The Effect of Wealth on Worker Productivity

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Abstract

We propose a theory that analyzes how a workers' asset holdings affect their job productivity. In a labor market with uninsurable risk, workers choose to direct their search to jobs that trade off productivity and wages against unemployment risk. Workers with low asset holdings have a *precautionary job search motive*, they direct their search to low productivity jobs because those offer a low risk at the cost of low productivity and a low wage. We show that such sorting occurs under a condition closely related to Decreasing Relative Risk Aversion and that the presence of consumption smoothing can reconcile the directed search model with negative duration dependence on wages, a robust empirical regularity that the canonical directed search model cannot rationalize. We calibrate the infinite horizon economy and find that this mechanism is quantitatively important. We evaluate a tax financed unemployment insurance (UI) scheme and how it affects welfare. Aggregate welfare is inverted U-shaped in benefits: the insurance effect UI dominates the incentive effects for low levels of benefits and vice versa for high benefits. Also, when UI increases, total production falls in the economy while worker productivity increases. Finally, we compare a one-off severance payment with per period benefits and find that per period benefits generate superior welfare.

Keywords. Unemployment Risk. Precautionary Savings. Precautionary Job Search. Sorting. Unemployment Insurance. Severance Pay. Directed Search. Duration Dependence.
JEL. C6. E2.

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1 Introduction

Unemployment risk is arguably the biggest risk workers face in their lifetime. Even if there is no market for unemployment insurance, workers nonetheless self-insure by accumulating assets while employed, in order to run those assets down when unemployed. This allows them to better smooth consumption. But, workers simultaneously also use the labor market to self-insure by applying for lower productivity jobs that have a higher job finding probability. We ask a basic question: what is the role of asset holdings and self-insurance for productivity? We then study the welfare implications of government mandated Unemployment Insurance (UI) benefits. There is of course a tension between the unemployed who receive UI benefits and the employed who pay for them through taxes. But there is now also a tension within the pool of the unemployed. We show that the unemployed with low asset levels benefit considerably more from a UI increases than the rich unemployed. We evaluate the effect that benefits have on total productivity.

We model the worker's savings and job search decision in a labor market where workers can direct their search towards jobs of different productivity, with firms posting wages to attract applicants. The worker's incentives are thus to trade off wages and job productivity against the probability of finding a job. Asset holdings crucially affect this tradeoff because the worker is less exposed to the consumption risk inherent in joblessness. In addition to the standard *precautionary savings motive* with asset-contingent consumption smoothing à la Bewley-Huggett-Aiyagari, workers now also counter unemployment risk by directing their search to jobs with a high matching probability and low productivity, call it a *precautionary job search motive*. Key in our analysis is the sorting of workers with different assets holdings into different productivity jobs. Our main objective is to analyze how the inequality inherent in a labor market with heterogeneous productivity jobs interacts with the inequality that results from asset accumulation, i.e., how the two precautionary motives interplay and affect the jobs productivity.

Our paper contributes to the literature on three fronts. First, our model reconciles the directed search model with the evidence on unemployment duration, both in theory and quantitatively. One of the most robust facts regarding unemployment is the *negative duration dependence* of unemployment on wages. Workers with higher wages tend to have shorter unemployment duration.[1] A major weakness of the canonical directed search model – and therefore a fundamental criticism of its broader applicability in explaining labor market frictions – is that it predicts the opposite, a positive duration dependence of unemployment on wages. Higher wages attract more applicants and therefore result in lower matching probabilities, i.e., longer unemployment duration. One of the contributions of this paper is to show negative duration dependence under directed search, due to the presence of consumption smoothing.

[1] See amongst many others, Heckman and Singer (1984a), Heckman and Singer (1984b), Honoré (1993), Van den Berg and Van Ours (1996), and Baley and Sepahsalari (2019). One of the most challenging research questions there is the extent to which the negative duration dependence is driven by genuine duration dependence, such as the depreciation of skills, or selection, where high wage workers have different job finding rates.

Workers who remain unemployed for longer run down their assets, and end up applying to low wage jobs, which induces negative duration dependence. The model therefore combines the moral hazard aspect of UI with a changing job finding probability due to asset decumulation. We believe that this is a new insight and offers an important empirical justification for the applicability of directed search models.

The second contribution is to show that workers with heterogeneous asset holdings sort into firms with heterogeneous productivities. This implies that equally skilled workers have different productivities, depending on their wealth holdings. The sorting happens despite the fact that there is no technological complementarity (supermodularity) between job productivity and worker skill. There is nonetheless a natural *preference complementarity* between firm productivity and worker assets because risk aversion generates different preferences for self insurance, with high asset holders trading off lower insurance for a higher productivity job. To establish the sorting in this model, we solve this as an allocation problem with risk aversion and therefore imperfectly transferable utility (ITU) as well as search frictions. It is the selection or sorting of workers into different productivity jobs that is responsible for the different matching probabilities of different asset holders, which we show occurs under a condition related to Decreasing Relative Risk Aversion. While directed search in the presence of risk aversion has been analyzed in the literature – most notably Acemoglu and Shimer (1999) and more recently Golosov et al. (2013) –, these are representative agent models without a non-degenerate distribution of assets.[2]

Our third contribution is the *quantitative* analysis of the model. This interaction between the distribution of assets and the incentives to search for different productivity jobs as well as smoothing consumption is not merely a theoretical artifact. We show that it is important quantitatively. We analyze the steady state of an infinite horizon version where workers and firms sort in each period. In the steady state, unemployed workers run down their assets, while at the same time moving their target from high to low productivity jobs. Employed workers run up their assets anticipating the eventual job loss resulting in necessity to insure against income loss while unemployed. Workers continuously move up and down the asset distribution, but the aggregate distribution of assets is stationary. We derive the ergodic distribution in this steady state as well as wages, savings, jobs search decisions (and unemployment), and the vacancy posting decision for every asset and productivity level. Unlike most existing work on unemployment insurance, we are able to incorporate the endogenous savings decision of the employed.[3] The novelty of our computational model is that we solve a sorting problem, with risk-aversion, in infinite horizon and with search frictions. In the process, we solve for the ergodic asset distribution as a state variable.

[2] Acemoglu and Shimer (1999) do consider a non-degenerate distribution when analyzing the case of CARA, which, as we show in this paper, is a knife-edge case with no sorting and where the asset distribution is indeterminate.

[3] The standard assumption in the literature is that employed workers values are constant (see for example Hopenhayn and Nicolini (1997), Shimer and Werning (2007) and Shimer and Werning (2008)). This is typically achieved by assuming that once employed, they do not face job separation, in conjunction with the assumption that discounting is exactly proportional to the return on assets. All this implies that workers in each period consume the return on their assets, keeping their asset holdings invariant.

We calibrate our model to the US economy and find that its features are quantitatively important. Workers direct their search towards jobs with different bundles of productivity and job finding probabilities. We find that the job finding probability of the low asset holders is 10% higher than that of the high asset holders. This establishes the important role of endogenous job finding rates and their interaction with the distribution of asset holdings.

In this setting we analyze the role of government mandated unemployment benefits. We have no pretense of analyzing a general mechanism design question where agents submit messages about their private asset holdings and receive benefits depending on their and all other agents' messages. This turns out to be a immensely complex problem with an infinite horizon and a continuum of heterogeneous agents. Rather, we analyze a realistic unemployment insurance institution where ex ante homogeneous workers with ex post heterogeneous (but time varying) asset holdings receive a constant benefit while unemployed and pay a constant tax rate on wage income while employed.

There are multiple channels through which benefits affect the equilibrium allocation and therefore welfare. We single out five equilibrium effects that result from an increase in benefits: 1. The unemployed worker is better insured and enjoys smoother consumption; 2. Because of better insurance prospects, workers with more wealth tend to sort into more productive jobs; Both of these effects affect welfare positively. The next effects are negative. 3. Higher wages reduce the firm's benefits and therefore job creation; 4. Higher benefits affect the sorting pattern with more workers applying for high productivity jobs, which uniformly leads to lower job finding probabilities and therefore higher unemployment; 5. Higher benefits increases the productivity of jobs but reduces the total production (extensive margin) and therefore lead to lower dividends. Some of these 5 equilibrium effects are present in other models, but the sorting mechanism and its effect on productivity is what distinguishes the mechanism here.

We are interested in what the net effect is of these countervailing forces on welfare. But there are also conflicts of interest between different agents. Not only are the unemployed broadly speaking better off from higher benefits than the employed, benefits have higher welfare effects for those with low assets. Overall, we find that nearly all workers, including those with high asset levels and those employed, have a preference for relatively high benefits. Depending on their asset holdings, the optimal benefit for the unemployed is between 60% and 47% of wages whereas it is less than 45% for the employed. When we aggregate the value functions across all agents, we find that welfare has an inverted U-shape in benefits where the optimal benefit level is higher for unemployed workers compared to the employed. A rise in UI from the laissez-faire economy is welfare increasing for all workers, but especially for the asset poor and the unemployed. In contrast, when the UI level get closer to the full replacement rate, the welfare falls for all workers, in particular the asset rich employed workers.

A novel feature of our model is the sorting between workers with different asset holdings and firms with different productivities. This implies that UI benefits affect the productivity of workers in the economy, through the allocation of workers to jobs of different productivities as well as through the firms' entry decision. This contrasts with models of homogeneous firms where a change in benefits leaves the average firm's productivity unaffected. We find that when UI benefits increase, average worker productivity increases, even though total production decreases. Higher benefits result in workers applying to more productive jobs, because they are better insured. But, this at the same time decreases the job finding probability and as a result, fewer workers find a job. Firms do not respond opening more vacancies because they see their profits reduced as benefits push up wages.

We also analyze the alternative policy scheme of severance pay, where a worker who becomes unemployed receives a lump sum payment instead of a per period UI benefit during the unemployment spell. Severance pay offers better incentives to search but less insurance in case the worker is unemployed longer than average. We find that the insurance effect dominates, and as result, per period benefits generate higher welfare than severance pay.

Related Literature. We are intellectually indebted to earlier work that has shaped our thinking on this topic. This paper is related to a large literature on unemployment risk and consumption smoothing. Danforth (1979) is one of the first to analyze search with risk averse workers in a partial equilibrium setting. Hopenhayn and Nicolini (1997), Shimer and Werning (2007) and Shimer and Werning (2008) analyze optimal unemployment insurance in a similar setting. Our paper is a general equilibrium search model with risk averse agents, closely related to Acemoglu and Shimer (1999). They either assume asset holdings are identical for all agents – thus they cannot address the role of inequality in assets – or that preferences satisfy CARA – in which case the asset distribution is indeterminate. The latter is is a knife-edge special case in our paper. The properties of equilibrium change completely when moving away from CARA, where there is no sorting and the allocation is indeterminate. We derive all our results from the fact that workers sort on assets and job productivity. Under CARA none of the implications for welfare or the impact of unemployment benefits would hold. We could endogenies asset and move beyond Acemoglu and Shimer (1999) only because we had the benefit of the results on sorting with risk averse agents in Legros and Newman (2007). The result is a directed search model with general preferences and an ergodic distribution of assets where we find that there are substantial effects on wages and the value functions (a feature that is hard to obtain in the most basic random search model, i.e., without search intensity or endogenous match formation).

Golosov et al. (2013) consider a similar setup to Acemoglu and Shimer (1999) with identical agents and analyze optimal taxation and benefits. Here, we focus on the distribution of assets and where the distribution of those assets is non-degenerate.

Our model follows in the footsteps of Krusell et al. (2010), who analyze the relation between asset dependent consumption-savings decisions and unemployment risk. Our focus is on directed rather than random search. This is not merely a semantic distinction. Directed search allows for the fact that the asset holdings affect the job finding probability. While Krusell et al. (2010) obtain a welfare function that is decreasing in benefits for asset rich workers, we get the opposite. This is because in the basic random search model the probability of job finding is exogenous for workers. Therefore when UI goes up, rich workers are disadvantaged: they pay higher taxes yet, they find jobs at lower rates and their consumption smoothing does not change much. Instead in our framework, all workers endogenously adjust their probability of job finding depending on the UI level. This leads to an increase of welfare in benefits. For the same reason of endogenous directed search, we also find that equilibrium job finding rates are increasing in assets and varying considerably, while they are constant in the basic random search without endogenous search intensity.

Our paper extolls the advantages of directed search framework to study unemployment. Given the difficulties to analyze sorting in the random search model (Shimer and Smith (2000)), there is little hope to address sorting on assets in random search with risk aversion. Recent work by Krusell et al. (2019) and Chaumont and Shi (2018) extends our setting (most notably with on-the-job search, absent in our paper), which is testament to the virtues of the directed search model compared to random search to study the asset distribution and the consumption-savings decision. The directed search model can take the results in Krusell et al. (2010) a step further with the aim of building a model that is not only versatile enough to address canonical macroeconomic questions but that also has the properties that the random search model lacks.

Our directed search setup is complex – it has risk averse agents, it involves a consumption-savings decision, there is sorting, and the economy is dynamic (infinite horizon) –, and the block-recursivity property (Menzio and Shi (2011)) does not apply because there is two sided heterogeneity, with firm productivity and worker asset holdings. Nonetheless, from the combination of directed search with two-sided heterogeneity (as in Eeckhout and Kircher (2010)), we can solve an assignment problem with risk aversion. We extend the analysis in Legros and Newman (2007) to derive the conditions for sorting. The novelty of our approach allows us to analyze an economy where the asset distribution is endogenous and where both savings and job search decisions depend on the worker's asset holdings. We can thus analyze how unemployment benefits affect workers' asset holdings and in turn the productivities of jobs they search for.

In the matching literature, our paper further relates to models with types that are endogenous to investment (see amongst others Peters and Siow (2002) and Cole et al. (2001)), and where matching incentives are derived from preferences coupled with market incompleteness rather than built into the technology (for example Legros and Newman (1996)).

This paper is also related to large literature that looks at the welfare impact of a change in UI in search and matching models with risk averse agents. Merz (1995), Andolfatto (1996) and den Haan et al. (2000) study the macroeconomic implications of search frictions in business cycle models, in an economy where a worker's idiosyncratic income shocks are fully insured. Krusell et al. (2010) nests the Diamond-Mortensen-Pissarides framework with asset dependent consumption savings decisions as in Bewley (1980), Huggett (1993) and Aiyagari (1994). This allows them to analyze the interaction of search frictions with the precautionary savings motive. The contribution of our paper is to take this one step further. We introduce endogenous job search that allows workers to implicitly insure unemployment risk, the precautionary job search motive. We find that this is important quantitatively, and as a result, a change in unemployment insurance changes the workers' welfare by affecting their job search decision as well as the productivity of jobs they choose.

There is direct evidence in the literature for the main mechanism of our model, namely that higher asset holdings leads to prolonged job search. Card et al. (2007) find that a lump sum transfer of two months of salary reduces the job finding rate by 8-12%. These numbers are in line with what we find for our benchmark economy.[4] Chetty (2008) shows that the elasticity of the job finding rate with respect to unemployment benefits decreases with liquid wealth. And Browning and Crossley (2001) show that unemployment insurance improves consumption smoothing for poor agents, but not for rich ones. Herkenhoff (2013) and Herkenhoff et al. (2015) provide evidence for the effect of better credit access on lower job finding rates. Herkenhoff (2013) shows that through this channel, increased credit access leads to longer recessions and slower recoveries. And Herkenhoff et al. (2015) exploit credit tightening over the business cycle which leads to an increase in employment and a decrease in output and productivity. We believe our model is novel in providing a theoretical framework where this observed relation between asset holdings and job finding rates stems from a precautionary job search motive and firm heterogeneity.

Finally, in an interesting piece, Michelacci and Ruffo (2014) analyze a related question where workers are heterogeneous: how does optimal unemployment insurance vary over the life cycle. Because workers accumulate human capital, young workers have strong incentives to find a job, yet they do not have the means to smooth consumption. Instead, older workers have less incentives and can smooth consumption better. They focus on the role of human capital accumulation and to that end, assume that matching probabilities are exogenous.

This paper is organized as follow. In section 2 we lay out the model. In section 3 we derive the equilibrium allocation and the conditions under which there exists positive (negative) assortative matching. In section 4, we compute and quantitatively analyze the full infinite horizon model. We perform a benchmark calibration, and evaluate the effects of different benefit levels as well the welfare analysis and the comparison of per period benefits with severance payment. We conclude in section 5.

2 The Model

Time Horizon. This is a T-period economy in which agents make a joint consumption-savings and job search decision. Endowed with assets, in each period $t < T$, unemployed workers choose their consumption-savings level, as well as which job to search for. Our interest is in analyzing the infinite horizon setting $T \to \infty$ (Section 3.2). To gain insights into the mechanism and in order to derive analytical results, we first analyze the two-period model $T = 2$ (section 3.1), in which workers make

[4] See also Rendon (2006) and Lentz (2009) for related findings.

decisions only once at $t = 1$.

Agents. There is a measure one of workers. When they are unemployed they indexed by their heterogeneous asset holdings in period t, $a_t \in \mathcal{A} = [\underline{a}, \overline{a}] \subset \mathbb{R}_+$.[5] Let $F_u(a)$ denote the measure of unemployed workers with asset levels weakly below $a \in \mathcal{A}$ (with positive derivative $f_u(a)$. When they are employed, workers are indexed by both assets a and a wage w. Let $F_e(a, w)$ be the measure of employed workers with asset levels below a and wages below w. We denote the marginal over w by $F_e(a)$ (with positive derivative $f_e(a)$).[6] In order to reduce notation, we denote $F = (F_u(a), F_e(a, w))$. The distribution of asset holdings amongst unemployed and employed workers is endogenous. In the infinite horizon model we derive the ergodic distribution of assets. Each worker supplies her labor and can only apply to one job at a time. Firms are heterogeneous in their productivities y and each has one job. Let $y \in \mathcal{Y} = [\underline{y}, \overline{y}] \subset \mathbb{R}_+$ and assume the firm type is observable. $H(y)$ denotes the measure of firms in the economy and with a type weakly below y. The total measure of firms $H(\overline{y})$ is assumed large. H is assumed C^2 with strictly positive derivative h. Not all firms enter the market, nor are all firms searching for workers. The measure of firms that post vacancies is endogenous and denoted by $G(y)$ (with positive derivative $g(y)$).

Preferences and Technology. Workers are risk averse and their preferences are represented by the Von Neumann-Mergenstern utility function $u(c)$ over consumption level c, where $u : \mathbb{R}_+ \to \mathbb{R}_{++}$. We assume that u is increasing and concave: $u' > 0, u'' < 0$. Agents discount utility with factor $\beta < 1$. Savings can be invested in a risk free bond at a fixed rate $R = 1 + r > 1$. We assume that firms are owned by entrepreneurs who are risk neutral and who do not participate in the labor market.[7] Firms have one job and can post a vacancy at cost k. Output produced at a firm of type y is equal to y.

Search Technology. Job search is directed. Firms post a wage w and there is a search technology that governs the frictions. These frictions crucially depend on the degree of competition for jobs, as captured by the ratio of vacancies to unemployed workers, denoted by $\theta \in [0, \infty]$. This represents the relative supply and demand for jobs, as it determines the probability of a match for an unemployed worker denoted by $m(\theta)$, where $m : [0, \infty] \to [0, 1]$: the higher the value of θ, the easier it is for a worker to find a job, so m is a strictly increasing function: $m' > 0$. In contrast, the higher the ratio of firms to workers, the harder it is for a firm to fill its vacancy. We denote the probability that a

[5] For much of the paper we will drop the subscript t and in the recursive (two-period) formulation we refer to $a_t = a$ ($a_1 = a$) and $a_{t+1} = a'$ ($a_2 = a'$).

[6] These are not distributions since their total measure is not equal to one. Because the measure of workers is equal to one and all are either employed or unemployed, it is the case that $F_u(\overline{a}) + F_e(\overline{a}) = 1$ and $F_u(\overline{a})$ is equal to the unemployment rate.

[7] This approach does not affect any of the results since the dividend deterministically increases the workers' asset holdings and merely shifts the asset distribution. However in the infinite horizon version of the model we assume that profits are distributed as the risk free dividend of a mutual fund owned by all workers and that has all firms in its portfolio as in Golosov et al. (2013). This closes the model in order to analyze the welfare implications of changes in UI.

firm gets matched by $q(\theta)$, where $q : [0, \infty] \to [0, 1]$ is a strictly decreasing function, $q' < 0$. Since matching is always in pairs, the matching probability of workers must be consistent with those of firms, in particular, it must be the case that $q(\theta) = m(\theta)/\theta$. We also require the standard assumptions hold: m is twice continuously differentiable, strictly concave and has a strictly decreasing elasticity. The fact that we express the matching probability in terms of the ratio of firms to workers θ and not the number of unemployed workers and vacancies effectively means that we assume a matching technology that is constant returns. As the number of workers and firms doubles, the number of matches doubles, yet the matching probabilities remain unchanged.

As is inherent in the nature of directed search, there is a separate submarket for each firm-worker type pair. Heterogeneous firms and workers operate in different markets, while identical agents are in a common market. This permits workers to direct their search to those firms that offer the optimal terms (matching probability and wages) and it enables firms with vacancies to influence the search decision of workers by changing the terms of the wage offer. Whenever unemployed, a worker searches to find a job, and once employed she holds the job until the match is separated with exogenous probability λ.

Unemployment Benefits. We assume that all unemployed workers receive unemployment benefit b. The benefit b is financed by a budget balancing proportional tax τ on wages. This requires that the sum of all benefits b over the unemployed agents is equal the sum of all taxes levied on wage income τw. We also assume that the entire income for the unemployed comes from UI. For a given b, the government sets τ to balance its period-by-period budget constraint:

$$ub = \tau \int w(a) f_e(a) da. \tag{1}$$

Profits and Dividends. Due to the sorting with firms of heterogenous productivity, all firms except the marginal firm make profits. The hedonic profit schedule that clears the market is increasing in the firm type: higher productivity firms make higher profits.[8] We assume that consumers own an equal share of the equity of all firms.[9] This assumption implies that all workers regardless of their employment status receive a dividend d every period. This enables the welfare analysis to take into account the impact of a change in unemployment benefits on profitability of firms.

Actions and Timing. In period $t < T$, workers choose their consumption-savings bundle as well as the job search decision. A worker enters period t with assets Ra_t chosen in period $t - 1$. The worker then chooses the assets a_{t+1} saved. In period t, firms y post wages w_{t+1}, and the worker chooses which

[8]This could be reconciled with zero profits by adding an earlier stage: firms pay an entry cost before the realization of their type. In equilibrium, the expected profits equate the entry cost.

[9]That is, no consumer holds the claims to the profit of an individual job but she holds the claim to an identical share of the aggregate profit. This is to avoid that an employed worker holds a short position in her own job in order to hedge against the risk of separation. Also, in Appendix D we consider alternative distributions of profits, including profits that are taxed in order to finance UI benefits and profits that are redistributed to workers in proportion to their asset holdings.

submarket (y_{t+1}, w_{t+1}) to search in. Even if firms with different productivity y_{t+1} offer the same wage w_{t+1}, in directed search they operate in different markets. Given the behavior of all other firms and applying workers, this market has a tightness θ_t.

Period t's consumption is contingent on the saved assets and on the labor market outcome. A worker carries over last period's assets with return R. If unemployed, her income is thus $Ra_t + b$ and it is $Ra_t + (1-\tau)w_t$ if employed. Her consumption is equal to this income net of here savings for next period a_{t+1}: $c_{e,t} = Ra_t - a_{t+1} + (1-\tau)w_t$ when employed and $c_{u,t} = Ra_t - a_{t+1} + b$ when unemployed. Within the same period t, a two-stage directed search extensive form game determines the labor market outcome for unemployed workers and vacant firms.

Firms first simultaneously announce wages w_{t+1} that will be paid starting in the next period: $w_{t+1} \in \mathcal{W} = [\underline{w}, \overline{w}] \subset \mathbb{R}_+$. We restrict the contract space to invariant wages. After observing all wage-firm type pairs (w_{t+1}, y_{t+1}), the workers then choose which pair to apply to. Denote by $P(y_{t+1}, w_{t+1})$ and $Q(a_t, a_{t+1}, y_{t+1}, w_{t+1})$ the distribution of actions by firms and workers: $P(y_{t+1}, w_{t+1})$ is the measure of firms that offer a productivity-wage pair below (y_{t+1}, w_{t+1}) and $Q(a_t, a_{t+1}, y_{t+1}, w_{t+1})$ is the measure of workers with assets below a_t who save less than a_{t+1} and who apply for productivity-wage pairs below (y_{t+1}, w_{t+1}). We impose that those distributions of actions are consistent with the initial distributions of types $G(y)$ and $F_u(a)$, i.e., that there is market clearing. In particular, it must be the case that $P_{\mathcal{Y}}(\cdot) = G(\cdot)$ and $Q_{\mathcal{A}} = F_u(\cdot)$, where $P_{\mathcal{Y}}$ and $Q_{\mathcal{A}}$ are the marginal distributions. This ensures that the allocation is measure preserving.

Value Functions and Equilibrium. Denote by $U(a_t)$ the value of being unemployed in period t with asset level a_t and by $E(a_t)$ the value of being employed.[10] The unemployed worker simultaneously chooses how much assets a_{t+1} to save for next period, and which submarket y_{t+1}, w_{t+1} to search in. The choice of the submarket determines the wage w_{t+1} but also the market tightness θ_t and hence the matching probability $m(\theta_t)$. The employed worker with assets a_t chooses how much to save a_{t+1}.

We can then write

$$U(a_t) = \max_{a_{t+1}, y_{t+1}, w_{t+1}} \{u(c_{u,t}) + \beta\left[m(\theta_t)E(a_{t+1}, w_{t+1}) + (1 - m(\theta_t))U(a_{t+1})\right]\} \tag{2}$$

$$\text{s.t.} \quad c_{u,t} = Ra_t - a_{t+1} + b + d \;\text{ and }\; a_{t+1} \geq \underline{a}$$

$$E(a_t, w_t) = \max_{a_{t+1}} \{u(c_{e,t}) + \beta[\lambda U(a_{t+1}) + (1-\lambda)E(a_{t+1}, w_{t+1})]\} \tag{3}$$

$$\text{s.t.} \quad c_{e,t} = Ra_t - a_{t+1} + (1-\tau)w_t + d \;\text{ and }\; a_{t+1} \geq \underline{a}.$$

[10] More precisely, $U(a_t, a_{t+1}, y_{t+1}, w_{t+1}, P, Q, F)$ is the value of an unemployed worker with assets a_t who saves a_{t+1}, who applies to a job y_{t+1} with wage w_{t+1} and who anticipates a distribution of offers P and a distribution of jobs Q, and when the asset distributions are given by F. Of course, the worker does not care about the productivity y_{t+1} and only about the wage w_{t+1}, but the submarket is indexed by the bundle y_{t+1}, w_{t+1} because different firm types y_{t+1} may offer the same wage, y_{t+1} formally enters in the value function of the worker. For notational convenience, we restrict the argument of the value function to the variable that indexes the type $U(a_t)$, i.e., the heterogeneity that is relevant for sorting: a_t. Likewise $E(a_t) = E(a_t, a_{t+1}, y_{t+1}, w_{t+1}, P, Q, F)$.

All workers' savings are limited by a borrowing constraint, $\underline{a}$, which measures the incompleteness of the credit market.

The continuation value to the firm of productivity y that posts a vacancy is denoted by $V(y_t)$:[11]

$$V(y_t) = -k + \max_{w_{t+1}} \beta[q(\theta_t)J(y_{t+1}, w_{t+1}) + (1 - q(\theta_t))V(y_{t+1})]\}, \tag{4}$$

where $J(y_{t+1}, w_{t+1})$ as well as the market tightness θ_t depend on the firm's choice w_{t+1}. For notational simplicity, we write $\theta_t(w_{t+1}) = \theta_t$. $V(y_{t+1})$ is the steady state continuation value when the job is not filled. At a cost k, the firm announces a vacancy and commits to a wage w_{t+1} that it will pay starting next period in the case of a match. Like workers, firms discount the future at rate β. $J(y_t, w_t)$ is the value of a filled job for a firm with productivity y_t when paying a wage w_t:

$$J(y_t, w_t) = y - w_t + \beta[\lambda V(y_{t+1}) + (1 - \lambda)J(y_{t+1}, w_{t+1})]. \tag{5}$$

In the infinite horizon version of the model, we focus on the ergodic steady state where the distribution is time invariant, but individual workers' assets, consumption, and labor market choices evolve. The firm's choices are time-invariant, so $J(y_t, w_t) = J(y_{t+1}, w_{t+1})$ and $V(y_t) = V(y_{t+1})$. In the two-period version of the model, we will use the shorthand notation $U_2 = u(c_{u,2})$ and $E_2 = u(c_{e,2})$, and where for all $t > 2$, $a_t = 0, U(a_t) = 0, E(a_t, w_t) = 0$, $V = 0$ and $J = 0$.

The matching of asset holders to firms is now fully described by the optimization decision of firms of type y to post wages and of unemployed workers of asset holdings a_t to choose which submarkets to enter, together with market clearing. Next, we formalize this in the equilibrium concept. Just like in the standard Becker (1973) assignment problem, the competition by heterogenous agents on both sides of the market is mediated by a hedonic price schedule. In the assignment game, that price is the wage schedule. Here, this equilibrium object is the tightness of each submarket, which in turn is determined by the wage. Like in the sorting problem with directed search in Eeckhout and Kircher (2010), the market clearing condition is adjusted for the fact that match formation is stochastic and dependent on the tightness in each market.

We adopt the equilibrium concept used by Acemoglu and Shimer (1999). To accommodate the two sided heterogeneity of firm productivity and worker assets, we will use the version of their equilibrium adjusted by Eeckhout and Kircher (2010) to allow for two-sided heterogeneity and a continuum of agents. They consider the Acemoglu and Shimer (1999) setup as a large game where each individual's payoff is determined only by her own action and the distribution of actions in the economy, which consists of the optimal choices of each of the individuals in the distribution.[12]

[11] For the firm the type y is fixed, so $y_t = y_{t+1}$. We nonetheless use a time subscript because for the worker the choice of submarket y_t, w_t evolves.

[12] The queue length θ is a function of the distribution of offers P and visiting decisions Q. Written explicitly, θ_{PQ} : $\mathcal{Y} \times \mathcal{W} \to [0, \infty]$ is the expected queue length at each productivity-wage combination (y, w). Then along the support of the firms' wage setting distribution, $\theta_{PQ} = dQ_{\mathcal{YW}}/dP$ is given by the Radon-Nikodym derivative, where $Q_{\mathcal{YW}}$ is the marginal

In line with the literature on directed search (see for example McAfee (1993), Acemoglu and Shimer (1999)), we impose restrictions on the beliefs about off equilibrium path behavior. In the current setup, beliefs about the queue length corresponding to firm or worker choices that do not occur in equilibrium are not defined. Therefore, we define those off equilibrium path beliefs corresponding to the notion of subgame perfection.[13] Firms expect workers to queue up for jobs as long as it is profitable for them to do so given the options they have on the equilibrium path. Formally, this defines the queue length over the entire domain as: $\theta(a,w) = \sup\{\theta \in \mathbb{R}_+ : \exists a, m(\theta)[y - w \geq \max_{y,w \in \text{supp}P} U(a,y,w,P,Q)]\}$. In all other cases, the queue length is zero.

This now permits us to define equilibrium. When time is finite, the equilibrium can be defined recursively starting from an initial asset distribution. In the infinite horizon economy, we solve for the stationary asset distribution. In each period, an equilibrium is a pair of distributions (P,Q) such that the following conditions hold: 1. Worker optimality: $(a_t, a_{t+1}, y_t, w_t) \in \text{supp } Q$ only if it maximizes (2) and (3) for a_t; 2. Firm optimality: $(y_t, w_t) \in \text{supp } P$ only if w' maximizes (4) and (5) for y.

This is a matching problem with a non-linear pairwise Pareto frontier. Existence is established in Legros and Newman (2007) and Kaneko (1982). Jerez (2014) establishes the existence of an equilibrium in a directed search model with a continuum of agents and a general matching technology.

The (measure preserving) market clearing condition is particularly transparent when matching is monotone. Then there is one-to-one matching of a to y, which we represent by a function $\mu : \mathcal{A} \to \mathcal{Y}$. Under positive assortative matching (PAM), $\mu'(y)$ is positive and it is negative under negative assortative matching (NAM). Under PAM high asset workers match with high productivity jobs, and the market clearing condition can be written as:

$$\int_a^{\overline{a}} \theta(s) f_u(s) ds = \int_{\mu(a)}^{\overline{y}} g(s) ds. \tag{6}$$

3 The Equilibrium Allocation

We first analyze a simple two period model. The objective is to provide us with insights into how the per period allocation of asset holders to firms works. We then turn to the infinite horizon model, where we focus attention on the steady state and where we lay the ground for the calibration and policy exercise. For the purpose of the theory results in this section, we assume benefits, vacancy posting costs, and dividends are zero: $b = 0, k = 0, d = 0$.[14] Benefits, vacancy posting costs, and dividends are important for the calibration in the infinite horizon model, but do not add any insights in understanding the mechanism of the equilibrium allocation.

distribution of Q with respect to $\mathcal{Y}$ and $\mathcal{W}$.

[13] Peters (1997) and Peters (2000) provide micro foundations for a version of this model where this assumption is indeed justified as the limit of deviations in a finite game.

[14] Zero dividends to consumers implicitly means that firms are owned by absentee investors.

3.1 The two-period model

We first analyze the decentralized equilibrium allocation in the two period model where all workers are initially unemployed. Let there be an exogenously given initial distribution of assets $G(a_1)$, which caries over to Ra_1 in period 1. With $T = 2$, there is only a consumption/savings-search $(a_2; w_2, y_2)$ decision in period 1. In the final period, consumption is determined by period's savings decision and the outcome of the job search. The value of both employment and unemployment are therefore equal to the utility of consumption in the respective states: $E(a_2) = u(Ra_2 + w_2)$, where $a_3 = 0$ and $U(a_2) = u(Ra_2)$. Then we can then rewrite (2) after substituting for (3) as:[15]

$$U(a_1) = \max_{a_2,y_2,w_2} \{u(Ra_1 - a_2) + \beta [m(\theta_1)u(Ra_2 + w_2) + (1 - m(\theta_1))u(Ra_2)]\}, \tag{7}$$

where the market tightness θ_1 is a function of the choice of submarket (y_2, w_2). The consumption is thus completely pinned down by the savings choice a_2 and the labor market choice (y_2, w_2), i.e., which submarket to search in, resulting in a matching probability $m(\theta_1)$. The expected payoff to a firm y posting a vacancy with posted wage w_2 is (where obviously $y_1 = y_2$)

$$V(y_1) = \max_{w_2} \beta q(\theta_1)(y_2 - w_2), \tag{8}$$

from (4) since $J_2 = y_2 - w_2$, and the continuation value is zero, and where, again, the market tightness θ_1 is a function of the posted wage w_2.

The firm's problem is to set wages w to maximize expected profits $V(y)$. The consumer's problem is to maximize expected utility from consumption while simultaneously making an optimal search decision. We can therefore summarize the joint worker and firm optimization as:

$$\max_{a_2,y_2,w_2} \{u(Ra_1 - a_2) + \beta [m(\theta_1)u(Ra_2 + w_2) + (1 - m(\theta_1)\, u(Ra_2)]\} \tag{9}$$

$$\text{s.t. } V = \max_{w_2} \beta q(\theta_1)(y_2 - w_2), \tag{10}$$

Given $w_2 = y_2 - \frac{V}{\beta q(\theta_1)}$ from (10) we can write this joint optimization problem as a single optimization after substituting for w_2. This is the standard solution method for directed search problems. With the wage w_2 substituted out, optimality now follows from the optimal choice of the queue length θ_1, since the posted wage directed determines the queue length. With risk averse preferences, we can write this problem as a matching problem with a non-linear Pareto frontier denoted by $U(a_1, y, V)$ (in what follows we use $y = y_2$). This denotes the value to the worker when matched with a firm y to which it

[15] We drop the time subscript $t = 1$ of the value functions. The period 2 values are either zero or we substitute them by the period payoff.

leaves the value V, and where the optimal choice is now over (a_2, θ_1) :

$$U(a_1, y, V) = \max_{a_2, \theta_1} u(Ra_1 - a_2) + \beta \left[m(\theta_1) u \left(Ra_2 + y - \frac{V}{\beta q(\theta_1)} \right) + (1 - m(\theta_1))\, u(Ra_2) \right] \tag{11}$$

Then the solution to the maximization problem is $a_2^\star, \theta_1^\star$ and satisfies:

$$-u'(Ra_1 - a_2) + \beta R \left[m(\theta_1) u' \left(Ra_2 + y - \frac{V}{\beta q(\theta_1)} \right) + (1 - m(\theta_1))\, u'(Ra_2) \right] = 0 \tag{12}$$

$$\beta m(\theta_1)' \left[u \left(Ra_2 + y - \frac{V}{\beta q(\theta_1)} \right) - u(Ra_2) \right] + \beta u' \left(Ra_2 + y - \frac{V}{\beta q(\theta_1)} \right) \frac{\theta_1 q'(\theta_1) V}{\beta q(\theta_1)} = 0. \tag{13}$$

The optimal savings behavior and optimal job search simultaneously imply a matching decision. That is, a worker a effectively chooses a firm y. We can now analyze this allocation problem with a non-linear frontier $U(a_1, y, V)$, where a_2 and θ_1 are chosen endogenously. We use the standard solution method for an assignment problem. The worker takes the firm payoff $V(y)$ as given (typically called the hedonic price schedule) and chooses the firm type y that maximizes her expected utility. From the first order condition, the optimal y therefore satisfies $U_y + U_V \frac{\partial V}{\partial y} = 0$. This implies:

$$\beta m u' \left(Ra_2 + y - \frac{V}{\beta q(\theta_1)} \right) \left(1 - \frac{V'}{\beta q} \right) = 0. \tag{14}$$

where the effect of y and V on U through a_2 and θ_1 is zero from the envelope theorem: $\frac{\partial U(a_2)}{\partial a} = 0$, $\frac{\partial U(\theta_1)}{\partial \theta} = 0$ imposed by equations (12) and (13). The details of the derivation of the partial derivatives are in the Appendix.

We want to ascertain under which circumstances there is monotone matching of asset holdings a_1 in job productivities y. This is now a matching problem $U(a_1, y, V)$ where a type a_1 chooses the optimal y, given optimizing behavior regarding a_2 and θ_1. The allocation is denoted by $a_1 = \mu(y)$. Then the total cross derivative of U with respect to a_1 and y is positive provided[16]

$$\frac{d^2 U}{da_1 dy} = U_{a_1 y} + U_{a_1 V} \frac{\partial V}{\partial y} = U_{a_1 y} - U_{a_1 V} \frac{U_y}{U_V} > 0, \tag{16}$$

where we use the first order condition to substitute for $\frac{\partial V}{\partial y}$. This sorting condition can be derived from the the second order condition, and therefore ensures that this solution is also a global maximum. In addition, for a *given distribution of types* it also ensures uniqueness (see Legros and Newman (2007) and Chade et al. (2017)). Therefore, there will be Positive Assortative Matching in types a_1, y provided

[16] As is conventional, we use subscripts for partial derivatives, for example $U_{a_1 y} = \frac{\partial^2 U(a_1, y, V)}{\partial a \partial y}$ is the cross-partial derivative of the value function U with respect to a and y, where $a = a_1$. In other words, equation (16) is short for:

$$\frac{d^2 U(a_1, y, V)}{da dy} = U_{ay}(a_1, y, V) + U_{aV}(a_1, y, V) \frac{\partial V}{\partial y} = U_{ay}(a_1, y, V) - U_{aV}(a_1, y, V) \frac{U_y(a_1, y, V)}{U_V(a_1, y, V)} > 0. \tag{15}$$

$U_{a_1 y} > \frac{U_y}{U_V} U_{a_1 V}$. The next Proposition establishes under which conditions on the primitives (preferences and technology) this is satisfied:

Proposition 1 *Workers with higher initial asset levels a_1 will apply for higher productivity jobs y provided*

$$\frac{u'(c_{e,2}) - u'(Ra_2)}{u(c_{e,2}) - u(Ra_2)} < \frac{u''(c_{e,2})}{u'(c_{e,2})}, \tag{U}$$

Where $c_{e,2} = Ra_2 + y - \frac{V}{\beta q(\theta_1)}$. Moreover, the equilibrium is unique.

Proof. In Appendix. ■

This Proposition establishes under what conditions of the utility function agents with higher levels of assets will choose more risky jobs. In addition to the solution being positively assorted under the condition, the allocation is unique in the two period version. This follows from the fact that the inequality in condition (**U**) is strict and the fact that this is effectively a static problem with exogenous types.

The condition does not immediately allow for a straightforward interpretation, and in the next two results we characterize the properties. First, we show that within the class of Hyperbolic Absolute Risk Aversion (HARA) utility functions, the condition is satisfied whenever absolute risk aversion is decreasing (DARA).

Proposition 2 *Consider the class of utility functions with Hyperbolic Absolute Risk Aversion (HARA):*

$$u(c) = \frac{1-\gamma}{\gamma}\left(\frac{\alpha c}{1-\gamma} + \beta\right)^{\gamma} \qquad \text{where } \alpha > 0,\ \beta + \frac{\alpha c}{1-\gamma} > 0.$$

*Then condition (**U**) holds whenever there is Decreasing Absolute Risk Aversion (DARA): $\gamma < 1$. It holds with opposite inequality when there is Increasing Absolute Risk Aversion (IARA): $\gamma > 1$.*

Proof. In Appendix. ■

A number of results for special cases of the HARA preferences immediately follow, including CRRA, logarithmic, CARA, risk neutrality and the quadratic.

Corollary 1 *Consider the class of HARA utility functions. Condition (**U**) holds:*

1. *under CRRA $u(c) = \frac{1-\gamma}{\gamma} c^{\gamma}$ $(\alpha = 1-\gamma, \gamma < 1, \beta = 0)$ and Log utility: : $u(c) = \log c$ $(CRRA, \gamma \to 0)$;*
2. *with equality under CARA $u(c) = 1 - e^{-\alpha c}$ $(\beta = 1, \gamma \to -\infty)$ and Risk Neutral $u(c) = \alpha c$ $(\gamma = 1)$;*
3. *with opposite inequality under Quadratic utility: $u(c) = -\frac{1}{2}(-\alpha c + \beta)^2$ $(\gamma = 2)$.*

Proof. In Appendix. ■

The results for HARA may indicate that condition (**U**) holds more generally. The answer is partially true. For small differences between the level of consumption when a job is obtained and the consumption of unemployment ($c_e - Ra' = w$ small), we can indeed completely generalize the characterization: when there is DARA, condition (**U**) is satisfied and high asset types choose high productivity jobs. This is proven in Proposition 3. However, for general utility functions beyond HARA and with wages w large, this characterization does not hold. In Example 1 in the Appendix, we show by counterexample that for w large, Decreasing Absolute Risk Aversion (DARA) is not sufficient for the condition to hold.

Proposition 3 *When w is small, condition (**U**) is satisfied for any utility function that exhibits Decreasing Absolute Risk Aversion (DARA), $-\frac{u''}{u'} < 0$, and thus has positive risk prudence, $u''' > 0$. Likewise, it holds with opposite inequality under IARA.*

Proof. In Appendix. ■

Condition (**U**) establishes that there are complementarities in the match value between a firm type y and a worker with assets a_1. In other words, the match value $U(a_1, y, V)$ between types a_1 and y is supermodular, and therefore the equilibrium allocation matches high asset workers with high productivity firms. While there are no technological complementarities (all workers are identically skilled), risk aversion and two-sided heterogeneity generates a *natural preference* complementarity between assets and job productivity.

The implication of this condition is that high asset workers apply for high productivity jobs, they earn higher wages, they have higher unemployment, they consume more and they have higher expected utility. Likewise, high productivity firms post higher wages, they attract higher asset workers, they have higher expected profits and they fill vacancies faster.

3.2 Infinite Horizon

We now consider the stationary equilibrium allocation in the infinite horizon version of the model. The per period allocation problem in the labor market is similar to the one analyzed for the two period model, with the exception of the continuation value. We derive a condition similar to the (**U**) condition, but now for the infinite horizon economy. Note though that this condition now involves value functions, i.e., endogenous objects and not just primitives such as utilities and consumption bundles.

Quantitatively we analyze the parameter configuration $\beta R < 1$. This implies that while employed, the consumption-savings decision varies with time, and we can thus incorporate precautionary savings by the employed who anticipate the possibility of becoming unemployed.[17] We show the following

[17] Traditionally, models such as ours with infinitely lived agents have been solved assuming $\beta R = 1$ together with $\lambda = 0$ (see amongst others Acemoglu and Shimer (1999), Shimer and Werning (2008), Hopenhayn and Nicolini (1997); the notable exception is Krusell et al. (2010)). Under the assumptions of this special case, the continuation value of employment can be derived the closed form, which we do in the Appendix.

result:

Proposition 4 *Then workers with higher initial asset levels a will apply for higher productivity jobs provided*

$$\frac{E_{a_{t+1}}(a_{t+1}, y) - U_{a_{t+1}}(a_{t+1})}{E(a_{t+1}, y) - U(a_{t+1})} < \frac{E_{a_{t+1}}(a_{t+1}, y)}{E_{w_{t+1}}(a_{t+1}, y)} \qquad (\mathbf{U}_\infty)$$

Proof. In Appendix. ■

The result in Proposition 1 thus generalizes to the case with an infinite horizon, albeit with two important caveats. The first caveat is that we cannot derive conditions on the primitives. In the next section we compute the equilibrium allocation with the corresponding ergodic distributions, and we verify whether along the equilibrium allocation condition $\mathbf{U}_\infty$ is satisfied.

The second caveat is that even though the equilibrium allocation satisfies positive sorting, we cannot guarantee that that positively assorted equilibrium allocation is unique.[18] While condition (16) guarantees uniqueness of the match surplus for a *given* surplus, the match surplus is *endogenous* and depends on the distribution of assets. Potentially there could therefore be multiple distributions of assets that give rise to different equilibrium actions – most notably different savings decisions – resulting in multiple ergodic asset distributions. Each distribution with corresponding equilibrium actions is a self-fulfilling prophecy. This type of multiplicity of steady states is common in other models with endogenous inequality (see amongst others Banerjee and Newman (1991), Banerjee and Newman (1993), Mookherjee and Ray (2002), and Mookherjee and Ray (2003)) and models of random search with two-sided heterogeneity (see Burdett and Coles (1997) and Shimer and Smith (2000)).

Unfortunately, with a continuous distribution of assets, there is no hope to find analytical solutions.[19] In absence of analytical solutions, in the quantitative analysis we therefore perform different exercises to ascertain whether the quantitative solution is unique or whether there is multiplicity. First, we start the numerical exercise from initial values of the asset distribution at opposite extremes. If there are multiple steady states, those extreme initial values are more likely to converge to different allocations. Second, we perturb the parameter estimates around the estimated equilibrium values to verify whether the equilibrium allocation is locally unique. In none of these robustness exercises we have found evidence of multiple steady states.

Duration Dependence. Our model has novel implications for duration dependence on wages. Under the canonical directed search model without precautionary savings, identical workers who apply to high wage jobs necessarily face longer unemployment duration to make them indifferent with low wage jobs that have shorter unemployment. This positive duration dependence on wages is considered

[18]We are grateful to one of the Referees for pointing out that multiple steady state equilibria are possible.

[19]Even in the case of simple examples with two types as in Burdett and Coles (1997) it is extremely hard to find analytical conditions for uniqueness.

counterfactual as in the data wages are found to exhibit negative duration dependence: workers with lower unemployment duration have higher wages.

In our directed search model with precautionary savings, we find that high wages jobs have shorter unemployment duration, the opposite duration dependence compared to the canonical model of directed search. This is because the hazard rate of finding a job is not constant but increasing: as workers are unemployed longer, they run down their assets and therefore apply to jobs with higher matching probability (and lower wages and lower productivity). In other words, for a given worker, in our model there is now negative duration dependence: workers with shorter unemployment duration tend to have higher wages (and also higher productive jobs). We state this finding formally, which follows immediately from Proposition 4:

Proposition 5 (Negative Duration Dependence) *Under condition ($\mathbf{U}_\infty$), the job productivity y and the wage w of a formed match decrease in the duration of unemployment of a given worker.*

This results is important because it addresses one of the main shortcomings of the canonical directed search model, namely the counterfactual prediction of positive duration dependence. What this result shows is that the precautionary savings are a force towards negative duration dependence.

This negative duration dependence holds for a given cohort of workers. Instead, within a given unemployment duration there is heterogeneity in assets. For those workers with the same unemployment duration in the cross section there is still positive duration dependence, for the same reason as in the canonical directed search model without precautionary savings: workers with high asset levels apply for high wage, high productivity jobs with high unemployment duration. In the quantitative exercise that we analyze below, we set out the answer the quantitative importance of each of the two effects.

4 Quantitative Exercise

We will now analyze the full model with ergodic asset and firm productivity distributions as well as with non-stationary savings by individual workers while unemployed and employed. The objective is to study welfare and the impact of unemployment benefits. The key feature of the model is the sorting of unemployed workers into different productivity jobs depending on their assets, just as in the simplified two period model. Now, with an infinite horizon, unemployed workers run down their assets while searching for a job in order to smooth consumption. In the process, as their assets decrease, they apply to the low productivity jobs with higher matching probability as a precautionary search motive. When on the job, they face a probability of exogenous separation. Anticipating the possibility of unemployment, workers therefore accumulate assets while working. This gives rise to a pattern of individual asset fluctuations in order to endogenously insure against unemployment risk.

Computationally, we derive the ergodic distribution of assets in this economy. The ergodic distribution is the time-invariant aggregate distribution where the asset holdings of individual workers are time-varying. In other words, individual changes cancel out in the aggregate.[20] It should be pointed out that a major technical innovation of our computation is the fact that the employed have a non-stationary policy function that reflects their precautionary savings behavior, i.e. $\beta R < 1$, unlike much of the exiting literature.[21] The non-stationarity of the savings decision of the employed is particularly demanding considering the endogenous sorting of workers to jobs with different productivities. It requires that the endogenous distribution of asset is a state variable and unlike most directed search models with savings, our problem is therefore not Block Recursive.

The detailed algorithm is explained in Appendix B. Broadly speaking, it works as follows. An efficient algorithm for any given level of benefit is to make a guess on (i) the dividend, (ii) the tax rate, (iii) the distribution of workers' assets (both employed and unemployed) and firms posting vacancies, (iv) the value of employment and unemployment and (v) the labor market clearing condition determining the productivity cut-off level of entry as well as its measure. Then we take the following six steps: 1. Given the distribution of unemployed workers and vacancies, the algorithm first sorts the top workers and firms in the first submarket and finds the job finding rate and wage for this submarket; then it finds the value of a vacancy in the next submarket, using the first order condition of the allocation problem with respect to productivity, and again calculates the job finding rate and the wage. 2. We continue at each subsequent submarket until we reach the boundary of at least one distribution and check if the labor market clears; If not, we change the cut-off point of firm entry. 3. We solve the consumer's dynamic non-linear programing problem. 4. We check the convergence of the distribution of firm types and worker assets (both employed and unemployed) and update them. 5. We check if the total tax revenue and benefits paid are equal. 6. We check whether our guess on the dividend is correct and update.

Our objective is to study the role of policy on the equilibrium allocation and on welfare. As unemployment insurance changes, both the incentives to save and accumulate assets and the job search behavior change. This also affects the allocation of workers to jobs of different productivities. Different asset holders have different preferences for insurance and therefore for benefits. We decompose the channels through which unemployment insurance affects the workers' welfare across the distribution.

The remainder of this section has five parts. First, we calibrate the baseline model with suitably chosen parameters and report its basic properties. Second, we analyze the equilibrium effect of Unemployment Insurance benefits. Third, we perform the welfare analysis and find the optimal UI policy. Fourth, we analyze the effect of UI benefits on worker productivity. And fifth, we consider severance pay as an alternative policy and compare its welfare properties relative to our the UI policy.

[20] As we stated following Proposition 4, there is no guarantee that the ergodic distribution is unique. Below, we report robustness checks to help establish that under our parameter configurations no other ergodic steady state exists.

[21] For computational reasons, we assume that there is no capital investment and that the interest rate is exogenous. Introducing both aspects does not affect the basic features of the directed search mechanism and its interaction with the consumption-savings decision, which is at the heart of our paper. In Appendix E we analyze a version of our model with capital investment and an endogenous interest rate.

4.1 Benchmark Calibration

We set one period to be a month. The utility function is $u(c) = \log(c)$, the output produced is y and the numeraire in this economy is one unit of output. Following Menzio and Shi (2011), we use the Constant Elasticity of Substitution (CES) matching rate function, $m(\theta) = \theta(1+\theta^{\gamma})^{\frac{-1}{\gamma}}$ and $q(\theta) = \frac{m(\theta)}{\theta}$. In the computational exercise we set the borrowing limit $\underline{a} = -1$ and the upper bound of the asset distribution to be larger than the equilibrium support of the asset distribution.

The interest rate r and the discount rate $1 - \frac{1}{\beta}$ determine the workers' savings decisions. Following Krusell et al. (2019) we set the discount factor β to be 0.997 which corresponds to approximately 3% annual discounting. The monthly interest rate is 0.002, equivalent to an annual interest rate of 2.5%. This implies that the annual median of wealth to wage ratio in the model is 0.48. This is comparable with the same figure from Panel Study of Income Dynamics (PSID) 2013 which is 0.47.

We set the flow value of the unemployment benefit $b = 0.8$, which in equilibrium is approximately 40% of average wages at steady state similar to Shimer (2005). We use the job finding and separation rates reported by Eeckhout and Lindenlaub (2019) and calculated from the Current Population Survey (CPS) to set the separation rate and to discipline the job finding rates. The monthly separation rate is 1.7%, implying an average employment duration of nearly 5 years. Then, we pick the elasticity of the matching function (γ) to target the average job-finding probability of 25% observed in the CPS. This implies an average unemployment duration of four months in the model which is similar to the average of 4.5 months in the data in the last four decades.[22] We set the cost of posting a vacancy (k) at 0.5, which means that at steady state the cost of a vacancy is 21% of the average productivity of active firms (see Shimer (2005)). This implies an unemployment rate of 6.4% at the steady state which is similar to the US unemployment rate between 1980 and 2020.[23]

We assume productivity y of potential entrants is uniformly distributed over $\mathcal{Y} = [2, 2.5]$. In equilibrium, only those firms with productivity $y \geq y^{\star}$ enter the market, where $y^{\star} \in \mathcal{Y}$ is determined endogenously.[24] The choice to parameterize the measure of firms at each productivity level as uniform is motivated by the employment distribution over industries in the PSID. Once we control for the probability of filling vacancies at steady states for different levels of productivities, this distribution is approximately uniform.[25]

Table 1 summarizes the externally chosen parameters and Table 2 reports the key endogenous

[22] https://fred.stlouisfed.org/series/UEMPMEAN

[23] The US unemployment rate between 1980 and 2020 is 6.2%: https://fred.stlouisfed.org/series/UNRATE

[24] In the case with homogenous firms, the free entry condition implies that the value of posting a vacancy is zero at steady state. However, with two-sided heterogeneity, free entry implies that the value of posting a vacancy for firms at the threshold is zero and that firms above the threshold have positive values for posting vacancies. Moreover, a change in unemployment benefit when there is no heterogeneity among firms only affects the number of vacancies created by firms while with heterogenous firms it affects the number of vacancies as well as the quality of vacancies by shifting the productivity threshold.

[25] To choose the domain of the productivities, we have been conservative and have chosen a limited domain. Ours is a model with identical workers, so we are modeling productivity for workers with the same education, experience and demographic characteristics (see Bonhomme et al. (2019)). A wider domain exacerbates our results even further.

	Definition	Value
β	discount factor	0.997
r	interest rate	0.002
b	replacement rate	0.80
λ	exogenous separation	0.017
k	cost of vacancy	0.50
γ	elasticity of matching function	0.40
$\underline{a}$	borrowing constraint	-1

Table 1: Externally Calibrated Parameters

moments in the ergodic steady state equilibrium. The unemployment rate of 6.4%, unemployment duration of four months as well as the average monthly matching probability are in line with the US averages in the last four decades. The median of wage to asset ratio is 4.83, close to the same figure from PSID 2013. At the steady state the total dividend pay-out is 7% of total production similar to the same figure in the US at 2019.

Further, the elasticity of the job finding rate to the market tightness is 0.36, which lies within the range of empirical estimates by Pissarides (2009) and Shimer (2005).[26] Moreover, the correlation between assets and the probability of job-finding is -0.89, indicating the lower hazard of job finding for richer unemployed workers. On average, a one percentage rise in assets is associated with a 0.04% decline in the probability of job finding.

u	unemp. dur.	avg. $m(\theta)$	avg. w	avg. a	med. $(\frac{a}{w})$
6.4%	4.06	24%	2.19	12.03	4.83

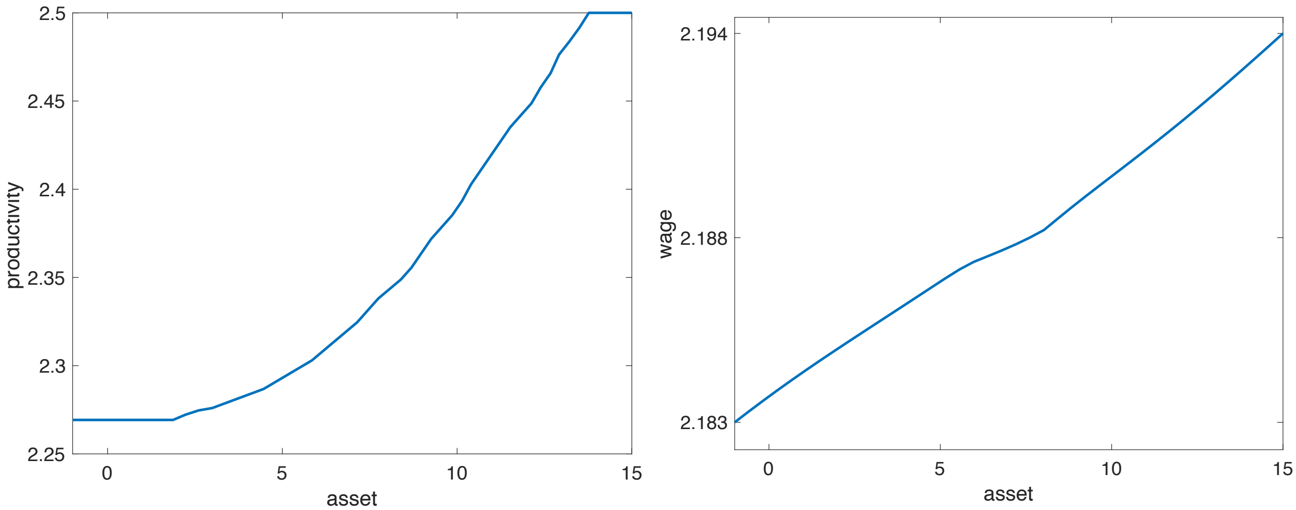
Table 2: Endogenous Outcomes

There is positive assortative matching between workers' asset holdings and firms' productivity: in equilibrium workers with a higher level of assets are matched with more productive firms.[27] Figure 1a shows the allocation of workers to firms in the labor market. There is relatively more mass at the bottom of the asset distribution for unemployed workers compared to employed workers, and less mass at the bottom of the productivity distribution, consistent with equation (6). The market clearing condition implies that all workers are allocated to submarkets while firms below a productivity threshold are staying out of the market. This threshold is obviously sensitive to different parameterizations of the model. In particular, below we will study the impact of a change in unemployment benefits on the

[26] Shimer (2005) reports 0.27 and Pissarides (2009) finds 0.50.

[27] From Proposition 2 we know that under log preferences there is indeed positive sorting in the two period model. Because we cannot solve the general model analytically, we guess the allocation is positively assorted and verify ex-post that the match surplus along the equilibrium allocation is indeed supermodular, and the condition in Proposition 4 is satisfied.

threshold and therefore on job creation. A higher threshold means more firms stay out of the market and hence fewer jobs are created.

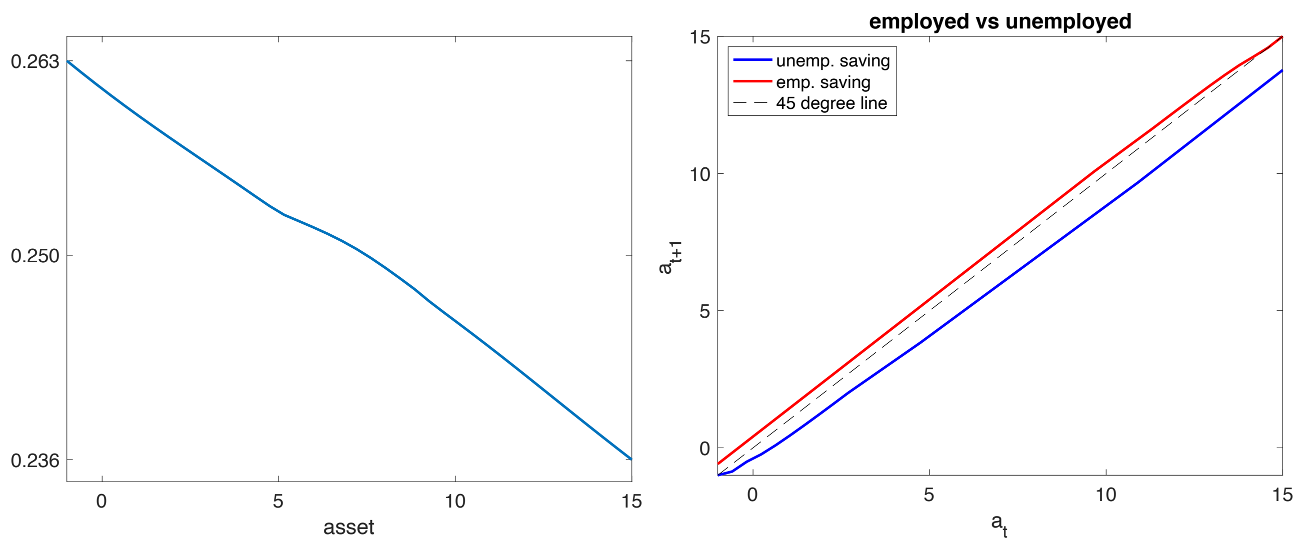


(a) Allocation of firms and workers in the labor market

(b) Wages by asset level at hiring

Figure 1: Equilibrium allocation and wages

Figure 1b depicts equilibrium wages for different asset levels. Firms with more productive jobs post higher wages. This decreases the vacancy to unemployment ratio θ and allows them to fill the vacancy with higher probability. Workers with more assets apply for the high wage jobs because they are able to insure better against unemployment. Their assets allow them to maintain a higher level of consumption so they can afford to apply for riskier jobs.



(a) Job finding probability $m(\theta(a))$

(b) Savings decision a_{t+1}

Figure 2: Policy functions of the unemployed

Quantitatively, the key aspect is the role that assets play in the productivity of *equally skilled* workers. Workers with higher assets apply for jobs with a substantially higher productivity than those with low assets (2.50 versus 2.26, Figure 1a). The reason why they are able to get those better jobs is that they take a substantially longer time to find a job than those with low asset holdings. As shown in Figure 2a, the monthly matching probability decreases from 26.5% for the low asset unemployed workers to just over 23% for those with high asset levels, or a 10% fall in the monthly job finding probability.[28] At any level of assets, unemployed workers deplete their assets and subsequently adjust their job search strategy. If they do not find a job this period and deplete their asset stock further, next period they apply for lower productivity jobs which they can get with a higher probability. This dependence of the job search decision on assets is absent in the basic random search model without search intensity: with random search, the probability of finding a job is the same for all workers regardless of their asset holding.[29]

This channel is also absent in models with homogenous firms, which has important implications for the role of UI benefits. A change in unemployment benefits only affects the measure of vacancies when all firms have identical productivity. In contrast, in our framework, a change in UI not only affects the measure of jobs created but also the productivity distribution of filled jobs and the productivity level in the economy even when the production function has constant returns to scale.

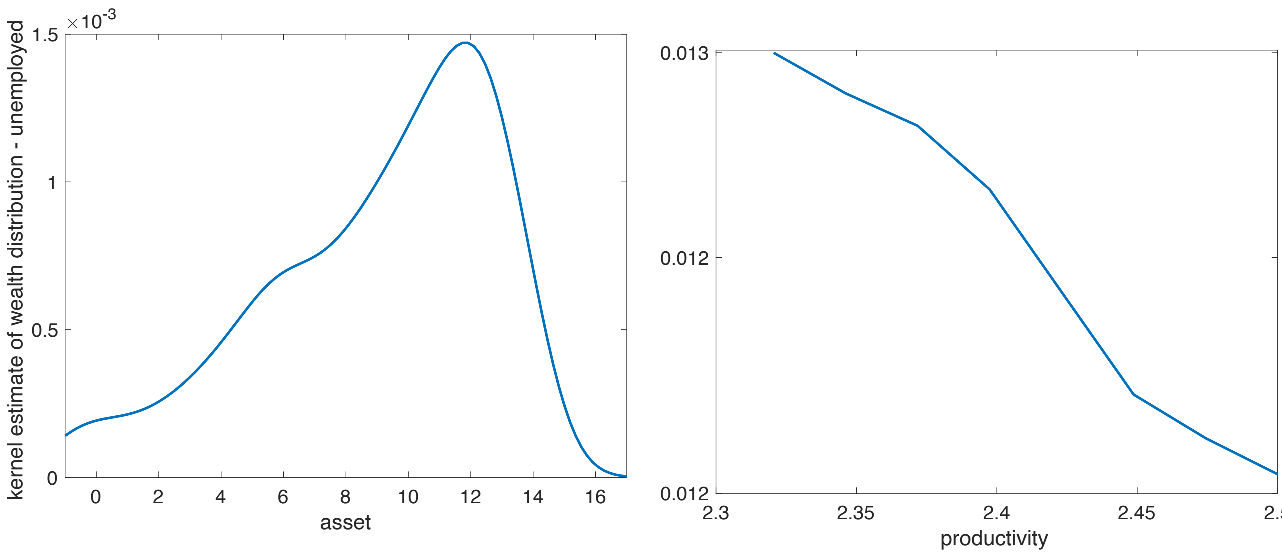
The endogenous matching probability explains why the wage function is increasing whereas it is mostly flat in the basic random search model. At first sight, the derivative of the wage function appears small. However, since the average duration of employment is around 59 periods (5 years), these small wage differences translate into big income differences over the duration of employment. In other words, workers choose submarkets with different probabilities of job finding, and different wages for the whole duration of employment. This is reflected in the fact that the equilibrium value of employment $E(a)$ shows large variation.

Interestingly, the dynamic nature of the problem now implies a time-varying job choice decision. A worker who fails to become employed sees their assets gradually deplete ($a_{t+1} < a_t$). But, the optimal search decision dictates application to less productive, lower wage jobs when assets are lower. As a result, over the duration of unemployment, workers will gradually apply for less productive, lower wage jobs that they get with higher probability. Instead, while employed, they gradually increase their assets. In Figure 2b, we see that savings by the employed is higher than that of the unemployed, and that the unemployed always deplete their savings (a_{t+1} is below the 45 degree line). The employed with low asset levels accumulate assets.

[28] The difference in job finding probabilities between low and high asset holders is higher for low levels of UI benefits that for high benefits. For instance, at $b = 0$ the monthly matching probability decreases from 36.6% to 27.7%.

[29] Extensions of the canonical random search model can also generate the dependence of job finding on asset holdings of unemployed workers. For instance, endogenous search effort may depend on asset holdings and therefore unemployed workers with different wealth holdings can have different probabilities of job findings in a random search model with search effort. We analyze this extension in Section C of the Appendix.

The probability of job finding is significantly lower for high asset holders. Those who are unlucky and do not find a job run down their assets in order to smooth consumption. In this process, they gradually apply to lower productivity jobs. As a result of this endogenous job finding probability, workers move down in their asset holdings during unemployment. This results in a stationary asset distribution for the unemployed that is depicted in Figure 3a.



(a) kernel estimate of asset distribution unemployed workers $f_u(a)$

(b) Productivity distribution vacant firms $g(y)$

Figure 3: Ergodic distributions (Densities)

On the firm side (Figure 3b), we observe a fatter left tail for the stationary distribution of firms posting vacancies compared to the distribution of firms in the population, which is uniform. High productivity firms have a higher option value of filling a vacancy, so they increase the probability of filling the vacancy by offering higher wages to the unemployed. Therefore more productive firms leave the pool of firms with a vacancy faster than less productive ones. As a result, in the steady state there are fewer high productive firms searching. In addition to the endogeneity of the vacancy distribution, also the marginal firm $y^\star$ is endogenous. This cutoff is thus a measure of job creation. Below when we analyze the impact of UI policy, we investigate how the equilibrium allocation (including job creation) is affected by unemployment benefits.

Duration Dependence. We now show quantitatively that the directed search model exhibits negative duration dependence. Workers with higher wages have shorter unemployment duration, as demonstrated in Figure 4. This finding reconciles the directed search model with one of the most robust facts regarding unemployment dynamics. Let us dig deeper and attempt to uncover how consumption smoothing leads to negative duration dependence, thus overturning the positive duration dependence inherent in directed search.

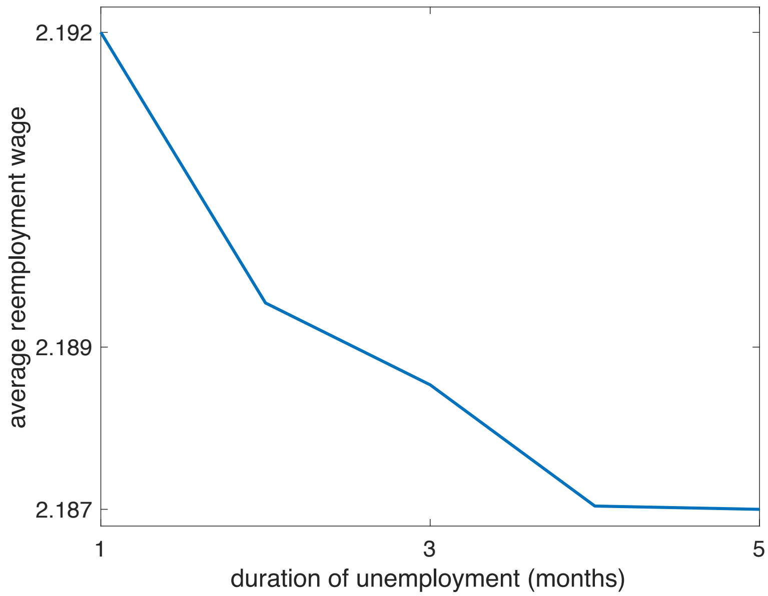
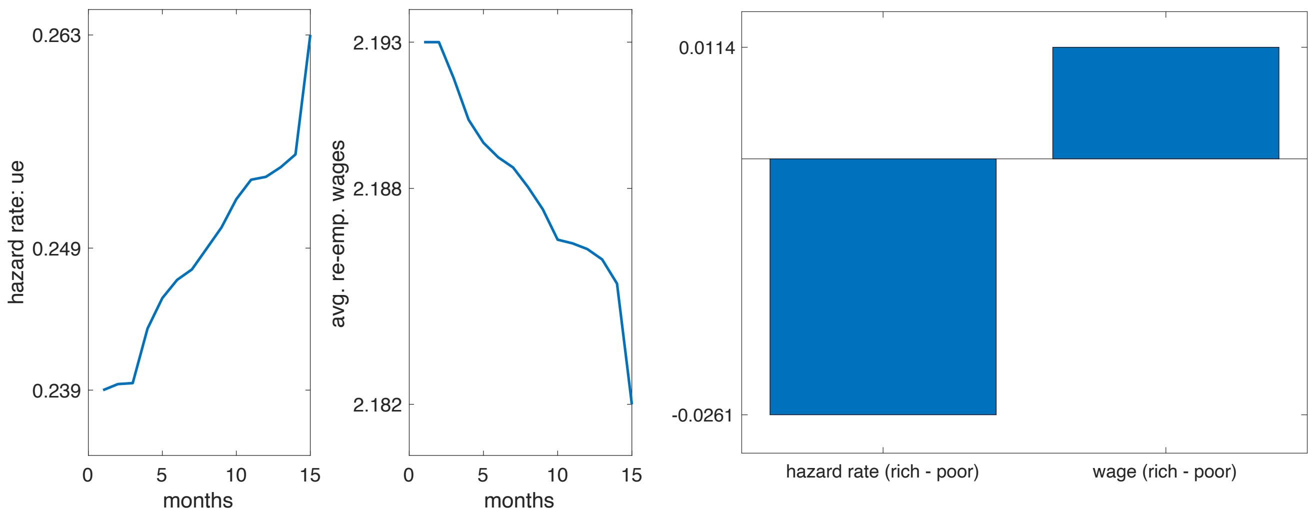


Figure 4: Unemployment duration & re-employment wages

In order to decompose the two opposing effects of negative duration dependence due to consumption smoothing and positive duration dependence due to directed search, we construct a simulation exercise that follows a cohort of newly unemployed workers with identical wealth. As we establish in Corollary 5, the likelihood of finding a job changes over time due to consumption smoothing. If two workers with the same wealth get unemployed at the same point in time, they have the same dissaving behavior but, the one who gets lucky and leaves unemployment first has a shorter unemployment spell as well as higher re-employment wages. This is because at the point the first to leave gets a job, they have depleted less assets due to shorter unemployment duration and therefore apply for higher productivity jobs with higher wages and lower job finding probabilities. Figure 5a depicts the job finding rate and wage for the cohort of workers who all initially entered with same wealth. We then keep track of the unemployment duration as well as their re-employment wages and hazard rate of leaving unemployment (the UE transition rate). The correlation coefficient between the hazard rate (which is the inverse of unemployment duration) and re-employment wages is -0.83.

However, this is not the full story. Just like in the canonical model of directed search, there is still positive duration dependence across workers in a given cohort. This implies that the high asset workers who seek higher wages also have a higher unemployment duration. This comes from the workers' indifference condition implying that better paid jobs attract more applicants and therefore it is harder for workers to obtain those jobs, while it is easier for firms to fill them. Figure 5b shows the difference between the rich and the poor in the hazard rate and the wage in the cross section. The rich have lower job finding rates and higher wages, which leads to positive duration dependence.

As illustrated in Figure 4, we find that the overall duration dependence that combines these two



(a) Across time

(b) Across workers

Figure 5: Duration dependence.

effects is negative. Unemployment duration is declining in wages, which establishes that in our calibration the consumption smoothing effect across time dominates the canonical directed search effect. This is consistent with the econometric literature on duration dependence (for a recent paper see Ahn and Hamilton (2019)) that finds that selection plays a dominant role over direct duration dependence such as skill depreciation. Typically there, the selection is explained by skill heterogeneity – workers with higher productivity find jobs faster. Here, we show that heterogeneity in asset holdings is an alternative determinant for the job finding rate and hence selection.

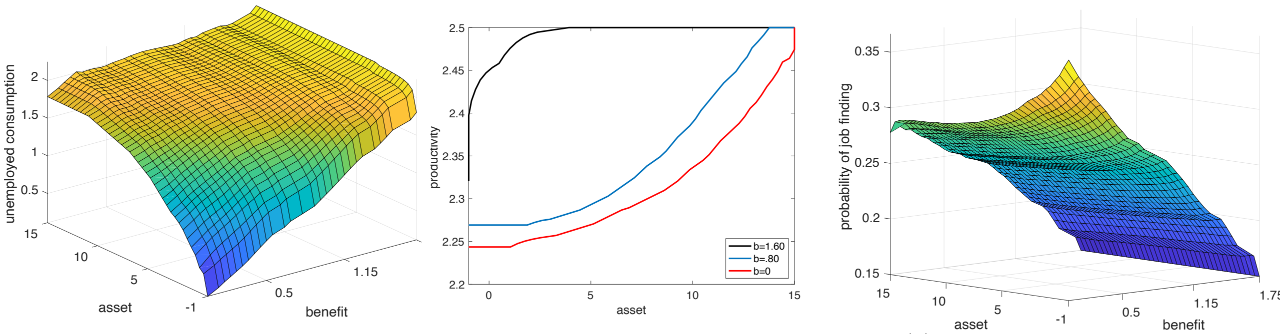
4.2 Equilibrium effects of Unemployment Insurance

We now study the impact of different unemployment benefits on the equilibrium allocation. In the absence of complete markets to insure the employment risk, we ask how changes in the government mandated unemployment insurance policy that is financed with income taxes affects welfare.[30] The direct impact of UI is that it allows workers to smooth consumption, as well as that it allows workers to apply for more productive jobs with lower job finding probabilities. However, UI will also affect welfare through various general equilibrium channels. In the first place, higher unemployment insurance reduces the firm's share of the match surplus. With a higher outside option, workers command a higher wage. This reduces job creation as only firms with higher productivity enter the market to post vacancies. This mechanism is similar to the one in Krusell et al. (2010) with random search, though now with two-sided heterogeneity. A change in unemployment benefits also moves the productivity threshold above which firms enter the market while with homogenous firms it only affects the measure of job

[30]Throughout we assume the budget is balanced.

openings but not the quality of jobs.

In our framework with heterogeneous productivity and sorting, direct insurance against unemployment affects the distribution of unemployed workers by influencing their saving decisions and therefore their allocation to jobs of different productivities. Guaranteed higher unemployment benefits, workers save less while employed, and as a result they hold fewer assets while unemployed. In addition to the savings decision, benefits also affect the workers' job search behavior. Since workers with different asset levels direct their search to firms of different productivity, higher benefits increase the unemployment rate (through a reduction in the matching probability) as well as increase the productivity of jobs that workers apply to. With less necessity to use their own assets for self insurance because of higher benefits, workers are more willing to take risk and will increasingly direct their search towards high productivity jobs that pay higher wages at the expense of lower matching probabilities.



(a) Consumption of unemployed (b) Allocation for different benefits (c) Job finding probability

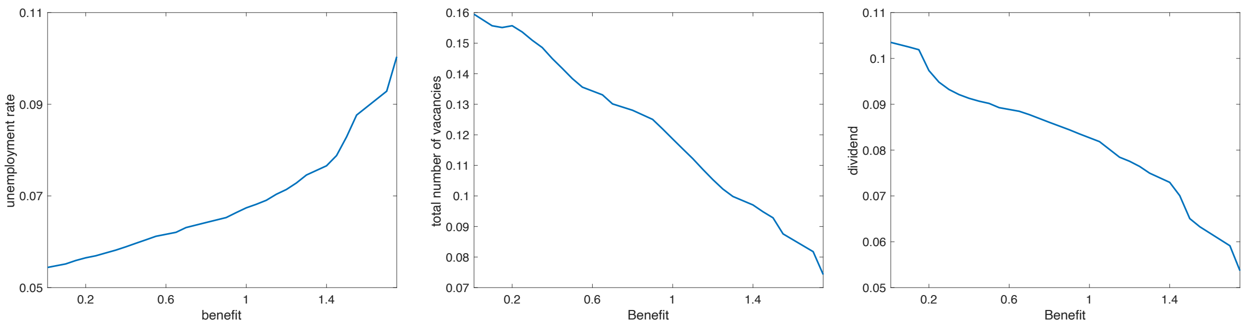
Figure 6: Consumption, equilibrium allocation, and job finding probability for different levels of benefits.

The general equilibrium effect of unemployment benefits are made explicit in the following series of Figures where in the benchmark economy we vary the benefits b between 0 and 1.75.[31] First, consider the impact on consumption of the unemployed (Figure 6a). For all asset holders, equilibrium consumption of the unemployed increases in benefits. The effect however is much more pronounced for the low asset holders. In fact, those with assets close to the borrowing constraint nearly exclusively consume the entire benefits. For the high asset holders, benefits have a much more moderate impact on consumption.

Figure 6b illustrates the impact of benefits on the job search behavior and the resulting equilibrium allocation. When benefits are higher, all workers direct their search to more productive, high paying jobs. As a result, for all asset levels, the allocation of assets to productivities shifts upwards as benefits increase. Moreover, higher levels of benefits not only shift the allocation function to the left (more productive jobs), but it also increases the entry threshold of firms. Moving from the laissez faire economy with zero benefits to an economy with a replacement rate of 80% increases the productivity

[31]The average wage is endogenous, and in our simulated economies this range of benefits corresponds to the range of 0% and 85% of average wages. Recall that in all counterfactual economies the government budget is balanced (total benefit is equal to total tax) and the firm dividends are distributed uniformly across all workers.

threshold of jobs by 3.5%, and increases the aggregate productivity of jobs by 2.5%. As benefits increase the productivity of the jobs, they also increases the competition for jobs and hence decrease the job finding probability (Figure 6c). This decrease is much more pronounced for the low asset holders. Benefits induce them to compete for higher productivity jobs.



(a) Unemployment rate (b) Total number of vacancies (c) Firms' dividend

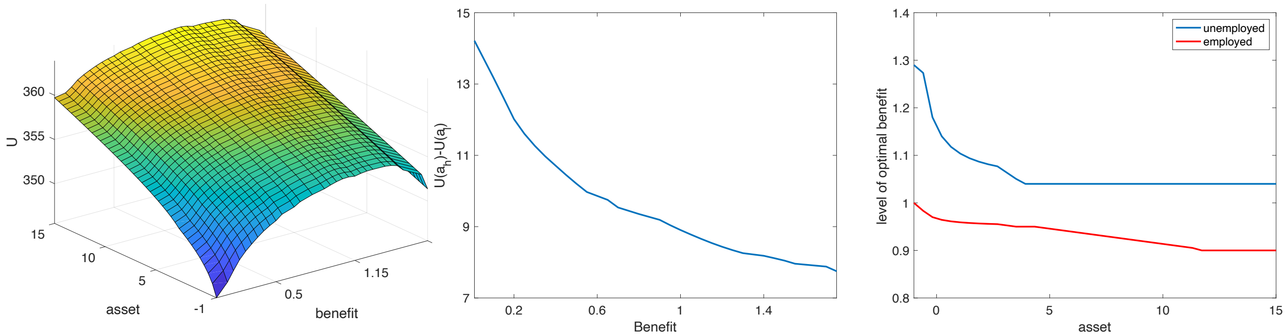
Figure 7: Equilibrium unemployment, vacancy creation and dividends for different levels of benefits.

Not surprisingly then, the impact of increased benefits is an increase in aggregate unemployment (Figure 7a). The unemployment rate goes up by almost 4 percentage points as benefits increase from 0 to 1.75. At the same time, the number of firms entering the market decreases only modestly: the cutoff $y^\star$ goes from 2.26 to 2.34. At the same time, there is a huge decline in the average number of vacancy rate in equilibrium, going from 0.16 to 0.08 (Figure 7b). Firms leave the market faster (because θ, the ratio of vacancies to unemployed searchers has fallen), hence the total equilibrium number of vacancies drops by half. The effect on job creation is therefore big. Moreover, a rise in UI means that workers have higher outside options and that increases wages which in turn reduces workers' consumption from firms dividends. The fall in the firms' dividend as a result of rise in UI is depicted in Figure 7c.

Higher benefit levels clearly pull the value of unemployment in opposing directions: search for better jobs and more consumption smoothing on the one hand, but lower vacancy creation, lower job finding rates and lower dividends on the other hand. To evaluate the overall impact, we look at the option value of unemployment as a function of assets and benefits. This is illustrated in Figure 8a.[32]

A rise in unemployment benefits affects the value of unemployment in the following ways: i) increasing consumption; ii) decreasing job finding probability; iii) decreasing firm entry; iv) decreasing dividends; and v) increasing gross wages.[33] While effect i) increases the value of unemployment, the next three are having the opposite effect. Higher benefits means more insurance and consumption smoothing during unemployment, but when UI goes up it also means that workers tend to apply for better paying jobs with a lower job finding probability. Moreover, higher benefits imply less entry of firms which contributes further to the lower job finding rates in the aggregate, and they imply higher

[32]Given log preferences, the variation in utility is nominally small, even if assets and benefits drop to zero.
[33]All this implies a reduction in net wages when the threshold of entry for firms in not changed.



(a) Value of unemployment

(b) Difference value unemployment

(c) Optimal Benefit by Assets

Figure 8: Value of unemployment.

wages because workers have a higher outside option which in turn reduces the surplus of firms and therefore the dividend workers receive.

The change in the value of unemployment depends on the level of asset holdings. While $U(a)$ has an inverted U-shape for all levels of asset holdings, the maximum value of unemployment is achieved at higher levels of UI for low asset holders compared to rich workers. However, the effect of benefits is much more pronounced for the asset poor unemployed workers. Because they have a high marginal utility of consumption, the insurance effect of unemployment benefits is much stronger. Figure 8b depicts the difference in value of unemployment for a high asset unemployed worker compared to a low asset holder for all benefit levels. When UI increases, this difference shrinks because high asset holders need less insurance while they suffer more from lower probabilities of job findings.

While it may not be immediately obvious from inspecting the three-dimensional figure, the value of unemployment attains its maximum at an interior benefit for the entire asset domain of the unemployed. Figure 8c depicts the level of benefits at which the value of unemployment and employment is maximized for each level of assets.

Low asset holders prefer higher benefits, and those preferred benefits are decreasing in assets. The unemployed with the highest assets prefer lower levels of benefits because they have a relatively low marginal utility of consumption. The negative impact of being taxed after becoming employed as well as the lower probability of job finding dominate the little extra marginal utility of consumption during unemployment and therefore they prefer lower levels of benefits. However, if these workers do not find jobs and deplete their asset during unemployment, they end up preferring higher levels of benefits when their assets run down and their marginal utility of consumption goes up. In other words, with fewer assets the relative importance of higher consumption increases compared to the probability of job finding. Figure 8c also plots the optimal benefit for the employed. At each levels of asset holdings, employed workers prefer lower levels of benefits. For them the negative impact of the taxes has a bigger impact relative to the positive insurance effect of the benefit (in the case of losing their jobs) compared to unemployed workers. Interestingly, as they build up their asset stock during employment, their need for external insurance falls and therefore they prefer lower levels of benefits.

Overall, this suggest that for high asset holders, the allocation and probability of job finding effect dominates the consumption smoothing effect. For workers who already have a high level of assets, an increase in UI does not affect their marginal utility of consumption much while it considerably reduces their probability of job finding. In contrast, low asset holders have high marginal utility of consumption. As mentioned, the highest unemployed asset holders prefer some benefits because life is so dire without any assets or benefits, and even the high asset holders have a positive probability of reaching that outcome.

4.3 Welfare

We now ask what the impact of UI benefits is on overall welfare. To study the welfare impact of a change in unemployment benefits, we compare steady states with different levels of UI. We measure welfare gains or losses by computing the percentage change in life time consumption required to give workers the steady state average lifetime utility. In our welfare analysis, we follow Krusell et al. (2010) and fix the distribution of workers' asset holdings at the benchmark economy. This implies that to compare the counterfactuals economies with the benchmark, we move all workers to a different economy which has a different level of UI but is otherwise identical to our benchmark, and measure the consumption losses or gains of workers across the asset distribution. We hold fixed the distribution of assets in the welfare calculation so that welfare is always compared from the perspective of the same agents. We can thus isolate the welfare effect from a change in the distribution of asset holding.[34]

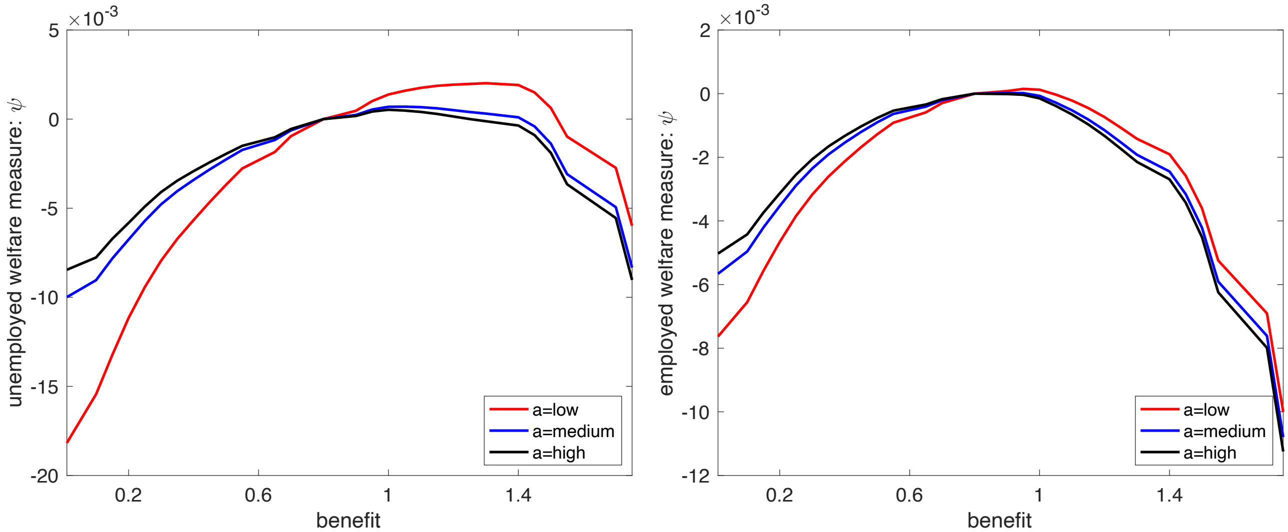
Denote our welfare measure in these comparisons by ψ. c_t is the consumption in the benchmark economy and $\hat{c}_t$ is the consumption in any of the counterfactual experiments.[35] Then, the welfare is calculated satisfying the following condition:

$$\mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t\log((1+\psi)c(a_t))\Big] = \mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t\log(\hat{c}(a_t))\Big]. \tag{17}$$

Figure 9 depicts the welfare measure for an unemployed and employed worker with low (P25), medium (P50) and high (P75) asset holdings at different economies. The net gain or loss of changing UI is heterogenous across the distribution of asset. The welfare of all unemployed workers is inverted U-shaped in benefits (Figure 9a). They all gain from moving to an economy with a higher level of benefit up to a certain level of UI and then their utility falls when UI increases further. In addition,

[34] We have repeated the entire welfare analysis with endogenous asset distributions, and find similar results. If anything, the qualitative findings are more pronounced. The results are available upon request.

[35] The value of consumption in the benchmark economy is $V_i = \mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t\log(c_i(a_t))\Big]$, and in the counterfactual economy is $\hat{V}_i = \mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t\log(\hat{c}_i(a_t))\Big]$ where $i \in \{u, e\}$, where the expectations operator is taken over the labor market uncertainty. The welfare gain or loss, ψ, can be calculated as $\psi_i = \exp[(1-\beta)(\hat{V}_i - V_i)] - 1$.



(a) unemployed workers

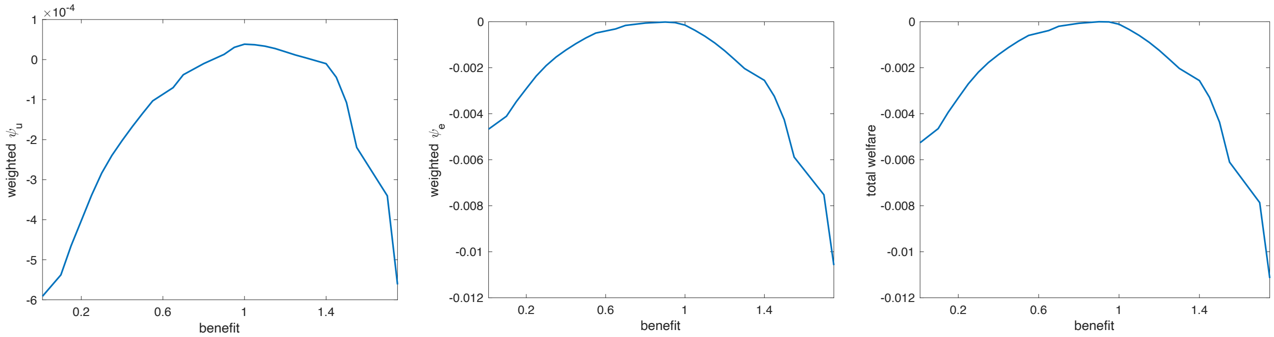
(b) employed workers

Figure 9: Welfare measure: ψ

maximal welfare is a decreasing function of asset holdings. Asset poor unemployed workers have a higher marginal utility of consumption and value insurance more. Therefore their welfare gain is higher than that of workers with a medium or high level of asset holdings. The asset rich unemployed workers care more about their employment probability as they have enough assets to smooth consumption, while poor unemployed workers care more about the direct insurance effect of UI as they have high marginal utility of consumption. That is why asset richer unemployed workers' welfare gain is less than that of other groups and also their welfare falls more when UI increases further.

The welfare gains or losses also differ across the distribution of asset holdings for employed workers (Figure 9b). Rich employed workers have lower welfare gains when UI goes down and face a higher welfare loss when UI increases. These workers have already high levels of asset holdings and can insure themselves well in case they lose their jobs. Instead, higher benefits mean also higher levels of taxes for them. In contrast, asset poor employed workers gain more welfare when UI increases from zero, since they value external insurance as they do not have enough assets to smooth their consumption in case they lose their jobs. However, their welfare is maximized close to the benchmark economy level of UI. Increasing UI more is welfare decreasing for these workers since it affects their net wages through taxation, while not providing substantially more insurance at the margin.

To calculate the aggregate measure of the welfare change, we integrate the welfare measure ψ over the distribution of asset holdings in the benchmark economy. This is depicted in Figure 10. Although the welfare function is inverted U-shape for both employed and unemployed workers, the welfare maximizing level of benefits for unemployed workers is nearly 25% higher than that of the employed. This again highlights the value of direct insurance for unemployed workers. Since employment consists of 90-95%



(a) Average value unemployed (b) Average value employed (c) Total welfare

Figure 10: Welfare

of the total labor force and unemployment only 5-10%, the aggregate welfare function has a similar shape to the one of the employed workers.

from $b = 0.8$ to $b =$	Total %	Unemp. %	Emp. %	$a_{u,l}$ %	$a_{u,m}$ %	$a_{u,h}$ %	$a_{e,l}$ %	$a_{e,m}$ %	$a_{e,h}$ %
0	-0.52	-0.06	-0.47	-1.82	-0.99	-0.85	-0.76	-0.57	-0.50
0.10	-0.48	-0.05	-0.42	-1.52	-0.92	-0.79	-0.66	-0.51	-0.46
0.35	-0.17	-0.02	-0.15	-0.66	-0.39	-0.34	-0.25	-0.19	-0.16
0.45	-0.11	-0.01	-0.09	-0.46	-0.28	-0.24	-0.16	-0.11	-0.09
0.65	-0.03	-0.01	-0.03	-0.18	-0.12	-0.10	-0.05	-0.03	-0.03
0.80	0	0	0	0	0	0	0	0	0
0.95	0.01	0.00	0.00	0.11	0.06	0.05	0.02	0.01	0.00
1.10	-0.05	0.00	-0.05	0.19	0.08	0.05	-0.01	-0.04	-0.06
1.30	-0.19	0.00	-0.19	0.21	0.04	-0.00	-0.13	-0.18	-0.20
1.55	-0.51	-0.01	-0.50	0.00	-0.20	-0.26	-0.43	-0.50	-0.52
1.70	-0.73	-0.03	-0.70	-0.22	-0.44	-0.50	-0.64	-0.71	-0.74

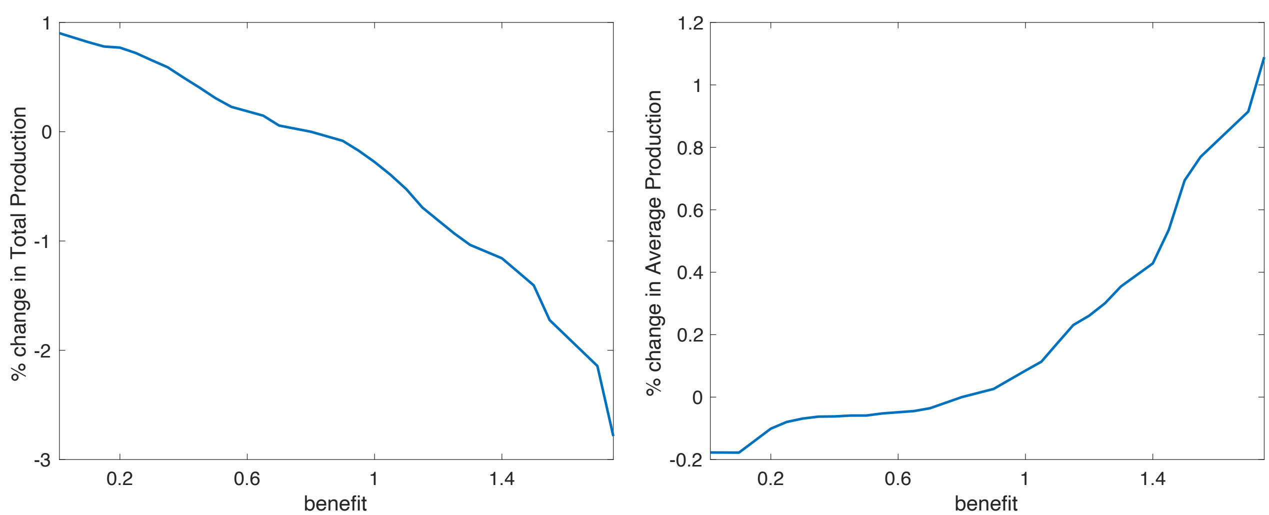
Table 3: Welfare change compared to benchmark economy (first three columns are welfare gains for all workers, the unemployed, and the employed. The last six columns are the welfare gains by an unemployed and employed worker with low(l), medium(m) and high(h) asset holding.

In Table 3 we report the welfare change for different benefit levels relative to the benchmark economy benefits of $b = 0.80$. Again, the net welfare gain is heterogenous across the distribution of workers. The last six columns aim to capture the heterogeneity for different asset holdings within the pool of employed and unemployed. A low asset unemployed worker gains more than 1.8% by moving from a Laissez-faire economy to the benchmark while there is less than a 0.9% gain for an asset rich unemployed worker. Increasing benefits further to 1.30, results in a 0.2% further rise of an asset poor unemployed worker compared to the benchmark economy while, it reduces the welfare of an asset rich unemployed worker.

A low asset employed worker gains 0.76% if they move from the Laissez-faire economy to the benchmark economy. An asset rich employed worker gain 75% of that amount of welfare in the same situation. For higher levels of benefits than the benchmark economy most employed workers start to experience a welfare loss. The losses are substantially higher for rich employed workers. They experience a welfare loss of more than 0.52% if they move from the benchmark to a counterfactual economy with twice more generous benefits while the same loss is 0.43% for poor employed workers.

4.4 The Effect of Benefits on Worker Productivity

A novel feature of our model compared to the models with identical firms is the impact of UI changes on worker productivity. When firms are homogenous the production technology is linear, a rise in benefits only affects the measure of firms entering the market, and leaves the productivity of jobs unaffected. However, in this framework, a rise in UI affects the productivity of workers because the allocation of workers to jobs changes, which in turn affects the firms' entry decision. Figure 11a depicts the percentage change in total output and 11b shows percentage change in average output per worker for different levels of benefits compared to the benchmark economy. By construction, at the benchmark $b = 0.8$, the change is zero.



(a) Total Output

(b) Productivity (Output per Worker)

Figure 11: Total Output and Productivity

We find that the total output is decreasing in benefits. When UI goes up, there are three countervailing forces: 1. all workers tend to apply to more productive jobs; 2. savings and therefore asset holdings are lower; and 3. the threshold of firms' entry moves up so lower productive firms do not find it profitable to enter the market. Since higher UI benefits are associated with less filled jobs, total production in the economy falls. That is depicted in Figure 11a. However, 11b shows that when UI increases workers tend to apply to more productive jobs indicating that the incentive effect dominates the effect of lower asset holdings. This shows the impact of higher asset holding as well as a higher entry threshold of firms. Therefore, labor productivity increases by up to 1.1% compared to the benchmark economy. Benefits affect productivity positively at the intensive margin as each worker is more productive, and it affects output produced negatively at the extensive margin as fewer workers hold a job. The net effect on total production is negative.

This measure of output per worker is not equal to the measure of welfare because it does not take into account the employment probability, which is decreasing with benefits, nor the measure of the unemployed. We know from the welfare calculations that welfare is inverted U-shape in benefits, indicating that benefits increase worker productivity of the asset rich, while the decrease in overall employment (just over four percentage point) is limited. The incentive effect is therefore key in understanding the change in welfare from an increase in benefits.

4.5 Severance Pay

So far, we have modeled the UI benefit by means of a per period payment, which is what most UI schemes consist of. An alternative UI regime is to offer a lump sum payment to the unemployed worker upon separation, call it severance pay.[36] This has been heralded as a UI scheme with better incentives to search for the unemployed, while at the same time offering insurance and a means to smooth consumption. Unlike per period benefits, workers now have to manage their assets to smooth consumption. The tradeoff between the standard benefit system and severance pay is that severance pay does not offer any form of income if a worker continues to be unemployed and their assets are depleted.

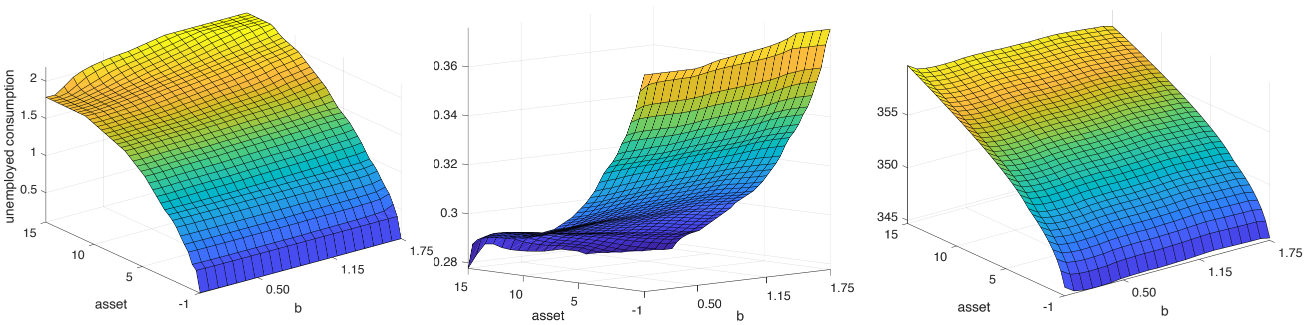
To that effect, in this section we assume workers get a lump sum transfer S upon separation and entry into unemployment and then receive nothing while being unemployed ($b = 0$) until they find a job again. We can rewrite the value function of the employed worker (the problem of other agents remains as before with $b = 0$):

$$E(a_t, w_t) = \max_{a_{t+1}} \{u(c_{e,t}) + \beta[\lambda U(a_{t+1} + S) + (1 - \lambda)E(a_{t+1}, w_{t+1})]\} \tag{18}$$
$$\text{s.t.} \quad c_{e,t} = Ra_t - a_{t+1} + (1 - \tau)w_t + d \;\; \text{and} \;\; a_{t+1} \geq \underline{a}.$$

As with the per period benefits b, the severance pay S is financed with a proportional wage tax and a balanced budget. To make the policies comparable, we express the severance pay S in terms of the equivalent expected benefit the average worker would receive under the per period benefit scheme, i.e., where $S = \frac{b}{m(\theta)}$. For example, if the average matching probability is $m(\theta) = 0.5$, and the per period

[36] We are grateful to Melvyn Coles and Pierre Cahuc for suggesting to analyze severance pay and its comparison with per period benefits.

benefit is $b = 0.8$, then the corresponding equivalent severance pay is $S = 1.6$.[37] Therefore, in order to be able to compare per period benefits with severance pay, in the figures we express the severance pay in terms of the corresponding $b(S)$ (=0.8 in this example) and not in terms of S (=1.6 in the example).



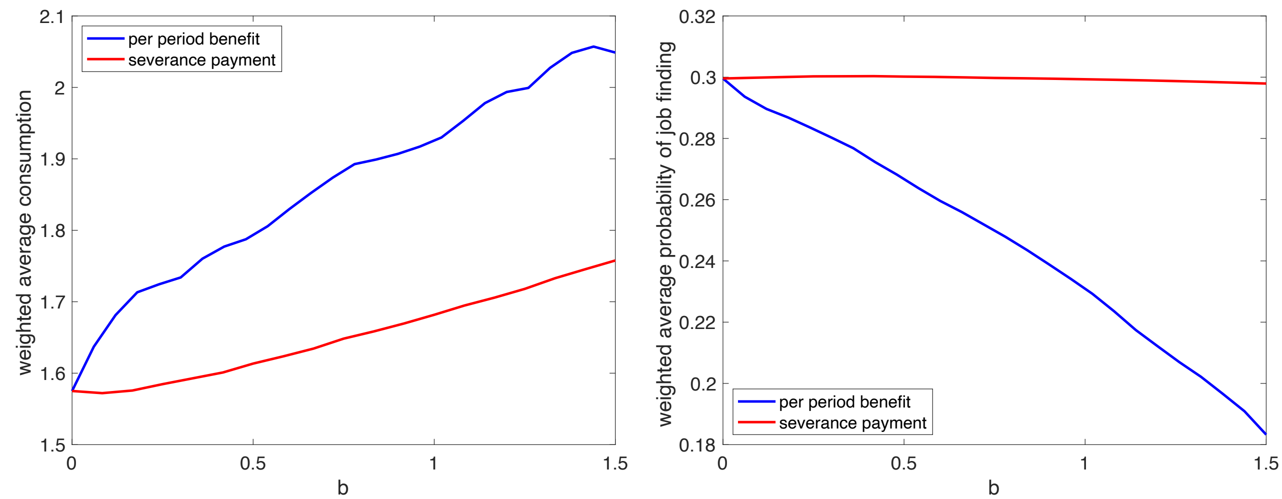
(a) Consumption of unemployed (b) Job finding probability (c) Value of unemployment

Figure 12: Consumption, job finding probability, and value of unemployment for different levels of severance payments and assets

Figure 12 depicts consumption, the job finding probability and the value of unemployment for different asset and for different levels of severance pay (expressed in b equivalence). A rise in severance pay does not significantly affect consumption and the probability of job finding: these two functions are flatter in severance pay for all asset levels compared to per period benefits. This is because severance pay is a one off payment upon becoming unemployed. Therefore its insurance power during joblessness is low and unemployed workers do not change by much their response in consumption or in job search behavior for different levels of severance pay. On the other hand, an increase in severance pay does increase taxes and reduces the net wage. Of course, workers dramatically change their behavior as assets deplete, much more so than with positive per period benefits. The effect that lower assets have on future consumption is reflected in the value of unemployment (Figure 12c): the value of unemployment as a function of severance payments is decreasing in severance pay for each asset level.

To analyze the welfare implications of severance pay, we calculate the welfare gain or loss of moving from our benchmark economy with per period benefit ($b = 0.8$), to any of the counterfactual severance pay economies. As in the previous Section, we compare utilities when we keep distribution of asset holdings from the benchmark economy to the counterfactual economies. We first compare the outcomes of per period benefits and severance payments, expressed in terms of the equivalent per period benefit b. Using the benchmark economy asset distribution, Figure 13 shows the change in average consumption and in the average job finding probability under the two policy regimes, per period benefits and severance pay. It is evident that per period benefits offer better consumption outcomes as benefits increase, but lower job finding probabilities. With higher benefits, workers apply for better jobs and hence have lower job finding rates. By contrast, unemployed workers who have received higher severance pay increase

[37] We calculate the severance pay to be equal to the average sum of all benefits received, given the expected duration of unemployment.



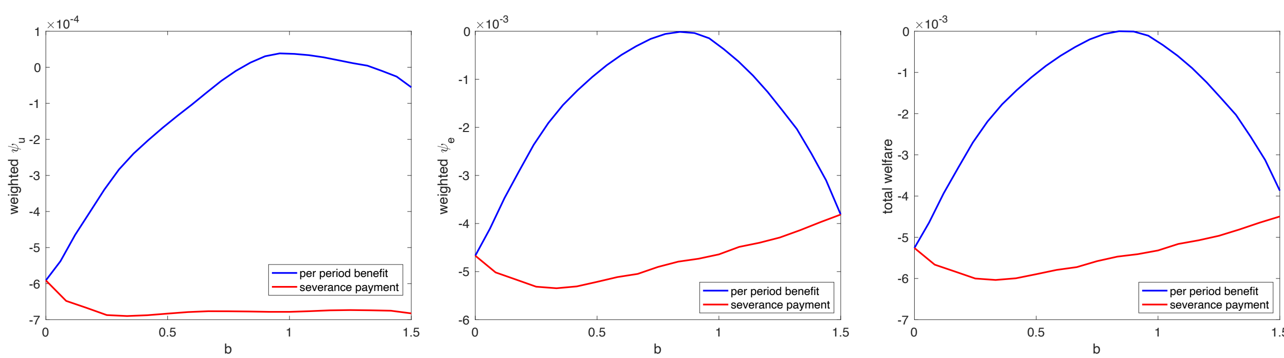
(a) Average consumption of unemployed workers

(b) Average job finding probability

Figure 13: Average weighted consumption and probability of job finding.

their consumption as well but less so because they anticipate the possibility of not receiving benefits if unemployment lasts longer than average. This also translates in a more moderate change in the job finding probability (Figure 13b) due to precautionary job search.

To make concrete the tradeoff between the insurance value of per period benefits and the incentive effects of severance pay, we evaluate the value of unemployment under both policies (this makes a comparison between Figure 12c to Figure 8a above).



(a) Average value unemployed

(b) Average value employed

(c) Total welfare

Figure 14: Comparing welfare effect of per period benefit and severance payment

Figure 14 shows the welfare gain and loss of workers under the different policy regimes (per period benefit and severance pay). A rise in per period benefits results in an inverted U shaped relation of workers' welfare. This effect comes from the trade-off between higher direct insurance and a lower probability of job findings. Instead, severance pay shows a flat welfare effect, slightly U shaped, increasing for sufficiently high severance pay, mainly for the employed workers. In contrast, per period benefit regime offers considerably higher amount of insurance. For lower levels of benefits especially, welfare increases in the per period regime. Eventually however, the negative effect on job finding kicks in and welfare function starts to decrease. Interestingly, the opposite happens with severance payment. When benefits increase further, the higher value of insurance dominates the mild fall in probability of job findings.

To conclude, the welfare gain of per period benefits dominates that of the severance pay regime. Severance pay has good incentive properties to induce unemployed workers to search, but the welfare value of insurance is poor. A worker who is unlucky and doesn't find a job and runs out of funds has no consumption under severance pay. With per period insurance, there is always some consumption. The insurance value dominates the incentive effect.

5 Conclusion

We have analyzed the effect of asset holdings on worker productivity in the presence of frictional job search. In the absence of complete insurance markets, workers have a precautionary search motive: the job search decision is an important source of self insurance for those with low assets levels. To analyze this, we solve a model with directed search and consumption smoothing where workers with high asset holdings sort into more productive jobs. Because asset holdings allow workers to smooth consumption, they can afford to face a substantially lower job finding probability. The difference in the job finding probability depends on the level of benefits. For our benchmark calibration, we find that the job finding rate for those with high assets is 10% lower, and it is 18% lower if benefits are zero. This is consistent with independent findings that the poor without liquid assets find jobs faster (Chetty (2008), Lise (2013), and Baley and Sepahsalari (2019)).

An important insight of our analysis is that the presence of consumption smoothing can address a major shortcoming of the canonical directed search model. When preferences are linear, the directed search predicts positive employment duration dependence on wages, which is counterfactual. Workers who direct their search to higher wage jobs face longer queues and thus stay unemployed longer. When workers are risk averse however, the unemployed deplete their assets while searching which forces them to apply to low wage jobs. This in turn induces negative duration dependence on wages. In the quantitative exercise we find that negative duration dependence dominates.

Key to the mechanism is that workers sort into firms with different levels of productivities based on their assets. Even if workers are identically skilled, there is nonetheless a preferences complementarity between assets and productivity. We derive conditions under which the model exhibit PAM or NAM and we use the sorting mechanism to solve for a non-degenerate distribution of assets in the infinite horizon problem.

In the quantitative analysis, we calibrate our model to the US economy and analyze the welfare effect of an income tax financed UI policy. Not only is there the usual conflict of interest between the unemployed who receive the benefits and those with a job who pay for it, there is also a conflict between the workers with assets and those without. Both receive benefits, but the rich can rely more on their savings for insurance. When we aggregate the welfare losses and gains over the distribution, we show that the welfare function is inverted U-shaped in benefits. At lower levels of benefits the insurance effect of UI dominates the incentive ones. However, when UI increases, workers tend to care more about the negative effect of insurance on job finding and their welfare gains start to diminish.

A novel feature of our model is the impact of UI benefits on workers productivity. UI affects the average productivity of workers through 1. the allocation of workers to jobs of different productivities; and 2. through the entry decision of firms. We show that for low UI benefit levels, an increase in benefits has no effect on average productivity per worker. However, for high enough benefit levels, a rise in benefits pushes up wages which in turn reduces the entry of firms with lower productivities. This increases average worker productivity, while at the same time it lowers total output.

Finally, we also compare per period benefits to a one off severance payment. Severance pay provides better job search incentives, but comes at the cost of poorer consumption smoothing. We find that per period benefits dominate severance pay. Workers value more the insurance effect than the search incentive effect.

Appendix A Proofs and Derivations

A.1 Partial Derivatives of U and a'

From equation (11) we calculate the derivates:

$$\begin{aligned}
U_y(a_1) &= \beta m u'(c_{e,2}) + U_{a_2}\frac{\partial a_2}{\partial y} + U_{\theta_1}\frac{\partial \theta_1}{\partial y} = \beta m u'(c_{e,2}) \\
U_a(a_1) &= u'(a_1 - a_2) + U_{a_2}\frac{\partial a_2}{\partial a_1} + U_{\theta}\frac{\partial \theta_1}{\partial a_1} = u'(a_1 - a_2) \\
U_V(a_1) &= \beta m u'(c_{e,2})\frac{-1}{\beta q} + U_{a_2}\frac{\partial a_2}{\partial V} + U_{\theta_1}\frac{\partial \theta_1}{\partial V} = \beta u'(c_{e,2})\frac{-\theta}{\beta} \\
U_{ay}(a_1) &= -u''(a_1 - a_2)\frac{\partial a_2}{\partial y} \\
U_{aV}(a_1) &= -u''(a_1 - a_2)\frac{\partial a_2}{\partial V}
\end{aligned}$$

where $c_{e,2} = Ra_2 + y - \frac{V}{\beta q(\theta_1)}$ and where $U_{a_2} = 0$ and $U_{\theta_1} = 0$ from the envelope theorem.

Denote the maximand of U by $\phi(a_2, \theta_1) = u(a_1 - a_2) + \beta\left[mu\left(c_{e,2}\right) + (1-m)\,u(Ra_2)\right]$, i.e., the objective function that is maximized with respect to $a2, \theta_1$. We calculate the derivative of a_2 using the implicit function theorem. For the problem to have a maximum, we require that the Hessian of the maximand is positive $|\mathbf{H}| > 0$ (recall that $\phi_{\theta_1\theta_1}$ is assumed negative), where:

$$|\mathbf{H}| = \begin{vmatrix} \phi_{a_2 a_2} & \phi_{a_2\theta_1} \\ \phi_{\theta_1 a_2} & \phi_{\theta_1\theta_1} \end{vmatrix}$$

Applying the implicit function theorem,

$$\frac{\partial a_2}{\partial y} = -\frac{\begin{vmatrix} \phi_{a_2 y} & \phi_{a_2\theta_1} \\ \phi_{\theta_1 y} & \phi_{\theta_1\theta_1} \end{vmatrix}}{|\mathbf{H}|} = \frac{\phi_{a_2 y}\phi_{\theta_1\theta_1} - \phi_{\theta_1 y}\phi_{a_2\theta_1}}{|\mathbf{H}|} \quad \text{and} \quad \frac{\partial a_2}{\partial V} = -\frac{\begin{vmatrix} \phi_{a_2 V} & \phi_{a_2\theta_1} \\ \phi_{\theta_1 V} & \phi_{\theta_1\theta_1} \end{vmatrix}}{|\mathbf{H}|} = \frac{\phi_{a_2 V}\phi_{\theta_1\theta_1} - \phi_{\theta_1 V}\phi_{a_2\theta_1}}{|\mathbf{H}|}$$

A.2 Proof of Proposition 1

Proof. $U_{a_1 y} > \frac{U_y}{U_V} U_{a_1 V}$ provided (where the partial derivatives of U are derived in the Appendix):

$$\begin{aligned}
-u''(a_1 - a_2)\frac{\partial a_2}{\partial y} &> \frac{\beta m u'(c_{e,2})}{\beta u'(c_{e,2})\frac{-\theta_1}{\beta}}\left(-u''(a_1 - a_2)\frac{\partial a_2}{\partial V}\right) \\
\frac{\partial a_2}{\partial y} &> -\beta q \frac{\partial a_2}{\partial V}
\end{aligned}$$

We obtain the expressions for $\frac{\partial a_2}{\partial y}$ and $\frac{\partial a_2}{\partial V}$ from the first order conditions (above). Then the condition for positive sorting of a on y becomes:

$$(\phi_{a_2 y} + \beta q \phi_{a_2 V})\, \phi_{\theta_1 \theta_1} \quad < \quad (\phi_{\theta_1 y} + \beta q \phi_{\theta_1 V})\, \phi_{a_2 \theta_1}$$

Observe that from the first order conditions to the maximization problem, we obtain the cross partials on ϕ. First, note that $\phi_{a_2 y} = -\beta q \phi_{a_2 V} = \beta R m u''(c_{e,2})$ so that the LHS is zero. This follows from the envelope theorem since ϕ is maximized with respect to a_2 and θ_1. Then we derive the following:

$$\begin{aligned}
\phi_{\theta_1 y} &= \beta m' u'(c_{e,2}) + \beta u''(c_{e,2}) \frac{\theta_1 q' V}{\beta q} \\
\phi_{\theta_1 V} &= \beta m' u'(c_{e,2}) \frac{-1}{\beta q} + \beta u'(c_{e,2}) \frac{\theta_1 q'}{\beta q} + \beta u''(c_{e,2}) \frac{-1}{\beta q} \frac{\theta_1 q' V}{\beta q} \\
&= \frac{-1}{\beta q} \phi_{\theta_1 y} + \beta u'(c_{e,2}) \frac{\theta_1 q'}{\beta q}
\end{aligned}$$

Therefore, the inequality can be written as:

$$0 \quad < \quad \beta u'(c_{e,2}) \theta_1 q' \phi_{a_2 \theta_1}$$

The term $\beta u'(c_{e,2})\theta_1 > 0$ but $q' < 0$, so the condition for positive sorting of a_1 on y is $\phi_{a_2\theta_1} < 0$. Equivalently:

$$\beta R \left(m'[u'(c_{e,2}) - u'(Ra_2)] + u''(c_{e,2}) \frac{\theta_1 q' V}{\beta q} \right) < 0.$$

From the first order condition $\phi_{\theta_1} = 0$ we obtain:

$$\frac{\theta_1 q' V}{\beta q} = -m' \frac{u(c_{e,2}) - u(Ra_2)}{u'(c_{e,2})}.$$

Substituting in the condition $\phi_{a_2\theta_1} < 0$:

$$m'[u'(c_{e,2}) - u'(Ra_2)] - u''(c_{e,2}) m' \frac{u(c_{e,2}) - u(Ra_2)}{u'(c_{e,2})} < 0,$$

or, noting that $m' > 0$,

$$u'(c_{e,2})[u'(c_{e,2}) - u'(Ra_2)] < u''(c_{e,2}) \left[u(c_{e,2}) - u(Ra_2)\right].$$

or alternatively

$$\frac{u'(c_{e,2}) - u'(Ra_2)}{u(c_{e,2}) - u(Ra_2)} < \frac{u''(c_{e,2})}{u'(c_{e,2})}.$$

Finally, uniqueness of the equilibrium allocation follows from the fact that the inequality in condition

(U) is strict and the fact that this is effectively a static problem with exogenous types. Legros and Newman (2007) establish uniqueness under condition (16), ensured by the measure-preserving market clearing condition, as long as types are exogenous. ■

A.3 Proof of Proposition 2

Proof. We calculate the derivatives:

$$
\begin{aligned}
u'(c) &= \alpha \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-1} \\
u''(c) &= -\alpha^2 \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-2}
\end{aligned}
$$

and condition (U) becomes (where $c = Ra_2$):

$$
\alpha \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-1} \left[\alpha \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-1} - \alpha \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-1} \right] <
$$
$$
-\alpha^2 \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma-2} \left[\frac{1-\gamma}{\gamma} \left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{\gamma} - \frac{1-\gamma}{\gamma} \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma} \right]
$$

and after dividing by α^2 and by $\left(\frac{\alpha c_e}{1-\gamma} + \beta \right)^{2\gamma-2}$, which under our assumptions are both positive, this implies:

$$
1 - \left(\frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \right)^{\gamma-1} < -\frac{1-\gamma}{\gamma} \left[1 - \left(\frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \right)^{\gamma} \right],
$$

or

$$
1 - x^{\gamma-1} < -\frac{1-\gamma}{\gamma} \left[1 - x^{\gamma} \right] \quad \text{where} \quad x = \frac{\frac{\alpha c}{1-\gamma} + \beta}{\frac{\alpha c_e}{1-\gamma} + \beta} \in (0,1).
$$

First consider $\gamma > 0$. After rearranging and multiplying by $\gamma x^{1-\gamma}$, which is positive for $\gamma > 0$:

$$
\begin{aligned}
x^{1-\gamma} - (\gamma + (1-\gamma)x) &< 0 \\
G(\gamma) - H(\gamma) &< 0.
\end{aligned}
$$

At $\gamma = 0$ and $\gamma = 1$ the expression is exactly zero, i.e., G and H cross at 0 and 1. Now, $G'(\gamma) = -x^{1-\gamma} \log x, H'(\gamma) = 1 - x$, and $G''(\gamma) = x^{1-\gamma}(\log x)^2 > 0, H''(\gamma) = 0$. Observe that $G(\gamma)$ is convex, $G''(\gamma) = x^{1-\gamma}(\log x)^2 > 0$, while $H(\gamma)$ is linear. As a result, for $\gamma \in (0,1)$ condition (U) holds with strict inequality. For $\gamma = 1$, (U) holds with equality and for $\gamma > 1$ it holds with opposite inequality.

Now consider $\gamma < 0$. Since we multiplied by $\gamma < 0$, condition (U) now implies that $G(\gamma) - H(\gamma) > 0$. Using the same logic, we establish that condition (U) holds for $\gamma < 0$.

This establishes that for a risk averse worker with HARA utility function, condition (U) holds strictly if and only if $\gamma < 1$, i.e., there is DARA. The condition holds with opposite inequality when there is IARA and $\gamma > 1$. ■

A.4 Proof of Corollary 1

Proof. All the cases can immediately be verified from Proposition 2, except for the case of CARA. There, $u'(c) = \alpha e^{-\alpha c}, u''(c) = -\alpha^2 e^{-\alpha c}$, so that condition (**U**) becomes:

$$\begin{aligned} \alpha e^{-\alpha c_e}\left(\alpha e^{-\alpha c_e} - \alpha e^{-\alpha c}\right) &\leq -\alpha^2 e^{-\alpha c_e}\left(1 - e^{-\alpha c_e} - 1 + e^{-\alpha c}\right) \\ e^{-\alpha c_e} - e^{-\alpha c} &\leq -\left(-e^{-\alpha c_e} + e^{-\alpha c}\right) \end{aligned}$$

which holds with equality. ■

A.5 Proof of Proposition 3

Proof. It is immediate that this condition is not satisfied when $u''' = 0$. To see this, observe that then $u'(c_{e,2}) - u'(Ra_2) = wu''(c_{e,2})$ and the condition (**U**) can be written as $u'(c_{e,2})wu''(c_{e,2}) < u''(c_{e,2})\left[u(c_{e,2}) - u(Ra_2)\right]$, or $u'(c_{e,2})w > u(c_{e,2}) - u(Ra_2)$. This condition only holds under convexity of u, and therefore is never satisfied for risk averse agents.

When $u''' < 0$, we have instead that $u'(c_{e,2}) - u'(Ra_2) > wu''(c_{e,2})$, so the left hand side is even smaller, and again, condition (**U**) implies $u'(c_{e,2})w > u(c_{e,2}) - u(Ra_2)$, which is not satisfied for risk averse agents.

Now consider $u''' > 0$. Then we can write the utility function and its derivative as

$$\begin{aligned} u(c) &= u(c_{e,2}) + u'(c_{e,2})(c - c_{e,2}) + \frac{u''(c_{e,2})}{2}(c - c_{e,2})^2 + \cdots \\ u'(c) &= u'(c_{e,2}) + u''(c_{e,2})(c - c_{e,2}) + \frac{u'''(c_{e,2})}{2}(c - c_{e,2})^2 + \cdots \end{aligned}$$

and therefore condition (**U**) becomes:

$$\begin{aligned} &u'(c_{e,2})\left[u''(c_{e,2})(c_{e,2} - c) - \frac{u'''(c_{e,2})}{2}(c_{e,2} - c)^2 + \frac{u''''(c_{e,2})}{6}(c_{e,2} - c)^3 - \cdots\right] < \\ &u''(c_{e,2})\left[u'(c_{e,2})(c_{e,2} - c) - \frac{u''(c_{e,2})}{2}(c_{e,2} - c)^2 + \frac{u'''(c_{e,2})}{6}(c_{e,2} - c)^3 - \cdots\right]. \end{aligned}$$

Canceling terms and dividing by $(c_{e,2} - c)^2$, this condition implies that at least for small $c_{e,2} - c = w$ implies

$$u'''(c_{e,2}) > \frac{u''(c_{e,2})^2}{u'(c_{e,2})}.$$

This is equivalent to requiring that the coefficient of risk aversion $A(c) = -\frac{u''}{u'}$ is decreasing, i.e., $A' = -\frac{u'''u' - (u'')^2}{u'^2}$ or $u''' > \frac{(u'')^2}{u'} = -u''A(c)$. ■

A.6 A Counter Example

Example 1 *Let w be large enough and find a u-function with u'''' suitably chosen such that the condition is not satisfied. Let $u(c)$ be defined as:*

$$u(c) = u(c_{e,2}) + u'(c_{e,2})(c - c_{e,2}) + \frac{u''(c_{e,2})}{2}(c - c_{e,2})^2 + \frac{u'''(c_{e,2})}{6}(c - c_{e,2})^3 + \frac{u''''(c_{e,2})}{24}(c - c_{e,2})^4.$$

Evaluating u at $c = Ra_2$ and observing that $c_{e,2} - Ra_2 = w$ we can then write

$$\begin{aligned} u(c_{e,2}) - u(Ra_2) &= u'(c_{e,2})w - \frac{1}{2}u''(c_{e,2})w^2 + \frac{1}{6}u'''(c_{e,2})w^3 - \frac{1}{24}u''''(c_{e,2})w^4 \\ u'(c_{e,2}) - u'(Ra_2) &= u''(c_{e,2})w - \frac{1}{2}u'''(c_{e,2})w^2 + \frac{1}{6}u''''(c_{e,2})w^3. \end{aligned}$$

*Now we can write condition (**U**) as (where u denotes $u(c_{e,2})$):*

$$\begin{aligned} u'\left[u''w - \frac{1}{2}u'''w^2 + \frac{1}{6}u''''w^3\right] &< u''\left[u'w - \frac{1}{2}u''w^2 + \frac{1}{6}u'''w^3 - \frac{1}{24}u''''w^4\right] \\ u'u''' &> u''^2 - \frac{1}{3}u''u'''w + \frac{1}{3}u''''w\left[u' + \frac{1}{4}u''w\right] \end{aligned}$$

Observe that $u'u''' > u''^2$ is the standard condition for Decreasing Absolute Risk Aversion. But for any $u''' > 0$, however large, we can find a utility function with $\frac{1}{3}u''''w\left[u' + \frac{1}{4}u''w\right]$ sufficiently large such that the inequality is not satisfied. For example, if $u' + \frac{1}{4}u''w > 0$ we can choose u'''' positive and large. Conversely, if $u' + \frac{1}{4}u''w < 0$ we can choose u'''' sufficiently negative such that the inequality does not hold.

A.7 Infinite Horizon: Special Case when $\beta R = 1$ and $\lambda = 0$

These assumptions imply that asset levels when employed are invariant in steady state equilibrium, since the employed workers consume a share of their assets exactly equal to the dividend. In that case, $a_{t+1} = \frac{a_t}{R} = \beta a_t$ and the value for employment is independent of $U(a_t)$. As a result, the employed worker's problem can be solved explicitly. The first-order condition of the employed worker is $u'(w + a_t - a_{t+1}) = \beta R E'(Ra_{t+1})$. With $\beta R = 1$ and $\lambda = 0$ the solution is $a_{t+1} = \frac{a_t}{R} = \beta a_t$ and we can explicitly write the value for employment:

$$E(a_t) = \frac{1}{1-\beta}u\left(w_t + (1-\beta)a_t\right).$$

We can then write the problem of the unemployed as:

$$U(a) = \max_{a_{t+1},\theta} \left\{ u(a_t - a_{t+1}) + \beta \left[m \frac{1}{1-\beta} u\left(w_t + (1-\beta)Ra_{t+1}\right) + (1-m)U(Ra_{t+1}) \right] \right\}$$

subject to the firm's value:

$$\begin{aligned} V(y) &= \max_{w_t} \{ q\,(y - w_t) + \beta(1-q)V(y) \} \\ &= \max_{w_t} \left\{ \frac{q}{1-\beta(1-q)} \left[y - w_t \right] \right\}. \end{aligned}$$

Using the standard technique in directed search, and similar to what we did in the two-period model, we substitute the wage and rewrite the problem as

$$U(a_t, y, V) = \max_{a_{t+1},\theta_t} \left\{ u(a_t - a_{t+1}) + \beta \left[m \frac{1}{1-\beta} u\left((1-\beta)Ra_{t+1} + y - V\left[-\beta\lambda + \frac{1-\beta(1-q)}{q}\left(\lambda + \frac{1}{\beta} - 1 \right) \right] \right) + (1-m)U(Ra_{t+1}) \right] \right\}. \tag{A.1}$$

A.8 Proof of Proposition 4

Proof. The (interior) solution $a_{t+1}(a_t, y, V), \theta_t(a_t, y, V)$ to the maximization problem satisfies:

$$\begin{aligned} -u'(c_{u,t}) + \beta[mE_{a_{t+1}}(a_{t+1}, y) + (1-m)U_{a_{t+1}}(a_{t+1})] &= 0 \\ m'[E(a_{t+1}, y) - U(a_{t+1})] + mE_{w_{t+1}}(a_{t+1}, y)\frac{\partial w_{t+1}}{\partial \theta_t} &= 0. \end{aligned}$$

Now we have monotone matching of a_t in y provided: $U_{a_t y} > \frac{U_y}{U_V} U_{a_t V}$.

$$\begin{aligned} U_y &= m\beta E_{w_{t+1}} \frac{\partial w_{t+1}}{\partial y} + U_{a_{t+1}} \frac{\partial a_{t+1}}{\partial y} + U_{\theta_t} \frac{\partial \theta_t}{\partial y} = m\beta E_{w_{t+1}} \\ U_{a_t} &= u'(c_{u,t}) + U_{a_{t+1}} \frac{\partial a_{t+1}}{\partial a_t} + U_{\theta_t} \frac{\partial \theta_t}{\partial a_t} = u'(c_{u,t}) \\ U_V &= m\beta E_{w_{t+1}} \frac{\partial w_{t+1}}{\partial V} + U_{a_{t+1}} \frac{\partial a_{t+1}}{\partial V} + U_{\theta_t} \frac{\partial \theta_t}{\partial V} = -m\beta E_{w_{t+1}} \left(-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q} \right) \\ U_{a_t y} &= -u''(a_t - a_{t+1}) \frac{\partial a_{t+1}}{\partial y} \\ U_{a_t V} &= -u''(a_t - a_{t+1}) \frac{\partial a_{t+1}}{\partial V} \end{aligned}$$

where $U_{a_{t+1}} = 0$ and $U_{\theta_t} = 0$ from the envelope theorem. Then:

$$\begin{aligned}
U_{a_t y} &> \frac{U_y}{U_V} U_{a_t V} \\
-u''(a_t - a_{t+1})\frac{\partial a_{t+1}}{\partial y} &> \frac{m\beta E_w(a_{t+1}, y)}{-m\beta E_w(a_{t+1}, y)\left(-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}\right)}(-u''(a_t - a_{t+1})\frac{\partial a_{t+1}}{\partial V}) \\
\frac{\partial a_{t+1}}{\partial y} &> -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\frac{\partial a_{t+1}}{\partial V}
\end{aligned}$$

Writing the Hessian $|\mathbf{H}| > 0$ as:

$$|\mathbf{H}| = \begin{vmatrix} \phi_{a_{t+1}a_{t+1}} & \phi_{a_{t+1}\theta_t} \\ \phi_{\theta_t a_{t+1}} & \phi_{\theta_t\theta_t} \end{vmatrix}$$

then

$$\frac{\partial a_{t+1}}{\partial y} = -\frac{\begin{vmatrix} \phi_{a_{t+1}y} & \phi_{a_{t+1}\theta_t} \\ \phi_{\theta_t y} & \phi_{\theta_t\theta_t} \end{vmatrix}}{|\mathbf{H}|} \quad \text{and} \quad \frac{\partial a_{t+1}}{\partial V} = -\frac{\begin{vmatrix} \phi_{a_{t+1}V} & \phi_{a_{t+1}\theta_t} \\ \phi_{\theta_t V} & \phi_{\theta_t\theta_t} \end{vmatrix}}{|\mathbf{H}|}$$

$$\begin{aligned}
\frac{\partial a_{t+1}}{\partial y} &> -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\frac{\partial a_{t+1}}{\partial V} \\
\phi_{a_{t+1}y}\phi_{\theta_t\theta_t} - \phi_{\theta_t y}\phi_{a_{t+1}\theta_t} &< -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\left(\phi_{a_{t+1}V}\phi_{\theta_t\theta_t} - \phi_{\theta_t V}\phi_{a_{t+1}\theta_t}\right) \\
\left(\phi_{a_{t+1}y} + \frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\phi_{a_{t+1}V}\right)\phi_{\theta_t\theta_t} &< \left(\phi_{\theta_t y} + \frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\phi_{\theta_t V}\right)\phi_{a_{t+1}\theta_t} \quad \text{(A.2)}
\end{aligned}$$

Observe that from the first order conditions to the (interior) maximization problem, we obtain the cross partials on ϕ. First, note that:

$$\phi_{a_{t+1}y} = -\frac{1}{-\beta\lambda + \frac{[1-\beta(1-\lambda)][1-\beta(1-q)]}{\beta q}}\phi_{a_{t+1}V}$$

so that the LHS is zero. Then we derive the expression for $\phi_{\theta_t y}$ and $\phi_{\theta_t V}$ while we note that $m'[E(a_{t+1})-$

$U(a_{t+1})] + qE_{w_{t+1}}\frac{\partial w_{t+1}}{\partial \theta_t}] = 0$, which implies:

$$\begin{aligned}
\phi_{\theta_t y} &= \beta m' E_{w_{t+1}}(a_{t+1}, y) + \beta E_{w_{t+1}w_{t+1}}(a_{t+1}, y)\frac{\partial w_{t+1}}{\partial y}\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{\beta q}V \\
\phi_{\theta_t V} &= \beta\frac{\partial w_{t+1}}{\partial V}\left(m' E_{w_{t+1}} + E_{w_{t+1}w_{t+1}}\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{\beta q}V\right) + \beta E_{w_{t+1}}\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{\beta q} \\
&= \left(\beta\lambda - \frac{[1-\beta(1-q)][1-\beta(1-\lambda)]}{\beta q}\right)\phi_{\theta_t y} + \beta E_{w_{t+1}}\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{\beta q}
\end{aligned}$$

the RHS reduces to:

$$\frac{(1-\beta)(1-\beta(1-\lambda))\theta_t q'}{q}E_{w_{t+1}}(a_{t+1}, y)\phi_{a_{t+1}\theta_t}$$

Therefore, the inequality (A.2) is satisfied provided $\phi_{a_{t+1}\theta_t} < 0$, since $q' < 0$:

$$\begin{aligned}
\phi_{a_{t+1}\theta_t} &= \beta m'[E_{a_{t+1}}(a_{t+1}, y) - U_{a_{t+1}}(a_{t+1})] + \beta m E_{a_{t+1},w}(a_{t+1}, y)\frac{\partial w_{t+1}}{\partial \theta_t} \\
&= \beta m'[E_{a_{t+1}}(a_{t+1}, y) - U_{a_{t+1}}(a_{t+1})] + \beta m E_{a_{t+1}}(a_{t+1}, y)\frac{-1}{mE_{w_{t+1}}(a_{t+1}, y)}m'[E(a_{t+1}, y) - U(a_{t+1})]
\end{aligned}$$

from the FOC for θ_t

$$\frac{\partial w_{t+1}}{\partial \theta_t} = \frac{-m'}{mE_{w_{t+1}}(a_{t+1}, y)}[E(a_{t+1}, y) - U(a_{t+1})]$$

Therefore $\phi_{a_{t+1}\theta_t} < 0$ provided

$$\frac{E_{a_{t+1}}(a_{t+1}, y) - U_{a_{t+1}}(a_{t+1})}{E(a_{t+1}, y) - U(a_{t+1})} < \frac{E_{a_{t+1}}(a_{t+1}, y)}{E_{w_{t+1}}(a_{t+1}, y)}$$

■

Appendix B Numerical Algorithm

Here we describe the algorithm used to solve the model. the algorithm consists of the following steps

1. Guess a tax rate, $\tau_{0,t}$.
2. Guess a dividend pay, $d_{0,t}$.
3. Guess the distribution of firms $G_0 = (G_{0,v}(y), G_{0,j}(y, w_t))$ with vacant and filled jobs, and guess the distribution of workers $F_0 = (F_{0,u}(a_t), F_{0,e}(a_t, w_t))$ who are either unemployed or employed.
4. Guess initial values of employment and unemployment: $U_0(a_t), E_0(a_t, w_t)$
   - Solve the problem of employed worker by Value Function Iteration (VFI), given the guess for $U_0(a_t)$, on a grid of (a_t, w_t).

- Find the saving policy of employed workers using the solution to VFI.

5. Labour Market clearing Loop: guess the threshold of firms' entry $y^\star$, for firms as well as the measure of entrant firms.

6. Sorting Loop: Given the guess on the entry level of firms, start sorting unemployed workers to the firm (asset to the productivity) from bottom to the top of distribution.
   Substitute equation (4) into the value of a filled job, equation (5), and rearranging we get

$$q(\theta_t) = \frac{[V(y)(1-\beta)+k][1-\beta(1-\lambda)]}{\beta(y_t - w_t - (1-\beta V))} \tag{B.1}$$

   - The value of a vacancy at the threshold of firm entry is zero. This is $V(y^\star) = 0$, where $y^\star$ is the productivity level below which firms do not enter the market.
   - Knowing the value of a vacancy at the bottom of the distribution and having a guess for $y^\star$, we can find the relation between θ and w.
   - Since we are sorting workers to the vacancies from the bottom to the top, this implies that the bottom unemployed workers in the asset distribution are constrained. Therefore, for these workers $a_t = a_{t+1}$. So, we can re-write equation (2) for constrained workers

$$U(a_t) = \frac{1}{1-\beta(1-m(\theta_t))}\Big[u(c_{u,t}) + \beta m(\theta_t) E(a_{t+1}, w_{t+1})\Big] \tag{B.2}$$

$$c_{u,t} = r a_t + b + d_t \tag{B.3}$$

   - Given the menu of tightness-wage bundles and the value $E(a_t, w_t)$ we solved in the last step, a worker can find its optimal submarket to apply for.
   - using the optimal market tightness, we can find the value of unemployment $U(a)$.
   - Using the optimal allocation condition and given the current value of vacancy and optimal choice of market tightness, we can find the value of posting a vacancy in the next submarket where the optimal matched firm type solves:

$$\beta m(\theta_t)\frac{\partial E(a_{t+1}, w_{t+1}(y))}{\partial w_{t+1}}\frac{\partial w_{t+1}}{\partial y} = \tag{B.4}$$

$$\beta m(\theta_t)\frac{\partial E(a_{t+1}, w_{t+1}(y))}{\partial w_{t+1}}\Big[1 - \Big(\frac{(1-\beta(1-q(\theta_t)))(1-\beta(1-\lambda))}{\beta q(\theta_t)} - \beta\Big)\frac{\partial V(y)}{\partial y}\Big] = 0 \tag{B.5}$$

   - Given the optimal market tightness chosen by unemployed worker, we allocate workers to market tightnesses from the distribution and construct $\theta_t(a_t, y) = \frac{v_{t,y}}{u_{t,a}}$. Here we keep track of the distribution.
   - In the next iteration of the sorting loop, using equation (B.1) and the value of vacancy we obtained through previous iteration using optimal allocation condition, we again solve for the optimal market tightness for unemployed worker with higher levels of asset holdings.

- When we move away from budget constrained, workers simultaneously choose optimal labour market decision ($\theta_t(a_t, y)$) as well as saving for next period (a_{t+1}). Since we started solving this problem from bottom to the top of distribution, for any level of a_t above the constraint we know the value of a_{t+1} which are below a_t. Therefore, the workers knows the value of depleting asset to a_{t+1} which is $U(a_{t+1})$.

7. By sorting workers to the firms from bottom to the top of distribution, three scenarios may happen

   a) An unemployed worker with a level of asset holding below the highest gets sorted to the highest productivity level. In this case, not all unemployed workers are allocated to the submarkets. Therefore, we update first by increasing the measure of entrant firms at the same level of entry and then by lowering the threshold of firms' entry $y^\star$ (go back to Step 5).

   b) A vacancy with a level of productivity below the highest gets sorted to the highest asset holding unemployed worker. In this case, high productivity firms do not get allocated to any vacancies. Therefore, we update first by decreasing the measure of entrant firms at the same level of entry and then by increasing the threshold of firms' entry $y^\star$ to make sure firms with high productivities all get allocated (go back to Step 5).

   c) The unemployed workers with highest levels of asset holdings gets sorted to the highest productivity vacancies. In this case the allocation of workers to the firms is such that the labour market is cleared.

8. Check the convergence of $U_0(a_t)$. If not converged go back to step 4 and update $U_0(a_t)$.

9. Using the policy functions for workers (job finding and saving for unemployed workers and saving for employed workers) and firms (job filling rates for firms with a vacancy) check the convergence of the distribution of workers and firms. If not converged, go back to step 3 and update the distributions.

10. Using the converged distribution of firms with filled $H_f(y)$, compute the total dividend paid by firms and compare it with previous guess.

$$d_t = \int [(y - w_t)h_f(y, w_t) - v_t(y)k]dy.$$

    If they are not similar, go back to step 2 and update the guess for dividend pay-out.

11. Use the distributions to compute the mass of workers with unemployment benefit entitlements and tax paid by employed workers to check if the government budget is balanced.

$$u_t b = \tau \int w_t(a_t) f_e(a_t) da.$$

If total benefit is higher than tax, go back to step 1 and increase τ_t, if total benefit is less than total tax, decrease τ_t.

12. Check ex-post if condition $\mathbf{U}_\infty$ holds.

Appendix C A random search model with search effort

In our model, the precautionary search motive acts through sorting into different jobs. A plausible alternative to our mechanism that generates job finding rates contingent on asset holdings is through search intensity. A worker who is heavily credit constrained will want to put in more search effort in order to increase the match finding probability, and thus smooth consumption. In this section we compare our two-period model presented in Section 3.1 with a two-period random search model with Nash bargaining and endogenous search effort.

We model the labor market with a Diamond-Mortensen-Pissarides random search set-up. The number of matches between unemployed workers and vacant jobs each period is determined by the aggregate matching function $M(\hat{s}, v)$, where $\hat{s}$ is aggregate search given by $\hat{s} = \int s(a) f_u(a) da$ where s is the search effort of an unemployed worker and $f_u(a)$ is the density of the distribution of unemployed workers over assets. Market tightness is defined as $\theta = \frac{v}{\hat{s}}$ (the ratio of the number of vacancies to the amount of aggregate search). The job finding rate per unit of search effort is $m(\theta) = \frac{M(\hat{s},v)}{\hat{s}}$. Therefore, an individual unemployed worker with asset level a, receives job offers at rate $s(a)m(\theta)$. The job filling rate for the firm is given as $q(\theta) = \frac{M(\hat{s},v)}{v}$. The law of motion for the unemployment rate is given by $u' = (1 - m(\theta)\bar{s})u$ where $\bar{s} = \frac{\hat{s}}{u}$ is the average search effort.

We further summarize the model as follows:

Workers. $e(\cdot)$ is a strictly convex cost function of search effort, $s(a)$ is individual search effort. The disutility from search is specified as in Christensen et al. (2005): $e(s) = \zeta \frac{s^{1+1/\rho}}{1+1/\rho}$ where ζ is a scaling function and $\rho > 0$ ensures strict convexity. We can write the value of an unemployed workers as

$$U = \max_{a_{t+1}, s_t} u(a_t - a_{t+1} + b) - e(s_t) + \beta[s_t m(\theta_t) u(Ra_{t+1} + w) + (1 - s_t m(\theta_t) u(Ra_{t+1} + b))]$$

This implies the worker's FOC:

$$\begin{aligned} u'(a_t - a_{t+1} + b) &= \beta R[s_t m(\theta_t) u'(Ra_{t+1} + w) + (1 - s_t m(\theta_t)) u'(Ra_{t+1} + b)] \\ e'(s_t) &= \beta m(\theta_t)[u(Ra_{t+1} + w) - u(Ra_{t+1} + b)] \end{aligned}$$

Firm. The firm value can be written as:

$$V = -k + \beta q(\theta_t) \int (y - w(a)) d\Lambda(a),$$

where $\Lambda(a) = \frac{s_t}{\bar{s}}\frac{f_u(a)}{u}$. $\Lambda(a)$ captures the fact that the unemployed workers search with different intensities depending on their asset level.

Worker Surplus.

$$S_w = u(Ra_{t+1} + w) - u(Ra_{t+1} + b)$$

Firm Surplus.

$$S_f = y - w + k$$

Nash Bargaining.

$$S = (u(Ra_{t+1} + w) - u(Ra_{t+1} + b))^{\gamma}(y - w)^{1-\gamma}$$

this implies

$$\log S = \gamma \log(u(Ra_{t+1} + w) - u(Ra_{t+1} + b)) + (1 - \gamma) \log(y - w)$$

to find the wage, maximize over w

$$\frac{\gamma u'(Ra_{t+1} + w)}{u(Ra_{t+1} + w) - u(Ra_{t+1} + b)} = \frac{1 - \gamma}{y - w} \Rightarrow \gamma u'(Ra_{t+1}+w)(y-w)-(1-\gamma)(u(Ra_{t+1}+w)-u(Ra_{t+1}+b)) = 0$$

Free Entry. The zero profit condition for firms satisfies:

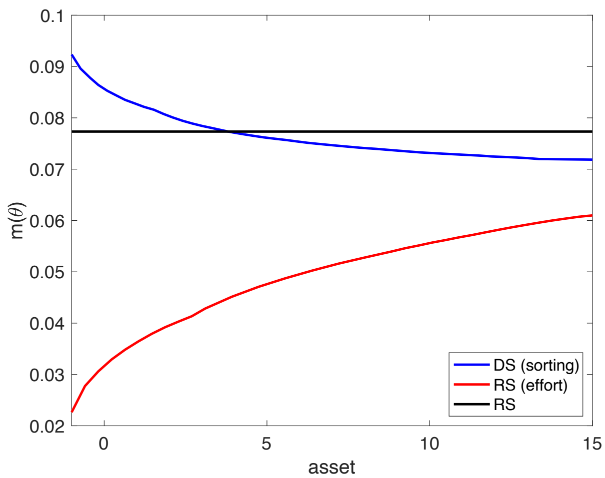
$$k = \beta q(\theta_t) \int (y - w) d\Lambda(a_t) \Rightarrow q(\theta_t) = \frac{k}{\beta \int (y - w) d\Lambda(a_t)}$$

having solved for wages and market tightness, now using low of motion, we can find the measure of unemployment and vacancy:

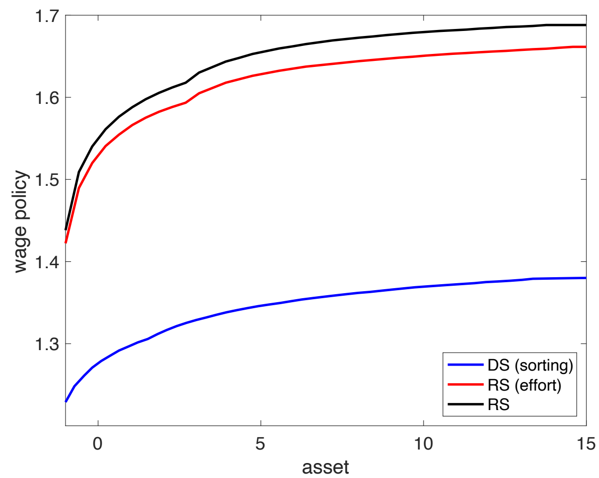
$$\begin{aligned}
u'(a_t - a_{t+1} + b) &= \beta R[s_t m(\theta_t) u'(Ra_{t+1} + w) + (1 - s_t m(\theta_t)) u'(Ra_{t+1} + b)] \\
e'(s) &= \beta m(\theta_t)[u(Ra_{t+1} + w) - u(Ra_{t+1} + b)] \\
\gamma u'(Ra_{t+1} + w)(y - w) &= (1 - \gamma)(u(Ra_{t+1} + w) - u(Ra_{t+1} + b)) \\
k &= \beta q(\theta_t) \int (y - w) d\Lambda(a_t)
\end{aligned}$$

To compare the heterogeneity in job finding rates and wages in a directed search model with sorting and a random search model with Nash bargaining, we parametrize the two-period version of our model using the value of parameters chosen in the benchmark calibration. We only change the cost of job creation (k) to get the same measure of vacancy creation in all three economies similar to our benchmark economy. Also, we set the level of productivity in the model with random search and random search with effort equal to the average level of productivity in the model with directed search and sorting.

In addition, we assume a lognormal distribution of asset holding – this is an equilibrium object in the infinite horizon version of the model while in a two period it is exogenous. Also for parameters regarding random search, we assume firms and workers have equal bargaining power – this is to ensure positive entry of firms given the value of parameters at the benchmark economy. Moreover, following Vejlin (2017) we set the the value of parameters in the search disutility function as $\zeta = \rho = 1$ implying that the disutility from search is given by $e(s) = \frac{1}{2}s^2$.



(a) Job Finding Probability



(b) Wage policy

Figure C.1: Sorting (benchmark) vs. Search Effort; Random vs. Directed Search

In Figure C.1 we compare sorting with search effort, and under two matching regimes, random versus directed search. In Figure 5.2a the probability of job finding is a decreasing function of the asset holdings in the directed search model. In contrast, the probability of job finding is increasing in wealth in the model with search effort and constant in the model just with Nash bargaining and no search effort. In the model with only random search all workers have the same probability of job findings no matter how poor or rich they are. However, with search effort, the effect of the job finding rate in the random search model is the opposite of that in the directed search model. Higher asset holdings result in a higher probability of job finding in the random search model with effort. These workers have lower marginal utility of consumption and are less concerned about consumption smoothing. Therefore, they are willing to exert more effort and pay the cost compared to poor workers. In contrast in directed search model higher asset holdings increase the ability of taking risk where richer workers apply for better paying jobs which are harder to get. Figure 5.2b shows that in all models (random and directed search) wages are increasing in assets.

Appendix D Alternative Distributions of Profits

Dividends as a Wage Subsidy. In the benchmark model the firms' dividend is equally distributed among all workers. In other words, we have implicitly assumed that all workers own an equal share of all firms. In this Section we depart from this assumption and propose a new way of redistribution of dividends. We assume that the firm's dividend is fully taxed and redistributed among unemployed workers to finance their benefits. Therefore, dividends are now a form of wage subsidy, because the tax burden is shifted away from wages in detriment of a tax on the profits of the firms. If the total dividend is not enough to finance a given level of UI, then we use a proportional tax on wages to cover what is lacking. B is the total benefit allocated to unemployed workers, $B = ub$. D is the total dividend of firms, $D = \int[y - w(y) - v(y)k]dy$. As a result, b and d are the level of benefit and dividend an unemployed worker receives, where we assume that the left over dividends (if any) are distributed equally among employed and unemployed workers. Therefore, there are two possible scenarios

1. if $D \geq B$: $c_{u,t} + a_{t+1} = b + Ra_t + D - B$ and $\tau = 0$.

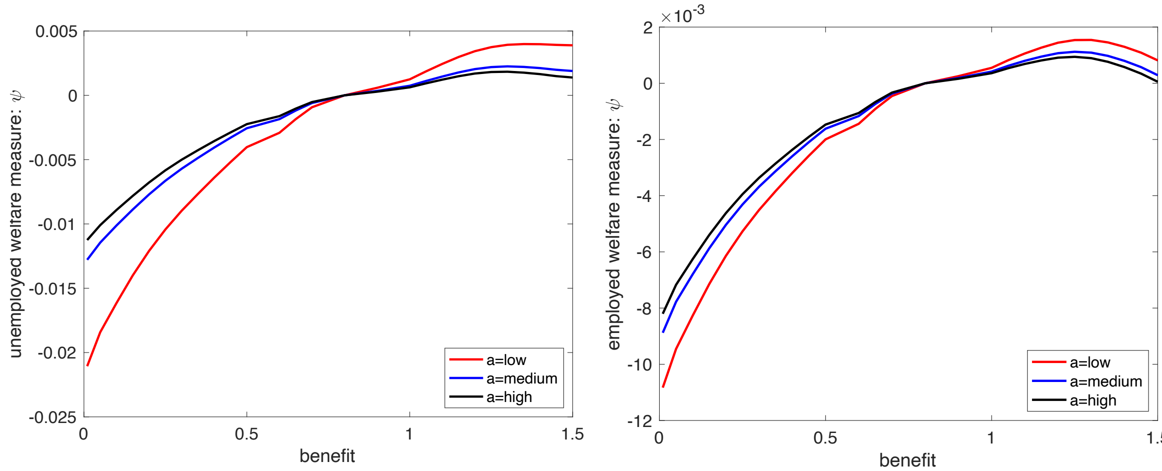
   $c_{e,t} + a_{t+1} = w + Ra_t + D - B$

2. if $D < B$: $c_{u,t} + a_{t+1} = b + Ra_t$ and $\tau > 0$ to finance $\frac{B-D}{u}$ for each unemployed

   $c_{e,t} + a_{t+1} = (1-\tau)w + Ra_t$

Using the benchmark calibration, up to $b = 1.25$ the total amount of dividend D under all counterfactual economies are higher than total amount of benefits distributed B. This implies that within this range, higher benefits are not associated with higher taxes for employed workers in this case. This is because the whole UI is now financed with dividends. The excess of dividend $(D - B)$ will also be distributed equally among all workers. Even though this excess is diminishing in benefits, it is always positive. When UI benefits increase, the net transfer to unemployed workers – consisting of benefits and excess dividends – increases. At the same time, the excess transfer to employed workers falls but they do not pay any taxes.

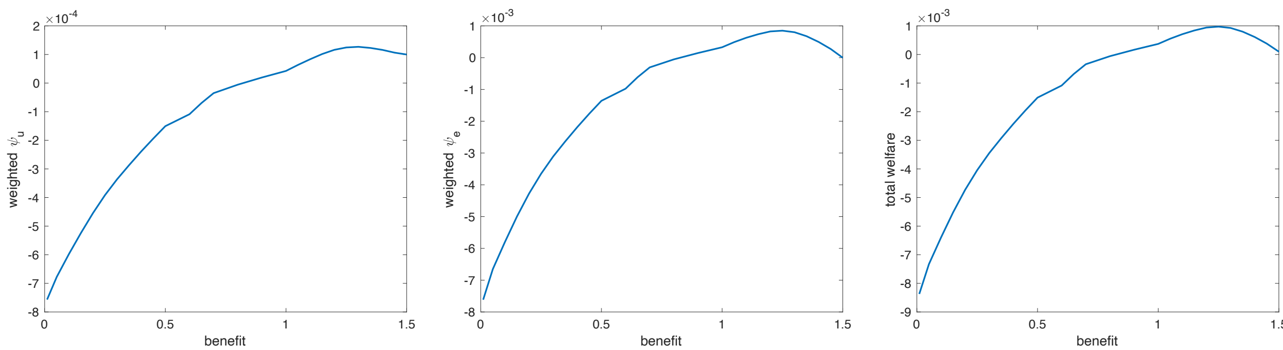
In Figure D.1 we plot the welfare for unemployed and employed workers of different assets, and in Figure D.2 we plot the average welfare. For benefits higher than 1.25, the total amount of dividend D is not sufficient enough to cover unemployment benefits. This implies that from this threshold of UI onward a proportional tax to wages is applied to cover what is lacking for financing benefits: $D - B < 0$. Interestingly that is exactly where the welfare function is maximized for employed workers and high asset unemployed ones. Still, asset poor unemployed workers prefer higher levels of UI no matter that this comes at the expense of higher taxes when they become employed. For asset rich unemployed and all employed workers, the negative effect of taxation on wages in their welfare function kicks in as soon as dividends are not enough to cover UI. These workers need less insurance than asset poor unemployed workers and therefore gain higher utility from higher levels of UI only when it is financed via dividends only.



(a) unemployed workers

(b) employed workers

Figure D.1: Welfare measure: ψ



(a) Average value unemployed

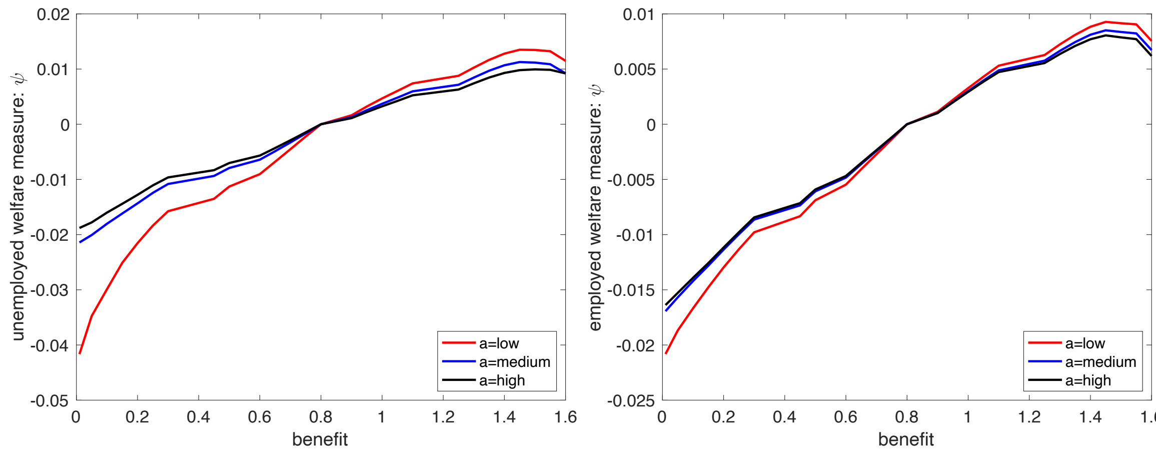
(b) Average value employed

(c) Total welfare

Figure D.2: Welfare

Dividends proportional to asset holdings. In this Section we consider yet another scheme to distribute profits: workers receive a share of the firms' dividend in proportion of their asset holdings. The idea is that assets are invested and that the return is proportional to the amount invested. We assume that workers with negative asset holdings do not receive any dividend while others depending on their position in the wealth distribution receive a share of the dividend. This complicates the numerical solution of the model further as we need to make a guess both on total amount of dividends distributed in the economy as well as on the share of each worker which is a function of the pdf of asset distribution. We keep the same benchmark economy as our main analysis in 4.1 where $b = 0.8$. The welfare results are presented below in Figures D.3 and D.4.

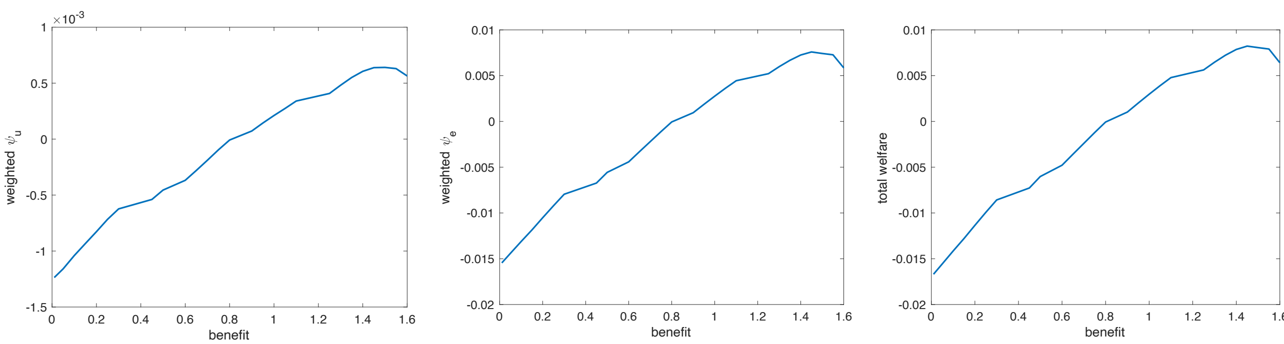
The welfare schedules are still inverted U-shape in benefits, but interestingly the workers' welfare is maximized at a much higher level of UI compared to the regime where all workers are assumed to own an equal share of total dividend. Higher benefits mean lower levels of dividends and also lower



(a) unemployed workers (b) employed workers

Figure D.3: Welfare measure: ψ

levels of savings. However, the new dividend regime incentivizes workers to save more to be able to get a higher share of dividends. For higher UI benefits, workers reduce their savings, but they do so less than in the constant dividend regime. Workers receive a higher share of dividends while they need less self-insurance because of higher benefits. As a result, relatively higher savings of workers increase their consumption which is welfare improving. Therefore, the negative effect of lower job finding rates kicks in at higher values of UI benefits.



(a) Average value unemployed (b) Average value employed (c) Total welfare

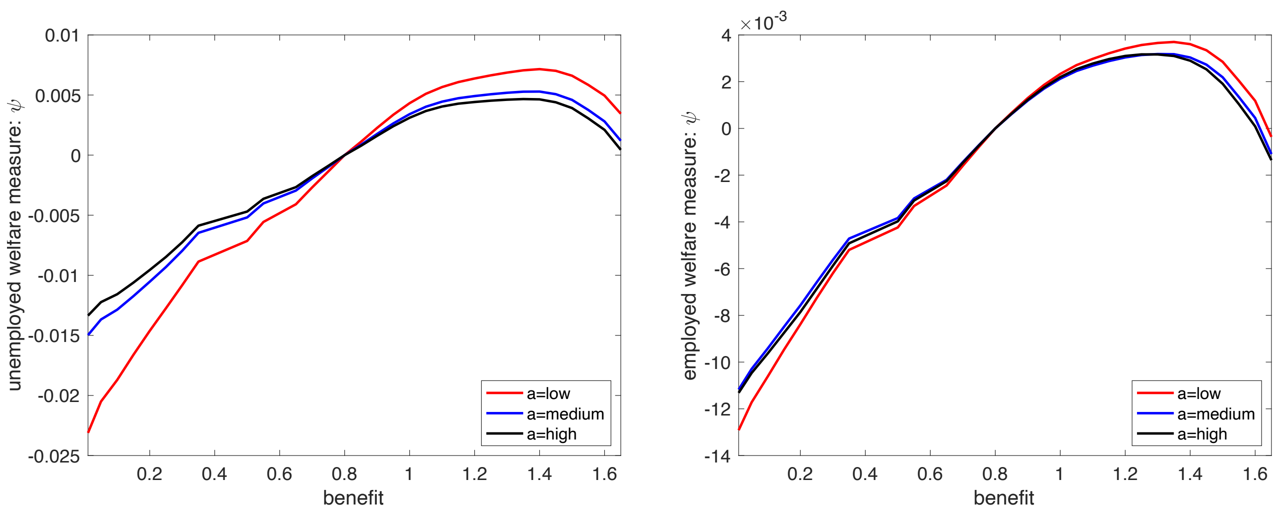
Figure D.4: Welfare

Appendix E Capital and endogenous interest rate

In this Section we assume that each employed worker produces $yf(\kappa)$ where y is the firm specific productivity, $f(\cdot)$ is an increasing and strictly concave production function and κ is the capital stock supplied by workers. We assume that $yf(\kappa) = y\kappa^{\alpha}$. To be able to compare our results with the benchmark analysis we use the same value of parameters (with the exception of endogenised interest rate, r) as the ones in section 4.1. Moreover, we choose α such that the range of output in this economy to be similar to that in the benchmark economy. We assume for this part that the borrowing constraint is zero and workers use their savings to lend them to firms for production. The firm first order condition implies that $r = yf'(\kappa)$ and for the equilibrium capital $\kappa^\star$, the firm's per period dividend is equal to $d = zf(\kappa^\star) - r\kappa^\star - w$, where r is the interest rate. We re-write the value of a filled job as

$$J(y, w) = \max_{\kappa} \; y\kappa^\alpha - r\kappa - w + \beta[\lambda V(y) + (1-\lambda)J(y, w)]. \tag{E.1}$$

We repeat the welfare exercise from Section 4.3. The only difference here is that the interest rate changes in the counterfactual economies.



(a) unemployed workers

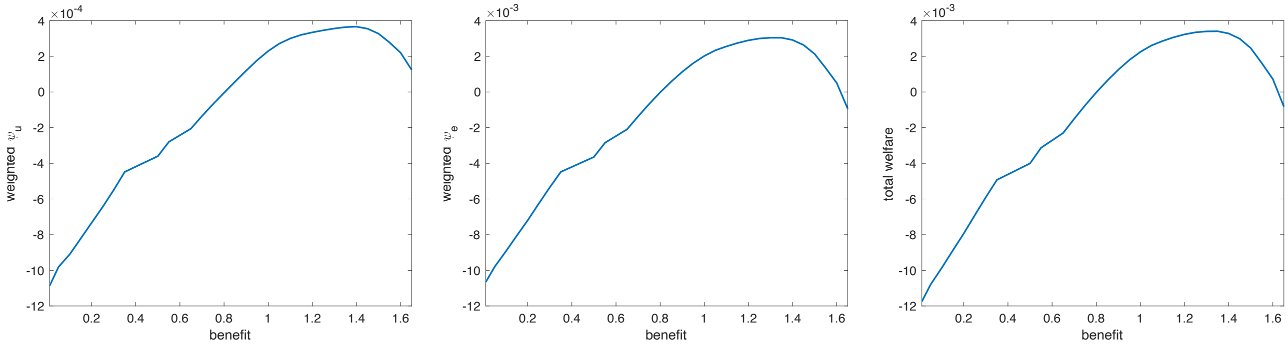
(b) employed workers

Figure E.1: Welfare measure: ψ when capital is endogenous

Qualitatively, the welfare results with an endogenous interest rate are similar to the benchmark economy. The lower interest rate at each benefit level compared to the benchmark analysis results in lower levels of saving. This magnifies the importance of external insurance and that is why the maximum welfare for both employed and unemployed workers are achieved at higher levels. But apart from that the shape of welfare function for all employed and unemployed workers remain qualitatively the same to our main exercise where we assumed a small open economy.

Appendix F Change in productivity

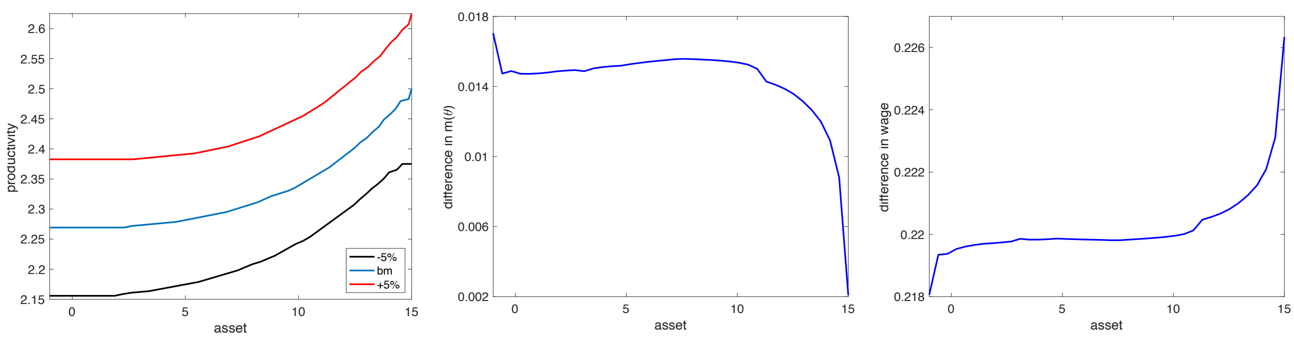
In this Section, we evaluate the impact of a change in the productivity distribution. We shift the distribution of productivities 5% to the right and left relative to the benchmark economy. Figure F.1a



(a) Average value unemployed (b) Average value employed (c) Total welfare

Figure E.2: Welfare when capital is endogenous

shows the change in allocation of workers to firms. When the productivities rise, the entire allocation moves up, including the threshold. There are some changes in the endogenous outcomes, in particular the probability of job finding and wages, all of which are mediated through the sorting of workers to firms.



(a) Allocation (b) Difference in prob. of job finding (c) Difference in wages

Figure F.1: Change in productivity distribution

Higher productivities means a higher probability of job finding and higher wages, as depicted in Figure F.1b and F.1c. They shows the difference in policy functions when the distribution of productivities shift 10% (5% in each direction). Although both the job finding probability and wages are higher when the distribution of productivities shifts up, the effect is heterogenous across the distribution of assets. The change in probability of job finding is least for high asset holders compared to asset poor unemployed workers. In contrast, the change in wages is positively correlated with asset holdings. This implies that when productivities increase, asset rich workers are more willing to take higher risks and apply for disproportionally higher wages while their probability of job finding does not increase as much. In contrast, poor workers apply for jobs they can get with a considerably higher probability but the wage of those jobs do not increase only a little.

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