

GROWTH AND STRUCTURAL CHANGE IN A DYNAMIC LAGAKOS-WAUGH MODEL

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Growth and Structural Change in a Dynamic Lagakos-Waugh Model ^{*}

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Abstract

This paper proposes a dual growth model built on a mechanism of self-selection whereby heterogeneous workers choose their optimal sectors based on comparative advantage. It shows that economic growth shifts workers' comparative advantage, and this shift induces rural-urban structural change. Following this mechanism, the model shows that average individual productivity in agriculture increases, while that in the non-agriculture sector decreases during structural change. Findings from simulations suggests an inverse correlation between the speed of structural change and dispersion of productivity across workers, and present improved predictions on transitional dynamics compared to the standard neoclassical growth model. The analysis of wage dynamics suggests that inequality over time does not necessarily follow an inverted-U curve when structural transformation takes place.

JEL Categories: J24, J31, O11, O15, O40

Keywords: structural change, self-selection, labor productivity, wage dynamics

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1 Introduction

This paper studies economic growth, structural change and inequality for a small open dual economy, in which heterogeneous workers choose their optimal sectors based on comparative advantage. In the paper, I will show how economic growth interacts with rural-urban structural change given this self-selection mechanism, and analyze sectoral labor productivity, wage distributions and inequality as structural transformation takes place.

Existing theoretical studies of economic growth, especially those considering multi-sector growth and structural change, largely focus on aggregate issues and tend to abstract from labor heterogeneities. Although this simplification is often useful, it limits the implications with respect to sectoral labor productivity and could potentially result in misleading predictions about transitional dynamics during structural transformation. Moreover, this simplification limits the scope for analyzing the dynamics of wage distributions and inequality over time.

This paper proposes a dual growth model with heterogeneous workers. The point of departure and central pivot of the new model lays in its micro-foundation in the labor market, namely a self-selection mechanism. This mechanism was first presented by [Roy \(1951\)](#) and applied to a dual economy by [Lagakos and Waugh \(2013\)](#). [Lagakos and Waugh's](#) model studies cross-country differences in labor productivity based on the equilibrium of a Roy-type labor market. In the Lagakos-Waugh model (LW model henceforth), a rational worker endowed with different levels of skills in agricultural and non-agricultural production will choose the sector where he or she has a comparative advantage, in order to maximize labor income. This mechanism determines the rural-urban labor allocation and production in equilibrium.

The model proposed in this paper departs from [Lagakos and Waugh](#) on two aspects. First, I embed their static equilibrium in a growth framework and introduce a dynamic version of the LW model. In the dynamic LW model, workers' comparative advantage will shift over time, since the technologies of production and relative factor abundance across sectors are changing during the process of economic growth. This endogenous shift of comparative advantage results in rural-urban labor migration.

Second, [Lagakos and Waugh](#) consider a closed economy, and show how non-homothetic preferences and the relative price of agriculture relate to workers' self-selection and sectoral allocations. In this paper, I assume a small open economy environment, in order to appropriately model the development process of countries that are small in geographical area and economic weight, but open to international trade. With this assumption, the model will focus more on the production side, such as capital accumulation and technical progress, rather than demand and the nature of preferences, when explaining the driving forces of structural transformation.

The model leads to two important findings on the path of structural change. The first finding concerns the dynamics of average individual productivity over the course of structural change. It predicts that average individual productivity in agriculture increases over time, while that in the non-agriculture sector decreases. This prediction accords with a fact that is observed from cross-country evidence (e.g. [Caselli, 2005](#)). In poor countries with undeveloped overall productivity, most workers, despite their low productivity, have to work in agriculture in order to supply subsistence foods. Therefore, a low average level of productivity in agriculture is commonly observed. In contrast, rich countries have low employment shares in agriculture and the majority of workers, skilled and unskilled, have migrated to the non-agriculture sector. In these countries, people continuing to select rural work must be those with the highest relative productivity in the agriculture sector, while skills are divergent across workers and have a relatively low average level in non-agriculture.

The second finding leads to an innovative point of view on explaining the speed of structural change. Explanations for the speed of structural change in the traditional models are often based on the assumption of exogenous technical progress for the two sectors. Other determinants of the speed in the existing literature include the nature of preferences, particularly the issue of subsistence consumption, and the rate of factor accumulation on the supply side. The innovation of this paper is to link the speed with the extent of heterogeneity in workers' skills, or the dispersion of productivity distributions.

Ample evidence based on measuring workers' educational attainment, as in [Castelló and Doménech \(2002\)](#) and [Castelló-Climent \(2010\)](#) among

others, suggests wide cross-country differences in the dispersion of labor skills. Though the existing literature has studied how this affects economic growth and inequality, the relationship between the dispersion and structural change remains unclear. For example, [Castelló and Doménech's](#) measures show that many emerging economies, especially those in East Asia, have relatively low inequality in labor skills during recent decades, due to their broad-based primary education. And meanwhile, these countries experienced fast declines of output and employment shares in agriculture (see [Young, 1995](#); [Collins and Bosworth, 1996](#); [Üngör, 2011](#)). No existing theory has ever related these two facts and asked if low skill inequality across workers accelerates labor reallocation. This paper models their relationship for the first time. Findings suggest an inverse correlation between the speed of structural change and the relative dispersion (rather than the absolute dispersion) of labor skills distributed across workers. That is, the higher the relative skill dispersion, the slower is structural transformation. Intuitively, for economies with high dispersion of labor skills across individuals, relatively unproductive workers in the rural sector often find themselves reluctant to transfer into the non-agriculture sector, because they are hardly competitive in urban jobs. On the contrary, structural change is accelerated in countries where sectoral productivity across individuals is similar, since workers are better substitutes in this case.

The steady state of the modeled dual economy is characterized as an asymptotic balanced growth path, where the agriculture sector asymptotically disappears, with only the most productive workers remaining in the sector. And the economy is dominated by a steadily growing non-agriculture sector, reflecting the modern economic structure in developed countries.

The transitional dynamics of the model are simulated and discussed in detail. Findings from numerical simulations suggest improved predictions on interest rates, growth rates and the speed of convergence, compared to the standard Ramsey model, which is criticized in [King and Rebelo \(1993\)](#), among others, because of its counterfactual transitional dynamics.

Another noteworthy merit of the dynamic LW model is to allow the study of wage distribution and inequality evolutions, commonly overlooked in growth and structural change models. In this paper, I will make

use of a special case with a closed-form solution to show how the self-selection behavior of heterogeneous workers reshapes the wage distribution in each sector and determines inequality as structural change takes place. The analytical special case implies a constant level of inequality over time. Though other results are possible, given different assumptions on productivity distributions, this argument does suggest that the path of inequality during structural transformation does not necessarily follow the inverted-U curve hypothesized by [Kuznets \(1955\)](#).

The paper will proceed as follows. Section 2 will review the existing literature relating to this paper. Section 3 will lay out the model and introduce the mechanism of self-selection. The dynamic equilibrium of the dual economy will be derived in this section. Section 4 provides further insights into the model in terms of structural change, labor productivity and long run growth paths. The transitional dynamics of the model are simulated in Section 5. Section 6 discusses the dynamics of wage distributions and inequality over time. Conclusions will be drawn in section 7.

2 Relation to existing literature

Studies on structural change for a dual economy, referring to a transitional economy with asymmetric structures in the traditional and modern sectors, include the well-known paper by [Lewis \(1954\)](#), who argued that a surplus of unproductive rural workers migrate out of the agriculture sector for higher wages in the urban sector, and this migration results in structural transformation and a higher level of aggregate output. Later work following this tradition often embeds a two-sector structure in a small open economy, and derives a static equilibrium based on the equalization of rural and urban wages. Comparative statics are commonly applied for further insights into their equilibria. Some of these studies make use of a [Rybczynski effect](#) for their analyses: an increase in the stock of one factor expands the sector that uses this factor relatively intensively ([Rybczynski, 1955](#)). Therefore, migration out of agriculture is often explained by capital accumulation in the urban sector. Some of these studies of dualism and structural change are reviewed in [Temple \(2005a\)](#).

Another strand of literature uses multi-sector growth models to de-

rive dynamic paths for structural transformation in closed economies. [Kongsamut, Rebelo and Xie \(2001\)](#) develop a three-sector growth model for a closed economy with a path of structural change that arises from non-homothetic preferences. They define a generalized balanced growth path to investigate structural change and long-run growth. But to obtain the balanced growth path, their model requires a tight link between parameters in the utility function and the production functions. [Ngai and Pissarides \(2007\)](#) propose a multi-sector growth model under less restrictive conditions and argue that employment increases in the sectors with low growth of total factor productivity. [Acemoglu and Guerrieri \(2008\)](#) develop a model that focuses on the production side and emphasize the impact of capital deepening and factor proportions on structural change.

Multi-sector growth models often tend to yield richer and more realistic predictions regarding transitional dynamics, compared to the one-sector neoclassical growth model. [Robertson \(1999\)](#) sets up a dual growth model that implies relatively small changes in interest rates when an economy is undergoing growth, and argues that a stable path for interest rates can be obtained in the dual economy when the agriculture sector uses labor more intensively than the non-agriculture sector. This outcome is verified here in the presence of the self-selection of heterogeneous workers.

Other versions of multi-sector growth models from the recent literature include [Caselli and Coleman \(2001\)](#), [Lucas \(2004\)](#), [Hayashi and Prescott \(2008\)](#), [Buera and Kaboski \(2009\)](#), [Duarte and Restuccia \(2010\)](#), [Dekle and Vandenbroucke \(2012\)](#), [Michaels, Rauch and Redding \(2012\)](#), [Uy, Yi and Zhang \(2013\)](#) and [Herrendorf, Rogerson and Ákos Valentinyi \(2014\)](#), among others. A contribution especially related to this paper is that by [Roe, Smith and Saraçoğlu \(2010\)](#) who recast static multi-sector models within the neoclassical growth framework and analyze economic growth in multi-sector dynamic general equilibrium models.

The other key ingredient of this paper relative to the previous literature is the mechanism of self-selection in the labor market. This mechanism is originally proposed in [Roy \(1951\)](#) as an occupational choice between 'fishing' and 'hunting'. Individuals in the Roy model choose an occupation to maximize their labor income based on comparative advantage.

[Lagakos and Waugh \(2013\)](#) build on Roy's self-selection mechanism.

They show the equilibrium of a Roy-type dual economy, in which heterogeneous workers select the optimal sector with the highest labor income. Based on this equilibrium, they derive the rural-urban labor allocation and sectoral productivity for the economy, and emphasize the role of self-selection in explaining the cross-country labor productivity differences seen in the data. In this paper, I make use of the same mechanism as [Lagakos and Waugh](#) to derive the labor allocation and sectoral productivity, but embed it in a growth model to approach some further insights from a dynamic perspective, including rural-urban structural transformation, wage dynamics and inequality over time.

Another recent application of the self-selection mechanism that shares some similarities with this paper is [Young \(2013\)](#). [Young](#) analyzes rural-urban bi-directional migration and inequality with an extended version of the LW model. His model considers the likelihood that workers are skilled or unskilled by measuring their educational attainment and derives an equilibrium of rural-urban labor allocation based on this probability. The equilibrium suggests that skilled workers tend to choose to work in the urban sector and earn more than those who choose the rural sector. This paper has similar topics, namely rural-urban structural change and inequality, but it extends the LW model through its analysis of a dynamic adjustment to long-run equilibrium.

A self-selection or Roy-type model is also recast in [Kuralbayeva and Stefanski \(2013\)](#) for a small open economy with a manufacturing sector and a non-manufacturing sector. Their model suggests that structural transformation induced by resource windfalls results in higher productivity in manufacturing and low productivity in non-manufacturing. Based on the model, they account for the productivity differences between resource-rich and resource-poor countries, and argue that low productivity in resource-rich countries is not the consequence of the contraction of the productive manufacturing sector.

3 The model

3.1 Setup of the model

The environment of the modeled economy is a small open economy with two production sectors, the agriculture (rural) sector and the non-agriculture (urban) sector, denoted by a and m respectively. There are no international capital flows. Households inelastically provide labor and each worker, indexed by i , is heterogeneous and endowed with a distinct specification of sectoral productivity, denoted by a vector $\{z_a^i, z_m^i\}$, which remains unchanging over time. Labor mobility across sectors is free and the total amount of labor, $L(t)$, grows at an exogenous rate, n . The two production sectors are characterized by neoclassical production functions with constant returns to scale. In per capita terms, they are expressed as

$$\begin{aligned}y_a(t) &= F(k_l(t), A_a(t) \cdot e_a(t)) \\y_m(t) &= G(k(t), A_m(t) \cdot e_m(t))\end{aligned}\tag{1}$$

Standard assumptions are applied and the time notation, t , will be omitted henceforth when no confusion is caused.¹ y_a and y_m are the per capita outputs of final goods, normalized by total labor (i.e. $y_a \equiv \frac{Y_a}{L}$ and $y_m \equiv \frac{Y_m}{L}$). y_a and y_m are internationally exchangeable at a relative price, p , with the non-agricultural good as the numéraire. In a small open environment, the relative price is exogenously determined by the general equilibrium of world markets, and hence the modeled country is a price-taker. It is considered fixed throughout the paper for analytical simplicity, but the model can easily accommodate other price patterns. The total output per worker, $y \equiv \frac{Y}{L}$, is therefore given by $p \cdot y_a + y_m$. k_l denotes land per capita (i.e. $k_l \equiv \frac{K_l}{L}$), where land is used specifically for the agricultural sector, and the aggregate stock of land, K_l , is fixed and normalized to unity. k is the stock of physical capital per worker specifically for the non-agriculture sector. A_a and A_m are the labor-augmenting efficiency terms growing at exogenous rates g_a and g_m respectively.

Workers are heterogeneous in productivity, but they are perfect substitutes at fixed ratios. The labor-related inputs in the two sectors are e_a and

¹Standard assumptions include twice differentiability, Inada conditions and positive and diminishing returns to each input.

e_m , denoting effective labor in per capita terms. The total amount of effective labor in one sector is defined as an aggregation of the corresponding sectoral individual productivity levels provided by those working in the sector. The per capita effective worker inputs are then defined as

$$\begin{aligned} e_a &\equiv \sum_{i \in \Omega_a} \frac{z_a^i}{L} \\ e_m &\equiv \sum_{i \in \Omega_m} \frac{z_m^i}{L} \end{aligned} \quad (2)$$

where Ω_a and Ω_m are the sets of workers in the two sectors.

3.2 Occupational self-selection

In a conventional dual economy model, homogeneous workers earn identical wages in either sector in equilibrium and thus they have no sectoral preference. In contrast, if workers are endowed with heterogeneous productivity, two facts are straightforwardly implied. First, within one particular sector, a relatively productive worker is likely to earn more than the less skilled. Second, from a cross-sector perspective, since workers have a comparative advantage in a particular sector, they will have some sectoral preference. In other words, workers always have a chance to self-select a sector that optimizes their income. These facts are modeled with the mechanism of self-selection in this paper, as in [Lagakos and Waugh \(2013\)](#).

Within one sector, workers devote their corresponding sectoral productivity as effective labor inputs, and each unit of effective labor or productivity is paid an identical factor price. Each unit of agricultural productivity receives a factor price ζ_a and in the non-agriculture sector each unit of effective labor is paid ζ_m . Define the ratio

$$q \equiv \frac{\zeta_a}{\zeta_m}$$

as the rural-urban differential of effective wages and the comparative productivity of a worker, i , is represented by the ratio $\frac{z_m^i}{z_a^i}$. A worker is said to have a (strict) comparative advantage in agriculture if $\frac{z_m^i}{z_a^i} < q$, and *vice versa*.

Workers are rational and they know their own comparative advantage all the time, so at each instant, they choose the sector that offers them the highest wage. The labor income of a worker, i , is then given by

$$w^i = \max \left\{ \zeta_a \cdot z_a^i, \zeta_m \cdot z_m^i \right\} \quad (3)$$

depending on the worker's comparative advantage.

Proposition 1 *A worker, i , chooses to supply labor in the agriculture sector if and only if*

$$\frac{z_m^i}{z_a^i} \leq q; \forall i$$

and chooses the non-agriculture sector otherwise.

Proposition 1 describes how the mechanism of self-selection allocates workers with heterogeneous productivity into optimal sectors. It claims that the necessary and sufficient condition for individuals to select a sector is owning a comparative advantage in the sector. An alternative interpretation of the proposition, from the perspective of structural transformation, is the migration condition. A person who initially works in agriculture remains in the sector as long as he or she retains a comparative advantage in agriculture. But once this comparative advantage alters from agriculture to non-agriculture, a migration decision will be made in order to maximize labor income. From this perspective, the effective wage differential is, therefore, acting as the migration margin of the labor transition.

Proposition 1 determines the employment shares across sectors as

$$\begin{aligned} l_a &= \text{Prob} \left\{ \frac{z_m^i}{z_a^i} \leq q \right\} \\ l_m &= \text{Prob} \left\{ \frac{z_m^i}{z_a^i} > q \right\} \end{aligned} \quad (4)$$

where a greater q implies that a larger proportion of workers have a comparative advantage in agriculture, and thus a higher rural employment share.

In the competitive economy, q is endogenously derived in equilibrium under the condition that factors receive their marginal products as payments. That is, ζ_a and ζ_m are the marginal products of effective labor in

the two sectors, $p \cdot F'_{e_a}$ and G'_{e_m} respectively, and thus the effective wage differential satisfies

$$q = \frac{p \cdot F'_{e_a}}{G'_{e_m}} \quad (5)$$

Similarly, competitive markets equate the returns to the other inputs to their marginal products:

$$\begin{aligned} r_l &= F'_{k_l} \\ r &= G'_k - \delta \end{aligned} \quad (6)$$

where r_l is the land rental, r is the return to physical capital or interest rate and δ denotes the depreciation rate. Solving the system of equations from (1) to (6) yields the static Roy equilibrium for a dual economy.

3.3 From Roy to Ramsey

The model remains static so far. The following subsection lays out a dynamic treatment to offer further insights into the LW model from a growth perspective. In the dynamic equilibrium, the mechanism of self-selection remains the central pivot.

Individuals in the economy are assumed to be the Ramsey type, and have the same intertemporal consumption preferences and time preference. In order to solve the intertemporal consumption problem with a representative-agent approach, the economy, in spite of the labor heterogeneities, is treated as an aggregation of large households that own land, capital and labor, and keep the returns to their assets. Thus, the intertemporal decision-making on consumption is centralized to the household level, while at each moment in time, members of households claim their own fractions of labor income. Since individuals have identical intertemporal consumption preferences, households would choose to consume the same fraction of their wealth at each instant, and hence behave as duplications of the household with average assets and income, or the representative household, which makes the representative-agent approach valid given heterogeneous workers.

The Ramsey problem of the representative household has two steps following the methodology in [Roe et al. \(2010, Chapters 3 and 4\)](#). Firstly, the representative household intertemporally chooses the consumption level

of a composite good, c , that maximizes the discounted lifetime utility

$$U = \int_0^{\infty} u(c) \cdot e^{-(\rho-n)t} dt$$

subject to a budget constraint

$$\dot{k} = y - \phi - (n + \delta) \cdot k \quad (7)$$

and a non-zero initial capital stock, where ρ is the rate of time preference, $u(\cdot)$ is the instantaneous felicity function taking the neoclassical form, and ϕ is the expenditure of the consumed goods.²

Secondly, given the selected consumption level, \bar{c} , the household decides the combination of the two goods to minimize the expenditure at each moment in time:

$$\phi \equiv \min_{c_a, c_m} \{p \cdot c_a + c_m : v(c_a, c_m) \geq \bar{c}\} \quad (8)$$

where $v(\cdot)$ is the utility function that decides preferences over the two goods, satisfying homogeneous of degree one or the Stone-Geary form.

In a closed economy, domestic output of each good must be equal to its consumption plus accumulation, so that the relative price is determined endogenously. However, this constraint is no longer valid in a small open economy, where the relative price is fixed by the worldwide general equilibrium, and hence domestic output and consumption on each good are likely to differ, as goods can be traded internationally at the world price. Usually consumption on one good will be met partly by imports of that good, while the other good will be exported to keep the trade account balanced.

Solving the present-value Hamiltonian yields the corresponding Euler equation for the intertemporal choice of consumption (see Appendix A):

$$\frac{\dot{c}}{c} = -\frac{1}{\epsilon} \cdot (r - \rho) \quad (9)$$

where $\epsilon \equiv \frac{u''(\phi) \cdot \phi}{u'(\phi)}$ is the elasticity of marginal utility with respect to con-

²The model is considered in per capita terms for now. Following the usual treatment, $\rho - n > 0$ is assumed to ensure a bounded utility integral in the case without technical progress, and a corresponding transversality condition $\lim_{t \rightarrow \infty} k \cdot \exp\left\{-\int_0^t [r(\tau) - n] d\tau\right\} = 0$ is applied.

sumption of the composite good. Note that consumption preferences over the two final goods, i.e. $v(c_a, c_m)$, do not enter the Euler equation in any forms. It means that, given the exogenous relative price and free trade environment, analyses on sectoral allocations and aggregate outcomes can be obtained without specifying a functional form for the nature of preferences.³

The equilibrium of the dynamic LW model is therefore defined as a time series sequence in continuous form

$$\left\{ k, c, y, y_a, y_m, q, l_a, l_m, e_a, e_m, r_l, r, w^i \right\}_{t \in [0, \infty)}$$

that satisfies the intratemporal equations (1) to (6) and intertemporal equations (7) and (9), given a certain specification of initial endowments. The remaining parts of this paper will be focused on this dynamic equilibrium.

3.4 Model assumptions

In order to provide analytical insights into equilibrium behavior, some assumptions are applied for the rest of the paper. First of all, the two individual productivity endowments for each worker, $\{z_a^i, z_m^i\}$, are assumed to be independently drawn from the continuous Fréchet distributions

$$H_a(z_a) = e^{-(\kappa_a \cdot z_a)^{-\theta}}$$

$$H_m(z_m) = e^{-(\kappa_m \cdot z_m)^{-\theta}}$$

where $H_a(z_a)$ and $H_m(z_m)$ are the cumulative density functions for their distributions. This two-parameter specification of the Fréchet distribution (or the Type-2 extreme value distribution) is also applied to characterize labor productivity in [Eaton and Kortum \(2002\)](#) and [Redding \(2012\)](#), among others. θ is the shape parameter representing the dispersion of individual labor productivity for both sectors. A smaller θ implies a higher diversity of productivity. κ_a and κ_m are the scale parameters that control the average levels of individual productivity for the two sectors respectively. For example, a smaller κ_a implies a higher average level of agricultural productivity.

³But assuming a particular preference function will make it possible to look at consumption relative to domestic production of each good, and thus the evolution of imports and exports, which is not the main focus of this paper.

It is noteworthy that a smaller κ_a would also imply a higher variance due to an enlarging scale of the distribution, but a normalized measure of dispersion (such as an inequality measure) would be unchanged. Assuming the same shape parameter across sectors, as in [Eaton and Kortum \(2002\)](#), [Kuralbayeva and Stefanski \(2013\)](#) and the analytical part of [Lagakos and Waugh \(2013\)](#), certainly restricts the analysis to a special case, but it allows the model to be tractable when analyzing structural change and wage distributions over time.

The second assumption is that the production technologies of the two sectors take the Cobb-Douglas forms

$$y_a = k_l^\alpha \cdot (A_a \cdot e_a)^{1-\alpha}$$

$$y_m = k^\beta \cdot (A_m \cdot e_m)^{1-\beta}$$

where α and β are the output elasticities of land and physical capital respectively.

Finally, the felicity function $u(c)$ follows the constant-relative-risk-aversion (CRRA) form

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

where $-\sigma$ is the elasticity of marginal utility with respect to consumption.

4 Model analysis

4.1 Structural change

A distinctive equilibrium behavior of dual growth models is rural-urban structural transformation: the non-agriculture sector expands relative to agriculture when an economy is undergoing development. In the dynamic LW model, this transformation involves the mechanism of self-selection, whereby workers gradually give up agricultural work and migrate to the non-agriculture sector over time, due to the shift of their comparative advantage.

Under the distributional assumptions for labor productivity, proposition 1 yields a closed-form solution for the employment shares across

sectors (see Appendix B):

$$l_a = \frac{1}{(\tilde{\kappa} \cdot q)^{-\theta} + 1}$$

$$l_m = \frac{1}{(\tilde{\kappa} \cdot q)^{\theta} + 1} \quad (10)$$

where $\tilde{\kappa} \equiv \frac{\kappa_m}{\kappa_a}$, denoting the ratio of the scale parameters for non-agriculture and agriculture. Similar results for the employment shares in equilibrium are derived in [Lagakos and Waugh](#), who draw individual productivity from one-parameter Fréchet distributions. From a static perspective, equation (10) implies that the distributions of sectoral productivity determine the labor allocation, given a specific value of q . The labor allocation relies on the rural-urban differential of the scale parameters, $\tilde{\kappa}$, with a greater ratio indicating a larger employment share in agriculture.

The supply of effective labor in each sector can be calculated as the conditional expectations of corresponding productivity given the labor allocation (see Appendix B):

$$e_a = \frac{1}{\kappa_a} \cdot l_a^{\frac{\theta-1}{\theta}} \cdot \gamma$$

$$e_m = \frac{1}{\kappa_m} \cdot l_m^{\frac{\theta-1}{\theta}} \cdot \gamma \quad (11)$$

where γ is a constant given by the Gamma function $\Gamma\left(\frac{\theta-1}{\theta}\right)$.

A rural-urban transition is straightforwardly implied from equation (10), since q is the only variable changing over time. Solving for the path of q from equation (5) yields lemma 1.

Lemma 1 *The rural-urban ratio of effective wages, q , decreases toward zero along the equilibrium path, under reasonable conditions on rates of technical progress.*

Proof. The path of q is endogenously determined in equilibrium following equation (5). Substituting marginal products of effective workers into equation (5) yields the following equilibrium condition:

$$k_l^\alpha \cdot k^{-\beta} \cdot \frac{A_a^{1-\alpha}}{A_m^{1-\beta}} = \frac{(1-\beta)}{p \cdot (1-\alpha)} \cdot \frac{e_a^\alpha}{e_m^\beta} \cdot q \quad (12)$$

Taking logs and differentiating with respect to time, the left side of the

equation decreases over time under the following condition for technical progress:

$$g_a < \frac{1-\beta}{1-\alpha} \cdot g_m + \frac{\alpha}{1-\alpha} \cdot n + \frac{\beta}{1-\alpha} \cdot g_k \quad (13)$$

where $g_k \equiv \frac{\dot{k}}{k}$ is the rate of capital accumulation. This condition is considered to hold throughout the paper, since it is reasonable for most economies and consistent with growth accounting exercises in [Bosworth and Collins \(2008\)](#) and [Dekle and Vandenbroucke \(2010\)](#), among others. In equation (12), e_a is a decreasing function of q , while e_m is increasing in q . Hence, only a declining path of q makes the right side of the equation decrease over time. ■

Lemma 1 has two implications for rural-urban structural change. The first is for the labor allocation, or the ‘quantity’ of workers. Equation (10) shows that the employment share in agriculture is positively correlated with the effective wage differential, and hence the employment share of the rural sector decreases as the effective wage differential converges toward zero. Workers originally in the rural sector are driven out because their comparative advantage shifts from agriculture to non-agriculture as q declines. Intuitively, a decreasing q implies the wage paid for each unit of effective labor in agriculture grows slower than that in the non-agriculture sector during economic growth. Consequently, the rural sector, as a relatively worse-off sector, is driving labor out, while the better-off urban sector attracts more workers, even those with relatively low productivity in non-agriculture.

The left side of equation (12) decomposes the driving force of structural transformation into three fundamental components. The first term shows the effect of scarce land: land per capita declines as population grows, crowding labor out of agriculture. The second term represents the impact of physical capital accumulation. As a result of capital accumulation over time, the non-agriculture sector that uses capital as an input expands. The last term shows the effect of uneven technical progress across sectors: the sector with relatively faster growth of technology would certainly see an increase in its relative marginal product and absorb workers into the sector. However, because of the first two mechanisms, structural transformation from agriculture to non-agriculture can sometimes take place even

if, as discussed in [Martin and Mitra \(2001\)](#), the rate of technical progress is slower outside agriculture in some economies, as long as the condition in (13) is satisfied.

As in [Lagakos and Waugh](#), equation (10) yields a log-linear relationship in the ratio of the employment shares for the two sectors:

$$\log \frac{l_a}{l_m} = \theta \cdot \log (\bar{\kappa} \cdot q) \quad (14)$$

This relationship implies that, with a greater dispersion of productivity across individuals, namely a lower shape parameter θ , a larger decline in q is required to achieve a given change in labor allocation.

The second implication from lemma 1 is more novel and somewhat overlooked in the structural change literature. It concerns labor ‘quality’, namely the average levels of sectoral individual productivity or effective labor. Combining equations (10) and (11), the average levels of individual productivity in the two sectors are given by

$$\begin{aligned} \bar{z}_a &= \frac{e_a}{l_a} = \frac{1}{\kappa_a} \cdot l_a^{-\frac{1}{\theta}} \cdot \gamma \\ \bar{z}_m &= \frac{e_m}{l_m} = \frac{1}{\kappa_m} \cdot l_m^{-\frac{1}{\theta}} \cdot \gamma \end{aligned} \quad (15)$$

\bar{z}_a and \bar{z}_m have an inverse relationship with their corresponding employment shares, and hence they are negatively correlated with the effective wage differential, q . Since q approaches zero, the average individual productivity in agriculture would increase to a high value and that in non-agriculture would decline towards a value given by $\frac{\gamma}{\kappa_m}$. Intuitively, as argued in proposition 1, when q is decreasing, workers with relatively low comparative productivity in agriculture are forced to migrate to the urban sector for higher wages. Consequently, when skilled and unskilled workers have migrated to the urban sector, those who continue to select rural work must be highly productive in agriculture.

4.2 Relative labor productivity

Relative labor productivity (RLP) measures the rural-urban differential of the average product of labor, given by

$$\text{RLP} = \frac{y_m/l_m}{p \cdot y_a/l_a}$$

Recent calculations based on World Bank data by [Temple and Woessmann \(2006\)](#) show that RLP is greater than one in most countries and has a downward trend in recent decades, indicating the sectoral productivity of labor is universally higher in non-agriculture than agriculture, but this relative superiority is narrowing.

In the dynamic LW model, it might appear at first sight that RLP will be decreasing, since one implication of structural change is that average individual productivity in agriculture improves. However, this conjecture does not necessarily hold. First of all, it is essential to distinguish the concepts of the average individual productivity and the average sectoral productivity of labor. The definition of the former concept is given by equation (15), which concerns the average levels of the corresponding skills provided by those working in the sector. The latter is measured as the sectoral output averaged over the workers in the sector, which also depends on the technologies of production and factor abundance as well as workers' abilities. RLP refers to the ratio of the latter measures. Under the assumption of Cobb-Douglas production technologies, RLP can be derived from equation (5):

$$\text{RLP} = \frac{1 - \alpha}{1 - \beta} \cdot \frac{\bar{z}_m}{\bar{z}_a} \cdot \frac{1}{q} \quad (16)$$

Equation (16) indicates that RLP over time is determined by two offsetting elements, namely the rural-urban ratio of the average individual productivity of labor, $\frac{\bar{z}_m}{\bar{z}_a}$, and the effective wage differential, q . Since the average individual agricultural productivity increases while that of non-agriculture decreases, the first term declines over time. However, this impact is offset by the decrease of the effective wage differential. This decrease, as lemma 1 indicates, is due to uneven technical progress across sectors and capital accumulation in the non-agriculture sector. In other words, in spite of a decreasing average level of individual productivity,

capital accumulation and relatively faster technical growth (if assumed) in the urban sector can offset this disadvantage.

In the analytical special case based on the assumption of Fréchet productivity distributions, equation (15) yields $\frac{\bar{z}_m}{\bar{z}_a} = q$, implying a perfect offset. Substituting into equation (16), the RLP predicted by the model is a constant given by $\frac{1-\alpha}{1-\beta}$. The result would accord with the first implication in [Temple and Woessmann's \(2006\)](#) calculations, if labor is used more intensively in the agriculture sector than non-agriculture. However, a constant RLP contrasts with their second result that RLP has often declined. This contradiction is due to the distributional assumption on individual productivity, where the same shape parameter is applied to the two sectors. If this assumption is replaced, a downward RLP is feasible in the dynamic LW model, even when the Cobb-Douglas production functions are retained. However, the lack of a closed-form solution for the employment shares in that case makes the model less tractable, especially when simulating dynamic paths with optimizing households.

4.3 Steady state

We now consider whether the long-run equilibrium of the dynamic LW model is compatible with a steady state. A steady state in growth models is mostly considered, on the basis of the well-known Kaldor facts, as a balanced growth path (BGP), where all per capita variables grow at a fixed rate and the interest rate is constant.

A standard BGP is unlikely in a dual economy model due to the co-existence of two sectors and labor migration across sectors over time. However, in a small open economy, this effect will be small in the long run when almost all workers have migrated to the non-agriculture sector. Based on this fact, an asymptotic balanced growth path (ABGP) can be defined to describe the long run growth path of a dual economy.

Definition 1 *An asymptotic balanced growth path is a long run equilibrium trajectory where the agriculture sector becomes infinitesimal and the economy dominated by the non-agriculture sector asymptotically grows at a constant rate and yields an asymptotically unchanging interest rate.*

To derive the steady state, the intertemporal equilibrium conditions are re-written in terms of efficiency units that are constant in the long

run, following the usual treatment. Original per capita variables are normalized by the efficiency level of the non-agriculture sector, A_m , since the agriculture sector asymptotically disappears in the long run. Variables in efficiency units are denoted as $\hat{y} \equiv \frac{y}{A_m}$, $\hat{k} \equiv \frac{k}{A_m}$ and $\hat{c} \equiv \frac{c}{A_m}$.

In the presence of technical progress, the intertemporal budget constraint (7) and the Euler equation (9) become

$$\dot{\hat{k}} = \hat{y} - \hat{c} - (n + \delta + g_m) \cdot \hat{k} \quad (17)$$

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{1}{\sigma} (r - \rho - \sigma \cdot g_m) \quad (18)$$

The steady state is approached when the changes of \hat{k} and \hat{c} are infinitesimal.⁴

Proposition 2 *The asymptotic balanced growth path is the unique long run steady state of the economy. The ABGP is approached if and only if $\frac{\dot{\hat{k}}}{\hat{k}} \rightarrow 0$, $\frac{\dot{\hat{c}}}{\hat{c}} \rightarrow 0$ and $q \rightarrow 0$.*

Proof. See Appendix C ■

Proposition 2 implies that the steady-state labor allocation $\{l_a^*, l_m^*\}$ approaches $\{0, 1\}$ and thus the effective labor inputs $\{e_a^*, e_m^*\}$ become $\left\{0, \frac{\gamma}{\kappa_m}\right\}$ due to equations (10) and (11). It further implies the following steady-state levels of capital and consumption per efficiency unit:

$$\hat{k}^* \rightarrow \left(\frac{\rho + \delta + \sigma \cdot g_m}{\beta} \right)^{\frac{1}{\beta-1}} \cdot e_m^*$$

$$\hat{c}^* \rightarrow \hat{k}^{*\beta} \cdot e_m^{*1-\beta} - (n + \delta + g_m) \cdot \hat{k}^*$$

As an economy converges to its ABGP, the process of structural change approaches its end and dualism disappears. The agriculture sector asymptotically shrinks to zero, except that land holds a small amount of labor with high comparative productivity in agriculture. The country, as a modern economy, is dominated by the non-agriculture sector, and grows at a constant rate approximately equal to g_m .

⁴Note that the parameter restriction for the utility integral to converge is $\rho - n > (1 - \sigma) \cdot g_m$ in this case.

5 Simulations

5.1 Methodology and parameter values

This section simulates the transitional dynamics of the model above to show how a dual economy with heterogeneous labor converges to its steady state. The simulations will display some important features of rural-urban structural change, especially showing how productivity distribution influences the speed of structural change. Dynamic paths for capital accumulation, consumption, saving and output will be described, and comparisons with the one-sector model will be made (when applicable), to show how the dynamic LW model differs from the standard growth model with respect to transitional dynamics.

The dynamic paths are obtained by solving the nonlinear dynamic equations from (1) to (9) numerically with a relaxation algorithm, developed by [Trimborn, Koch and Steger \(2008\)](#). A numerical solution is applied to study the transitional dynamics due to the absence of closed-form solutions and the infeasibility of a straightforward phase diagram.

A specification of parameter values is given in Table 1. Basic parameters (in the first row) that are also used in a standard Ramsey model take values as in [Barro and Sala-i-Martin \(2003\)](#), for the sake of comparisons. The output elasticities of land and capital are chosen to accord with the fact that agriculture uses labor slightly more intensively than the non-agriculture sector does, as in [Robertson \(1999\)](#). The rate of technical progress in the non-agriculture sector is assumed slightly faster than that of agriculture, following some growth accounting literature for developing countries (e.g. [Bosworth and Collins, 2008](#)). The shape parameter for the distributions of productivity endowments is chosen based on the values calibrated in [Kuralbayeva and Stefanski \(2013\)](#) using the US Current Population Survey (2009) and cross-country data. The scale parameters for the distributions are assumed to accord with the fact, following a calibration result in [Lagakos and Waugh \(2013\)](#), that the variation of labor productivity in non-agriculture is greater than that in agriculture. Meanwhile, the average level of productivity in non-agriculture is assumed to be higher than that in agriculture. In addition, the relative price of the agriculture product is fixed throughout and normalized to 0.5. The initial stock of physical capital is set to be 10 percent of its steady-state value and the

Table 1: Parameter assumptions for simulations

ρ	δ	σ	n
0.02	0.05	3	0.01
α, β	g_a, g_m	θ	κ_a, κ_m
0.3, 0.4	0.018, 0.020	2	2, 1

initial employment share of agriculture is assumed to be 80 percent.

5.2 Structural change

The dynamics of structural change are displayed in figure 1. Panel (a) shows the declining path of the effective wage differential, q , following lemma 1. q is greater than one at the early stage, implying a higher payment to each unit of productivity in the agriculture sector. However, this superiority is gradually overtaken by the non-agriculture sector during economic growth, shifting workers' comparative advantage from agriculture to non-agriculture. As a consequence, people initially working for the agriculture sector move to the non-agriculture sector over time, as displayed in panel (b). In the baseline case (i.e. $\theta = 2$, shown with the solid line), it takes approximately 90 years to reduce the agricultural employment share from 80 percent to 20 percent. Afterwards, four more decades are needed before 90 percent of workers are located in the urban sector.

The speed of structural transformation in the dynamic LW model has a tight link with the dispersion of productivity across workers, besides other standard determinants such as technical progress and capital accumulation. The log-linear relation in equation (14) implies that, if θ is lower, meaning a higher variance of labor productivity, the economy requires a larger change in q to obtain a given change in labor allocation. However, this does not necessarily affirm that a lower θ leads to slower structural change, since q is endogenous and a different θ may affect the dynamics of q as well. The dashed line in panel (a) shows that, with $\theta = 1.2$ rather than $\theta = 2$, q starts from a higher initial value than the baseline case and declines at a faster rate, partly offsetting the effect implied in equation (14). The dashed line in panel (b) shows the overall impact of lower θ on the speed of structural change, confirming that a higher dispersion of productivity results in slower structural change. It indicates that

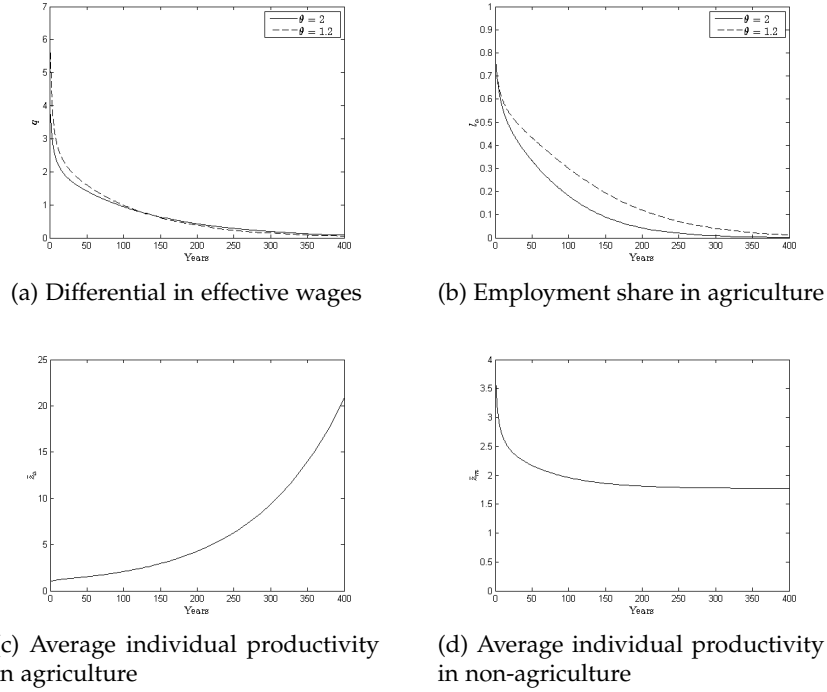


Figure 1: Structural change

five more decades are required, compared to the baseline case, to reduce the employment share in agriculture from 80 percent to 20 percent. In contrast, a higher θ or a smaller variance of individual productivity will, *ceteris paribus*, serve to accelerate structural transformation.

Another implication from equation (14) is that the value of $\tilde{\kappa}$ will not influence the speed of structural transformation, if it does not affect the dynamics of q .⁵ This conjecture is verified by a series of numerical simulations, showing that different values of $\tilde{\kappa}$, due to various κ_a and κ_m , yield the same path for $\frac{\dot{q}}{q}$ and thus employment shares. In other words, only a different level of relative dispersion, rather than absolute dispersion, in productivity across individuals alters the speed of structural change over time, and the higher relative dispersion, the slower is structural change.

The average level of individual productivity in agriculture, shown in panel (c), increases over time as implied by equation (15). It increases from an original value of 1 to approximately 3 when 90 percent of peo-

⁵Differentiating equation (14) with respect to time, the $\tilde{\kappa}$ outside of q drops.

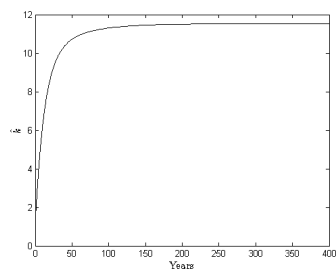
ple choose to work for the non-agriculture sector. Afterwards, it sharply increases beyond 10 at the stage that the rural sector asymptotically disappears, indicating the fact that only those who have extremely high comparative advantage in agriculture remain in the agriculture sector in the long run. On the contrary, panel (d) shows the downward trend of average individual productivity in the non-agriculture sector. It drops from 3.6 to its steady-state value 1.77, given by $\frac{\gamma}{\kappa_m}$, since almost all workers, including those with low productivity in non-agriculture, have transferred to the sector.

5.3 Capital, consumption and output

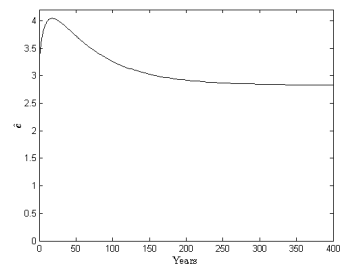
Figure 2(a) displays the transitional dynamics of physical capital in efficiency units from the initial stock, $\hat{k}(0)$, to the steady-state value, \hat{k}^* . The trajectory of \hat{k} shares a similar shape with the one-sector scenario (i.e. the model simulated in [Barro and Sala-i-Martin](#), Chapter 2, henceforth). The accumulation of capital starts with a relatively fast speed and takes a short period to complete its first half-life from $\hat{k}(0)$ to \hat{k}^* . However, this speed slows down during the second half-life. It shows that more than 200 years are needed to approach the end of convergence, contrasting with the one-sector counterfactual that closes this gap within 50 years. The slow convergence in the dual economy case implies the benefits from capital accumulation would not shrink and disappear within a short period even when a low output-capital elasticity is assumed.

Panels (b) and (c) illustrate consumption and output in efficiency units respectively. Contrasting with the one-sector case, \hat{c} and \hat{y} in this model experience overshooting at the early stage and then converge towards their steady-state values from above. Taking consumption as an example, the hump shape of \hat{c} implies the growth rate of consumption per efficiency unit, $\frac{\dot{\hat{c}}}{\hat{c}}$, is negative during the latter period of convergence. Following the Euler equation (18), this negative growth for consumption per efficiency unit occurs when the return to capital is smaller than $\rho + \sigma \cdot g_m$.

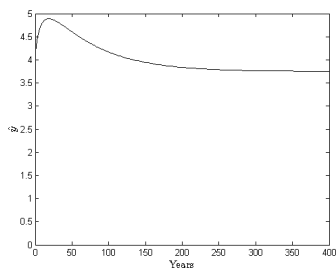
In spite of the negative growth of \hat{c} and \hat{y} , consumption and output per capita are always on an upward trend, as shown in panels (d) and (e) in logarithms. The decreasing paths of \hat{c} and \hat{y} simply mean that the growth rates of consumption and output per capita, implied by the slopes of pan-



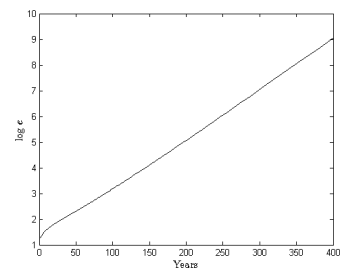
(a) Capital per efficiency unit



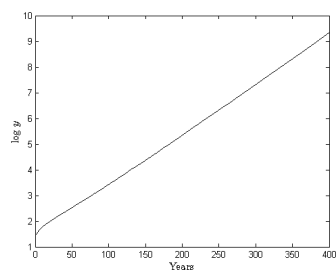
(b) Consumption per efficiency unit



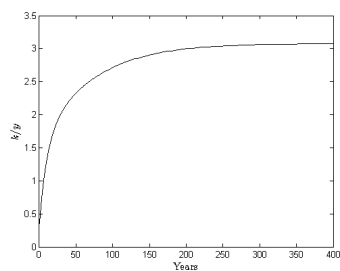
(c) Output per efficiency unit



(d) Log consumption per capita



(e) Log output per capita



(f) Capital-output ratio

Figure 2: Capital, consumption and output

els (d) and (e), are lower than the technical progress in the non-agriculture sector g_m and they approach g_m from below in the later period, whereas in the one-sector case, they are always greater than g_m , and converge to it from above. The pattern of consumption, as well as output, in the dynamic LW model suggests that individuals tend to consume less and save a larger fraction of income in the later period, compared to the standard one-sector model.

Panel (f) shows the behavior of the capital-output ratio. It has been argued in the literature that the movement of this ratio is relatively steady during economic growth (e.g. [Kaldor, 1963](#)), whereas the Ramsey model, under standard assumptions, predicts that the ratio sharply increases and approaches its steady state within the first 50 years. The dynamic LW model improves this prediction and implies a more stable movement. It suggests that more than 300 years are required for the capital-output ratio to approach its steady state.

5.4 Interest rate and saving rate

The one-sector Ramsey model makes counterfactual predictions on interest rates with a standard output-capital elasticity, as criticized in [King and Rebelo \(1993\)](#) and [Barro and Sala-i-Martin \(2003\)](#) among others, where paths with high initial values and a sharp fall are presented, since a low output-capital elasticity makes the effect of diminishing returns on capital set in quickly.⁶ The interest rate in the dynamic LW model is displayed in figure 3(a). It starts at approximately 0.21 and drops to a value that is lower than $\rho + \sigma \cdot g_m$ in the early period because of fast capital accumulation. At this stage, the consumption level in efficiency units surges and experiences an overshooting. Afterwards, the interest rate gently recovers, since workers are gradually transferred into the urban sector from the rural sector, partly offsetting the impact of diminishing returns to capital. Correspondingly, households choose to save more of their income and consumption per efficiency unit falls.

In the steady state, the interest rate levels off at an asymptotic value of 0.08, given by $\rho + \sigma \cdot g_m$. Throughout the whole process of development,

⁶The Ramsey model would predict initial interest rates to be more than 60 percent, or even higher than 100 percent when calibrated for the Japanese post-war experience as in [King and Rebelo \(1993\)](#).

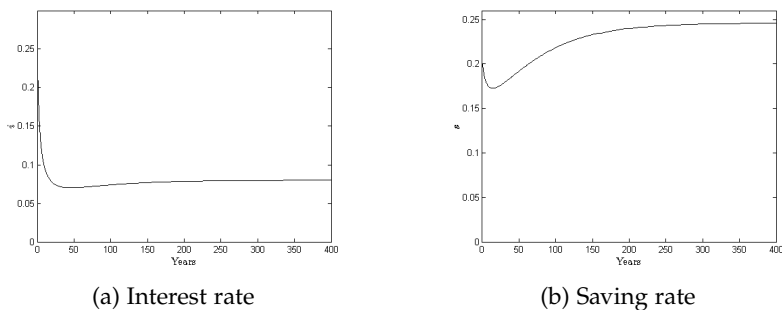


Figure 3: Interest rate and saving rate

the predicted interest rate is within a range between 0.08 and 0.21 and has a relatively steady path, which is more reasonable compared to the one-sector Ramsey model.

Panel (b) reports the saving rate, which is predicted to lie between 0.17 to 0.25. Corresponding to the interest rate path, the saving rate shows an undershooting at the early stage and experiences an upward trend afterwards, following the rebound of the interest rate. Undershooting dynamics, as well as overshooting cases, of saving rates are also presented and discussed in [Smetters \(2003\)](#), who analyzes the properties of saving rates in the one-sector Ramsey model with CES production technologies. [Smetters](#) finds that either undershooting or overshooting paths can exist with a reasonably parameterized one-sector economy, while this model shows that undershooting (overshooting) paths also exist in a dual economy during structural transformation.

5.5 Growth rate and speed of convergence

Figure 4(a) sketches the slope of figure 2(e), showing the growth rate of the economy, $\frac{\dot{y}}{y}$. The growth rate at the initial stage is relatively high at around 5.5 percent per year, and approaches 2 percent as the economy converges to its steady state. Unlike the result in the one-sector Ramsey model, where implausibly high growth rates are implied for early periods, the prediction in the dynamic LW model is more consistent with cross-country evidence.

The predictions on the speed of convergence, which captures how much the gap between the current and the steady state is closing over time,

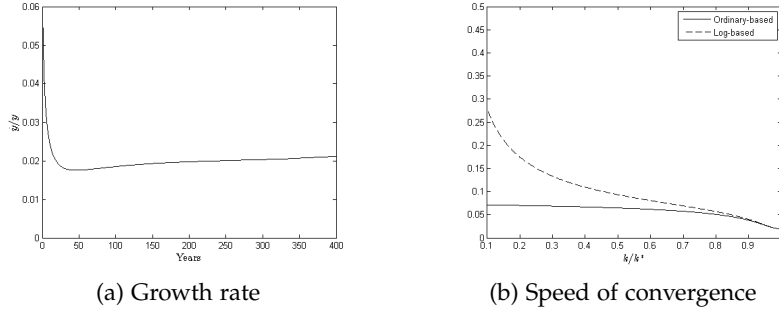


Figure 4: Growth rate and speed of convergence

are improved as well. In the one-sector model, some unrealistically high speeds of convergence are predicted, especially when the capital stock is far away from its steady state.

Following the definitions in [Mathunjwa and Temple \(2007\)](#), the speed of convergence for the economy is measured by two approaches here:

$$\Lambda_{\hat{k}}^1 = \frac{\dot{\hat{k}}}{\hat{k}^* - \hat{k}}$$

$$\Lambda_{\hat{k}}^2 = \frac{\dot{\ln \hat{k}}}{\ln \hat{k}^* - \ln \hat{k}}$$

where $\Lambda_{\hat{k}}^1$ is named the ordinary-variable based measure and $\Lambda_{\hat{k}}^2$ is the log-variable based measure.

Panel (b) of figure 4 demonstrates the above two measures. In the neighborhood of the steady state, the speed of convergence is 0.016, which could be analytically obtained via linearization or log-linearization, given that agriculture asymptotically disappears. When capital is far away from its steady state, the ordinary-variable based measure predicts a speed below 0.07, indicating less than 7 percent of the gap between \hat{k} and \hat{k}^* vanishes in one year. This prediction of a low convergence speed is consistent with cross-country growth data. The log-variable based measure indicates a higher convergence speed when capital is far away from the steady state. However, this prediction has been greatly improved compared to the same measure in the one-sector model.

6 Wages and inequality

6.1 Dynamics of the wage distributions

This section provides analysis of the dynamics of the wage distributions and inequality during growth and rural-urban transition. In the dynamic LW model, the self-selection behavior of heterogeneous workers reshapes the wage distributions over time from two aspects, both implied in equation (3). First, the self-selection mechanism alters the productivity distribution of those who choose to work in each sector, which influences the shape of the wage distribution. The second aspect is the change of the sectoral payment to each unit of productivity, which determines the scale of the wage distribution in each sector.

Distributions of sectoral productivity in the two sectors are conditionally drawn from the distributions of the productivity endowments of workers, $\{z_a^i, z_m^i\}$, on the basis of the necessary and sufficient condition in proposition 1: the distribution of workers' productivity in one sector is evaluated as the distribution of the corresponding productivity of those who have a comparative advantage in the sector. The cumulative density functions of the productivity distributions for the two sectors are then given by (see Appendix D)

$$\begin{aligned} H_a^*(z_a) &= e^{-\left(\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot z_a\right)^{-\theta}} \\ H_m^*(z_m) &= e^{-\left(\kappa_m \cdot l_m^{\frac{1}{\theta}} \cdot z_m\right)^{-\theta}} \end{aligned} \quad (19)$$

It can be observed from equation (19) that workers' productivity in each sector follows a Fréchet distribution with a shape parameter θ and a scale parameter $\kappa_a \cdot l_a^{\frac{1}{\theta}}$ or $\kappa_m \cdot l_m^{\frac{1}{\theta}}$. The shape parameter is identical to the one for individual productivity endowments, while the scale parameters, where the impact of self-selection enters, are augmented by the sectoral employment share, raised to the power of $\frac{1}{\theta}$.

Figure 5 plots the dynamics of the productivity distributions in both sectors, when the employment share in agriculture is 0.8, 0.5 and 0.2 respectively. In the agriculture sector, a shrinking employment share over time leads to a decreasing scale parameter of the distribution, indicating an increasing average level of productivity over time, as shown in panel

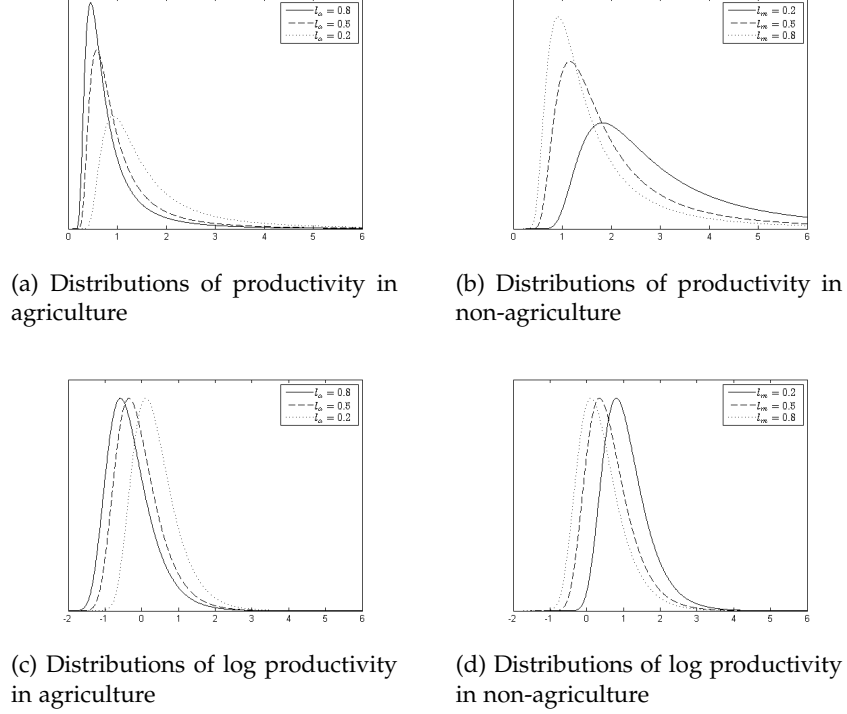


Figure 5: Distributions of workers' productivity over time

(a), according with the implication from equation (15). But from the figure, it would appear that the productivity distribution tends to be more divergent due to an extended scale. Nevertheless, the normalized, or relative, dispersion will remain unchanging because of the constant shape parameter. This can be seen from the distribution of log productivity in agriculture, shown in panel (c), where the distribution only shifts to the right without any shape change. Opposite implications can be obtained from the productivity distribution for the non-agriculture sector, namely a left-shifting distribution of productivity, as less productive workers enter the sector over time.

Now the second determinant of wages is considered, by substituting $z_a = \frac{w_a}{\zeta_a}$ and $z_m = \frac{w_m}{\zeta_m}$ into (19), yielding the cumulative density functions of the wage distributions for both sectors:

$$H_a^o(w_a) = e^{-\left(\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot \zeta_a^{-1} \cdot w_a\right)^{-\theta}}$$

$$H_m^o(w_m) = e^{-\left(\kappa_m \cdot l_m^{\frac{1}{\theta}} \cdot \zeta_m^{-1} \cdot w_m\right)^{-\theta}} \quad (20)$$

It shows that the wage distributions in the two sectors are also characterized by the Fréchet form with the same shape parameter θ and scale parameters $\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot \zeta_a^{-1}$ and $\kappa_m \cdot l_m^{\frac{1}{\theta}} \cdot \zeta_m^{-1}$ respectively. The sectoral effective wages enter into the scale parameters. In this analytical special case, it can be proved that

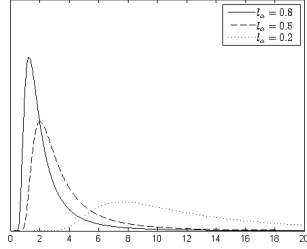
$$\frac{\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot \zeta_a^{-1}}{\kappa_m \cdot l_m^{\frac{1}{\theta}} \cdot \zeta_m^{-1}} = \frac{1}{\bar{\kappa}} \cdot (\bar{\kappa} \cdot q) \cdot \frac{1}{q} = 1$$

The same value of the scale parameters in the two sectors leads to identical wage distributions across sectors. Since the scale parameter for the agriculture sector, $\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot \zeta_a^{-1}$, is clearly decreasing over time, the wage distributions of the two sectors will shift to the right, as the first two plots in figure 6 display. Figures 6(c) and 6(d) show the distributions of log wages for the two sectors. Similar to log productivity, they only change locations horizontally, and the log-variances of labor income for the two sectors are constant.

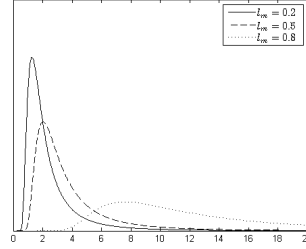
This section has analytically shown how the mechanism of self-selection shapes wage distributions over time, which makes use of a simplified assumption on productivity endowments. However, the result that the two sectors have identical wage distributions is clearly a special case. Other outcomes are possible when the distributional assumption is replaced, but the model will become significantly less tractable in that case, due to the absence of a closed-form solution for the labor allocation.

6.2 Inequality over time

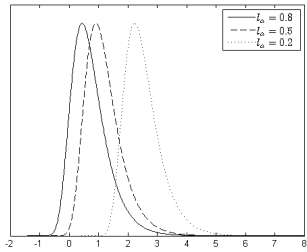
The relationship between growth and inequality is often discussed in terms of the Kuznets hypothesis. The hypothesis, proposed by [Kuznets \(1955\)](#) and subsequently investigated by [Robinson \(1976\)](#), [Fields \(1979\)](#) and [Temple \(2005b\)](#) among others, claims that when an economy is undergoing development, income inequality over time follows an inverted-U shape, which is widening at the early stage and narrowing later on. This point of view seems especially natural if economic dualism and structural change are presumed. A low level of inequality exists before and after structural



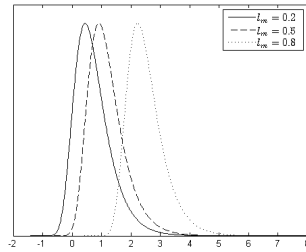
(a) Distributions of wages in agriculture



(b) Distributions of wages in non-agriculture



(c) Distributions of log wages in agriculture



(d) Distributions of log wages in non-agriculture

Figure 6: Wage distributions over time

change, since people are all poor at the initial stage and all become rich finally, while inequality widens when the economy is undergoing transition, because workers receive diverse income in different sectors. However, this theory requires a strong assumption that the within-sector wage is identical (or at least similar) for everyone. This is obviously not supported by the data, especially in the non-agriculture sector. When this assumption is replaced by a different approach to within-sector wages, e.g. the dynamic LW model, the inverted-U does not necessarily exist and a different path for inequality might be suggested.

In the dynamic LW model, the path of inequality is endogenously determined as the mechanism of self-selection reshapes the wage distributions across sectors over time. Two aspects contribute to the dynamics of inequality, implied in equation (3), namely the effective wage differential across sectors and the sectoral productivity distributions. As lemma 1 indicates, the effective wage differential, q , approaches zero. This movement of q implies that the gap between rural-urban effective wages is widening

to infinity. This increasing gap in sectoral effective wages has two offsetting effects on inequality. At first sight, an increasing gap of effective wages across sectors directly contributes to aggravating inequality. However, this increasing gap would activate self-selection and reallocate workers with less comparative productivity in agriculture into the urban sector for higher wages, which reshapes the sectoral productivity distributions and partly balances out the effect of the increasing effective wage gap.

In the analytical special case emphasized here, inequality is constant over time for two reasons. First, the inequality level within each sector remains unchanging, since the distribution of log wages shifts to the right without any shape transformation. Second, the two sectors have exactly the same wage distributions, thus the inequality level must be equal across sectors, as well as the whole economy. This implication can also be obtained from a closed-form solution for the Gini coefficients. Appendix E shows that the Gini coefficients for both sectors are constant over time at $2^{\frac{1}{\theta}} - 1$, only depending on the shape parameter θ .

The finding of constant inequality over time is obviously a special result based on a relatively restrictive assumption. Richer implications on inequality are possible, given various specifications of productivity distributions. However, this special case does show that the inequality path is not necessarily an inverted-U curve, even in a transitional dual economy. This argument is consistent with some recent empirical literature on the relationship between growth and inequality that shows little support for the Kuznets curve (e.g. [Angeles, 2010](#)).

7 Conclusion

In this paper, I have presented a dual growth model built on [Lagakos and Waugh \(2013\)](#) with its focus on the self-selection mechanism whereby workers choose their optimal sectors based on comparative advantage. This mechanism motivates structural transformation from agriculture to non-agriculture over time because of the shift of workers' comparative advantage during economic growth. Based on the new model, the paper offers a novel dimension to view the dynamics of structural change, labor productivity, wage distributions and inequality for dual economies.

Following the self-selection mechanism, the model presents a dynamic

trajectory of rural-urban structural change, and shows that structural change raises average individual productivity in the rural sector and lowers that in the urban sector.

A main contribution of this paper is to build a link between structural transformation and the distribution of labor productivity. It suggests that the speed of structural change is influenced by the relative dispersion of productivity across workers, with a higher dispersion leading to a slower rural-urban transition.

Relative labor productivity in this model is constant over time, based on the assumption of Fréchet productivity distributions with the same shape parameter. However, a more general version of this model assuming an alternative form of individual productivity could yield a declining path for relative labor productivity to accord with the evidence in [Temple and Woessmann \(2006\)](#).

The model predicts a unique steady state in the long run where dualism asymptotically disappears and the economy, dominated by the non-agriculture sector, moves towards an asymptotic balanced growth path. Numerical simulations for the model suggest that the transitional dynamics of the dynamic LW model are more compatible with reality than the standard Ramsey model. In particular, the LW model significantly improves the predictions on interest rates, growth rates and the convergence speed.

The dynamics of wages and inequality are analyzed based on a special case, which implies the same wage distribution across sectors, and leads to constant aggregate inequality over time. Although other outcomes are possible given an alternative specification of individual productivity endowments, the special case shows that the inequality path over the course of structural transformation is not necessarily an inverted-U curve.

A potential direction for future research could be to revise the distributional assumption to find a path of relative labor productivity that is more consistent with recent evidence. In addition, a more general approach to sectoral productivity will lead to richer implications in terms of wage dynamics, so that further insights into income inequality will be possible.

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Appendix A Derivation of the Euler equation

The Ramsey problem of the representative household need to be solved backwards in two steps. First comes the second step: given the composite good consumed at each instant, \bar{c} , the household chooses the consumption bundle (c_a, c_m) to minimize the expenditure as in (8). The expenditure function is separable in the relative price p and \bar{c} , if $v(c_a, c_m)$ is homogeneous of degree one.⁷ To prove, rearrange the constraint in equation (8)

$$\begin{aligned}
 \phi &\equiv \min_{c_a, c_m} \{ p \cdot c_a + c_m : v(c_a, c_m) \geq \bar{c} \} \\
 &= \min_{c_a, c_m} \left\{ p \cdot c_a + c_m : v\left(\frac{c_a}{\bar{c}}, \frac{c_m}{\bar{c}}\right) \geq 1 \right\} \\
 &= \bar{c} \cdot \min_{c_a, c_m} \left\{ p \cdot \frac{c_a}{\bar{c}} + \frac{c_m}{\bar{c}} : v\left(\frac{c_a}{\bar{c}}, \frac{c_m}{\bar{c}}\right) \geq 1 \right\} \\
 &= \bar{c} \cdot \min_{\tilde{c}_a, \tilde{c}_m} \{ p \cdot \tilde{c}_a + \tilde{c}_m : v(\tilde{c}_a, \tilde{c}_m) \geq 1 \} \\
 &= \bar{c} \cdot J(p)
 \end{aligned}$$

where $J(p)$ represents the price index of the composite good derived from the expenditure minimization problem. Note that the second line makes use of the property of homogeneity.

For the first step of the optimization problem, given that the expenditure is separable as $\phi = c \cdot J(p)$, the household's intertemporal problem becomes:

$$\max_c \int_0^{\infty} u(c) \cdot e^{-(\rho-n)t} dt$$

subject to the budget constraint

$$\dot{k} = y - c \cdot J(p) - (n + \delta) \cdot k$$

The present-value Hamiltonian is given by

$$\mathcal{H} = u(c) \cdot e^{-(\rho-n)t} + \mu \cdot [y - c \cdot J(p) - (n + \delta) \cdot k]$$

where μ is the co-state variable. The first order conditions include

⁷It can be easily extended to the Stone-Geary case, since it gives rise to the Stone-Geary linear expenditure system (see [Chung, 1994](#)).

$$\frac{\partial \mathcal{H}}{\partial c} = u(c) \cdot e^{-(\rho-n) \cdot t} - \mu \cdot J(p) = 0 \quad (21)$$

$$\frac{\partial \mathcal{H}}{\partial k} = \mu \cdot (G'_k - \delta - n) = \mu \cdot (r - n) = -\dot{\mu} \quad (22)$$

and the transversality condition is $\lim_{t \rightarrow \infty} k \cdot \mu = 0$.

Taking logs and differentiating with respect to time, equation (21) becomes

$$\frac{u''(c) \cdot \dot{c}}{u'(c)} - (\rho - n) - \frac{\dot{\mu}}{\mu} - \frac{J'(p) \cdot \dot{p}}{J(p)} = 0$$

Rearranging it and combining with equation (22) obtains

$$\frac{\dot{c}}{c} = -\frac{1}{\epsilon} \cdot \left(r - \rho - \frac{J'(p) \cdot \dot{p}}{J(p)} \right)$$

Since the relative price is considered fixed exogenously, i.e. $\dot{p} = 0$, the Euler equation is given as (9). [Roe et al. \(2010\)](#) derive similar results for a closed economy, and a discrete-form analogue is introduced in [Hayashi and Prescott \(2008\)](#).

Appendix B Derivations of labor inputs

Following proposition 1, the employment share in the agriculture sector is given by the probability that workers have a comparative advantage in agriculture (i.e. $\left\{ \frac{z_m^i}{z_a^i} \leq q \right\}$). Given that workers' productivity endowments, $\{z_a^i, z_m^i\}$, are independently drawn from two continuous Fréchet distributions, the agricultural labor share can be analytically worked out via a double integral:

$$\begin{aligned}
l_a &= \text{Prob} \left\{ \frac{z_m}{z_a} \leq q \right\} = \iint_{z_a \cdot q \geq z_m} dH_a(z_a) dH_m(z_m) \\
&= \int_0^\infty \left[\int_0^{z_a \cdot q} dH_m(z_m) \right] dH_a(z_a) \\
&= \int_0^\infty e^{-(\kappa_m \cdot z_a \cdot q)^{-\theta}} dH_a(z_a) \\
&= \int_0^\infty \theta \cdot \kappa_a \cdot (\kappa_a \cdot z_a)^{-\theta-1} \cdot e^{-(\kappa_a \cdot z_a)^{-\theta} - (\kappa_m \cdot z_a \cdot q)^{-\theta}} dz_a \\
&= \int_0^\infty e^{-(\kappa_a \cdot z_a)^{-\theta} - (\kappa_m \cdot z_a \cdot q)^{-\theta}} d \left[-(\kappa_a \cdot z_a)^{-\theta} \right] \\
&= \int_0^\infty e^{-(\kappa_a \cdot z_a)^{-\theta} \cdot [(\tilde{\kappa} \cdot q)^{-\theta} + 1]} d \left[-(\kappa_a \cdot z_a)^{-\theta} \right] \\
&= \frac{1}{(\tilde{\kappa} \cdot q)^{-\theta} + 1}
\end{aligned}$$

Evaluating a similar integral or simply computing $1 - l_a$ yields the labor share in the non-agriculture sector. [Lagakos and Waugh \(2013\)](#) derive the labor allocation in equilibrium using the same approach, but their results are based on the assumption of one-parameter Fréchet distributions for productivity.

The effective labor in each sector is calculated as the conditional expectation of the corresponding productivity distribution. The effective labor

in the agriculture sector is given by

$$\begin{aligned}
e_a &= \iint_{\Omega_a} z_a d\Omega_a \\
&= \iint_{z_a \cdot q \geq z_m} z_a dH_a(z_a) dH_m(z_m) \\
&= \int_0^\infty z_a \cdot \left[\int_0^{z_a \cdot q} dH_m(z_m) \right] dH_a(z_a) \\
&= \int_0^\infty z_a \cdot e^{-(\kappa_m \cdot z_a \cdot q)^{-\theta}} dH_a(z_a) \\
&= \int_0^\infty z_a \cdot \theta \cdot \kappa_a \cdot (\kappa_a \cdot z_a)^{-\theta-1} \cdot e^{-(\kappa_a \cdot z_a)^{-\theta} - (\kappa_m \cdot z_a \cdot q)^{-\theta}} dz_a \\
&= \int_0^\infty z_a \cdot \theta \cdot \kappa_a \cdot (\kappa_a \cdot z_a)^{-\theta-1} \cdot e^{-(\kappa_a \cdot z_a)^{-\theta} \cdot [(\bar{\kappa} \cdot q)^{-\theta} + 1]} dz_a \\
&= \int_0^\infty z_a \cdot \theta \cdot \kappa_a \cdot (\kappa_a \cdot z_a)^{-\theta-1} \cdot e^{-(\kappa_a \cdot z_a)^{-\theta} \cdot l_a^{-1}} dz_a \\
&= l_a \cdot \int_0^\infty z_a \cdot \theta \cdot \kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot \left(\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot z_a \right)^{-\theta-1} \cdot e^{-\left(\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot z_a \right)^{-\theta}} dz_a
\end{aligned}$$

Notice that the integral part in the last step is to evaluate the expectation of a Fréchet distribution with a scale parameter $\kappa_a \cdot l_a^{\frac{1}{\theta}}$, and this expectation is given by $\frac{1}{\kappa_a} \cdot l_a^{-\frac{1}{\theta}} \cdot \gamma$, where γ is the Gamma function evaluated at $\frac{\theta-1}{\theta}$. Therefore, the effective labor in the agriculture sector is

$$e_a = \frac{1}{\kappa_a} \cdot l_a^{-\frac{\theta-1}{\theta}} \cdot \gamma$$

as given in equation (11). A similar calculation can obtain the effective labor in the non-agriculture sector. An alternative argument, via computing the average individual productivity for workers, is provided in [Lagakos and Waugh](#), which will yield the same result.

Appendix C Proof of proposition 2

Existence: Similar to the standard Ramsey model, a steady state requires the changes of \hat{k} and \hat{c} are infinitely close to zero. Those conditions are satisfied in the ABGP where $\frac{\dot{\hat{k}}}{\hat{k}} \rightarrow 0, \frac{\dot{\hat{c}}}{\hat{c}} \rightarrow 0$ and $q \rightarrow 0$.

Uniqueness: Assume there exists another steady state where $\frac{\dot{\hat{k}}}{\hat{k}} \rightarrow 0, \frac{\dot{\hat{c}}}{\hat{c}} \rightarrow 0$

but $q \rightarrow 0$. Equation (18) implies that $\frac{\dot{\kappa}}{\dot{\bar{\kappa}}}$ must equal $\frac{\dot{e}_m}{\dot{e}_m}$, when $\frac{\dot{\bar{c}}}{\dot{\bar{c}}} \rightarrow 0$. Since $\frac{\dot{\kappa}}{\dot{\bar{\kappa}}} \rightarrow 0$ is required, $\frac{\dot{e}_m}{\dot{e}_m} \rightarrow 0$ implies $\frac{\dot{q}}{q} \rightarrow 0$. A situation that $\frac{\dot{q}}{q} \rightarrow 0$ where $q \rightarrow 0$ violates lemma 1.

Appendix D Distribution of sectoral productivity

Workers choose the agriculture sector based on the condition $\left\{ \frac{z_m^i}{z_a^i} \leq q \right\}$. Hence the distribution of productivity in agriculture is evaluated at the range of $z_m \in [0, z_a \cdot q]$. The probability density function is given by

$$\begin{aligned}
h_a^*(z_a) &= \frac{1}{l_a} \cdot \int_0^{z_a \cdot q} h_a(z_a) \cdot h_m(z_m) dz_m \\
&= \frac{1}{l_a} \cdot \theta \cdot \kappa_a \cdot (\kappa_a \cdot z_a)^{-\theta-1} \cdot e^{-(\kappa_a \cdot z_a)^{-\theta}} \cdot e^{-(\kappa_m \cdot z_a \cdot q)^{-\theta}} \\
&= \frac{1}{l_a} \cdot \theta \cdot \kappa_a \cdot (\kappa_a \cdot z_a)^{-\theta-1} \cdot e^{-(\kappa_a \cdot z_a)^{-\theta} \cdot [(\bar{\kappa} \cdot q)^{-\theta} + 1]} \\
&= \frac{1}{l_a} \cdot \theta \cdot \kappa_a \cdot (\kappa_a \cdot z_a)^{-\theta-1} \cdot e^{-(\kappa_a \cdot z_a)^{-\theta}} \cdot l_a^{-1} \\
&= \theta \cdot \kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot \left(\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot z_a \right)^{-\theta-1} \cdot e^{-\left(\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot z_a \right)^{-\theta}}
\end{aligned}$$

where $h_a(z_a)$ and $h_m(z_m)$ are the corresponding probability density functions for $H_a(z_a)$ and $H_m(z_m)$ respectively and $\frac{1}{l_a}$ is applied to ensure the probability density function integrates to one. The corresponding cumulative density function of $h_a^*(z_a)$ is

$$H_a^*(z_a) = e^{-\left(\kappa_a \cdot l_a^{\frac{1}{\theta}} \cdot z_a \right)^{-\theta}}$$

as shown in equation (19). The distribution in the non-agriculture sector, $H_m^*(z_m)$ can be derived from a similar argument.

Appendix E Within-sector Gini coefficients

The within-sector Gini coefficients for the two sectors are derived from their Lorenz curves. The Lorenz curve is graphed as $[\Phi(x), \Psi(x)]$, where $\Phi(x)$ denotes the population share up to the worker(s) with x units of

productivity and $\Psi(x)$ is the income share for these workers. In the agriculture sector, $\Phi(x)$ is given by

$$\Phi(x) = \frac{1}{l_a} \cdot \int_0^x \int_0^{z_a \cdot q} dH_m(z_m) dH_a(z_a)$$

and $\Psi(x)$ is evaluated as

$$\begin{aligned} \Psi(x) &= \frac{\int_0^x \int_0^{z_a \cdot q} \zeta_a \cdot z_a dH_m(z_m) dH_a(z_a)}{\zeta_a \cdot e_a} \\ &= \frac{1}{e_a} \cdot \int_0^x \int_0^{z_a \cdot q} z_a dH_m(z_m) dH_a(z_a) \end{aligned}$$

The Gini coefficient for each sector is then defined as $1 - 2 \int \Psi(x) d\Phi(x)$ or $2 \int \Phi(x) d\Psi(x) - 1$. In the agriculture sector,

$$\begin{aligned} \int \Phi(x) d\Psi(x) &= \int \Phi(x) \Psi'(x) dx \\ &= \frac{1}{l_a \cdot e_a} \cdot \int_0^\infty \left[\int_0^x \int_0^{z_a \cdot q} dH_m(z_m) dH_a(z_a) \cdot x \cdot H'_a(x) \cdot \int_0^{x \cdot q} dH_m(z_m) \right] dx \\ &= \frac{1}{l_a \cdot e_a} \cdot \int_0^\infty \frac{e^{-(\kappa_a \cdot x)^{-\theta} \cdot [(\tilde{\kappa} \cdot q)^{-\theta} + 1]}}{(\tilde{\kappa} \cdot q)^{-\theta} + 1} \cdot x \cdot \theta \cdot \kappa_a \cdot (\kappa_a \cdot x)^{-\theta-1} \cdot e^{-(\kappa_a \cdot x)^{-\theta} \cdot [(\tilde{\kappa} \cdot q)^{-\theta} + 1]} dx \\ &= \frac{1}{e_a} \cdot \int_0^\infty x \cdot \theta \cdot \kappa_a \cdot (\kappa_a \cdot x)^{-\theta-1} \cdot e^{-(\kappa_a \cdot x)^{-\theta} \cdot 2 \cdot [(\tilde{\kappa} \cdot q)^{-\theta} + 1]} dx \\ &= \frac{1}{e_a} \cdot \frac{l_a}{2} \cdot \int_0^\infty x \cdot \theta \cdot \kappa_a \cdot 2^{-\frac{1}{\theta}} \cdot l_a^{\frac{1}{\theta}} \cdot \left(\kappa_a \cdot 2^{-\frac{1}{\theta}} \cdot l_a^{\frac{1}{\theta}} \cdot x \right)^{-\theta-1} \cdot e^{-\left(\kappa_a \cdot 2^{-\frac{1}{\theta}} \cdot l_a^{\frac{1}{\theta}} \cdot x \right)^{-\theta}} dx \\ &= \frac{1}{e_a} \cdot 2^{\frac{1}{\theta}-1} \cdot l_a^{\frac{\theta-1}{\theta}} \cdot \frac{1}{\kappa_a} \cdot \gamma \\ &= 2^{\frac{1}{\theta}-1} \cdot \frac{e_a}{e_a} = 2^{\frac{1}{\theta}-1} \end{aligned}$$

Hence the within-sector Gini coefficient for the agriculture sector is $2^{\frac{1}{\theta}} - 1$, which only depends on the shape parameter θ and remains constant over time. A similar calculation yields the Lorenz curve and the same Gini coefficient for the non-agriculture sector, reflecting the outcome that wage distributions in the two sectors are identical in equilibrium.