-p_\alpha-p_\omega)(1-\delta)}{p_\omega(1-c-\varepsilon)(1-\delta\varepsilon)}\right).
\end{aligned}$$

Second, we rewrite the definition of procrastination and calculate the comparative statics. procrastination is derived from inequality (4). For $p_\alpha = 0$, $\underline{n}_{FL}$ is constant: For $p_\alpha = 0$ and $p_\omega < \overline{p}_\omega$, we have $\underline{n}_{FL} = \infty$. For $p_\alpha = 0$ and $p_\omega \geq \overline{p}_\omega$, we have $\underline{n}_{FL} = 0$. Thus, we assume

$p_\alpha > 0$ for the remainder of the proof. We rewrite (4) as follows:

$$
\begin{aligned}
& p_\omega((0,0,0,n_{FL})) \geq \overline{p}_\omega\left(p_\alpha(0,0,0,n_{FL})\right) \\
\Leftrightarrow\quad & \frac{p_\omega}{p_\omega + p_\alpha \varepsilon^{n_{FL}} + 1 - p_\omega - p_\alpha} \\
& \geq \frac{c}{(1-\delta\varepsilon-\delta c)}\left[\frac{1-\delta}{1-\varepsilon} + \frac{p_\alpha \varepsilon^{n_{FL}}}{p_\omega + p_\alpha\varepsilon^{n_{FL}} + 1 - p_\omega - p_\alpha}\frac{\delta(2-\delta-\delta\varepsilon)}{1-\delta}\right] \\
\Leftrightarrow\quad & \varepsilon^{n_{FL}} \leq \frac{1-\delta}{p_\alpha\,(1-\delta\varepsilon)}\left[p_\omega(1-\varepsilon)\left(\frac{1}{c} - \frac{\delta}{1-\delta\varepsilon}\right) - (1-p_\alpha)\,\frac{1-\delta}{1-\delta\varepsilon}\right] \\
\Leftrightarrow\quad & n_{FL} \geq \frac{1}{\ln(\varepsilon)}\ln\left(\frac{(1-\delta)\left[p_\omega(1-\varepsilon)(1-\delta\varepsilon-\delta c) - c(1-\delta)(1-p_\alpha)\right]}{cp_\alpha(1-\delta\varepsilon)^2}\right) := \tilde{n}_{FL}. \qquad (11)
\end{aligned}
$$

Note that $\underline{n}_{FL}$ is the minimal step function satisfying condition (11). We derive monotonicity properties of $\underline{n}_{FL}$, by taking the derivatives of the right side of inequality (11). The relevant range is $\underline{p}_\omega \leq \boldsymbol{p} < \overline{p}_\omega$, since outside of this range the function is constant. Consequently, if the derivative with respect to a variable is positive (negative) in this range, $\overline{n}_{FL}$ weakly increases (decreases) in this variable. The derivatives are given by

$$
\begin{aligned}
\frac{\partial}{\partial p_\omega}\tilde{n}_{FL} &= \frac{(1-\varepsilon)(1-\delta c-\delta\varepsilon)}{\ln(\varepsilon)\left[p_\omega(1-\varepsilon)(1-\delta\varepsilon-\delta c) - c(1-\delta)(1-p_\alpha)\right]} \leq 0, \\
\frac{\partial}{\partial p_\alpha}\tilde{n}_{FL} &= -\frac{p_\omega(1-\varepsilon)(1-\delta\varepsilon-\delta c) - c(1-\delta)}{p_\alpha \ln(\epsilon)\left[p_\omega(1-\varepsilon)(1-\delta\varepsilon-\delta c) - c(1-p_\alpha)(1-\delta)\right]} \geq 0, \\
\frac{\partial}{\partial c}\tilde{n}_{FL} &= -\frac{p_\omega(1-\varepsilon)(1-\delta\varepsilon)}{c\ln(\varepsilon)\left[p_\omega(1-\varepsilon)(1-\delta\epsilon-\delta c) - c(1-\delta)(1-p_\alpha)\right]} \geq 0, \\
\frac{\partial}{\partial \delta}\tilde{n}_{FL} &= -\frac{(1-\varepsilon)\left[p_\omega(1-\varepsilon)(1-\delta\varepsilon-\delta c) + p_\omega c(1-\delta) - 2c(1-\delta)(1-p_\alpha)\right]}{(1-\delta)(1-\delta\varepsilon)\ln(\varepsilon)\left[p_\omega(1-\varepsilon)(1-\delta\varepsilon-\delta c) - c(1-p_\alpha)(1-\delta)\right]}, \\
\frac{\partial}{\partial \varepsilon}\tilde{n}_{FL} &= \frac{p_\omega \ln(\varepsilon)\left[1+\delta(1-c-2\varepsilon)\right]}{p_\omega(1-\varepsilon)(1-\delta\epsilon-\delta c) - c(1-p_\alpha)(1-\delta)} - \frac{2\delta\ln(\varepsilon)}{1-\delta\varepsilon} \\
&\quad - \frac{1}{\epsilon}\ln\left[\frac{(1-\delta)\left[p_\omega(1-\varepsilon)(1-\delta\varepsilon-\delta c) - c(1-p_\alpha)(1-\delta)\right]}{cp_\alpha(1-\delta\epsilon)^2}\right] \geq 0.
\end{aligned}
$$

For the derivative with respect to δ, note that the sign is characterized by

$$
p_\omega \geq \frac{2c(1-\delta)}{(1-\varepsilon)(1-\delta\varepsilon) + c(1-2\delta+\delta\varepsilon)}(1-p_\alpha),
$$

which is a line through $\{p_\alpha = 1\}$ in the belief triangle, with a steeper negative slope than $\underline{p}_\omega$ has. The derivative is negative between $\underline{p}_\omega$ and this line, and positive above this line. Thus, procrastination $\underline{n}_{FL}$ decreases in patience δ for low values of confidence p_α but increases for high values. The cross partial with respect to δ and p_α is positive:

$$\frac{\partial}{\partial\delta\partial p_\alpha}\tilde{n}_{FL} = -\frac{cp_\omega(1-\varepsilon)(1-c-\varepsilon)}{\left[c(1-\delta)(1-p_\alpha) - p_\omega(1-\varepsilon)(1-\delta\varepsilon-\delta c)\right]^2 \ln(\varepsilon)} \geq 0.$$

□

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