# The Cyclical Behavior of Equilibrium Unemployment and Vacancies – A Comment

Simon Burgess Hélène Turon

Discussion Paper No. 05/573

January 2005

Department of Economics University of Bristol 8 Woodland Road Bristol BS8 1TN

## The Cyclical Behavior of Equilibrium Unemployment and Vacancies – A Comment

Simon Burgess<sup>\*</sup>and Hélène Turon<sup>†</sup>

January 2005

#### Abstract

The Mortensen-Pissarides model is an attractive model because it is tractable, delivers some intuitive comparative statics and permits policy analysis. However, Shimer (2005) shows that the model generates far too little volatility in its key variables – unemployment and vacancies – relative to the variation in the shock variables. Shimer identifies the flexibility of wages as the key issue. In this Comment, we show that it is possible to generate sufficient volatility in unemployment and vacancies whilst retaining the standard wage determination process. We set out a model with two important changes from the Mortensen-Pissarides approach: job search by the employed is allowed, and the vacancy creation condition is changed to allow churning of workers. Calibrating the model to UK data, we show that our model can produce volatility in the unemployment and vacancy series to match the data; we confirm for the UK that the Mortensen-Pissarides model cannot, as shown by Shimer for the US.

Keywords: Unemployment, on-the-job search, worker flows, job flows, matching.

JEL classification: J64.

<sup>\*</sup>University of Bristol, CEPR and IZA.

<sup>&</sup>lt;sup>†</sup>University of Bristol and IZA. Corresponding author: Helene.Turon-Lacarrieu@bristol.ac.uk. Department of Economics, University of Bristol, 8 Woodland Road, Bristol BS81TN. We would like to acknowledge financial support from the European Union for the DAEUP CEPR network and from the Leverhulme Trust for Helene's Special Research Fellowship. We are grateful to Sonia Bhalotra, David de Meza, Christian Heafke, Fabien Postel-Vinay, Marisa Ratto, Silvia Sonderegger and seminar participants at the DAEUP, Royal Economic Society, EEA and EALE conferences, at the IZA/SOLE Meeting and at University Pompeu Fabra for useful comments.

## 1 Introduction

The Mortensen-Pissarides model of the search and matching process is now firmly established in the toolkit for analysing labour markets and the macroeconomy (Mortensen and Pissarides, 1994, 1999a). It is an attractive model because it is tractable, delivers some intuitive comparative statics and permits policy analysis. However, Shimer (2005) shows that in one major aspect the standard model does not fit the facts. This is that the model generates far too little volatility in its key variables – unemployment and vacancies – relative to the variation in the shock variables. Shimer identifies the flexibility of wages as the key issue<sup>1</sup>.

In this Comment, we show that it is possible to generate sufficient volatility in unemployment and vacancies whilst retaining the standard wage determination process. We set out a model<sup>2</sup> with two important changes from the Mortensen-Pissarides approach: job search by the employed is allowed<sup>3</sup>, and the vacancy creation condition is changed to allow "churning" of workers<sup>4</sup>. That is, firms may keep jobs open when workers leave either to retire or to take another job, and find replacement workers. Both these features are quantitatively significant in real labour markets. Calibrating the model to UK data, we show that our model produces volatility in the unemployment and vacancy series to match the data; we confirm for the UK that the Mortensen-Pissarides model cannot, as

 $<sup>^1{\</sup>rm Hall}$  (2003b) notes the same problem, and proposes a model of sectoral shocks to generate sufficient volatility in unemployment and vacancies.

<sup>&</sup>lt;sup>2</sup>We have developed this more fully elsewhere (Burgess and Turon, 2004).

<sup>&</sup>lt;sup>3</sup>Pissarides (1994), Shimer (2003) and Eriksson and Gottfries (2002) also set up unemployment models with on-the-job search but keep job destruction exogenous and do not allow firms' churning.

<sup>&</sup>lt;sup>4</sup>This is defined and measured in Burgess, Lane and Stevens (2000). It is essentially the excess of worker flows over job flows. The re-filling of jobs left by other workers yields worker flows but no job flows.

shown by Shimer with US data.

Our model delivers a greater volatility of the V-U ratio for two reasons. First, our vacancy creation condition leads to a more volatile labour market tightness. Second, allowing for on-the-job search releases the tie between labour market tightness and the V-U ratio (this is detailed below), thereby allowing this ratio to be much more volatile than the tightness.

We assume the arrival rate of potential new vacancies to be *finite*, differing from the standard model. These job "ideas" are created over a range of idiosyncratic productivities. The state of the business cycle, embodied in the value of the common component of productivity, together with the tightness of the labour market, determine the threshold idiosyncratic productivity below which it is not worthwhile for a firm to turn this "idea" into a vacant job. In a boom, three factors operate in our model to raise vacancies. First, as in the standard Mortensen-Pissarides (hereafter M-P) model, an increase in the aggregate price raises the value of all jobs and so generates more vacancies. Second, in our model with heterogeneous vacancy creation the threshold value of a viable 'idea' declines in a boom. Thus more 'ideas' are turned into jobs in a boom, increasing vacancies further. The third factor is a limited vacancy chain. Retiring workers leave open job slots, some of which firms find it worthwhile to re-advertise. Without employed job search, these vacancy chains are of maximum length one, but still exceed the length of zero assumed in the M-P model. This creates yet further vacancies in a boom. Thus, the assumption of heterogeneous vacancy creation induces a greater increase in vacancies in a boom relative to the M-P model both through the greater range of 'ideas' turned into jobs, and through the creation of limited vacancy chains.

Once we also allow for employed job searchers, labour market tightness and the unemployment-vacancy ratio are not equivalent. The standard model essentially implies a link between the volatility of shocks and the volatility of labour market tightness. The key point is that it is the restriction that only the unemployed can search that forces the equivalence between tightness and the unemployment-vacancy ratio. Once this is relaxed, this gives the system another degree of freedom, and in this model that works to impart greater volatility to the unemployment-vacancy ratio. This works as follows: the boom raises labour market tightness as measured by the ratio of vacancies (V) to total job seekers (J). Total job seekers are the unemployed (U) plus the employed job searchers. We can write the labour market tightness, or V-J ratio as  $(V/J) \equiv (V/U) * (U/J)$ . With endogenous employed job search, the ratio of unemployed to all job seekers, the U-J ratio, is counter-cyclical, falling in booms as more of the employed find job search profitable and unemployment declines. This is very clear in the data – see Burgess (1993). Thus the rise in the V-J ratio translates into a fall in the U-J ratio and hence a much larger rise in the V-U ratio. Employed job search also accentuates the vacancy chain process, further increasing volatility.

In the next section we set out our model, and summarise the standard M-P approach. Section 3 briefly introduces the UK data on unemployment, vacancies and productivity shocks. We confirm the spirit of Shimer's finding for UK data: the standard deviation of the vacancy-unemployment ratio relative to trend is approximately 28 times that of productivity relative to its trend. We calibrate three models to UK data: the standard MP model; a model with heterogeneous vacancy creation but no on-the job search (Model I); and a model with heterogeneous vacancies and on-the-job search added (Model II). This isolates the impact of each of our two key assumptions on the volatilities of interest here. We show that the implied elasticity of the V-U ratio is around 1 in the M-P model, 4.0 in Model I, and 12.2 in Model II, with both our vacancy creation rule and employed job search. Our data yield an elasticity of 12.4. Section 4 concludes.

## 2 Models of Search and Matching

Our model builds on the standard M-P framework<sup>5</sup>, but differs from it with respect to job search and job creation; it is developed in greater detail in Burgess and Turon (2004). First, we incorporate on-the-job search, with an endogenous fraction of the employed finding it worthwhile to search. Second, we allow for heterogeneous vacancy creation and a finite (rather than infinite) supply rate of potential new jobs per period. We do this for the following reasons. We want to allow some of the jobs quit from to be re-advertised rather than destroyed<sup>6</sup>. If all new vacancies were created at the highest idiosyncratic productivity, it would never be optimal to re-advertise an on-going job with a lower productivity. Therefore we assume heterogeneous vacancy creation. We also assume a finite supply of potential jobs, as in the presence of an infinite supply of potential jobs ("ideas") per period, firms would reject all but the highest idiosyncratic productivity ideas and again there would be no re-advertisement. Given these

 $<sup>^5 \</sup>mathrm{See}$  Pissarides (2000) or Mortensen and Pissarides (1999a) for the derivation of the original model.

 $<sup>^{6}\,\</sup>mathrm{In}$  the standard set-up, total separations equal total job destruction.

assumptions, firms find it optimal to maintain and re-advertise an endogenous fraction of jobs that workers quit from.

Potential new 'ideas' for jobs are born at a *finite* rate of *jcr* per period and their value is distributed over the range  $(-\sigma; \sigma)$  according to a cumulative distribution function  $F(\epsilon)$ . The value of the output produced by firms is composed of two parts: the aggregate component, p and the idiosyncratic value of the idea,  $\epsilon$ . When  $\epsilon$  is above an endogenous threshold T it is worthwhile opening a vacancy. The value of  $\epsilon$  is subject to idiosyncratic shocks occurring at rate  $\lambda$ , anticipated by both workers and firms. The post-shock distribution of  $\epsilon$  is also  $F(\epsilon)$ . If the post-shock idiosyncratic productivity falls below an endogenous threshold R, the job is destroyed and the worker becomes unemployed; we show that R < T. If the idiosyncratic productivity is below an endogenous threshold S, workers find it worthwhile to search on the job. Whether a worker searches on the job or not only depends on the idiosyncratic productivity of the job,  $\epsilon$ .

Following Jovanovic (1979), we assume that the job match is an experience good, so the idiosyncratic productivity is unknown to the worker at the time of the match and is only discovered once she takes the job. It is not possible for firms advertising vacant jobs with a high productivity to signal it to job seekers because they cannot commit to the promised wage once the worker is employed (because of wage renegotiation). Because  $\epsilon$  is unknown *ex ante*, employed job seekers sample *all* the available vacancies, and accept all offers rather than just those better than their current match. Therefore, neither the matching probability nor the expected value of employment in the next job depend on the value of  $\epsilon$  in their current job.

#### 2.1 Bellman equations

We denote labour market tightness by  $\theta$ , and an unemployed searcher's matching probability as  $\theta q(\theta)$ . We assume that employed job seekers have a matching probability equal to a times the matching probability of unemployed job searchers. The per-period cost of employed job search is denoted k. We include a constant probability of leaving the labour force, from employment, and entering the labour force, through unemployment, denoted l. The Bellman equations for employed and unemployed workers are as follows ( $E_o(\epsilon)$  denotes employed workers searching on the job,  $E_n(\epsilon)$  non-searching employed workers and Uunemployed workers):

$$rE_{i}(\epsilon) = w_{i}(\epsilon) + l(U - E_{i}(\epsilon)) +$$

$$+\lambda \cdot \left[\int_{R}^{\sigma} E(x)dF(x) + U.F(R) - E_{i}(\epsilon)\right]$$

$$+I_{o} \cdot \left[a\theta q(\theta) \left(\int_{T}^{\sigma} E(x)dF_{V}(x) - E_{o}(\epsilon)\right) - k\right]$$

$$rU = b + \theta q(\theta) \left(\int_{T}^{\sigma} E(x)dF_{V}(x) - U\right)$$
(2)

where i = o, n when  $\epsilon$  is in the range (R; S) or (S; T) respectively and  $I_o$  equals 1 if the worker searches, i.e. if  $\epsilon$  is in the range (R; S) and 0 otherwise. r is the discount rate, b is the per-period sum of the unemployment benefit and the value of leisure, net of job search costs.

The Bellman equations for filled jobs,  $J_{o}(\epsilon)$  ( $J_{n}(\epsilon)$  respectively) when the

worker is (respectively is not) searching on the job, are:

$$rJ_{i}(\epsilon) = p + \epsilon - w_{i}(\epsilon) + l \cdot (V_{i}(\epsilon) - J_{i}(\epsilon))$$

$$+\lambda \cdot \left[ \int_{R}^{\sigma} J(x) dF(x) - J_{i}(\epsilon) \right]$$

$$+I_{o} \cdot \left[ a\theta q\left(\theta\right) \left(V_{o}\left(\epsilon\right) - J_{o}\left(\epsilon\right)\right) \right]$$

$$(3)$$

For values of  $\epsilon$  in the interval (T; S), the firm expects the job to become vacant again with probability  $a\theta q(\theta)$  (the matching probability for employed workers). It also expects the worker to leave the labour force with probability  $l^7$ . When the job becomes vacant it keeps its level of idiosyncratic productivity  $\epsilon$ , because this defines the job and is not attached to the worker or the worker-job match. The wage negotiated with a worker continuing job search,  $w_o(\epsilon)$ , will be different from the wage negotiated with a worker who stops searching,  $w_n(\epsilon)$  (see section 2.2). Since we assume that  $\epsilon$  is unobserved to the worker until the match actually takes place and that no signalling can take place, all vacant jobs have the same probability of being matched, irrespective of their idiosyncratic productivity  $\epsilon$ . The Bellman equations for vacant jobs are:

$$rV_{i}(\epsilon) = -c + q(\theta) \left(J_{i}(\epsilon) - V_{i}(\epsilon)\right) + \lambda \left[\int_{T}^{\sigma} V(x) dF(x) - V_{i}(\epsilon)\right]$$

where c is the per-period cost of opening a vacancy.

Note that the distribution of idiosyncratic productivities  $\epsilon$  in vacant jobs,  $F_V(.)$ , is different from the distribution of  $\epsilon$  following an idiosyncratic shock, F(.). Reasons for this are detailed below and the derivation of the density

<sup>&</sup>lt;sup>7</sup>Note that in case of retirement or quit to another job, the firm plans to re-advertise the job. So separations and job destruction are different and labour force exits are not a form of exogenous job destruction: most jobs left by a retiree are re-advertised.

functions is given in Burgess and Turon  $(2004)^8$ . Filled jobs with idiosyncratic productivities in the range (T; S) are quit from and re-advertised at a rate  $(l + a\theta q(\theta))$  whereas filled jobs with idiosyncratic productivities in the range  $(S;\sigma)$  are quit from and re-advertised at a rate l; filled jobs with  $\epsilon$  in the range (R;T) are quit from at rate  $(l + a\theta q(\theta))$  but once quit from are destroyed. Vacant jobs are all matched at the same rate,  $q(\theta)$ . The distribution of idiosyncratic productivity  $\epsilon$  will hence be different between vacant jobs  $F_V(.)$  and filled jobs  $F_E(.)$ . Although the job-to-job quit rate and the retirement rate are of similar magnitude on average, the retirement rate l is much lower than the quit rate  $a\theta q(\theta)$  for a given idiosyncratic productivity (by a factor 30 in our base calibration). This is because workers retire from the whole employment stock whereas they quit from only a small fraction of it (6%) in our base calibration). So the distribution of  $\epsilon$  in vacant jobs  $F_V(.)$  is highly concentrated in the interval (T; S) compared to the distribution of filled jobs  $F_E(.)$ . In our base calibration, about half the vacancy stock is in this range against 6% of the employment stock.

#### 2.2 Surplus and wage bargaining

The wage is negotiated at the time of matching and is re-negotiated after idiosyncratic shocks. The firm knows that the worker will search if the idiosyncratic productivity is below S, and this influences the wage setting process. The wage rate is determined by Nash bargaining between worker and firm, as in the Mortensen-Pissarides framework<sup>9</sup>. However, in their framework all new

 $<sup>^{8}</sup>$  The fact that the model implies these three different distributions instead of just one in the standard setting prevents us from having a simple closed form solution to our model.

 $<sup>^{9}</sup>$  There has been a growing literature on alternative models of wage determination over the past decade, particularly models with wage-posting games (see Mortensen and Pissarides

matches are formed at the maximum idiosyncratic productivity, leading to a single wage rate at the match. In our setup, new matches occur over a range of idiosyncratic productivities, namely  $(T; \sigma)$ . The idiosyncratic productivity of a job,  $\epsilon$ , is unknown to the worker at the time of the match, and is only revealed when she starts in the job. Before the match actually occurs, there is no wage offer apart from a knowledge of the renegotiation rule. The worker always finds it worthwhile to take up the job offer: for unemployed searchers because  $E_o(T)$ is greater than U, and for employed job seekers because necessarily the expected value of alternative employment net of search costs is greater than the value of her current employment (otherwise she would not search). It may be that the employed job seeker is unlucky when she finds a new job in that she experiences a wage drop<sup>10</sup>, but her *expected* returns to search were still positive *ex ante*.

From the equations above, we see that the surplus from a match between a vacant job and a job seeker will have a different expression for jobs in which the worker carries on searching and in jobs where the worker stops searching:

$$S_{i}(\epsilon) = J_{i}(\epsilon) - V_{i}(\epsilon) + E_{i}(\epsilon) - U$$
(4)

Because wage negotiation occurs once the worker is in the job, we assume that the worker's outside option is unemployment in both cases. It is never optimal for a firm to attempt to retain a worker with an outside offer by making a counter-offer. The potential outside offer hence never becomes a new outside option for the worker in the wage bargaining process as it would in Postel-Vinay (1999b) for a survey). Here, wage dispersion is obtained with Nash bargaining wage determination.

 $<sup>^{10}</sup>$ Nickell (2002, Table 7, p.21) reports that over 10% of job movers with no intervening spell of unemployment experience a wage drop of over 10%, with data on British men over the period 1982-1996.

and Robin (2002). A worker who has quit her previous job does not have the option to go back to it, hence employed and unemployed job seekers are offered the same wage rates when hired. Pissarides (1994, p.465) and Shimer (2004, p.5) make the same assumption, based on the impossibility of returning to the old employer for a worker who quits a job and on the impossibility to commit to a long-term contract to attract an employed worker for the prospective firm.

The two wage rates  $w_o(\epsilon)$  and  $w_n(\epsilon)$  resulting from the Nash bargaining will satisfy the following conditions:

$$\beta \left( J_{i}\left( \epsilon \right) - V_{i}\left( \epsilon \right) \right) = \left( 1 - \beta \right) \left( E_{i}\left( \epsilon \right) - U \right)$$

where  $\beta$  is the worker's share of the surplus<sup>11</sup>.

In jobs with low idiosyncratic productivity, there are expected benefits to job search for the worker as the expected value of a future job is high enough compared to the value of the current job to more than offset the search costs k. For the firm, employed job search represents a cost as it expects to have to readvertise the vacancy in the near future, and the value of a vacant job is always lower than the value of the filled job for the same productivity. These two facts imply that the wage rate for workers engaged in on-the-job search,  $w_o(\epsilon)$ , is lower than the wage rate they would get if they were not searching,  $w_n(\epsilon)$ , over the range of productivities where workers do decide to search on-the-job, (T; S).

Firms with idiosyncratic productivities in the range (T; S) are aware that

<sup>&</sup>lt;sup>11</sup>Shimer (2004) shows that, in his model of on-the-job search, surplus sharing is not generally equivalent to the Nash solution. In our setting, however, because we assume  $\epsilon$  to be unobserved before an offer is accepted, neither the quit rate nor the expected value of employment in the next job for employed job seekers depend on the level of their current wage. It follows that, for employed job seekers, raising  $w_o$  by  $\Delta w_o$  will increase the worker's side of the surplus by  $[r + l + a\theta q(\theta) + \lambda] \cdot \Delta w_o$  and lower the firm's side of the surplus by the same amount. The Nash solution will hence coincide with the surplus-sharing rule, in our context.

the workers they hire engage in on-the-job search. They are not tempted to stop them from doing so by offering them a higher wage because their search creates an expected benefit, that they enjoy a share of, through the Nash bargaining wage determination. In other words, as the worker expects to find a better job, the value of which more than offsets the search costs, she is better off searching. Some of this benefit from search (but not all) is taken away from her in the wage determination as she gets paid a lower wage than she would have, had she not searched on the job. The firm anticipates having to pay advertising costs when the worker quits, so would be worse off if the search decision did not affect the wage rate. However, because the Nash bargaining leads to a lower wage when the worker searches on the job, the firm is in fact better off if the worker does search as the wage difference more than offsets the anticipated advertising costs<sup>12</sup>. Furthermore, firms cannot afford to retain workers who have an outside offer in hand: as the new job's  $\epsilon$  is unobserved until the worker actually starts in the new job, the minimum wage that the old firm would have to pay to retain the worker is a wage that matches the expected wage  $E_V\{w\}$ that the worker anticipates. In fact, in all our simulations, even the firm with the highest productivity in the range where workers engage in on-the-job search  $(\epsilon = S)$  cannot afford to offer such a wage, i.e. re-opening a vacancy is more profitable than retaining the worker with a wage offer of  $E_V\{w\}$ .

<sup>&</sup>lt;sup>12</sup>Shimer (2004) argues that surplus sharing may be inefficient as, in some cases, the firm is better off raising the wage in order to reduce quitting. This can occur in his model of on-thejob search where the quitting probability decreases continuously as the wage rate increases. In our context, however, the quitting probability is either 0 or  $a\theta q(\theta)$ . The minimum wage that the firm would have to pay to stop the worker from searching is  $w_n(S)$ . However, in the range of productivities (R; S), the firm is better off paying  $w_o(\epsilon)$  and let the worker search on the job. So it will not attempt to discourage the worker to search.

#### 2.3 Equilibrium

Matches between searching workers and vacant jobs occur at a rate determined by the matching function, which we assume to exhibit constant returns to scale. The pool of job searchers comprises all the unemployed job seekers, u, plus the employed workers engaged in on-the-job search, oj, counted in terms of efficiency units:

Number of matches = 
$$\xi . (u + a \cdot oj)^{(1-\alpha)} v^{\alpha}$$
 (5)

where  $\xi$  is the matching efficiency,  $\alpha$  the matching elasticity with respect to vacant jobs and v the stock of vacancies. If we denote  $\theta$  the labour market tightness:

$$\theta = \frac{v}{u + a.oj} \tag{6}$$

we have the following expressions for the workers'  $(\theta q(\theta))$  and vacancies'  $(q(\theta))$ matching probabilities:

$$\theta q\left(\theta\right) = \xi \cdot \theta^{\alpha} \tag{7}$$

$$q(\theta) = \xi \cdot \theta^{\alpha - 1} \tag{8}$$

The labour force is assumed constant and normalised to 1. The equations that determine the model equilibrium are the job creation condition (9), the job destruction condition (10) and the on-the-job search threshold condition (11) as well as the flow equations for filled jobs and vacant jobs over each range of idiosyncratic productivities. As in the standard model, vacant jobs are created until rents are exhausted. What is different here is that all vacancies but the marginal one will make a positive  $profit^{13}$ .

$$V_o(T) = 0 \tag{9}$$

$$S_o(R) = 0 \tag{10}$$

$$S_o(S) = S_n(S) \tag{11}$$

The value of R is less than T because the value of a filled job is positive at T and the function  $J_o(.)$  is increasing. S is the idiosyncratic productivity at which *both* workers and firms are indifferent between the worker continuing or stopping search. The derivation of the model is given in Burgess and Turon (2004).

#### 2.4 Standard Mortensen-Pissarides Model

The standard M-P model is set out in many places, initially in Mortensen and Pissarides (1994), and also in Pissarides (2000). Here we simply highlight the differences between it and the model set out above. First, workers are not allowed to search on the job in their model, so the only case when matches are dissolved is job destruction. Second, in the M-P model, there is an infinite supply of job "ideas" at the top productivity  $\sigma$ . These are turned into vacancies until the exhaustion of rents, i.e. until the value of vacancies is zero. In this setting, vacant jobs are homogeneous and there is no uncertainty about the idiosyncratic value of a job at the time of the match, as this is necessarily  $\sigma$ .

<sup>&</sup>lt;sup>13</sup>In Mortensen and Pissarides (1994), all new jobs were created at the top idiosyncratic productivity - for which the profits from a vacancy is zero. Here, jobs are created over a range of idiosyncratic productivities  $(T;\sigma)$  and the profits from a vacancy are zero at T and positive over the rest of the range. In den Haan, Haefke and Ramey (2001, pp. 8-10), new matches are 'accepted' by worker and firm as long as the relationship-specific productivity is greater than some threshold for which the joint surplus of the match is zero. Blanchard and Diamond (1989, p.9) already suggested that, in the short run, the profits from a vacancy were not necessarily zero.

From then on, it is subject to idiosyncratic shocks that will affect the value of the match and lead to wage renegotiation, as in our setting.

The Bellman equations for the MP model are:

$$rE(\epsilon) = w(\epsilon) + l(U - E(\epsilon)) + \lambda \left[ \int_{R}^{\sigma} E(x)dF(x) + U.F(R) - E(\epsilon) \right] (12)$$

$$rU = b + \theta q (\theta) \cdot (E(\sigma) - U)$$
(13)

$$rJ(\epsilon) = p + \epsilon - w(\epsilon) + l(V - J(\epsilon)) + \lambda \left[ \int_{R}^{b} J(x)dF(x) - J_{n}(\epsilon) \right]$$
(14)

$$rV = -c + q(\theta) \left( J(\sigma) - V \right)$$
(15)

For comparability purposes, we have added exogenous labour force flows to the standard M-P model. This is in order to keep the same calibrated parameters, particularly for the value of the arrival rate of idiosyncratic shocks,  $\lambda$ , while keeping a calibrated unemployment rate that is similar to that simulated by our model. The equilibrium conditions for this model are the following job creation and job destruction conditions:

$$V = 0 \tag{16}$$

$$J(R) = 0 \tag{17}$$

### **3** Results

In this section we first show that Shimer's claim about relative volatility between aggregate productivity and the vacancy to unemployment ratio (V-U hereafter) holds true for UK data. We then show with simulations that, although the standard M-P model cannot mimic the cyclical volatility of the V-U ratio, our modified version does. That is, our model leads to a much higher elasticity of the V-U ratio with respect to aggregate productivity (12 times greater) and hence is able to replicate the actual volatility of this ratio. It would be necessary to assume very high productivity shocks in the M-P framework to match the empirical volatility of the V-U ratio. To illustrate the impact of each of the two main differences between our framework and M-P, we carry out simulations of our model with heterogenous vacancies but without on-the-job search (Model I) and of our model with both heterogeneous vacancies and on-the-job search (Model II) separately.

We take long series on unemployment, vacancies and productivity<sup>14</sup>. Following Shimer (2005), we construct  $log(X/X^T)$  where  $X^T$  is a smoothed series of X using the Hodrick-Prescott filter. The standard deviation and maximum - minimum differences are reported in the first two columns of Table 1. The standard deviation of the V-U ratio is about 28 times higher than that for productivity. For unemployment and for vacancies it is respectively 12 and 19 times higher. Thus the UK also sees much greater volatility of these labour market magnitudes than productivity. We also estimate the empirical elasticity of each variable with respect to productivity, since this is what we compare with the calibrated models. The estimates are reported in column 3 of Table 1. This is computed from a dynamic bivariate regression of  $log(X/X^T)$  on  $log(prod/y/prod/y^T)$ . The regression is set up in error-correction form with 9 lags of  $\Delta log(X/X^T)^{15}$  and of  $\Delta log(prod/y/prod/y^T)$ , and seasonal dumnies.

<sup>&</sup>lt;sup>14</sup>'Unemployment' is all unemployed claimants, measured on a consistent basis through various definition changes; 'vacancies' is all job vacancies notified to Jobcentres; productivity is real output per filled job. All are quarterly series.

 $<sup>^{15}</sup>$  13 lags were used for unemployment. As we look at the volatility of the various variables in terms of deviations of their logarithms from trend and as we allow for dynamics in their time series behaviour, the elasticities in the fourth column do not necessarily add up.

The V-U ratio has a long-run elasticity of 12.4 with respect to productivity.

	Emj	pirical Volatil	ities	Simulated Elasticities			
	Std. dev.	$\max$ - $\min$	Elasticity	M-P model	Model I	Model II	
Productivity	0.0127	0.0733	-				
V/U Ratio	0.3653	1.9889	12.42	1	4.0	12.2	
Unemployment	0.1521	0.7209	-22.05	-0.5	-1.6	-4.3	
Vacancies	0.2434	1.3355	10.98	0.5	1.9	2.6	
Tightness				1	4.0	4.5	
U/J ratio				0	0	-3.5	

Table 1: Empirical and Simulated Features of UK Data

Notes:

1. Series are quarterly and run 1967.1 to 1998.4

2. For each variable X we analyse  $log(X/X^T)$  where  $X^T$  is a smoothed series of X using the Hodrick-Prescott filter with the recommended smoothing parameter value of 1600. The descriptive statistics refer to this variable. Tightness and the U/J ratio are not available empirically, as J is not easily measured.

3. The elasticity is the long-run elasticity of  $(X/X^T)$  with respect to  $prod/y/prod/y^T$ . The elasticity of the V-U ratio with respect to productivity does not equal the difference between the vacancy elasticity and the unemployment elasticity for two reasons. First, the V-U ratio is detrended itself and is not equal to the ratio of detrended V and detrended U. Second, the elasticity is a long-run coefficient from a dynamic regression, not a simple coefficient in a bivariate regression.

4. M-P: Mortensen-Pissarides (1994); Model I: our model without on-the-job search; Model II: our model with on-the-job search.

Turning to the simulations, we report in Burgess and Turon (2004) the details of the calibration, where we set the model parameters to mirror reality in terms of the stocks and flows in the labour market in the UK. We keep the same values of the parameters to simulate the three models. We look at the impact of a change in the aggregate price component p by comparing steadystates for different values of p, embodying once-and-for-all shocks to aggregate activity<sup>16</sup>. The resulting changes in the key variables are given in Table A.1 in the Appendix, and the implied elasticities in the right panel of Table 1<sup>17</sup>.

 $<sup>^{16}</sup>$  This comparative statics exercise is less informative than a dynamic simulation of a stochastic version of the model, as in Pissarides (2000) or Shimer (2005, p.14), but the greater complexity of our model means that this is not feasible.

<sup>&</sup>lt;sup>17</sup>As we calculate the various elasticities as changes in the variables concerned  $(\eta_{X,Y} = \frac{\Delta X}{\Delta Y} \cdot \frac{Y}{X})$  rather than differentiations  $(\eta_{X,Y} = \frac{\partial X}{\partial Y} \cdot \frac{Y}{X})$ , they do not add up:  $\eta_{V/U,p} = \frac{\Delta(V/U)}{(V/U)_0} \cdot \frac{p_0}{\Delta p} = (\eta_{V,p} - \eta_{U,p}) \cdot \frac{U_0}{U_1}$  where  $\eta_{X,Y}$  denotes the elasticity of X with respect to Y.

As already shown by Shimer (2005), the M-P model predicts an elasticity of the V-U ratio with respect to productivity<sup>18</sup> much smaller than the one observed in the data (1 in our simulation). However, simulations of our Model I predict this elasticity to be 4.0. When we allow for employed job search, i.e. in Model II, the predicted elasticity of the V-U ratio with respect to aggregate productivity becomes 12.2 and matches its empirical counterpart well.

So both features that distinguish our setup from the standard MP framework, namely our vacancy creation condition and the presence of on-the-job search, increase the model's ability to generate volatility in the V-U ratio. Looking at unemployment and vacancies separately, going from the M-P model to our model I increases their volatilities by a factor 3 and 4 respectively, while going from our model I to our model II, i.e. allowing for employed job search, increases these volatilities by a factor 3 and 1.4 respectively. The elasticities of unemployment and vacancies with respect to aggregate productivity predicted by Model II, i.e. -4.3 and 2.6 respectively, are still lower than those observed in our UK data, i.e. -22 and 11 respectively. However, the volatility of the V-U ratio in Model II mirrors real facts very well.

Another way of decomposing the V-U ratio is to look at the behaviour of the V-J ratio (where J is the total number of job seekers), which represents the labour market tightness, and of the U-J ratio, which represents the fraction of job seekers who are unemployed. This latter ratio always equals 1 in both the MP model and our Model I as only unemployed workers are allowed to search. Consequently, the V-J and V-U ratio coincide in these two models. Simulated elasticities of the V-J and U-J ratios in the three different models are displayed

In our simulations, the last term,  $\frac{U_0}{U_1}$ , is substantially away from 1, particularly in Model II. <sup>18</sup>Shimer (2005) refers more specifically to the elasticity of the V-U ratio with respect to p-b, i.e. a measure of the replacement ratio. Here, we report results with respect to p as they are easier to compare with their empirical counterpart. Simulated elasticities with respect to p-b for the three models however show a similar pattern to that presented here, although less marked (with a value of 7.4 in Model II).

in the last two lines of Table 1. Introducing the different vacancy creation condition, (going from the M-P model to Model I), causes a large increase in the elasticity of the labour market tightness with respect to productivity (from 1 to 4), while further allowing for employed job search has a much smaller impact, increasing this elasticity from 4 to 4.5. Clearly, the elasticity of the U-J ratio is unaffected by the different vacancy creation condition, as in this model  $J \equiv U$ . However, allowing for employed job search in Model II changes the elasticity of the U-J ratio with respect to productivity to -3.5, contributing substantially to the volatility of the V-U ratio. So allowing for worker churning strongly increases the sensitivity of tightness to the cycle; allowing further for on-the-job search increases that and makes the V-U ratio considerably more volatile than tightness.

Shimer (2005) argues that the failure of the M-P model to mimic the empirical volatility of the V-U ratio is rooted in the lack of wage rigidity embodied in this model, which itself comes from the assumption of Nash bargaining wage determination. While he convincingly makes the point that more wage rigidity would indeed generate greater volatility in the V-U ratio, our simulations show that such volatility can also be generated in a framework where wages are determined by Nash bargaining and are as volatile as in the M-P model. Indeed, both our models in fact predict less wage rigidity than the M-P model. Having said that, within the context of the model described here, greater on-the-job search (induced by a lower cost, k) implies greater wage rigidity. The intuition for this is as follows. As noted above, on-the-job search yields a prospective benefit to the worker that is shared with the firm through a lower wage. In the boom increased employed job search extends this effect and reduces wages, partially offsetting the straightforward positive effect of the boom on wages.

## 4 Conclusion

There are good reasons for the popularity of the Mortensen-Pissarides model of the labour market. However, as Shimer (2005) stresses, it fails to replicate evidence on the cyclical sensitivity of the vacancy-unemployment ratio for the US. We show above that the same applies to the UK. We propose an extension of their framework in which vacancies are heterogeneous and employed job search is allowed. This therefore incorporates two well-documented features of labour markets - worker churning and job-to-job quits. We show in this comment that adopting these two assumptions is one way of resolving the puzzle highlighted by Shimer. Furthermore, this can be done within the standard assumptions on wage setting, and therefore this route offers an alternative to Shimer's and Hall's focus on wage determination. Our simulations show that each of our two assumptions causes a substantial rise in the cyclical volatility of the V-U ratio.

This approach also suggests that we need to interpret the vacancy-unemployment ratio differently. With employed job search this ratio does not measure the tightness of the labour market. It measures the outcome of the joint processes determining labour market tightness and the composition of job seekers. Including this in the modelling approach is one promising way of reconciling theory and evidence.

## Appendix

		Model I		Model II		MP model	
		Base	Boom	Base	Boom	Base	Boom
Market tightness	θ	0.36	0.50	0.47	0.68	1.83	2.01
Unemp. job seekers	u	0.147	0.124	0.088	0.050	0.149	0.141
Un. outflow rate	$\theta q(\theta)$	0.36	0.43	0.41	0.50	0.81	0.85
Emp. job seekers	oj	0	0	0.063	0.070	0	0
Vacancies	v	0.052	0.062	0.092	0.116	0.272	0.285
V/U ratio		0.36	0.50	1.05	2.32	1.83	2.01
U/J ratio		1	1	0.45	0.29	1	1

Table A.1: Cyclical volatility of key variables in the three models

Note: A higher value of p by 10% simulates a boom.

### References

- Blanchard, O. and Diamond, P. (1989), 'The Beveridge curve', Brookings Papers on Economic Activity, pp. 1-60.
- [2] Burgess, S. (1993), 'A model of competition between employed and unemployed job searchers: An application to the unemployment outflow rate in Britain', Economic Journal, pp. 1190-1204.
- [3] Burgess, S. and Turon, H. (2004), 'Worker Flows, Job Flows and Unemployment in a Matching Model', IZA Discussion Paper # 753.
- [4] Den Haan, W., Haefke, C. and Ramey, G. (2001), 'Shocks and institutions in a job matching model', NBER Working Paper # 8463.
- [5] Eriksson, S. and Gottfries, N. (2002), 'Ranking of job applicants, on-thejob search and persistent unemployment' Working Paper Series, Uppsala University, Department of Economics.
- [6] Hall, R. (2003a) 'Wage determination and employment fluctuations', mimeo.
- [7] Hall, R. (2003b) 'A model of aggregate fluctuations', mimeo.
- [8] Jovanovic (1979), 'Job matching and the theory of turnover', Journal of Political Economy, vol. 87, pp. 972-990.
- [9] Mortensen, D. and Pissarides, C. (1994), 'Job creation and job destruction in the theory of unemployment', Review of Economic Studies, vol. 61, pp. 397-416.
- [10] Mortensen, D. and Pissarides, C. (1999a), 'Job reallocation, employment fluctuations and unemployment' in Handbook of Macroeconomics by J. Taylor and M. Woodford (eds), vol. 1, pp. 1137-1170.
- [11] Mortensen, D. and Pissarides, C. (1999b), 'New developments in models of search in the labor market' in Handbook of Labor Economics by O. Ashenfelter and D. Card (eds), vol. 3, pp. 2567-2627.
- [12] Nickell, S. (2002), 'A picture of job insecurity facing British men', Economic Journal, vol. 112, pp. 1-27.
- [13] Pissarides, C. (1994), 'Search unemployment with on-the-job search', Review of Economic Studies, vol. 61, pp. 457-475.
- [14] Pissarides, C. (2000), 'Equilibrium unemployment theory', Blackwell, Oxford.
- [15] Postel-Vinay, F. and Robin, J.M. (2002), 'Equilibrium wage dispersion with worker and employer heterogeneity', Econometrica, vol. 70, pp. 2295-2350.
- [16] Shimer, R. (2005), 'The cyclical behavior of equilibrium unemployment and vacancies: evidence and theory', American Economic Review, forthcoming
- [17] Shimer, R. (2003), 'Dynamics in a model with on-the-job search', mimeo.
- [18] Shimer, R. (2004), 'On-the-job search, bargaining, and wage dispersion', mimeo.