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Francesco Giovannoni
Miltiadis Makris

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Department of Economics
University of Bristol
8 Woodland Road
Bristol BS8 1TN
United Kingdom

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Miltiadis Makris

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Abstract

We consider auctions where bidders care about the reputational effects of their bidding and argue that the amount of information that is disclosed at the end of the auction will influence bidding. Our analysis focuses on several bid disclosure rules that capture all of the realistic cases. We show that bidders distort their bidding in a way that conforms to stylized facts about takeovers/licence auctions. Also, we rank the disclosure rules in terms of the expected revenues they generate and find that, under certain conditions, full disclosure will not be optimal.

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1. Introduction

In this paper we investigate a class of auctions where bidders have reputational concerns. We consider auctions where a single, indivisible object is for sale in the standard independent private-values (IPV) setting. To this setting, we add reputational concerns for bidders by assuming that each bidder has a payoff-relevant type which is correlated with the bidder's valuation of the object, with both being her private information. One example is that where bidders are managers of firms in an auction for some takeover target and each bidder's type is her "quality" as a manager. This quality is correlated with her valuation because higher quality managers are better at extracting value from their acquisition or because they have more expertise in determining the valuation itself. Whenever managerial quality affects the managers' private valuations of the takeover target, bidding behavior will provide a signal of the manager's quality to a future job market for managers. Consequently, managers' bidding behavior will be affected by how much of the bidding process will be publicly disclosed at the end of the auction. We restrict attention to the cases where (a) auctions are either sealed-bid or descending for any number of bidders, or ascending (with the auction stopping when there is only one remaining bidder willing to buy) with two bidders,² and (b) the (labor) market after the auction has ended is perfectly competitive.³

²As we shall see, reputational incentives introduce issues that are reminiscent of those found in common value auctions. Thus, information released *during* an ascending auction with more than two bidders is important for the bidding behavior of remaining bidders. By excluding this case, we focus on the implications for bidding behavior of information released *at the end* of auctions. Nevertheless, we note that our results regarding over- or under-bidding (Proposition 3) would still hold in such setting.

³Of course, an environment where the post-auction market is imperfectly competitive and/or there is a common-value component in the bidders' valuations is worth investigating for a full understanding of reputational bidding but we view this as a starting point that clarifies the crucial role of various disclosure rules in an otherwise standard setting.

Different auctions may imply different kinds of bids-disclosure and this will have a decisive impact on bidder's incentives. For example, if a bidder's valuation is very low, then in a Dutch auction it is unlikely to be disclosed, whereas it would be certainly disclosed in a (sealed-bid) auction where all bids are disclosed at the end of the auction. We focus on four different disclosure rules. For each of these rules, the identity of the winner and of the bidders whose bids are revealed are always disclosed, as it would be natural in most conceivable applications. We have disclosure rule \mathcal{A} (for "all"), where all the bids are revealed; disclosure rule \mathcal{N} (for "none") where none of the bids are disclosed; disclosure rule \mathcal{W} (for "winner") where only the winning bid is disclosed - as in Dutch auctions - and disclosure rule \mathcal{S} (for "second") where only the highest losing bid is disclosed, as in a second-price sealed-bid auction where the price is disclosed. Conditional on revealing the winner's identity, other disclosure rules are still possible but we believe the rules above cover all the realistic cases. For the case of two bidders, these are *all* the possible cases.

Our analysis begins by characterizing Perfect Bayesian Nash Equilibria in pure strategies where bidding functions are symmetric and strictly increasing. We show that bidding functions are analogous to the ones in the absence of reputational incentives, after using what we call the bidders' effective valuations in the place of their valuations. These effective valuations take into account reputational effects and depend on the disclosure rule. This analogy implies revenue equivalence for auctions with different price mechanisms, but the same disclosure rule.

We then proceed to show that in this framework, for any disclosure rule, bidders will over- or under- bid depending on their reputational incentives. Therefore, recalling our example, in an environment where high valuations are perceived as signals of high managerial quality, managers with career concerns may consciously decide to bid too much.⁴ Further, we rank the different

⁴Burguet and McAfee (2009) argue that too much optimism on the value of the licenses might be at the heart of excessive bidding in telecommunication auctions, but our theory provides an alternative explanation that does

disclosure rules in terms of their expected revenues to the seller, conditional on the existence of symmetric and strictly increasing equilibria. The following are important consequences:

1. When bidders wish to be perceived to be as high (respectively low) a type as possible, a simplistic intuition might suggest that full revelation (respectively no revelation) of bids is expected-revenue maximizing. We show that this intuition is correct *only* under certain (sufficient) conditions. When these conditions are not satisfied, the disclosure rule that is revenue maximizing might actually be the opposite of the one basic intuition would suggest.
2. First-price sealed-bid auctions where only the price is disclosed (or Dutch auctions) and second-price sealed-bid auctions where only the price is disclosed (or ascending auctions with two bidders), utilize different disclosure rules. The former is a \mathcal{W} auction, while the latter is a \mathcal{S} auction. We show that their expected revenues differ and can be ranked. This is of particular interest given that it is common practice to disclose only the price and that in the standard framework without reputational concerns revenue equivalence obtains.

The organization of the paper is as follows. Section 2 discusses the related literature and Section 3 introduces the model. Section 4 characterizes equilibrium bidding functions and discusses expected revenues for given disclosure rules. Section 5 focuses on a comparative analysis of disclosure rules. Section 6 discusses the results and applies them to a couple of stylized models of licence auctions and corporate takeovers. Section 7 concludes and discusses future research. Most of our proofs are relegated to an appendix.

not require that bidders/managers systematically overestimate the value of their acquisitions.

2. Related Literature

There is a literature that deals with cases where reputational effects distort bidding behavior. Goeree (2003), Haile (2003), Das Varma (2003), Salmon and Wilson (2008) focus on the comparison of various price mechanisms *for a given disclosure rule*—whereas our main focus is on the comparison between various disclosure rules. Moreover, our revenue equivalence result for a given disclosure rule generalizes similar results in these papers for a wider range of disclosure rules.

The closest paper to ours is Katzman and Rhodes-Kropf (2008) but there are three important differences. First, they only consider - in terms of our terminology - second-price \mathcal{S} auctions versus first-price and second-price \mathcal{W} auctions. We consider $\mathcal{A}, \mathcal{W}, \mathcal{S}$ and \mathcal{N} auctions and emphasize that for a fixed disclosure rule, revenue equivalence obtains, thus showing that it is disclosure rules and *not* price mechanisms that affect expected revenues from a given auction. The second important difference is that in Katzman and Rhodes-Kropf (2008) the external incentives matter just for the winner while in our set-up bidding has reputational effects regardless of whether a bidder has won or lost the auction, as is natural in a signaling context. The final difference stems from the fact that, in our paper, reputational effects arise through a return that accrues to bidders *after* the conclusion of the auction (whether they have won or lost) through their interaction with a *third* party. This implies the time-additive separability in the payoffs between returns from the auction and reputational returns. This separability is not present in Katzman and Rhodes-Kropf (2008) (and in all the other papers cited above), as the payoff gross of the price paid in the auction accrues all in one instance in the future. The last two differences lie behind the difference in the revenue rankings between first-price and second-price auctions where only the price is disclosed (in our notation, \mathcal{W} and \mathcal{S} auctions respectively). Katzman

and Rhodes-Kropf (2008) cannot pin down an unambiguous result, while we can rank them (see Proposition 4II and 4III).

The paper by Molnar and Virag (2008) assumes additive separability between valuations and reputational payoffs, despite the fact that, similarly to the aforementioned papers, the payoff gross of the price paid in the auction accrues all in one instance in the future. There are still three major differences with their setting. First, in their paper, the question is one of optimal mechanism design, whereas we focus on standard auctions with realistic disclosure rules. Second, in their setting reputational incentives matter only for the winners. The final difference is that in our paper we have a more general formulation of the utility functions. A major implication is that in their framework \mathcal{W} or \mathcal{N} auctions always dominate \mathcal{S} auctions whereas this is not necessarily the case in our set up (again, see Proposition 4II and 4III).

Our main contribution to the literature on auctions is therefore to provide clear revenue rankings for all realistic disclosure rules in a context of pure reputational concerns where both winners and losers have reputational incentives. This is crucial if one wishes to understand bidding behavior in licence auctions or corporate takeovers.

3. The Model

We consider N bidders indexed by $i = 1, \dots, N$ who bid for a single unit of an indivisible object (or asset) and supply their services (or labor) in a perfectly competitive market that opens after the conclusion of the auction and the possible publication of (some of) its outcomes. All participants in this competitive market take information-contingent market-clearing wages as given and bidders supply their labor/services to future employers/firms at these wages. We assume free entry and exit of potential future employers, that the number of potential future employers is larger than N and that the reservation wage of bidders is zero. To simplify the

narrative we refer to any such post-auction interaction as the “after-market”.

We assume that bidders are risk neutral, have no budget constraint and face time additively separable utilities. We represent bidder i 's (expected) valuation for the object (or asset), with a realization $x_i \in [\underline{x}, \bar{x}] \equiv X \subset \mathbb{R}_+$, of the random variable X_i , with all the X_i distributed identically and independently across bidders according to the cdf F_X with a strictly positive density f_X which is continuously differentiable. A crucial element of our model is the function $V(\cdot)$, since $V(x_i)$ represents the returns from the bidder's services in the after-market *if it was publicly known* that her realized valuation is x_i . We assume that $V(\cdot)$ is common knowledge, while the bidders' valuations are their private information. We further assume that $V(\cdot)$ is strictly monotone and twice continuously differentiable.⁵ It will sometimes be convenient to refer to the random variable $V_i = V(X_i)$, with typical realization v_i and density (almost everywhere) $f_V(v_i)$. Let $y = \max_{j \neq i} \{x_j\}$ be the highest expected valuation amongst i 's competitors, which is distributed according to the cdf $G(y) \equiv F_X^{N-1}(y)$. Furthermore, $y_2 = \max_{j \neq i} (\{x_j\} / y)$ is the second highest expected valuation amongst i 's competitors. Let $L(y_2|y) \equiv \Pr(Y_2 \leq y_2 | Y = y)$.

The timing of events is as follows. First, an auction takes place. Then, some information (discussed below) about submitted bids and identities of corresponding bidders is publicly disclosed. Given this information, the after-market opens. Thus, when strategies are such that ties are zero probability events - which will be the case in the equilibria we will focus on - the payoff of bidder i given bids $\mathbf{b} = [b_1, \dots, b_N]$ is

$$(3.1) \quad \Pr[b_i > \max_{j \neq i} \{b_j\}](x_i - p(\mathbf{b})) + \delta \omega_i(\mathbf{b})$$

Here, $\Pr[b_i > \max_{j \neq i} \{b_j\}]$ and $p(\mathbf{b})$ denote, respectively, the probability of bidder i winning the object and the price paid upon winning given bids \mathbf{b} . In addition, $\delta > 0$ represents the discount

⁵Given the compactness of X , our assumptions on f_X and V imply that both are bounded and with bounded first derivatives. In addition, V must have a bounded second derivative.

factor (which is a measure of career concerns), and $\omega_i(\mathbf{b})$ denotes the expected wage earned in the after-market given bids \mathbf{b} . If the true valuation x_i was known to the after-market, then (with some abuse of notation) $\omega_i(\mathbf{b}) = V(x_i)$. In general, however, the expected wage will depend on the information that will be publicly disclosed at the end of the auction and the associated inferences of the after-market.

We will focus, throughout the paper, on symmetric and strictly increasing Perfect Bayesian Nash Equilibria in pure strategies, which we will refer to simply as “equilibria”. Restricting attention to symmetric equilibria follows the usual practice in the literature when the auction is symmetric as in our setup. Symmetric and strictly increasing equilibria are the only ones capable of guaranteeing efficiency in the sense of allocating the object for sale to the bidder with the highest valuation. Aside from this efficiency-driven motivation, we restrict attention to strictly increasing equilibria because we can show that for small enough and positive values of the discount factor, these are the only pure strategy symmetric PBN equilibria that may exist. Such equilibria will be represented by a strictly increasing bidding function $\beta(x_i)$. We analyze Perfect Bayesian equilibria rather than Bayesian Nash equilibria (as standard in the literature), because we need to impose credible restrictions on the after-market’s beliefs after equilibrium play in the auction. Regarding off-the-equilibrium path beliefs we make the following assumption, which we can show is compatible with the Universal Divinity refinement introduced by Banks and Sobel (1987).⁶

Assumption A (Beliefs) Let $\beta(\bullet)$ be an equilibrium of the auction under consideration. We

assume that in any such equilibrium, any bid lower than $\beta(\underline{x})$ is believed to come from

⁶Existence of strictly increasing, symmetric, pure strategy PBNE is discussed in Section 5. Proofs of non-existence of other symmetric, pure strategy PBNE and the fact that assumption A satisfies Universal Divinity are available in the Additional Propositions section of the Appendix

type $x_i = \underline{x}$ and any bid higher than $\beta(\bar{x})$ is believed to come from type $x_i = \bar{x}$. Further, if there is a bid b and a type \hat{x} such that $b \in (\lim_{x_i \rightarrow \hat{x}^-} \beta(x_i), \lim_{x_i \rightarrow \hat{x}^+} \beta(x_i))$ then b is believed to come from type $x_i = \hat{x}$.

Assumption A allows us to associate to each vector of bids \mathbf{b} a corresponding vector \mathbf{z} of types, which we refer to as *announcements*.⁷ Throughout the paper, we will assume that at the end of any auction, the identity of the participants and the identity ι of the winner are common knowledge. However, we will allow for the possibility that not all bids are disclosed. We represent this by assuming that, in equilibrium, individual i will expect that by submitting an announcement z_i , while everyone else reports her valuation truthfully, i.e. $\mathbf{z}_{-i} = \mathbf{x}_{-i}$, a subset $\phi(z_i, \mathbf{x}_{-i}) \subseteq (z_i, \mathbf{x}_{-i})$ of the announcements and the identities of the bidders who have submitted them will also be publicly disclosed at the end of auction. We will use the label ϕ to identify a particular disclosure rule and will refer to auctions with the disclosure rule ϕ as “ ϕ auctions”. Thus, in our set up, $\phi(z_i, \mathbf{x}_{-i})$ (plus the identities of the corresponding bidders and the winner) is the only information that is relevant to the after-market. In particular, for disclosure rule $\phi = \mathcal{A}$, where all the bids are revealed, $\mathcal{A}(z_i, \mathbf{x}_{-i}) = (z_i, \mathbf{x}_{-i})$; for disclosure rule $\phi = \mathcal{N}$, where none of the bids are disclosed, $\mathcal{N}(z_i, \mathbf{x}_{-i}) = \{\emptyset\}$; for disclosure rule $\phi = \mathcal{W}$, where only the winning bid is disclosed, $\mathcal{W}(z_i, \mathbf{x}_{-i}) = \max\{z_i, \mathbf{x}_{-i}\}$; for disclosure rule $\phi = \mathcal{S}$, where only the highest losing bid is disclosed, $\mathcal{S}(z_i, \mathbf{x}_{-i}) = \max\{z_i, \mathbf{x}_{-i} | j \neq \iota\}$.⁸

⁷That is, with z_i such that $\beta^{-1}(b_i) = z_i$ if $b_i \in \beta(X_i)$, $z_i = \underline{x}$ if $b_i < \beta(\underline{x})$, $z_i = \bar{x}$ if $b_i > \beta(\bar{x})$ and $z_i = \hat{x}$ if $b_i \in (\lim_{x \rightarrow \hat{x}^-} \beta(x), \lim_{x \rightarrow \hat{x}^+} \beta(x))$.

⁸Clearly, for \mathcal{A} and \mathcal{W} auctions the information on the identity of the winner is redundant as it can be recovered from the available information on bids and their corresponding bidders. However, for \mathcal{S} and \mathcal{N} auctions, it is not redundant. Results for these two auctions would be affected if the identity of the winner was not publicly disclosed. For instance, in \mathcal{N} auctions, not knowing also the identity of the winner would imply that the beliefs of the after-market coincide with the priors. We find the assumption that the winner’s identity is publicly disclosed quite

Let

$$M(y) = E_{F_X} [V(X_i) | X_i > y]$$

$$\Lambda(y) = E_{F_X} [V(X_i) | X_i < y]$$

Given any equilibrium bidding function β , we have that in \mathcal{A} auctions, the after-market's beliefs are that i 's type is z_i . Thus,

$$\omega_i^{\mathcal{A}}(\mathbf{b}) = V(z_i)$$

In \mathcal{W} auctions, we have instead

$$\omega_i^{\mathcal{W}}(\mathbf{b}) = \int_{\underline{x}}^{z_i} V(z_i) dG(y) + \int_{z_i}^{\bar{x}} \Lambda(y) dG(y)$$

because if i loses, then the winning bid reveals the winner's type to be y and thus the after-market's beliefs are that i 's type is below y , while if i wins then the after-market's beliefs are that i 's type is z_i . In \mathcal{N} auctions, the only information available is whether i has won the auction or not, and so

$$\omega_i^{\mathcal{N}}(\mathbf{b}) = \int_{\underline{x}}^{z_i} E_G[M(Y)] dG(y) + \int_{z_i}^{\bar{x}} E_G[\Lambda(Y)] dG(y)$$

For \mathcal{S} auctions, we have that if i wins, y is revealed and the expected wage earned by i conditional on y is $M(y)$. However, if i is not the winner, then i may either be the second-highest bidder (in which case z_i is revealed) or below the second-highest bidder (in which case y_2 is revealed). In these events, the conditional expected wage earned by i is $V(z_i)$ and $\Lambda(y_2)$, respectively. Therefore,

$$\omega_i^{\mathcal{S}}(\mathbf{b}) = \int_{\underline{x}}^{z_i} M(y) dG(y) + \int_{z_i}^{\bar{x}} \left[\int_{\underline{x}}^{z_i} V(z_i) dL(y_2|y) + \int_{z_i}^{\bar{x}} \Lambda(y_2) dL(y_2|y) \right] dG(y)$$

We leave this section by denoting with $v_i^\phi(y, z_i)$ the reputational returns of bidder i conditional on winning and with $v_{-i}^\phi(y, z_i)$ the reputational returns of bidder i conditional on losing. when natural to make, as it is satisfied in many real world auctions.

the highest opponent's valuation is y . We can then rewrite the expected wages as

$$\omega_i^\phi(\mathbf{b}) = \int_{\underline{x}}^{z_i} v_i^\phi(s, z_i) dG(s) + \int_{z_i}^{\bar{x}} v_{-i}^\phi(s, z_i) dG(s), \quad \phi = \mathcal{A}, \mathcal{W}, \mathcal{N}, \mathcal{S}$$

Simple inspection shows that the functions v^ϕ are twice differentiable in each argument.

4. Equilibrium Bidding Functions

The expected payoff for bidder i as a function of z_i and valuation x_i is

$$(4.1) \quad \int_{\underline{x}}^{z_i} (x_i + \delta v_i^\phi(s, z_i) - p(z_i, s)) dG(s) + \int_{z_i}^{\bar{x}} \delta v_{-i}^\phi(s, z_i) dG(s)$$

where $p(z_i, y) = \beta(z_i)$ in a first-price auction and $p(z_i, y) = \beta(y)$ in a second-price auction. We introduce now an important definition. Let $g(y) = (N-1) F_X^{N-2}(y) f_X(y)$ be the density of y .

Definition Let

$$\begin{aligned} \Psi^\phi(x_i, z_i) \equiv \\ x_i + \delta \left[v_i^\phi(z_i, z_i) - v_{-i}^\phi(z_i, z_i) + \frac{1}{g(z_i)} \left\{ \int_{\underline{x}}^{z_i} \frac{\partial}{\partial z_i} [v_i^\phi(s, z_i)] dG(s) + \int_{z_i}^{\bar{x}} \frac{\partial}{\partial z_i} [v_{-i}^\phi(s, z_i)] dG(s) \right\} \right] \end{aligned}$$

and

$$\psi^\phi(x_i) \equiv \Psi^\phi(x_i, x_i)$$

We call $\psi^\phi(x_i)$ the *effective valuation* for bidder i with valuation x_i who faces a disclosure rule ϕ .

Effective valuations can be obtained by maximizing the payoff of typical bidder i , (4.1), in a second-price auction, with respect to her announcement and requiring $z_i = x_i$ at the optimum. $\Psi^\phi(x_i, z_i)$ is the net welfare gain to a bidder of type x_i from winning (gross of payments) relative to the increase in the probability of winning, after increasing the announcement marginally over z_i , when the disclosure rule is ϕ . An effective valuation is such net welfare gain evaluated at

equilibrium. The component $v_i^\phi - v_{-i}^\phi$ captures the net reputational gain to the bidder from winning the auction, while the remaining term in the square brackets captures the additional reputational net gain from marginally increasing the announcement.

For example, compare the effective valuations in the \mathcal{A} and \mathcal{W} auction cases. It is easy to see that:

$$(4.2) \quad \psi^{\mathcal{A}}(x_i) = x_i + \delta \frac{V_x(x_i)}{g(x_i)}$$

$$(4.3) \quad \psi^{\mathcal{W}}(x_i) = x_i + \delta \left[V(x_i) - \Lambda(x_i) + \frac{G(x_i)}{g(x_i)} V_x(x_i) \right]$$

In \mathcal{A} auctions, other bidders' behavior has no impact on reputational returns and this is immediately apparent in the fact that $v_i^{\mathcal{A}} - v_{-i}^{\mathcal{A}} = V(x_i) - V(x_i) = 0$. On the other hand, with \mathcal{W} auctions winning or losing does matter for inferences about x_i because if i wins then x_i becomes known, while if i loses then the after-market believe x_i to be below the highest competing announcement (which is x_i at the margin between winning or losing) and $v_i^{\mathcal{W}}(x_i, x_i) - v_{-i}^{\mathcal{W}}(x_i, x_i) = V(x_i) - \Lambda(x_i)$. In addition, we have the reputational gain/loss (relative to the increase in the likelihood of winning the auction) from increasing marginally the perception of the after-market about bidder i 's type by means of increasing bidder i 's announcement marginally. For both \mathcal{A} and \mathcal{W} rules this relative gain/loss is always $\frac{V_x(x_i)}{g(x_i)}$ but the difference between \mathcal{A} and \mathcal{W} auctions is that with the former bids are always disclosed, while with the latter, such gain/loss only applies when i wins, which occurs with probability $G(x_i)$.

For our other disclosure rules, we have that⁹

$$(4.4) \quad \psi^{\mathcal{S}}(x_i) = x_i + \delta \left[M(x_i) - V(x_i) + \frac{1 - F_X(x_i)}{f_X(x_i)} [V_x(x_i) + (N-2)\Lambda_x(x_i)] \right]$$

$$(4.5) \quad \psi^{\mathcal{N}}(x_i) = x_i + \delta [E_G[M(Y)] - E_G[\Lambda(Y)]]$$

To state our first result we need the following two assumptions:

⁹Calculating $\psi^{\mathcal{S}}(x_i)$ is not so straightforward, and the details are available upon request.

Assumption B (Lower Bound Condition) For $N > 2$, there exists a positive scalar T such that

$$\left| \lim_{x_i \rightarrow \underline{x}^+} \frac{V_x(x_i)}{g(x_i)} \right| \leq T$$

This condition requires that marginal reputational incentives for the lowest type be bounded and is a necessary condition for the existence of equilibrium for \mathcal{A} auctions when $N > 2$, because in such auctions the lowest type will be revealed in equilibrium. By contrast, it is redundant for \mathcal{N} , \mathcal{S} and \mathcal{W} auctions, where the lowest type is never revealed in equilibrium. Assumption B and our assumptions about $V(x)$ guarantee that effective valuations are well-defined, bounded and differentiable. From now on, we slightly abuse notation by denoting $\lim_{x_i \rightarrow \underline{x}^+} \psi^\phi(x_i)$ with $\psi^\phi(\underline{x})$.¹⁰

Assumption C.

$$\psi^\phi(\underline{x}) \geq 0$$

Given Assumption B, a sufficient condition for $\psi^\phi(\underline{x}) \geq 0$, whatever the disclosure rule, is that \underline{x} is sufficiently high. We then have:

Proposition 1 *Assume A, B and C hold. If $\psi^\phi(x_i)$ is strictly increasing, then:*

1. *The equilibrium in second-price sealed-bid auctions with a disclosure rule ϕ , $\beta^{FP-\phi}$, is given by*

$$\beta^{SP-\phi}(x_i) = \psi^\phi(x_i), \quad x \in [\underline{x}, \bar{x}]$$

2. *The equilibrium in first-price sealed-bid auctions with a disclosure rule ϕ , $\beta^{SP-\phi}$, is given by*

$$\beta^{FP-\phi}(x_i) = E_G \left[\psi^\phi(Y) | Y < x_i \right], \quad x \in [\underline{x}, \bar{x}].$$

¹⁰For $N = 2$, $\psi^\phi(\underline{x})$ is necessarily well defined, given our assumptions, in particular, that $f_X(\underline{x}) > 0$. However, for $N > 2$, we will have that $g(\underline{x}) = (N - 1)F_X(\underline{x})^{N-2}f_X(\underline{x}) = 0$. It is in those cases that $\psi^\phi(\underline{x})$ should be interpreted as $\lim_{x_i \rightarrow \underline{x}^+} \psi^\phi(x_i)$.

Proof See the Additional Proofs sections of the Appendix ■

It is immediate to see the similarity between this result and that for the basic IPV set up since the only difference is that bidders use their *effective valuations* rather than their valuations. Two issues arise from the proposition above. The first is that $\psi^\phi(x_i)$ or $E_G[\psi^\phi(Y)|Y < x_i]$ are not guaranteed to be strictly increasing. The second is whether, conditional on equilibrium existence, revenue equivalence between standard price mechanisms still obtains in our set-up.

We briefly take up the equilibrium existence issue in Section 5 below, but with respect to the second issue, we can show that:

Proposition 2 *Consider a disclosure rule ϕ . Any equilibrium of any auction such that (a) the highest bidder wins, (b) no information about bids becomes public during the auction,¹¹ and (c) the expected payment of a bidder with the lowest valuation is zero, yields expected revenue to the seller equal to*

$$E_{F_2^{(N)}}[\psi^\phi(Y_2^{(N)})]$$

where $F_2^{(N)}$ is the cdf of the random variable $Y_2^{(N)}$ that represents the second-highest type amongst all bidders.

Proof. See the Additional Proofs sections of the Appendix ■

Since effective valuations depend on ϕ , Proposition 2 implies that standard auctions with the same price mechanism but different disclosure rules may have different expected revenues. Below, we investigate bidding functions in more detail and revenue rankings of disclosure rules.

5. Bidding Distortions and Revenue Comparisons of Disclosure Rules

We start with a property of effective valuations:

¹¹Recall footnotes 2 and 3.

Proposition 3 *Whenever $V_x > 0$ for all $x \in (\underline{x}, \bar{x})$, then $\psi^\phi(x) > x$ for all $x \in (\underline{x}, \bar{x})$, and $\psi^\phi(\underline{x}) \geq \underline{x}$ and $\psi^\phi(\bar{x}) \geq \bar{x}$, for all disclosure rules $\phi \in \{\mathcal{N}, \mathcal{A}, \mathcal{W}, \mathcal{S}\}$. Conversely if $V_x < 0$ for all $x \in (\underline{x}, \bar{x})$.*

Proof. Follows directly from the definitions of effective valuations and $M(x)$ and $\Lambda(x)$ above

■

This proposition simply states that if the reputational returns when one's bid is revealed in equilibrium, $V(x_i)$, are strictly increasing (respectively strictly decreasing) in x_i , we then have that with any of our disclosure rules here, there is almost everywhere overbidding (respectively underbidding). The reason is that bidders want the after-market to believe their valuations are high (respectively low).¹²

In our set up, existence of equilibrium is not guaranteed for all disclosure rules because the standard incentives when participating in an auction may conflict with reputational incentives. Existence is guaranteed for \mathcal{N} auctions: in this case the bidding functions are as in the standard IPV framework up to a constant. In addition, recall from (4.2)-(4.5) that effective valuations have two additive components, the first of which is the standard valuation.¹³ Therefore, the bidding function is strictly increasing when $\delta = 0$. This implies directly that, for the rest of the disclosure rules, since reputational components have bounded first derivatives, there is a range for small enough and positive discounting factors for which an equilibrium exists, under both first- and second-price auctions. Furthermore, we can provide, for any discount factor, sufficient conditions for equilibrium existence in both first- and second-price auctions with any N for disclosure rules \mathcal{W} and \mathcal{A} , and with $N = 2$ for \mathcal{S} auctions. These conditions are described in Proposition A1, in

¹²We say *almost everywhere*, because the only cases when there is no over/under-bidding for a type x_i are when

(i) $x_i = \underline{x}$, $\phi = \mathcal{A}$ and $\lim_{x_i \rightarrow \underline{x}^+} \frac{V_x(x_i)}{g(x_i)} = 0$, or (ii) $x_i = \underline{x}$ and $\phi = \mathcal{W}$ or (iii) $x_i = \bar{x}$ and $\phi = \mathcal{S}$.

¹³That is, we have $\psi^\phi(x) = x + \delta \tilde{\psi}^\phi(x)$, where $\tilde{\psi}^\phi(x)$ is the reputational component.

the appendix.¹⁴ Assuming thus existence of equilibrium, we compare next the various disclosure rules in terms of expected revenues in such equilibria. Denote by $ER(\phi)$ the expected revenue associated with a specific disclosure rule ϕ . The proposition below summarizes a complex list of results which we report in full in the appendix.

Proposition 4 *[summary] Assume existence of equilibrium. Then:*

- I For large enough N , $ER(\mathcal{A}) - ER(\mathcal{N}) > 0$ (respectively < 0) whenever $V_x > 0$ (respectively < 0)*
- II Whenever $V_x > 0$ for all $x \in (\underline{x}, \bar{x})$, then $ER(\mathcal{S}) > ER(\mathcal{W})$. Further, if $N = 2$ then $ER(\mathcal{A}) = ER(\mathcal{S})$ while if $N > 2$ then $ER(\mathcal{A}) > ER(\mathcal{S})$*
- III Whenever $V_x < 0$ for all $x \in (\underline{x}, \bar{x})$, then $ER(\mathcal{S}) < ER(\mathcal{W})$. Further, if $N = 2$ then $ER(\mathcal{A}) = ER(\mathcal{S})$ while if $N > 2$ then $ER(\mathcal{A}) < ER(\mathcal{S})$*
- IV $ER(\mathcal{W}) - ER(\mathcal{N})$ is positive (respectively zero, negative) if f_V is strictly increasing (respectively constant, strictly decreasing).*

Proof See Appendix ■

Given that reputational incentives lead to overbidding (respectively underbidding) when being perceived to be of a higher (respectively lower) type is favorable to the bidder, one might be tempted to think that the more information is disclosed, the more overbidding (respectively underbidding) one should expect and consequently, more (respectively less) expected revenue. This is not true if, for example, $F_X(x) = U[0, 1]$, $V(x) = \frac{1}{4}x^4$ and $N = 2$ because $ER(\mathcal{N}) -$

¹⁴ Among other things, Proposition A1 implies that existence for second-price ϕ auctions implies existence for first-price ϕ auctions. Moreover, when $V_x > 0$, existence for second-price \mathcal{A} auctions implies existence for second-price \mathcal{W} auctions and conversely when $V_x < 0$.

$ER(\mathcal{A}) = \frac{7}{16} - \frac{13}{30} = \frac{1}{240} > 0$. Thus, even if $V_x > 0$, more disclosure leads to less revenue.¹⁵ To understand why this may be, we focus on the case $V_x > 0$ as an entirely symmetric argument applies for the case $V_x < 0$. Recall that disclosure rules provide reputational incentives in two ways. The first reputational incentive is the one that gives us the simple intuition: if more bids are disclosed, more bidders are likely to see their type disclosed in equilibrium, and so reputational incentives to overbid increase. There is, however, also a reputational incentive that comes from the simple difference between winning and losing the auction because it may provide clues to each bidder's type. \mathcal{A} auctions and \mathcal{N} auctions are very different because the former generate only the first reputational incentive (knowing someone's bid is all you need to recover their type in equilibrium), while the latter generate only the second reputational incentive. There are no simple necessary and sufficient conditions that guarantee that one incentive dominates the other, but Proposition 4 does provide a simple set of sufficient conditions. Of course, being only sufficient, if these conditions are not satisfied, the reverse may occur.¹⁶

In an \mathcal{N} auction, the reputational incentives are bounded above by $\max_x (M(x) - \Lambda(x))$. The reputational component of expected revenues from \mathcal{A} auctions is $N(E_{F_X}(V(X)) - V(\underline{x}))$ which is strictly increasing and unbounded in N . Intuitively, what happens is that any bidder, from a reputational perspective, faces the same potential gains from having her type revealed no matter how many other bidders there are, and so the reputational component of expected payments from a given bidder is a constant. Of course, then, the reputational component of expected revenues increases by this expected payment whenever there is an additional bidder. This explains part I of Proposition 4.

¹⁵In the appendix, the proof of Proposition 4 provides the relevant formulas to make this a straightforward calculation. Note also that in this example, f_V is strictly decreasing.

¹⁶For example, if $V(x) = \frac{1}{10}x^{10}$ then \mathcal{N} auctions generate more expected revenue than \mathcal{A} auctions for $2 \leq N \leq 5$.

Regarding the comparison between \mathcal{A} and \mathcal{S} auctions, in parts II and III of Proposition 4, we note that when $V_x > 0$, with disclosure rule \mathcal{S} bidders with low valuations have a stronger relative incentive to overbid in equilibrium than under $\phi = \mathcal{A}$. The reason is that low types will be rewarded by their bid not being disclosed in case they win; such an additional incentive does not exist for \mathcal{A} auctions. Our result shows that the difference in incentives between disclosure rules cancel out in expectation when $N = 2$, whereas for $N > 2$ they work in favour of disclosure rule \mathcal{A} .¹⁷ Conversely, if $V_x < 0$.

With regards to the comparison between $\phi = \mathcal{S}$ and $\phi = \mathcal{W}$, the two types of auction reveal exactly one bid each, yet our results suggest that reputational incentives are stronger in the former. Low valuation bidders have a higher chance of being the highest loser than the winner while the difference between the probability of being the winner or the highest loser is not so significant for high valuation bidders. Thus, low valuation bidders have proportionately higher incentives to distort their bids in \mathcal{S} auctions.

In part IV, we complete our comparisons by considering \mathcal{N} and \mathcal{W} auctions. Consider the case where f_V is strictly increasing and $V_x > 0$. In this case, high realizations of x are more likely than low realizations. Also, in \mathcal{N} auctions overbidding is constant in x while in \mathcal{W} auctions high types overbid more than low types. Thus, a distribution of valuations that puts more weight on high realizations than low ones will have a greater impact on revenue in \mathcal{W} auctions than on revenue in \mathcal{N} auctions. Obviously, if f_V is strictly decreasing the reverse obtains.¹⁸

¹⁷More specifically, when $N > 2$, a very low type knows that her type is still unlikely to be revealed in an \mathcal{S} auction while very likely when $N = 2$, and so the incentives to overbid are smaller for the former case. Thus, it is not surprising that the difference in expected revenues between \mathcal{A} auction (where reputational incentives for a given bidder are constant in N) and \mathcal{S} auctions (where they are strictly decreasing in N) is strictly increasing in N .

¹⁸The intuition for the cases where $V_x < 0$ follows along similar lines, if one recalls that in this case we have underbidding and that f_V strictly increasing (respectively, strictly decreasing) now implies that high realizations

6. Discussion

Proposition 4 and the analysis above give us the opportunity for several corollaries which we summarize next.

In the previous section we emphasized the ex-ante trade-offs that an auctioneer must confront when choosing a disclosure rule. Thus, when $V_x > 0$, a government that mainly wants to guarantee an efficient allocation of an asset it owns and sells might prefer \mathcal{N} auctions, but when f_V is strictly increasing, equilibrium existence is not a problem for \mathcal{A} auctions and bidders are expected to play such equilibria, a government that puts a lot of emphasis on revenue generation will choose \mathcal{A} auctions.

Secondly, Proposition 4 highlights that full disclosure may be dominated by other disclosure rules. When $V_x < 0$, \mathcal{A} auctions are revenue dominated by auctions with disclosure rules \mathcal{W}, \mathcal{S} and, when f_V is strictly decreasing, \mathcal{N} . Quite surprisingly, this may also be possible when $V_x > 0$, as long as f_V is strictly decreasing and N is not too large, as the example immediately below Proposition 4 demonstrates.¹⁹

Finally, consider a first-price and a second-price sealed-bid auction where only the price, the corresponding bidder and winner are disclosed. The former is a \mathcal{W} auction while the latter is a \mathcal{S} auction. Thus, from Proposition 4, whenever $V_x > 0$ (respectively $V_x < 0$) for all $x \in (\underline{x}, \bar{x})$ and an equilibrium exists, the second-price auction generates more (less) expected revenues than the first-price auction. The linkage principle - obtained for single-object auctions by Milgrom and Weber (1982) - has been broadly interpreted as implying that more public information raises revenues. This corollary here could be interpreted, as a failure of such interpretation of the

of x are less (respectively, more) likely.

¹⁹By the same token, Proposition 4 clarifies that in the presence of under-bidding, no transparency may not be revenue maximizing either, as \mathcal{N} auctions may be dominated by \mathcal{W} auctions, for example.

linkage principle in an environment where valuations are independent but there are reputational effects.²⁰

6.1. Licence Acquisitions and Corporate Takeovers (I)

We apply our findings to an example motivated by recent telecommunication auctions and corporate takeovers. Bidders are the firms' managers who have career concerns and are involved in the takeover of another firm or licence acquisition.²¹ Managers are trying to determine the value of the target/licence for their firm and higher ability managers are those more capable of asset evaluation. To model this, we assume that for each bidding firm, the private valuation is w_i .²² This valuation is unknown to everyone and firm i 's manager receives a private signal θ_i on it. How good a signal this is depends on the manager's quality γ_i . We follow Ottaviani and Sorensen's (2006) multiplicative linear experiment by assuming that all these random variables

²⁰For a similar argument in multi-unit sequential auction with unit-demands and interdependent types/signals see Mezzetti, et al. (2008).

²¹It is well worth emphasizing that in the example we explicitly interpret bidders as agent-managers working on behalf of a principal (the owners/shareholders). This raises the issue of whether shareholders have possible explicit incentives in place to counteract the implicit incentive of signaling to the after-market. Depending on the available instruments, shareholders might be able to alleviate the effects on profits of their manager's implicit incentives. We do not model the possibility of explicit counter-incentives but our discussion remains valid as long as implicit incentives cannot be completely eliminated. This seems to be a realistic assumption as any explicit contract designed for counter-incentives would have to be able to quantify and verify precisely how much bidding was distorted. For a similar argument, see Maldoom (2005, pp. 582).

²²Börger and Dustmann (2005, pp. 557) argue that in the context of the UK 3G auctions, the assumption of private valuations is reasonable as "...all relevant information had already reached the public domain and that no firm had important insider information, except for information that concerned only its own situation, with no immediate relevance for other firms."

are defined on the unit interval, and that the joint density is given by

$$f_{W,\Theta,\Gamma}(w_i, \theta_i, \gamma_i) = \left(\gamma_i \left(w_i - \frac{1}{2} \right) \left(\theta_i - \frac{1}{2} \right) + 1 \right) f_W(w_i) f_\Gamma(\gamma_i)$$

where f_W and f_Γ are well-defined densities of the true valuation and managerial ability, respectively, and positive everywhere in their supports. This captures the idea that with probability γ_i the signal θ_i is informative about w_i , while with probability $(1 - \gamma_i)$ the signal is pure noise.

Denote with $x_i = X(\theta_i)$ the expected valuation of bidder i after having observed the signal θ_i , which can be shown to be strictly increasing. We also define here $V(x_i)$ to be the expected quality of bidder i conditional on her having observed a signal $X^{-1}(x_i)$. If we assume that the density f_W has mean $\mu \neq \frac{1}{2}$ and variance ξ_w , while the density f_Γ has mean τ and variance ξ_γ , then the multiplicative linear experiment can be shown to imply that

$$V(x_i) = \tau - \frac{\xi_\gamma (2\mu - 1) (\mu - x_i)}{2\tau\xi_w}$$

$$f_V(v_i) = 2 \frac{\xi_\gamma^2}{(\tau^2 - \tau v_i + \xi_\gamma)^3 |2\mu - 1|} 1_{\left[\tau - \frac{\xi_\gamma |2\mu - 1|}{4 - \tau |2\mu - 1|}, \tau + \frac{\xi_\gamma |2\mu - 1|}{4 + \tau |2\mu - 1|} \right]}(v_i)$$

where $1_{[v_L, v_H]}(v_i) = 1$ if $v_i \in [v_L, v_H]$ and 0 otherwise. $V(x_i)$ is strictly increasing in x_i whenever $\mu > \frac{1}{2}$ and strictly decreasing in x_i whenever $\mu < \frac{1}{2}$.

Thus, in this context, a high signal (or expected valuation given monotonicity of $X(\cdot)$) is interpreted as high expertise when $\mu > \frac{1}{2}$ and as low expertise when $\mu < \frac{1}{2}$.²³ Therefore, inferences about managerial ability depend on prior beliefs about w_i . When these are optimistic (i.e. $\mu > 1/2$), a high expected valuation x_i also leads to the inference of a high γ_i because it conforms to the prior beliefs. Conversely, when prior beliefs are pessimistic (i.e. $\mu < 1/2$). For us, this is reminiscent of the discussion of bidding behavior in recent telecommunication auctions.

²³Note that if $\mu = \frac{1}{2}$, then for any θ_i the after-market would not be able to make any additional inferences about the manager's own expertise, which is why we do not allow for this case. Formally, if $\mu = \frac{1}{2}$, then the conditional on signal density of ability is equal to the unconstrained density $f_\Gamma(\gamma_i)$.

Many have argued for the presence of overbidding because there was a lot of hype about the value of these licences.²⁴ In the context of our example here, the pre-auction hype could be interpreted as an optimistic prior belief on the part of the after-market. Under the multiplicative expertise model, this would lead to the inference that a high (expected) valuation is also an indicator of high managerial ability and our model would predict overbidding. Here managers do not overbid because they are too optimistic, but in order to pander to the public's optimistic beliefs. Finally, given that f_V is strictly increasing, Proposition 4 implies:

$$ER(\mathcal{A}) \geq ER(\mathcal{S}) > ER(\mathcal{W}) > ER(\mathcal{N}) \text{ if } \mu > \frac{1}{2}$$

$$ER(\mathcal{W}) > ER(\mathcal{S}) \geq ER(\mathcal{A}) \text{ and } ER(\mathcal{W}) > ER(\mathcal{N}) \text{ if } \mu < \frac{1}{2}$$

where weak inequalities become equalities for $N = 2$ and strict inequalities for $N > 2$.

6.2. Licence Acquisitions and Corporate Takeovers (II)

Consider again the takeover of a targeted firm but assume that the ability/quality of the firms' managers affects their valuation of the target. This would typically be the case when the winning bidder's manager will be in charge of the newly acquired firms.

To model this, we assume that for each bidding firm, the private valuation of the takeover target is known to the bidder, and for bidder i equals x_i . We also assume that this valuation is a strictly increasing function of the quality γ_i of the bidding firm's management: $x_i = X(\gamma_i)$.²⁵

²⁴For instance, Burguet and McAfee (2009).

²⁵For example, we could have that the valuation for firm i is a function of the manager's quality γ_i and of some intrinsic characteristic s of the target. This characteristic s influences all bidders' valuations equally, but no bidder has specific private information about it since due diligence leaves all perspective buyers relatively well informed. Thus,

$$x_i \equiv X(\gamma_i) = \int_{-\infty}^{\infty} h(s, \gamma_i) f_S(s) ds$$

where $h(\bullet)$ is a function common to all bidders and strictly increasing in γ_i , while s is a common value component,

This might be because the bidding firm and the takeover target will have complementary assets. For example, the bidder may be a large pharmaceutical conglomerate bidding for a small biotech firm that has produced a new drug. The logic behind the takeover is that the bidder can bring in its marketing, sales and regulatory expertise which are beyond the biotech firm's ability. So, the bidder's valuation represents its management's ability to make the most of the new drug.²⁶ Let us now identify here the quality of manager i , Γ_i , with the variable V_i introduced in Section 3 of the paper; that is, $f_V(v_i) \equiv f_\Gamma(\gamma_i)$ and $X^{-1}(x_i) \equiv V(x_i)$. We then have that $V(x_i)$ is strictly increasing, due to the assumed properties of $X(\cdot)$. Thus, recalling Proposition 4, revenue rankings in this model depend on the properties of f_V .

Andrade, et al. (2001) provide evidence that firms overbid in takeovers and mergers, while Yim (2013) surveys the previous literature and provides evidence that younger CEOs are more keen to do takeovers and mergers. She argues that higher career concerns for younger CEOs must be involved, because managers get rents from strictly increasing the size of their firms. Our theory suggests that career concerns may be behind overbidding behavior, but rather than arguing that rents are involved, we show that bidding managers send signals about their ability to future potential employers through such takeovers.²⁷ In particular, the result in Proposition independent of γ_i , with its density f_S assumed to be common knowledge.

²⁶Rhodes-Knopf and Robinson (2008) argue that takeovers and mergers most often arise because of the sort of complementarities described above. One can, however, conceive of cases where the acquiring firm has been unsuccessfully trying to develop a product for a market and finally decides to give up and to obtain instead a better product by acquiring a smaller but more successful competitor. Microsoft's decision to bid for Yahoo after unsuccessful attempts to develop its own portal can represent such a case. In such situations, the assets of the two firms are substitutes and we should expect a strictly decreasing $X(\cdot)$.

²⁷Note that our theory does not necessarily predict overbidding. Indeed, the example here describes both a scenario where one would expect overbidding and one where one would expect underbidding. Further, the (ego-) rents theory does not predict that bidding distortions will depend on specific disclosure rules, while our theory

3 suggests that in takeovers managers will tend to overbid whenever the assets of the bidding firms and the target are complements.

7. Conclusions

This paper studies auctions where bidders have reputational concerns. We show how disclosure rules and not price mechanisms are crucial in this context and discuss the relative implications of using different disclosure rules for maximizing the seller's expected revenue. Also, these results shed some light on the perceived overbidding that has occurred in telecommunication auctions and corporate takeovers in the past.

Future research should consider a more complex environment where the job market for managers/bidders benchmarks a manager's type with that of another. Therefore, a bidder's reputational returns would depend on the expected valuations of other bidders as well, even in the case where the after-market was certain post-auction of the bidders' expected valuations.

Also, a crucial assumption in our model is that the after-market is aware of the identity of the bidders. With endogenous participation, such assumption would no longer be warranted and it would be interesting to examine how reputational concerns would affect it.

University of Bristol, U.K.; University of Southampton, U.K.

8. Appendix: Proofs

Throughout the proofs, we remove the subscript i whenever there is no risk of confusion. Also, monotonicity statements should be understood in the strict sense. The following Lemma will be used extensively in proving our results.

does. All these differences are potentially testable, but such a task is beyond the scope of the current paper, and is left for future research.

Lemma 1 If $V_x(x) > (<) 0$, for any $x \in (\underline{x}, \bar{x})$, then $\Lambda_x(x)$ and $M_x(x) > (<) 0$.

Proof. Differentiating $M(x)$ and $\Lambda(x)$ we have $M_x(x) = \frac{f_X(x)}{1-F_X(x)}[M(x) - V(x)]$ and $\Lambda_x(x) = \frac{f_X(x)}{F_X(x)}[V(x) - \Lambda(x)]$. The lemma then follows directly from the fact that if $V_x(x) > 0$ for any $x \in (\underline{x}, \bar{x})$, then $M(x) > V(x) > \Lambda(x)$, and vice versa ■

Proposition A1 Suppose assumptions **A**, **B**, and **C** hold. Let $G_V = (F_V)^{N-1}$ and $\bar{G}_V = (1 - F_V)^{N-1}$

1. Whenever $V_x > 0$ for all $x \in (\underline{x}, \bar{x})$, then equilibrium for \mathcal{W} (respectively \mathcal{A}) second-price auctions is guaranteed if G_V is log-concave (respectively iff G_V is concave).
2. Whenever $V_x < 0$ for all $x \in (\underline{x}, \bar{x})$, then equilibrium for \mathcal{W} (respectively \mathcal{A}) second-price auctions is guaranteed if \bar{G}_V is log-convex (respectively iff \bar{G}_V is convex).
3. If $V_x > 0$ and $N = 2$, then equilibrium for \mathcal{W} , \mathcal{S} and \mathcal{A} second-price auctions is guaranteed if $1 - F_X(x)$ is log convex and $V(x)$ is convex.
4. If $V_x < 0$ and $N = 2$, then equilibrium for \mathcal{W} , \mathcal{S} and \mathcal{A} second-price auctions is guaranteed if $F_X(x)$ is log convex and $V(x)$ is concave.
5. For any disclosure rule, an equilibrium in a first-price auction is guaranteed if an equilibrium exists in a second-price auction.

Proof. We can show that Proposition A1 is a consequence of the following:

Lemma A1 a. $\tilde{\psi}^{\mathcal{W}}(x)$ is strictly increasing if G_V is log concave when $V_x > 0$, and if \bar{G}_V is log convex when $V_x < 0$.

b. $\tilde{\psi}^{\mathcal{A}}(x)$ is strictly increasing if G_V is concave when $V_x > 0$, and if \bar{G}_V is convex when $V_x < 0$.¹

¹The conditions for \mathcal{A} auctions are actually necessary and sufficient.

c. $\tilde{\psi}^{\mathcal{S}}(x)$ is strictly increasing when $N = 2$, if $1 - F_V$ is log convex when $V_x > 0$,

and if F_V is log concave when $V_x < 0$.

Proof: Recall that $\tilde{\psi}^\phi(x) = \psi^\phi(x) - x$ is the reputational component of the effective valuation. Recall also that $G_V(v) = F_V(v)^{N-1}$, $g_V(v) = \frac{dG_V(v)}{dv}$, $\bar{G}_V(v) = (1 - F_V(v))^{N-1}$, $\bar{g}_V(v) = \frac{d\bar{G}_V(v)}{dv}$ and define

$$\Lambda_V(\nu) = E_{F_V}[V|V \leq \nu]$$

$$M_V(\nu) = E_{F_V}[V|V \geq \nu]$$

Suppose first $V_x > 0$. If so, then notice that $F_V(v) = F_X(V^{-1}(v))$ and thus, for $v = V(x)$ (recalling that for \mathcal{S} auctions we only consider the $N = 2$ case) we get

$$\begin{aligned}\tilde{\psi}^{\mathcal{A}}(x) &= \frac{V_x(x)}{g(x)} = \frac{1}{g_V(v)} \equiv \tilde{\psi}^{\mathcal{A}}(v) \\ \tilde{\psi}^{\mathcal{W}}(x) &= V(x) - \Lambda(x) + G(x) \frac{V_x(x)}{g(x)} = v - \Lambda_V(\nu) + \frac{G_V(v)}{g_V(v)} \equiv \tilde{\psi}^{\mathcal{W}}(v) \\ \tilde{\psi}^{\mathcal{S}}(x) &= M(x) - V(x) + (1 - F_X(x)) \frac{V_x(x)}{f_X(x)} = M_V(\nu) - v + \frac{1 - F_V(v)}{f_V(v)} \\ &\equiv \tilde{\psi}^{\mathcal{S}}(v)\end{aligned}$$

and so the sign of $\tilde{\psi}_x^\phi(x)$ equals the sign of $\tilde{\psi}_v^\phi(v)$. Now, for \mathcal{W} auctions, $\frac{G_V(v)}{g_V(v)}$ is strictly increasing in v iff $G_V(v)$ is log concave. This condition implies that $F_V(v)$ is also log concave, which in turn implies that $v - \Lambda_V(\nu)$ is strictly increasing in v (Bagnoli and Bergstrom 2005, Corollary 1 and Lemma 1). For \mathcal{A} auctions, concavity of $G_V(v)$ is clearly necessary and sufficient for $\tilde{\psi}^{\mathcal{A}}(v)$ to be strictly increasing in v . Finally, for \mathcal{S} auctions, $\frac{1 - F_V(v)}{f_V(v)}$ is strictly increasing in v iff $1 - F_V(v)$ is log convex. It can be shown that log convexity of $1 - F_V(v)$ implies that (Bagnoli and Bergstrom 2005, Theorem 4 and an immediate reverse implication of Lemma 2) $M_V(\nu) - v$ is strictly increasing in v .

Suppose now $V_x < 0$. If so, then notice that $F_V(v) = 1 - F_X(V^{-1}(v))$ and thus, for $v = V(x)$ (recalling that for \mathcal{S} auctions we only consider the $N = 2$ case) we get

$$\begin{aligned}\tilde{\psi}^{\mathcal{A}}(x) &= \tilde{\psi}^{\mathcal{A}}(x) = \frac{V_x(x)}{g(x)} = \frac{1}{\bar{g}_V(v)} \equiv \tilde{\psi}^{\mathcal{A}}(v) \\ \tilde{\psi}^{\mathcal{W}}(x) &= V(x) - \Lambda(x) + G(x) \frac{V_x(x)}{g(x)} = v - M_V(v) + \frac{\bar{G}_V(v)}{\bar{g}_V(v)} \equiv \tilde{\psi}^{\mathcal{W}}(v) \\ \tilde{\psi}^{\mathcal{S}}(x) &= \Lambda_V(v) - v + \frac{1 - \bar{F}_V(v)}{\bar{f}_V(v)} \equiv \tilde{\psi}^{\mathcal{S}}(v)\end{aligned}$$

and so the sign of $\tilde{\psi}_x^\phi(x)$ equals the sign of $-\tilde{\psi}_v^\phi(v)$. Now, for \mathcal{W} auctions, $\frac{\bar{G}_V(v)}{\bar{g}_V(v)}$ is strictly decreasing in v iff $\bar{G}_V(v)$ is log convex. This condition implies that $1 - F_V(v)$ is also log convex, which in turn implies that $v - M_V(v)$ is strictly decreasing in v . For \mathcal{A} auctions, convexity of $\bar{G}_V(v)$ clearly is clearly necessary and sufficient for $\tilde{\psi}^{\mathcal{A}}(v)$ to be strictly decreasing in v . Finally, for \mathcal{S} auctions, $\frac{1 - \bar{F}_V(v)}{\bar{f}_V(v)}$ is strictly decreasing in v iff $1 - \bar{F}_V(v) = F_V(v)$ is log concave. But this implies that (Bagnoli and Bergstrom 2005, Corollary 1 and Lemma 1) $\Lambda_V(v) - v$ is strictly decreasing in v . ■

Lemma A1 proves directly parts 1. and 2. of Proposition A1. Part 3 also derives from Lemma A when $N = 2$ because, following an argument similar to that in Theorem 7 in Bagnoli and Bergstrom (2005), the log convexity of $1 - F_V$ is implied by the log convexity of $1 - F_X(x)$ and the convexity of $V(x)$. A similar argument provides the proof for part 4. Finally, part 5 follows from noting that we have that

$$\frac{d(E_G[\psi^\phi(Y)|Y < x_i])}{dx_i} = \frac{g(x_i)}{G(x_i)} \left(\psi^\phi(x_i) - E_G[\psi^\phi(Y)|Y < x_i] \right)$$

which is then positive for all x_i if $\psi^\phi(x_i)$ is strictly increasing in x_i ■

We now provide a complete statement and proof of Proposition 4. The order of the statements is somewhat different from the main text because it is convenient to follow the order below in the proof.

Proposition 4 (complete) Assume equilibrium existence. Then:

I $ER(\mathcal{W}) - ER(\mathcal{N})$ is positive (respectively zero, negative) if f_V is increasing (respectively constant, decreasing)

II Whenever $V_x > 0$ for all $x \in (\underline{x}, \bar{x})$ then

a.

$$ER(\mathcal{A}) - ER(\mathcal{W}) > 0; ER(\mathcal{S}) - ER(\mathcal{W}) > 0$$

$$ER(\mathcal{A}) - ER(\mathcal{S}) > 0 \text{ if } N > 2 \text{ while } ER(\mathcal{A}) - ER(\mathcal{S}) = 0 \text{ if } N = 2$$

b. For large enough N , $ER(\mathcal{A}) - ER(\mathcal{N}) > 0$.

III Whenever $V_x < 0$ for all $x \in (\underline{x}, \bar{x})$ then all the inequalities in II are reversed.

Proof. Given our bidding functions are separable between a non-reputational component and a reputational component, and given that the former is the same across disclosure rules, we can restrict attention to the reputational component of expected revenue for each disclosure rule. This is defined as $\widetilde{ER}(\phi)$. It will also prove convenient to consider an additional disclosure rule \mathcal{NW} , where all bids are disclosed except for the winner's. This disclosure rule is not of particular interest per se (although \mathcal{NW} and \mathcal{S} are equivalent for $N = 2$), but it will prove useful in the proofs. Indeed we begin our analysis, with the following:

Lemma 2 Assume equilibrium existence. Then:

$$\widetilde{ER}(\mathcal{A}) = \widetilde{ER}(\mathcal{NW})$$

Proof. It is easy to show that

$$\psi^{\mathcal{NW}}(x) = x + M(x) - V(x) + (1 - G(x)) \frac{V_x(x)}{G(x)}$$

and so

$$\widetilde{ER}(\mathcal{A}) - \widetilde{ER}(\mathcal{NW}) = N \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^x \left[V(y) - M(y) + \frac{G(y)}{g(y)} V_x(y) \right] dG(y) dF_X(x)$$

We begin by noting that

$$\int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^x \frac{G(y)}{g(y)} V_x(y) dG(y) dF_X(x) = \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^x [V(x) - V(y)] dG(y) dF_X(x).$$

So,

$$\begin{aligned} \widetilde{ER}(\mathcal{A}) - \widetilde{ER}(\mathcal{NW}) &= N \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^x [V(x) - M(y)] dG(y) dF_X(x) \\ &= N \int_{\underline{x}}^{\bar{x}} V(x) G(x) dF_X(x) - N \int_{\underline{x}}^{\bar{x}} \left(\frac{1}{1 - F_X(x)} \int_x^{\bar{x}} V(s) dF_X(s) \right) (1 - F_X(x)) dG(x) \\ &= N \int_{\underline{x}}^{\bar{x}} V(x) G(x) dF_X(x) - N \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^s V(s) dG(x) dF_X(s) = 0 \end{aligned}$$

as desired ■

I. We will need first to prove the following: if $V(\bullet)$ is increasing or decreasing, then $M(x) - \Lambda(x)$ has the opposite monotonicity of f_V . To prove this, note first from Jewitt (2004) that $E_{F_X}[X|X \geq x] - E_{F_X}[X|X < x]$ has the opposite monotonicity of f_X . Note now that if $V(\bullet)$ is increasing, with $\nu \equiv V(x)$, then $M(x) - \Lambda(x) \equiv M_\nu(\nu) - \Lambda_\nu(\nu)$ and so $M(x) - \Lambda(x)$ has the opposite monotonicity of f_V . Conversely, if $V(\bullet)$ is decreasing then $M(x) - \Lambda(x) = \Lambda_\nu(\nu) - M_\nu(\nu)$ and since $\frac{d\nu}{dx} < 0$ by assumption, we have then that the monotonicity of $M(x) - \Lambda(x)$ has the same sign as the monotonicity of $M_\nu(\nu) - \Lambda_\nu(\nu)$ and thus the opposite monotonicity of f_V . Given this, we compare expected revenues for disclosure rules $\phi = \mathcal{W}$ versus $\phi = \mathcal{N}$. We know from Lemma 2 that $\widetilde{ER}(\mathcal{NW}) = \widetilde{ER}(\mathcal{A})$. This means that

$$\widetilde{ER}(\mathcal{W}) = \widetilde{ER}(\mathcal{W}) + \widetilde{ER}(\mathcal{NW}) - \widetilde{ER}(\mathcal{A}) = \int_{\underline{x}}^{\bar{x}} [M(y) - \Lambda(y)] dF_2^{(N)}(y),$$

So, we have that

$$\begin{aligned}
\widetilde{ER}(\mathcal{W}) - \widetilde{ER}(\mathcal{N}) &= \int_{\underline{x}}^{\bar{x}} [M(y) - \Lambda(y)] \left(f_2^{(N)}(y) - f_1^{(N-1)}(y) \right) dy \\
&= \int_{\underline{x}}^{\bar{x}} \left(F_1^{(N-1)}(y) - F_2^{(N)}(y) \right) (M_x(y) - \Lambda_x(y)) dy. \\
&= (N-1) \int_{\underline{x}}^{\bar{x}} \left(F_X^N(y) - F_X^{N-1}(y) \right) (M_x(y) - \Lambda_x(y)) dy < 0 \text{ a.e.}
\end{aligned}$$

So, we have, after recalling our result above on the properties of $M_x(y) - \Lambda_x(y)$ that $\widetilde{ER}(\mathcal{W}) - \widetilde{ER}(\mathcal{N}) > 0 (=, <) 0$ if f_V is increasing (uniform, decreasing).

IIa (and corresponding III).

We provide the proof by comparing $E_G[\psi^\phi(Y)|Y < x]$ across for the relevant rules for \mathcal{A} vs \mathcal{W} and for $\mathcal{N}\mathcal{W}$ vs. \mathcal{S} . This establishes, together with Lemma 2, that for FP auctions with V increasing, \mathcal{A} provide higher revenues than \mathcal{W} and \mathcal{S} respectively. For the comparison between \mathcal{S} and \mathcal{W} , on the other hand, our result only applies to expected revenues. We begin with the comparison between \mathcal{A} and \mathcal{W} .

$$E_G[\psi^{\mathcal{A}}(Y)|Y < x] - E_G[\psi^{\mathcal{W}}(Y)|Y < x] = \frac{1}{G(x)} \int_{\underline{x}}^x \left[\Lambda(y) - V(y) + \frac{1-G(y)}{g(y)} V_x(y) \right] dG(y).$$

But

$$\int_{\underline{x}}^x \frac{1-G(y)}{g(y)} V_x(y) dG(y) = \int_{\underline{x}}^x V(y) dG(y) + (1-G(x)) V(x) - V(\underline{x}),$$

and so we have that

$$\int_{\underline{x}}^x \left[\Lambda(y) - V(y) + \frac{1-G(y)}{g(y)} V_x(y) \right] dG(y) = \int_{\underline{x}}^x \Lambda(y) dG(y) + \int_x^{\bar{x}} V(x) dG(y) - V(\underline{x}).$$

Clearly, if $V(\bullet)$ is increasing then $V(x) > V(\underline{x})$ and $\Lambda(y) > V(\underline{x})$ for any $x, y > \underline{x}$, and conversely if $V(\bullet)$ is decreasing. The result follows directly.

Now we consider the comparison between \mathcal{A} and \mathcal{S} . Given Lemma 2, this requires a comparison between \mathcal{NW} and \mathcal{S} .

$$\begin{aligned} & E_G [\psi^{\mathcal{NW}}(Y)|Y < x] - E_G [\psi^{\mathcal{S}}(Y)|Y < x] \\ &= \frac{1}{G(x)} \int_{\underline{x}}^x \left[\frac{1-G(y)}{g(y)} V_x(y) - \frac{1-F_X(y)}{f_X(y)} [V_x(y) + (N-2)\Lambda_x(y)] \right] dG(y) \end{aligned}$$

We already know from the previous comparison that

$$\int_{\underline{x}}^x \frac{1-G(y)}{g(y)} V_x(y) dG(y) = \int_{\underline{x}}^x V(y) dG(y) + (1-G(x)) V(x) - V(\underline{x})$$

Now, for $N > 2$ we have,

$$\begin{aligned} & \int_{\underline{x}}^x \frac{1-F_X(y)}{f_X(y)} V_x(y) dG(y) \\ &= \int_{\underline{x}}^x V(y) dG(y) + (N-1) \left[(1-F_X(x)) F_X(x)^{N-2} V(x) - \int_{\underline{x}}^x V(y) (1-F_X(y)) dF_X(y)^{N-2} \right] \end{aligned}$$

Finally, from Lemma 1,

$$\int_{\underline{x}}^x (N-2) \Lambda_x(y) \frac{1-F_X(y)}{f_X(y)} dG(y) = (N-1) \int_{\underline{x}}^x [V(y) - \Lambda(y)] (1-F_X(y)) dF_X(y)^{N-2}$$

which gives us

$$\begin{aligned} & \int_{\underline{x}}^x \left[\frac{1-G(y)}{g(y)} V_x(y) - \frac{1-F_X(y)}{f_X(y)} [V_x(y) + (N-2)\Lambda_x(y)] \right] dG(y) \\ &= (1-G(x)) V(x) - (N-1) \left((1-F_X(x)) V(x) F_X(x)^{N-2} - \int_{\underline{x}}^x \Lambda(y) (1-F_X(y)) dF_X(y)^{N-2} \right) - V(\underline{x}) \\ &= \int_{\underline{x}}^x \Lambda(y) dF_2^{(N-1)}(y) + \int_x^{\bar{x}} V(x) dF_2^{(N-1)}(y) - V(\underline{x}) \end{aligned}$$

The above is positive for $V_x > 0$ since then $V(x) > V(\underline{x})$ and $\Lambda(x) > V(\underline{x})$ for any $x > \underline{x}$.

Conversely if $V_x < 0$.

Finally, we focus on the comparison between \mathcal{S} and \mathcal{W} .

Recall that $\widetilde{ER}(\mathcal{W}) = N \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^x [M(y) - \Lambda(y)] dG(y) dF_X(x)$ from above. Now,

$$\widetilde{ER}(\mathcal{S}) - \widetilde{ER}(\mathcal{W}) = N \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^x \left[\Lambda(y) - V(y) + \frac{1-F_X(y)}{f_X(y)} (V_x(y) + (N-2)\Lambda_x(y)) \right] dG(y) dF_X(x)$$

But

$$N \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^x \left[\Lambda(y) - V(y) + \frac{1 - F_X(y)}{f_X(y)} V_x(y) \right] dG(y) dF_X(x) = \int_{\underline{x}}^{\bar{x}} [V(s) + \Lambda(s)] dF_2^{(N)}(s) - 2 \int_{\underline{x}}^{\bar{x}} V(s) dF_3^{(N)}(s)$$

while

$$N \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^x \left[\frac{1 - F_X(y)}{f_X(y)} (N - 2) \Lambda_x(y) \right] dG(y) dF_X(x) = 2 \int_{\underline{x}}^{\bar{x}} [V(s) - \Lambda(s)] dF_3^{(N)}(s),$$

Thus,

$$\begin{aligned} \widetilde{ER}(\mathcal{S}) - \widetilde{ER}(\mathcal{W}) &= \int_{\underline{x}}^{\bar{x}} [V(s) + \Lambda(s)] dF_2^{(N)}(s) - 2 \int_{\underline{x}}^{\bar{x}} \Lambda(s) dF_3^{(N)}(s) \\ &> 2 \int_{\underline{x}}^{\bar{x}} \Lambda(s) dF_2^{(N)}(s) - 2 \int_{\underline{x}}^{\bar{x}} \Lambda(s) dF_3^{(N)}(s) = 2 \int_{\underline{x}}^{\bar{x}} \left(F_3^{(N)}(s) - F_2^{(N)}(s) \right) \Lambda_x(s) ds > 0 \end{aligned}$$

with the first inequality above being true if $V_x > 0$ and hence (from Lemma 1) $\Lambda_x > 0$. The last equality follows from integration by parts. The argument is symmetric if $V_x < 0$. The above proves the result for $N > 2$. Note that if $N = 2$, however, then Lemma 2 implies that $ER(\mathcal{A}) = ER(\mathcal{S})$.

IIb (and corresponding III) . We know that

$$\widetilde{ER}(\mathcal{A}) = N \int_{\underline{x}}^{\bar{x}} [V(x) - V(\underline{x})] dF_X(x) \text{ and } \widetilde{ER}(\mathcal{N}) = \int_{\underline{x}}^{\bar{x}} [M(y) - \Lambda(y)] dF_X^{N-1}(y)$$

The first integral is clearly positive and unboundedly increasing in N , if $V(\bullet)$ is increasing. Conversely, if $V(\bullet)$ is decreasing. For the second integral, if $V(\bullet)$ is increasing, then $M(y) > \Lambda(y)$ almost everywhere, and hence $\widetilde{ER}(\mathcal{N})$ is bounded from above by $\max_y \{M(y) - \Lambda(y)\}$. Conversely, if $V(\bullet)$ is decreasing $\widetilde{ER}(\mathcal{N})$ is bounded from below by $\min_y \{M(y) - \Lambda(y)\}$. The proof follows immediately ■

In the next section of this appendix, we begin by showing how effective valuations for \mathcal{S} auctions are calculated and then provide proofs of Propositions 1, 2 and A in the paper. In the following section, we provide a rationale for focusing on strictly increasing bidding functions (propositions B and C) and show (proposition D) how these equilibria satisfy Banks and Sobel (1987) Universal Divinity.

9. Additional Proofs

Effective valuation for \mathcal{S} auctions. We have:

$$v_l^{\mathcal{S}}(y, z) = M(y)$$

$$v_{-l}^{\mathcal{S}}(y, z) = \int_{\underline{x}}^z V(z) dL(y_2|y) + \int_z^{\bar{x}} \Lambda(y_2) dL(y_2|y)$$

where

$$l(y_2|y) = \begin{cases} \frac{(N-2)F_X(y_2)^{N-3}f_X(y_2)}{F_X(y)^{N-2}} & \text{if } \underline{x} \leq y_2 \leq y \\ 0 & \text{if } \bar{x} \geq y_2 > y \end{cases}$$

$$L(y_2|y) = \begin{cases} \frac{F_X(y_2)^{N-2}}{F_X(y)^{N-2}} & \text{if } \underline{x} \leq y_2 \leq y \\ 1 & \text{if } \bar{x} \geq y_2 > y \end{cases}.$$

Given the above

$$\frac{\partial v_l(y, z)}{\partial z} = 0$$

$$\frac{\partial v_{-l}(y, z)}{\partial z} = L(z|y) V_x(z) + V(z) l(z|y) - \Lambda(z) l(z|y)$$

so that

$$\Psi^{\mathcal{S}}(x, x) \equiv \psi^{\mathcal{S}}(x) = x + M(x) - V(x)L(x|x) - \int_x^{\bar{x}} \Lambda(y_2) dL(y_2|x)$$

$$+ \frac{V_x(x)}{g(x)} \int_x^{\bar{x}} L(x|s) dG(s) + \frac{[V(x) - \Lambda(x)]}{g(x)} \int_x^{\bar{x}} l(x|s) dG(s)$$

But

$$L(x|x) = 1$$

and

$$\int_x^{\bar{x}} \Lambda(y_2) dL(y_2|x) = 0.$$

Also

$$\begin{aligned}
\int_x^{\bar{x}} L(x|s) dG(s) &= \int_x^{\bar{x}} \frac{F_X(x)^{N-2}}{F_X(s)^{N-2}} dF_X(s)^{N-1} \\
&= (N-1) F_X(x)^{N-2} (1 - F_X(x)) \\
&= \frac{(1 - F_X(x)) g(x)}{f_X(x)}
\end{aligned}$$

and

$$\begin{aligned}
\int_{x_i}^{\bar{x}} l(x|s) dG(s) &= \int_x^{\bar{x}} \frac{(N-2) F_X(x)^{N-3} f_X(x)}{F_X(s)^{N-2}} dF_X(s)^{N-1} \\
&= (N-2) \frac{g(x)}{F_X(x)} (1 - F_X(x_i)).
\end{aligned}$$

Using these results we have

$$\psi^S(x) = x + M(x) - V(x) + \frac{[V(x) + (N-2) \Lambda_x(x)]}{f_X(x)} (1 - F_X(x))$$

where we have used

$$\Lambda_x(x) = \frac{f_X(x)}{F_X(x)} [V(x) - \Lambda(x)]$$

■

Proof of Proposition 1

We save on notation by omitting the subscript i throughout. Note first, that given that $\psi^\phi(x)$ is strictly increasing, $\beta^{SP-\phi}(x) = \psi^\phi(x)$ as well as $\beta^{FP-\phi}(x) = E_G[\beta^{SP-\phi}(Y) | Y < x]$ are strictly increasing in x .

Also, define

$$\overline{\Psi}^\phi(x, y, z) = x + v_\ell^\phi(y, z) - v_{-\ell}^\phi(y, z)$$

and note that

$$W^{m-\phi}(x, z) \equiv \int_{\underline{x}}^z [\overline{\Psi}^\phi(x, y, z) - p^{m-\phi}(y, z)] dG(y) + \int_{\underline{x}}^{\overline{x}} v_{-\ell}^\phi(y, z) dG(y)$$

is the expected profit of type x from bidding $\beta^{m-\phi}(z)$ where $p^{FP-\phi}(y, z) = \beta^{FP-\phi}(z)$ is the price in a first-price auction and $p^{SP-\phi}(y, z) = \beta^{SP-\phi}(y)$ is the price in a second-price auction. We begin our proof with first-price auctions

Part (a): FP auctions

Suppose now that a symmetric and strictly increasing equilibrium β exists. Note then that in a such an equilibrium

$$\int_{\underline{x}}^{\beta^{-1}(b)} [\overline{\Psi}^\phi(x, y, \beta^{-1}(b)) - b] dG(y) + \int_{\underline{x}}^{\overline{x}} v_{-\ell}^\phi(y, \beta^{-1}(b)) dG(y)$$

is the expected profit of type x from bidding $b \geq 0$, with, by assumption A, $\beta^{-1}(b) \equiv \overline{x}$ if $b \geq \beta(\overline{x})$, $\beta^{-1}(b) \equiv \underline{x}$ if $0 \leq b \leq \beta(\underline{x})$ and $\beta^{-1}(b) \equiv \hat{x}$ if $b \in (\lim_{x_i \rightarrow \hat{x}^-} \beta^{m-\phi}(x_i), \lim_{x_i \rightarrow \hat{x}^+} \beta^{m-\phi}(x_i))$.

Moreover, β , being strictly increasing, is almost everywhere differentiable. The first-order condition (FOC) for a maximum of the expected profit of type x is (except in points of non-

differentiability of $\beta(\cdot)$)

$$\begin{aligned}
& \{(\overline{\Psi}^\phi(x, \beta^{-1}(b), \beta^{-1}(b)) - b)g(\beta^{-1}(b)) + \\
& + \int_{\underline{x}}^{\beta^{-1}(b)} \frac{\partial}{\partial z} v_t^\phi(y, \beta^{-1}(b)) dG(y) + \\
& + \int_{\beta^{-1}(b)}^{\bar{x}} \frac{\partial}{\partial z} v_{-t}^\phi(y, \beta^{-1}(b)) dG(y)\} \frac{1}{\beta'(\beta^{-1}(b))} \\
& = G(\beta^{-1}(b)).
\end{aligned}$$

So, if β is a symmetric and strictly increasing equilibrium, then it must be that $b = \beta(x)$, with

$\beta(x) > 0$ for any $x > \underline{x}$, and hence

$$\begin{aligned}
& [\overline{\Psi}^\phi(x, x, x) - \beta(x)]g(x) + \\
& + \int_{\underline{x}}^x \frac{\partial}{\partial z} v_t^\phi(y, x) dG(y) + \int_x^{\bar{x}} \frac{\partial}{\partial z} v_{-t}^\phi(y, x) dG(y) \\
& = \beta'(x)G(x)
\end{aligned}$$

almost everywhere in $x \in (\underline{x}, \bar{x}]$.

One can easily see that if β is a symmetric and strictly increasing equilibrium, then it must be continuous: if \hat{x} was a jump point then bidding $\lim_{x \rightarrow \hat{x}^-} \beta(x)$ is preferred to bidding $\lim_{x \rightarrow \hat{x}^+} \beta(x)$ by bidder of type \hat{x} (resp. $\hat{x} + \varepsilon$, where ε is arbitrarily small) when $\lim_{x \rightarrow \hat{x}^+} \beta(x) = \beta(\hat{x})$ (resp. when $\lim_{x \rightarrow \hat{x}^-} \beta(x) = \beta(\hat{x})$); such deviation does not have an effect on the auction's outcome and the reputational return, but leads to lower price upon winning. Note also that in any symmetric and strictly increasing equilibrium, $\beta(\underline{x})G(\underline{x}) = 0$. Continuity of β , and hence $\beta(x)G(x)$, implies, therefore, that the differential equation

$$\begin{aligned}
& \overline{\Psi}^\phi(x, x, x)g(x) + \int_{\underline{x}}^x \frac{\partial}{\partial z} v_t^\phi(y, x) dG(y) + \int_x^{\bar{x}} \frac{\partial}{\partial z} v_{-t}^\phi(y, x) dG(y) \\
& = \frac{d[\beta(x)G(x)]}{dx}, \quad x \in (\underline{x}, \bar{x}]
\end{aligned}$$

with the boundary condition $\beta(\underline{x})G(\underline{x}) = 0$ has unique, for any $x \in [\underline{x}, \bar{x}]$, solution the proposed equilibrium, $\beta^{FP-\phi}$.²

It remains thus to show that $\beta^{FP-\phi}$ is indeed an equilibrium. To this end, note first that, given that competitors deploy $\beta^{FP-\phi}$, any bidder is indifferent over any bid weakly lower than $\beta^{FP-\phi}(\underline{x})$. Also, any bidder strictly prefers $\beta^{FP-\phi}(\bar{x})$ to any higher bid. Recall that $W^{FP-\phi}(x, z)$ is the expected profit of type x from bidding $\beta^{FP-\phi}(z)$. We have that

$$\begin{aligned} W_2^{FP-\phi}(x, z) &= [\bar{\Psi}^\phi(x, z, z) - \beta^{FP-\phi}(z)]g(z) - \frac{d\beta^{FP-\phi}(z)}{dz}G(z) \\ &\quad + \int_{\underline{x}}^z \frac{\partial}{\partial z} v_{-i}^\phi(y, z) dG(y) + \int_z^{\bar{x}} \frac{\partial}{\partial z} v_{-i}^\phi(y, z) dG(y). \end{aligned}$$

So,

$$\begin{aligned} W_2^{FP-\phi}(x, z) - W_2^{FP-\phi}(z, z) \\ = g(z)[\bar{\Psi}^\phi(x, z, z) - \bar{\Psi}^\phi(z, z, z)]. \end{aligned}$$

Given that $\bar{\Psi}^\phi(x, z, z)$ is strictly increasing in x , we have that if $z < x$ then $W_2^{FP-\phi}(x, z) > W_2^{FP-\phi}(z, z)$, and vice versa. That is, $W_2^{FP-\phi}(x, z)$ is strictly increasing in x . Note also that $\beta^{FP-\phi}$ satisfies $W_2^{FP-\phi}(z, z) = 0$ for any $z \in (\underline{x}, \bar{x}]$. These, in turn, imply that for any z and x such that $\underline{x} < z < x \leq \bar{x}$ we have $W_2^{FP-\phi}(x, z) > 0$, while for any z and x such that $\underline{x} \leq x < z \leq \bar{x}$ we have $W_2^{FP-\phi}(x, z) < 0$. Note also that for any $x > \underline{x}$ bidding $\beta^{FP-\phi}(\underline{x})$ is not optimal. To see this, note first that from the above we have that $W_2^{FP-\phi}(x, z) \geq 0$ for any $z = \underline{x} + \varepsilon$ where ε is arbitrarily small. So, it will be enough to show that $W^{FP-\phi}(x, \underline{x}) \leq \lim_{z \rightarrow \underline{x}^+} W^{FP-\phi}(x, z)$. This holds as an equality by continuity of $v_{-i}^\phi(y, z)$.

Thus, $z = x$ is indeed indeed a global maximum of $W^{FP-\phi}(x, z)$, for any $x \in [\underline{x}, \bar{x}]$, given that competitors deploy $\beta^{FP-\phi}$. Thus, $\beta^{FP-\phi}$ is an equilibrium.

²The presence of reputational concerns does not invalidate the standard argument which implies (if applied to our setting) that $\beta^{FP-\phi}(\underline{x}) = \psi^\phi(\underline{x})$.

Part (b): SP auctions

Note that if $\beta(\cdot)$ is a symmetric and strictly increasing equilibrium then the expected profit of type x from bidding $b \geq 0$ in such an equilibrium is

$$\int_{\underline{x}}^{\beta^{-1}(b)} [\bar{\Psi}^\phi(x, y, \beta^{-1}(b)) - \beta(y)] dG(y) + \int_{\underline{x}}^{\bar{x}} v_{-\iota}^\phi(y, \beta^{-1}(b)) dG(y)$$

with, by assumption A, $\beta^{-1}(b) = \bar{x}$ if $b \geq \beta(\bar{x})$, $\beta^{-1}(b) = \underline{x}$ if $0 \leq b \leq \beta(\underline{x})$, and, finally, $\beta^{-1}(b) \equiv \hat{x}$ if $b \in (\lim_{x_i \rightarrow \hat{x}^-} \beta^{m-\phi}(x_i), \lim_{x_i \rightarrow \hat{x}^+} \beta^{m-\phi}(x_i))$. The first-order condition (FOC) for a maximum of it is (except in points of non-differentiability of $\beta(\cdot)$)

$$\begin{aligned} & \{(\bar{\Psi}^\phi(x, \beta^{-1}(b), \beta^{-1}(b)) - \beta(\beta^{-1}(b)))g(\beta^{-1}(b)) + \\ & + \int_{\underline{x}}^{\beta^{-1}(b)} \frac{\partial}{\partial z} v_{-\iota}^\phi(y, \beta^{-1}(b)) dG(y) + \int_{\beta^{-1}(b)}^{\bar{x}} \frac{\partial}{\partial z} v_{-\iota}^\phi(y, \beta^{-1}(b)) dG(y)\} \frac{1}{\beta'(\beta^{-1}(b))} \\ & = 0 \end{aligned}$$

If β is a symmetric and strictly increasing equilibrium then it must be that $b = \beta(x)$, with $\beta(x) > 0$ for any $x > \underline{x}$, and thereby,

$$\begin{aligned} & [\bar{\Psi}^\phi(x, x, x) - \beta(x)]g(x) + \\ & + \int_{\underline{x}}^x \frac{\partial}{\partial z} v_{-\iota}^\phi(y, x) dG(y) + \int_x^{\bar{x}} \frac{\partial}{\partial z} v_{-\iota}^\phi(y, x) dG(y) = 0, \end{aligned}$$

almost everywhere, which implies $\beta(x) = \psi^\phi(x)$, for $x \in (\underline{x}, \bar{x}]$. Note now that monotonicity requires also that $\beta(\underline{x}) \leq \psi^\phi(\underline{x})$. In addition, a necessary condition for $\beta(\underline{x})$ to be part of a symmetric and strictly increasing equilibrium is that

$$\begin{aligned} & [\bar{\Psi}^\phi(\underline{x}, \underline{x}, \underline{x}) - \beta(\underline{x})]g(\underline{x}) + \\ & + \int_{\underline{x}}^{\bar{x}} \frac{\partial}{\partial z} v_{-\iota}^\phi(y, \underline{x}) dG(y) \leq 0 \implies \beta(\underline{x}) \geq \psi^\phi(\underline{x}). \end{aligned}$$

Thus, by continuity of $\psi^\phi(x)$, if β is a symmetric and strictly increasing equilibrium then it must be given by $\beta^{SP-\phi}(x)$ for any $x \in [\underline{x}, \bar{x}]$.

Next we show that $\beta^{SP-\phi}$ is indeed an equilibrium. To this end, note, first, that, given that competitors deploy $\beta^{SP-\phi}$, any bidder is indifferent over any bid weakly lower than $\beta^{SP-\phi}(\underline{x})$. Also, any bidder is indifferent over any bid weakly higher than $\beta^{SP-\phi}(\bar{x})$. Recall that $W^{SP-\phi}(x, z)$ is the expected profit of type x from bidding $\beta^{SP-\phi}(z)$. We have that

$$\begin{aligned} W_2^{SP-\phi}(x, z) &= [\bar{\Psi}^\phi(x, z, z) - \beta^{SP-\phi}(z)]g(z) + \int_{\underline{x}}^z \frac{\partial}{\partial z} v_l^\phi(y, z) dG(y) \\ &\quad + \int_z^{\bar{x}} \frac{\partial}{\partial z} v_{-l}^\phi(y, z) dG(y) \end{aligned}$$

So,

$$\begin{aligned} W_2^{SP-\phi}(x, z) - W_2^{SP-\phi}(z, z) \\ = g(z)[\bar{\Psi}^\phi(x, z, z) - \bar{\Psi}^\phi(z, z, z)]. \end{aligned}$$

Given that $\bar{\Psi}^\phi(x, z, z)$ is strictly increasing in x , we have that if $z < x$ then $W_2^{SP-\phi}(x, z) > W_2^{SP-\phi}(z, z)$, and vice versa. That is, $W_2^{SP-\phi}(x, z)$ is strictly increasing in x . Note also that $\beta^{SP-\phi}$ satisfies $W_2^{SP-\phi}(z, z) = 0$ for any $z \in (\underline{x}, \bar{x}]$. These, in turn, imply that for any z and x such that $\underline{x} < z < x \leq \bar{x}$ we have $W_2^{SP-\phi}(x, z) > 0$, while for any z and x such that $\underline{x} \leq x < z < \bar{x}$ we have $W_2^{SP-\phi}(x, z) < 0$. Next, notice that, after using a similar argument to that we used in part (a), for any $x > \underline{x}$ bidding $\beta^{SP-\phi}(\underline{x})$ is not optimal. Thus, $z = x$ is indeed a global maximum of $W^{SP-\phi}(x, z)$ for any $x \in [\underline{x}, \bar{x}]$, given that competitors deploy $\beta^{SP-\phi}$. Thus, $\beta^{SP-\phi}$ is an equilibrium. ■

Proof of Proposition 2

Fix a disclosure rule ϕ . Consider a standard auction form m and fix a symmetric and strictly increasing equilibrium β of m . The expected payoff to bidder i with valuation x from submitting

a bid that corresponds to type z , i.e. $\beta(z)$, is

$$\begin{aligned} & \int_{\underline{x}}^z \bar{\Psi}^\phi(x, y, z) dG(y) + \int_{\underline{x}}^{\bar{x}} v_{-i}^\phi(y, z) dG(y) - \pi^{m-\phi}(z) \\ &= \int_{\underline{x}}^z \left[x + v_i^\phi(y, z) - v_{-i}^\phi(y, z) \right] g(y) dy + \int_{\underline{x}}^{\bar{x}} v_{-i}^\phi(y, z) g(y) dy - \pi^{m-\phi}(z) \end{aligned}$$

where $\pi^{m-\phi}(z^i)$ is the expected payment as a result of an implicit bid z^i in auction $m-\phi$, which is assumed to be a differentiable function. Note that it is independent of all bidders' valuations, while it depends implicitly on the other players' strategy β . Now, the FOC in maximizing the above is

$$\begin{aligned} & \int_{\underline{x}}^z \left[\frac{\partial}{\partial z} v_i^\phi(y, z) - \frac{\partial}{\partial z} v_{-i}^\phi(y, z) \right] g(y) dy \\ &+ \left[x + v_i^\phi(z, z) - v_{-i}^\phi(z, z) \right] g(z) \\ &+ \int_{\underline{x}}^{\bar{x}} \frac{\partial}{\partial z} v_{-i}^\phi(y, z) g(y) dy - \pi_z^{m-\phi}(z) = 0. \end{aligned}$$

In a symmetric and strictly increasing equilibrium it is optimal to report $z = x$ and so we have

$$\begin{aligned} \pi_x^{m-\phi}(x) &= \left[x + v_i^\phi(x, x) - v_{-i}^\phi(x, x) \right] g(x) \\ &+ \int_{\underline{x}}^x \frac{\partial}{\partial z} v_i^\phi(y, x) dG(y) + \int_x^{\bar{x}} \frac{\partial}{\partial z} v_{-i}^\phi(y, x) dG(y) \end{aligned}$$

and thus

$$\begin{aligned} \pi^{m-\phi}(x) &= \pi^{m-\phi}(\underline{x}) + \int_{\underline{x}}^x \left[y + v_i^\phi(y, y) - v_{-i}^\phi(y, y) \right] dG(y) \\ &+ \int_{\underline{x}}^x \left[\int_{\underline{x}}^y \frac{\partial}{\partial z} v_i^\phi(s, y) dG(s) + \int_y^{\bar{x}} \frac{\partial}{\partial z} v_{-i}^\phi(s, y) dG(s) \right] dy \\ &= G(x) E_G \left[\psi^\phi \left(Y_1^{(N-1)} \right) | Y_1^{(N-1)} < x \right], \end{aligned}$$

where we have used that, by assumption, $\pi^{m-\phi}(\underline{x}) = 0$. Note that this is a differentiable function of x , which is also independent of the particular auction form m . The proof is complete by noting

that the expected revenue must be

$$\begin{aligned}
& N \int_{\underline{x}}^{\bar{x}} \pi^{m-\phi}(x) dF(x) \\
&= N \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^x \psi^\phi(y) dG(y) dF(x) \\
&= N \int_{\underline{x}}^{\bar{x}} (1 - F(y)) \psi^\phi(y) g(y) dy \\
&= E_{F_2^{(N)}} \left[\psi^\phi \left(Y_2^{(N)} \right) \right],
\end{aligned}$$

as desired. ■

10. Additional Propositions

Let $a^\phi(b_i; \beta)$ be the expected wage earned in the (competitive) after-market when bidder i submits a bid b_i and expects that her competitors employ the strategy β , and the after-market expects (and uses for inference purposes) that all bidders use β . We note that this expected wage is common for every bidder who bids b_i , if strategy β is commonly anticipated. Given off-equilibrium beliefs of the after-market and a disclosure rule ϕ , $a^\phi(b; \beta)$ depends on bid b and strategy β on the part of i 's competitors - in particular, for $\phi \neq \mathcal{A}$, on the "pooling" intervals, i.e. the types who submit the same bids according to β . To see this, suppose, for example, that β has a single "pooling" interval $[x_l, x_h]$ (which could also be open, and hence β have discontinuities); that is, $\beta(x) = b^p$ for any $x \in [x_l, x_h]$. Suppose also that $b = \beta(z) < \beta(x_l)$. We have

$$\begin{aligned}
a^{\mathcal{A}}(b_i; \beta) &= V(z) \\
a^{\mathcal{W}}(b_i; \beta) &= \int_{\underline{x}}^z V(z) dG(y) + \int_z^{x_l} \Lambda(y) dG(y) + \int_{x_h}^{\bar{x}} \Lambda(y) dG(y) \\
&\quad + \int_{x_l}^{x_h} E_{F_X}[V(X) \mid b_w = b^p, i \neq \iota] dG(y)
\end{aligned}$$

where $E_{F_X}[V(X) \mid b_w = b^p, i \neq \iota]$ is the expected V of bidder i conditional on the winning bid (b^p) being equal to b^p and bidder i not being disclosed as the winner.³ Similarly (but with more cumbersome exposition) for all other cases (of disclosure rule, bidding functions and on- or off-equilibrium bids). Note that $a^\phi(b_i; \beta)$ is bounded by our assumption that $V(\cdot), g(\cdot)$ and $f(\cdot)$ are bounded over $[\underline{x}, \bar{x}]$.

Our focus on strictly increasing symmetric PBNE is justified by the following two propositions.

Proposition B Conditional on the typical bidder expecting that competitors adopt a common bidding strategy β and the after-market expects (and uses for inference purposes) that all bidders use β , the best response correspondence of the bidder is non-decreasing.⁴

Proof We show first that the corresponding returns to the typical bidder from higher bids are non-decreasing with her type. In more detail, the expected payoff of the typical bidder from bidding b is

$$\pi(b; \beta)x - \hat{\pi}(b; \beta) + \delta a^\phi(b; \beta)$$

where $\pi(b; \beta)$ is the expected winning probability and $\hat{\pi}(b; \beta)$ is the expected price paid, given own bid b and anticipated competitors' strategy β . Note then that

$$\begin{aligned} (10.1) \quad & \{\pi(b'; \beta)x' - \hat{\pi}(b'; \beta) + \delta a^\phi(b'; \beta) - [\pi(b; \beta)x' - \hat{\pi}(b; \beta) + \delta a^\phi(b; \beta)]\} \\ & - \{\pi(b'; \beta)x - \hat{\pi}(b'; \beta) + \delta a^\phi(b'; \beta) - [\pi(b; \beta)x - \hat{\pi}(b; \beta) + \delta a^\phi(b; \beta)]\} \\ & = [\pi(b'; \beta) - \pi(b; \beta)](x' - x) \geq 0 \text{ if } b' > b \text{ and } x' > x \end{aligned}$$

³The exposition of this expectation is cumbersome without adding much to our understanding of what follows, and hence is omitted.

⁴Let $BR(x)$ be the set of payoff-maximising bids (given the competitors' strategy and the after-market's inferences). $BR(x)$ is non-decreasing when, for $x' > x$, (a) if $b \notin BR(x')$ and $b \in BR(x)$ then $b' \notin BR(x')$ for all $b' < b$ such that $b' \in BR(x)$, (b) if $b \notin BR(x)$ and $b \in BR(x')$ then $b > b'$ for all $b' \in BR(x)$, and (c) if $b \notin BR(x)$ and $b < \max BR(x)$ then $b \notin BR(x')$.

Therefore, the returns to the typical bidder from higher bids are non-decreasing with her type. This property ensures that the best response correspondence of the typical bidder to any bidding strategy deployed by all her competitors is non-decreasing, as:

$$\begin{aligned} \pi(b'; \beta)x - \widehat{\pi}(b'; \beta) + \delta a^\phi(b'; \beta) - [\pi(b; \beta)x - \widehat{\pi}(b; \beta) + \delta a^\phi(b; \beta)] &\geq 0 \implies \\ \pi(b'; \beta)x' - \widehat{\pi}(b'; \beta) + \delta a^\phi(b'; \beta) - [\pi(b; \beta)x' - \widehat{\pi}(b; \beta) + \delta a^\phi(b; \beta)] &\geq 0 \\ \text{if } x' > x \text{ and } b' > b. \end{aligned}$$

■

The above Proposition does not imply that there is no best reaction function which is strictly decreasing over some range of types. However, it does imply that even if there is such a reaction function, there is always a payoff-equivalent non-decreasing best response function; for instance, the lower envelope of the best-response correspondence. This motivates our focus on non-decreasing symmetric equilibrium bid strategies. We then have:

Proposition C: There are no symmetric PBNE equilibria with non-decreasing bidding functions that involve (partial) pooling, as $\delta \rightarrow 0$

Proof: Let b^p be the bid of a pooling interval with end-types x_l and x_h , $x_l < x_h$. The pooling interval is allowed to be (semi-) open, and thus the end-points are allowed to be discontinuity points. We note also that we do not specify off-equilibrium beliefs, as it is not needed for our result.

Define $\sigma_T(y, \mathbf{x}_{-i}; \beta) = \{j \neq i \mid \beta(x_j) = \beta(y), x_j \leq y\}$.

The payoff of type x from bidding b^p is then

$$\int_{\underline{x}}^{x_l} [x - p^m(b^p, \beta(y))] dG(y) + \int_{x_l}^{x_h} [x - b^p] E_{F_X} \left[\frac{1}{1 + |\sigma_T(y, \mathbf{X}_{-i})|} \right] dG(y) + \delta a^\phi(b_p; \beta)$$

where, with $b^p > \beta(y)$, recall, $p^{FP}(b^p, \beta(y)) = b^p$ and $p^{FP}(b^p, \beta(y)) = \beta(y)$. The payoff of type x from bidding $b' \downarrow b^p$ (i.e. a bid higher than but very close to b^p) is

$$\int_{\underline{x}}^{x_l} [x - p^m(b^p, \beta(y))] dG(y) + \int_{x_l}^{x_h} [x - b^p] dG(y) + \delta \lim_{b' \downarrow b^p} a^\phi(b'; \beta)$$

where we note that in general $a^\phi(b_p; \beta) \neq \lim_{b' \downarrow b^p} a^\phi(b'; \beta)$ because the latter takes into account that there is no draw at bid $b' \downarrow b^p$. The payoff of type x from bidding $b'' \uparrow b^p$ (i.e. a bid lower than but very close to b^p) is

$$\int_{\underline{x}}^{x_l} [x - p^m(b^p, \beta(y))] dG(y) + \delta \lim_{b'' \uparrow b^p} a^\phi(b''; \beta)$$

where we note that in general $a^\phi(b_p; \beta) \neq \lim_{b'' \uparrow b^p} a^\phi(b''; \beta)$ because the latter takes into account that there is no draw at bid $b'' \uparrow b^p$.

We thus have that there are no profitable deviations (for all b^p -pooling types) from b^p iff

$$b^p \geq x_h + \delta \frac{\lim_{b' \downarrow b^p} a^\phi(b'; \beta) - a^\phi(b^p; \beta)}{\int_{x_l}^{x_h} [1 - E_{FX}[\frac{1}{1+|\sigma_T(y, \mathbf{X}_{-i})|}]] dG(y)}$$

and

$$b^p \leq x_l + \delta \frac{a^\phi(b^p; \beta) - \lim_{b'' \uparrow b^p} a^\phi(b''; \beta)}{\int_{x_l}^{x_h} E_{FX}[\frac{1}{1+|\sigma_T(y, \mathbf{X}_{-i})|}] dG(y)}$$

Clearly, then, due to $x_l < x_h$ and that the fractions in the above formulae are bounded (given our assumptions that $V(\cdot)$, $f(\cdot)$ and $g(\cdot)$ are bounded on the domain $[x, \bar{x}]$) and independent of δ , we have the desired result. ■

Therefore, for small enough δ , the only symmetric non-decreasing equilibrium bidding strategy is strictly increasing, and as we have shown, in this case, the beliefs in Assumption A satisfy the UD criterion, and the symmetric equilibrium bidding function, under these beliefs, is continuous and as described in the main text of our paper.

The use of Assumption A is justified by the following proposition.

Proposition D Assumption A satisfies Universal Divinity.

Proof The utility of the typical bidder/sender (given that she anticipates the other bidders to deploy a common strictly increasing bidding function β) is

$$\int_{\underline{x}}^{\beta^{-1}(b)} (x - p(b, y)) dG(y) + \delta \int_{\underline{x}}^{\bar{x}} E_{F_X}[V(X) \mid \beta^{-1}(b), y, \phi] dG(y)$$

where b is the sender's action/bid, $G(y)$ is the cdf of the highest competing type, and $p(b, y) = b$ in first-price auctions while $p(b, y) = y$ in second-price auctions. In addition, $\beta^{-1}(b) \equiv \{t \in X : \beta(t') > b \text{ for any } t' > t \text{ when } b < \beta(\bar{x}), \text{ and } \beta(t') < b \text{ for any } t' < t \text{ when } b > \beta(\underline{x}), \text{ with } t' \in X\}$ (this allows for discontinuities in β). Furthermore, $E_{F_X}[V(x) \mid \beta^{-1}(b), y, \phi]$ is the "receiver's response" to disclosed bids, which, in turn, depends on the rule ϕ , the anticipated strategy β , the bid of the sender b and the type of the highest type among the $N - 1$ competitors of the sender.

Define the equilibrium payoff for the sender with

$$U(x) \equiv \int_{\underline{x}}^x (x - p(\beta(x), y)) dG(y) + \delta a^\phi(\beta(x); \beta)$$

Notice that incentive-compatibility (i.e. monotonicity of bidding function) implies that $\int_{\underline{x}}^{x'} (x' - p(\beta(x'), y)) dG(y) + \delta a^\phi(\beta(x'); \beta) \geq \int_{\underline{x}}^x (x' - p(\beta(x), y)) dG(y) + \delta a^\phi(\beta(x); \beta)$ and $\int_{\underline{x}}^x (x - p(\beta(x), y)) dG(y) + \delta a^\phi(\beta(x); \beta) \geq \int_{\underline{x}}^{x'} (x - p(\beta(x'), y)) dG(y) + \delta a^\phi(\beta(x'); \beta)$ for any x', x . Rewriting the incentive-compatibility constraints, by making use of the definition of $U(\cdot)$, we have that $G(x)(x' - x) \leq U(x') - U(x) \leq G(x')(x' - x)$. Dividing across sides with $x' - x$ and taking the limits as $x' \rightarrow x$ we clearly have that $U(x)$ is differentiable (with $U'(0)$ being the right derivative, $U'(1)$ being the left derivative), with

$$(10.2) \quad U'(x) = G(x).$$

Thus, crucially, $U(x)$ is continuous.

Let now $\widehat{a}^\phi(x, b; \beta)$ be the after-market return that leaves the sender with type x indifferent between sending her equilibrium message $\beta(x)$ or sending message b . That is,

$$\widehat{a}^\phi(x, b; \beta) = U(x) - \int_{\underline{x}}^{\beta^{-1}(b)} (x - b) dG(y).$$

It follows that $\widehat{a}^\phi(x, b; \beta)$ is everywhere differentiable (and continuous). In fact,

$$\widehat{a}_x^\phi(x, b; \beta) \equiv \frac{\partial \widehat{a}^\phi(x, b; \beta)}{\partial x} = G(x) - G(\beta^{-1}(b)).$$

So, we have that:

$$(10.3) \quad \widehat{a}_x^\phi(x, b; \beta) \begin{matrix} > \\ < \end{matrix} 0 \text{ if } b \begin{matrix} < \\ > \end{matrix} \beta(x).$$

The above result is very useful - combined with Universal Divinity (Banks and Sobel, 1987) - in restricting the set of admissible out-of-equilibrium beliefs of the receiver and thereby of symmetric and strictly increasing PBNE. In particular, suppose that $\beta(x)$ has a jump at x' and consider the out-of-equilibrium message $b \in (\beta(x' - \varepsilon), \beta(x' + \varepsilon))$ where $\varepsilon \gtrapprox 0$. Given (10.3) we have that if $x > x'$ (and hence $\beta(x) > b$) then $\widehat{a}_x^\phi(x, b; \beta) > 0$. Similarly, if $x < x'$ (and hence $\beta(x) < b$) then $\widehat{a}_x^\phi(x, b; \beta) < 0$. Therefore, by continuity of $\widehat{a}_x^\phi(x, b; \beta)$, we have that $x' = \arg \min_x \widehat{a}_x^\phi(x, b; \beta)$. Thus, for all actions $a \geq \widehat{a}_x^\phi(x, b; \beta)$ (under which sender- x would weakly prefer to deviate to b from $\beta(x)$) there exists a type (specifically type x') who strictly prefers to deviate to b from $\beta(x')$, due to $a \geq \widehat{a}_x^\phi(x, b; \beta) > \widehat{a}_x^\phi(x', b; \beta)$. So UD requires that upon observation of the deviation b the receiver places all probability on the event that the sender is of type x' .

Similarly, consider an out-of-equilibrium message $b > \beta(\bar{x})$. Given (10.3) we have that $\bar{x} = \arg \min_x \widehat{a}_x^\phi(x, b; \beta)$. Thus, for all actions $a \geq \widehat{a}_x^\phi(x, b; \beta)$ (under which sender- x would weakly prefer to deviate to b from $\beta(x)$) there exists a type (specifically type \bar{x}) who strictly prefers to deviate to b from $\beta(\bar{x})$, due to $a \geq \widehat{a}_x^\phi(x, b; \beta) > \widehat{a}_x^\phi(\bar{x}, b; \beta)$. So UD requires that upon observation of the deviation b the receiver places all probability on the event that the sender is of type \bar{x} .

Following analogous steps we have that UD requires that upon observation of a deviation $b < \beta(\underline{x})$ the receiver places all probability on the event that the sender is of type \underline{x} . ■

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