A Comparison of the Translog and Almost Ideal Demand Models

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Abstract. A version of the Translog demand system is compared with the Almost Ideal demand model within a time series setting, where variables are nonstationary, by testing both models for the theoretical demand propositions of "homogeneity, symmetry and negativity" and by comparing out of sample forecasting performance. Demographic age and income distributional effects are included in both models.

Keywords: Demand Equations, Age Demographics, Nonstationarity.

JEL Classification: C1, C3, D1.

1. Introduction

The most popular forms for demand functions, in empirical time series research, are the Almost Ideal Demand Model, AIDM, of Deaton and Muellbauer [5, 1980] and the Translog Model of Jorgenson, Lau and Stoker [10, 1982]. Both these systems display Diewert [6, 1974] flexibility, i.e., they do not impose unlikely constraints on demand elasticities. Recently, Lewbel and Ng [11, 2004], analyse the Translog model in a time series setting where some of the variables are non-stationary. They refer to their model as the Non-stationary Translog system, NTLOG, and we use the same term.

This paper attempts to choose between the two models on empirical grounds by testing both models, firstly, for the central theoretical propositions of demand theory such as homogeneity and symmetry, and secondly, by comparing forecasting performance outside sample values. An important extension of the models is the inclusion of indices for the effects of demographic age and income distributional changes, the indices being constructed from cross section data using the Family Expenditure Surveys. Lewbel and Ng [11, 2004] and Attfield [2, 2004] have shown the importance of including demographic variables in demand systems. The former in the NTLOG model and the latter in the AIDM. We also demonstrate that the models with the demographic and income distribution indices are preferable, on the grounds of satisfying demand propositions, to the standard AIDM without the indices.

Attfield [2, 2004], in related research, argues that the AIDM, allowing for demographic and income distribution effects, satisfies all the propositions of demand theory when estimated and tested in a time series setting where the variables are non-stationary. In the next section, section 2, we sketch the main results of Attfield [2, 2004] for the AIDM. Section 3 defines the NTLOG model, outlines the estimation

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procedure and presents estimation and testing results. Section 4 compares estimation results while dynamic out of sample forecasts from the models are compared in section 5. For testing and comparing the models aggregate data on the conventional commodity groups of Food, Alcohol & Tobacco, Clothing & Footwear and Fuel & Housing are used but section 6 applies the preferred model - the AIDM - to a set of time series expenditure data on Health, Communications, Recreation and Education. Section 7 concludes.

2. The Almost Ideal Demand Model

The standard AIDM is given by:

$$w_{hjt} = \alpha_{oj} + \sum_i \gamma_{ij} \ln p_{it} + \ln(x_{ht}/P_t^*)\beta_j \tag{1}$$

where w_{hjt} is the budget share for good j at time t for household h, x_{ht} is per-household total income, p_{it} is the price of commodity i at time t, and $\ln P_t^*$ is Stone's price index[1] which linearises the theoretical AIDM model, Deaton and Muellbauer [5, 1980, p.316], and the coefficient β_j is constant across all households. Suppose all households at a particular time are grouped into those with heads the same age and that there are $\mathbf{G}$ such age groups denoted by $\mathcal{G}_{gt}$, $g = 1, ..., \mathbf{G}$. Let $\xi_{gt} = n_{gt}/N_t$ be the proportion of households in age group $\mathcal{G}_{gt}$, n_{gt}, in the total number of households, N_t.

Assume that the constant α_{oj} subsumes a fixed effect for each age group in the population, which can be thought of as a taste parameter in the utility function, so that the intercept in (1) is given by:

$$\alpha_{oj} = \theta_{oj} + \theta_{gj}.$$

Then, budget shares of good j for household h are:

$$w_{hjt} = \frac{x_{hjt}}{x_{ht}}$$

where $x_{hjt} = p_{jt} q_{hjt}$ is expenditure on good j by household h, and $x_{ht} = \sum_j x_{hjt}$ is total expenditure on all goods by household h.

Aggregate budget shares for all households in group g are then:

$$w_{gjt} = \frac{x_{gjt}}{x_{gt}} = \frac{\sum_{h \in \mathcal{G}_{gt}} x_{hjt}}{\sum_{h \in \mathcal{G}_{gt}} x_{ht}} = \frac{\sum_{h \in \mathcal{G}_{gt}} x_{ht} w_{hjt}}{\sum_{h \in \mathcal{G}_{gt}} x_{ht}} = \theta_{oj} + \theta_{gj} + \sum_i \gamma_{ij} \ln p_{it} + z_{gt}\beta_j, \tag{2}$$

where z_{gt} is log real income per capita for age group g. Aggregation within an age group is along the same lines as the overall aggregation in Deaton and Muellbauer

[1] Stone's price index is defined as $\ln P_t^* = \sum_j w_{jt} \ln(p_{jt})$, where w_{jt} is the budget share for the jth commodity at time t aggregated across all households.

[5, 1980, p.314]. That is, we can assume that there is a component, say $\ln k_{gt}$, which reconciles the aggregation over levels with the aggregation over logarithms such that:

$$\ln k_{gt} = -\sum_h \left(\frac{x_{ht}}{x_{gt}}\right) \ln \left(\frac{x_{ht}}{x_{gt}}\right).$$

Deaton and Muellbauer [5, 1980, p.315] refer to $\ln k_{gt}$ as the log of Theil's [17, 1972] entropy measure of equality. Testing $\ln k_{gt}$ for a unit root, using pseudo-panels constructed from the FES and a pooled ADF test suggested by Ng & Perron [13, 1997], Attfield [2, p.6, 2004] found the null of a unit root could be rejected for all lags up to 3 in the ADF test with statistics 37.15, 19.59, 8.91, 4.01 for lags 0, 1, 2 and 3. With 4 lags the test statistic is 1.33. The critical 5% value under the standard normal is 1.65 so it is safe to assume that $\ln k_{gt}$ is stationary. If income were equal within the group, $\ln k_{gt}$ would be a constant but would not be the same constant across groups. Attfield [2, p.6, 2004] tested for equality of group means of $\ln k_{gt}$ - over time - using a Wald test. The result, 5783 with 65 degrees of freedom, rejects the null of equality at any conventional significance level.

Since the $\ln k_{gt}$ are stationary we assume that each is equal to a constant (its mean) plus a random error. The constant is absorbed into θ_{gj} and the random component into the equation error. This means that the estimates of each θ_{gj} contain a fixed age effect plus a measure of the inequality of the income distribution for that age cohort.

Now, aggregating over all G age groups gives:

$$w_{jt} = \frac{x_{jt}}{x_t} = \frac{\sum_g x_{gjt}}{\sum_g x_{gt}} \equiv \frac{\sum_g x_{gt} w_{gjt}}{\sum_g x_{gt}} = \theta_{oj} + \frac{\sum_g x_{gt}\theta_{gj}}{\sum_g x_{gt}} + \sum_i \gamma_{ij} \ln p_{it} + z_t \beta_j \qquad (3)$$

where z_t is the log of total real income per capita. The aggregation procedure is similar to that outlined above but now the discrepancy index is given by:

$$\ln k_t = -\sum_g \left(\frac{x_{gt}}{x_t}\right) \ln \left(\frac{x_{gt}}{x_t}\right).$$

We assume $\ln k_t$ is stationary[2] with mean 4.1104 and estimated standard error 0.0022. As in the case of age groups, we assume that $\ln k_t$ is a constant (its mean) plus a random error so that the constant is absorbed into the equation intercept and the error into the equation disturbance.

The second term in the final expression in (3) can be written:

$$\frac{\sum_g x_{gt}\theta_{gj}}{\sum_g x_{gt}} = \frac{\sum_g \frac{x_{gt}}{n_{gt}} \theta_{gj} \frac{n_{gt}}{N_t}}{\frac{\sum_g x_{gt}}{N_t}} = \sum_g \frac{\overline{x}_{gt}}{\overline{x}_t} \theta_{gj} \xi_{gt} \qquad (4)$$

[2] The pooled ADF test stistic is -1.96 with a 5% critical value of -1.98 so it is on the borderline of being non-stationary.

where $\overline{x}_{gt} = x_{gt}/n_{gt}$ is average total expenditure per household in age group g, $\overline{x}_t = x_t/N_t$ is average total expenditure across all households and $\xi_{gt} = n_{gt}/N_t$. The ratio of group means to overall means, $\overline{x}_{gt}/\ \overline{x}_t$, turns out to be stationary as ratios often are, e.g., the "great" ratios consumption/income and investment/output[3].

In its present form (4) is difficult to construct for researchers working with aggregate time series as although the population proportion variable is readily available the parameter θ_{gj} has to be estimated, and the variable x_{gt} obtained, from cross section sources. Since the ratio of means, $\overline{x}_{gt}/\ \overline{x}_t$, is stationary, we assume:

$$\frac{\overline{x}_{gt}}{\overline{x}_t} = \delta_g + v_{gt} \tag{5}$$

where v_{gt} is a random error. The parameter δ_g for each age group is assumed constant over time and can be directly estimated by least squares from the cross section data to give $\widehat{\delta}_g$ which is, of course, the sample mean of the ratio for the gth group. The null hypothesis that the mean of the ratio $\overline{x}_{gt}/\overline{x}_t$ is the same across all g, i.e., $\delta_g = c$, for all g, is comprehensively rejected by a Wald test with statistic 69540 with 65 degrees of freedom. Substituting (5) into (4) yields:

$$\frac{\widetilde{\sum_g x_{gt}\theta_{gj}}}{\sum_g x_{gt}} = \sum_g \widehat{\delta}_g \theta_{gj} \xi_{gt}. \tag{6}$$

Substituting the estimate in (6) into (3) results in:

$$w_{jt} = \theta_{oj} + \sum_g \widehat{\delta}_g \theta_{gj} \xi_{gt} + \sum_i \gamma_{ij} \ln p_{it} + z_t \beta_j \tag{7}$$

which contains stochastic trends associated with the demographic, income and price variables. The omission of the demographic variables could explain the "no cointegration" result of many demand studies. Lewbel and Ng [11, 2004], for example, show that for data for the USA, budget share demand systems which include z_t and logged prices do not cointegrate, i.e., have a non-stationary equation error. Attfield [2, 2004] discusses estimation of the parameters in equation (6) and we use these estimates directly to form a set of demographic indices for each commodity group, $\widetilde{I}_{jt}$, of the form:

$$\widetilde{I}_{jt} = \sum_g \widehat{\delta}_g \widetilde{\theta}_{gj} \xi_{gt}. \tag{8}$$

The construction of an index for $\ln k_{gt}$, the income distributional part of θ_{gj}, from cross section data is a suggestion of Deaton and Muellbauer [5, 1980, pp.314-315].

[3] The pooled test statistic is greater than 3.4 for all lags less than and including 4, so the null of a unit root in $\overline{x}_{gt}/\ \overline{x}_t$ is rejected.

2.1. Estimating the Almost Ideal Demand Model. We use the demographic indices calculated above with quarterly time series data from the ONS data bank[4] for the period 1971Q1 to 2001Q3 firstly, to test for the cointegrating rank of an AIDM model, and secondly, to estimate the parameters of the demand model. Annual series on age proportions, for each age group between 19 and 84 inclusive, were obtained from the Government Actuarial service but are only available from 1971 for each age group in the population[5].

The starting point is the structural demand model of equations (7) and (8) for each commodity group, j:

$$w_{jt} = \theta_{oj} + \psi_j \widetilde{I}_{jt} + \sum_i \gamma_{ji} lnp_{it} + z_t \beta_j + u_{jt} \tag{9}$$

where u_{jt} is a random error and we have aggregated over all G age groups and include a parameter, ψ_j, on the estimated demographic index $\widetilde{I}_{jt}$, firstly, to allow for any differences in magnitude between the cross section and time series data, and secondly, to allow a fixed linear relationship between the proportion of households in each age group, in the FES samples, and the proportion of each age group in the population in the ONS series. The budget shares and prices are ordered $j = 1, ..., 5$ for Food, Alcohol & Tobacco, Clothing & Footwear, Fuel & Housing and Other Goods.

In the time series data, we tested all variables for unit roots using the procedures by Ng and Perron [13, 1997] and Perron and Ng [16, 1996] which optimally choose the lag length for the ADF test. Their DF-GLS test for a unit root did not reject unit roots for any of the variables, including the demographic indices[6]. In the estimation procedures and system tests which follow, one equation has to be dropped because on the null hypothesis of a demand system, the “adding up” restriction leads to a singularity if all equations are used. Dropping the equation for Other Goods means that the demographic index for this commodity group does not appear in the system we are estimating and testing. Sufficient conditions for adding up to be satisfied are:

$$\sum_j^J \theta_{oj} = 1; \ \sum_j^J \gamma_{ji} = 0, \forall i; \ \sum_j^J \beta_j = 0$$

and:

$$\sum_j^J \psi_j \widetilde{I}_{jt} = 0. \tag{10}$$

[4] Quarterly, seasonally unadjusted, series on real and nominal expenditures for all categories of goods were obtained and aggregated into the five main groups Food, Alcohol & Tobacco, Clothing & Footwear, Fuel & Housing and Other Goods. Commodity price indices and total expenditures (income) were derived from these data sets. Prior to analysis, seasonal components were removed using seasonal dummies.

[5] Prior to 1971 population statistics are available for 5-yearly age groupings only. The annual population series were converted to quarterly using the logarithmic interpolation procedure.

[6] The 5% critical value for the test is -1.98 and the test statistics for all the variables in the time series data set were greater than -1.58.

Substituting the formulae for the indices in equation (8) into (10) yields:

$$\sum_g \left(\sum_j^J \psi_j \widetilde{\theta}_{gj} \right) \widehat{\delta}_g \xi_{gt} = 0$$

so that adding up is satisfied if $\left(\sum_j^J \psi_j \widetilde{\theta}_{gj} \right) = 0, \forall g$. We can therefore write ψ_J - the coefficient on the Other Goods demographic index which isn't estimated - as:

$$\psi_J^{(g)} = -\frac{\sum_j^{J-1} \psi_j \widetilde{\theta}_{gj}}{\widetilde{\theta}_{gJ}}, \; g = 1, ..., G \qquad (11)$$

where the superscript (g) denotes that there will be G "solutions" to the equations. For adding up the solutions, $\psi_J^{(g)}$, must all be equal. With the estimates $\widetilde{\theta}_{gj}$ and estimates of ψ_j, $j = 1, .., J-1$, this equality hypothesis can be tested. We report the result of the test below.

We tested for cointegration in the demand system using Johansen's [9, 1995] likelihood ratio procedures. That is, we test for the rank of the matrix π in the vector error correction model, (VECM):

$$\triangle x_t = \theta_o + \theta_1 \triangle x_{t-1} + ... + \theta_s \triangle x_{t-s} + \pi x_{t-1} + \zeta_t,$$

where x_t contains the set of 14 variables, i.e., 4 budget shares, 5 prices, 4 demographic indices and log real per capita income. To find the number of lags in first differences, s, we estimated an unrestricted equation in levels with lags 1 to 4. The BIC, Hannan-Quinn and Akaike information tests (obtained with PcGive [15, 2001]) gave results for lag lengths of 1, 4, and 4 respectively in levels (0, 3 and 3 in first differences). We therefore carried out the tests and subsequent estimation in a VECM with 3 lags in first differences. The trace test statistic for the null of 8 cointegrating vectors is 136.21 with a 5% critical value of 94.15 so we can reject 8 cointegrating vectors in favour of 9 or more. The λ-*max* statistic for the null of 8 is 39.84 with 5% critical value of 39.37 so that, with this statistic, we can also reject 8 in favour of 9 cointegrating vectors. The trace test statistic for the null of 9 cointegrating vectors is 96.36 with a 5% critical value of 68.52 so we can reject 9 cointegrating vectors in favour of 10 or more but the λ-*max* statistic for the null of 9 is 30.39 with 5% critical value of 33.46 so that we cannot reject 9 in favour of 10[7]. Taking evidence from both test statistics together we accept the null of 9 cointegrating vectors.

With $p = 14$ in the model and $r = 9$ cointegrating equations it follows that the rank of the $p \times p$ matrix π is equal to r so that we can write:

$$\pi = \gamma \alpha'$$

where γ is $p \times r$ and α is $p \times r$ and is the matrix of cointegrating coefficients.

[7] Critical values were obtained from Osterwald-Lenum [14, 1992]

To identify and estimate the cointegrating equations we need some structure on the relations. Since there are $r = 9$ cointegrating relations we can always write:

$$\pi = \beta\alpha' = \beta G^{-1} G\alpha'$$

where G is any $r \times r$ nonsingular matrix. Therefore, to identify the coefficients of the demand equations we need at least 9 restrictions on each equation. Consider the following structural definition of the cointegrating vectors, α' :

$$\begin{array}{cccccccccccccc}
w_{1t} & w_{2t} & w_{3t} & w_{4t} & \ln p_{1t} & \ln p_{2t} & \ln p_{3t} & \ln p_{4t} & \widetilde{I}_{1t} & \widetilde{I}_{2t} & \widetilde{I}_{3t} & \ln p_{5t} & \widetilde{I}_{4t} & z_t \\
-1 & 0 & 0 & 0 & \gamma_{11} & \gamma_{21} & \gamma_{31} & \gamma_{41} & \psi_1 & 0 & 0 & \gamma_{51} & 0 & \beta_1 \\
0 & -1 & 0 & 0 & \gamma_{12} & \gamma_{22} & \gamma_{32} & \gamma_{42} & 0 & \psi_2 & 0 & \gamma_{52} & 0 & \beta_2 \\
0 & 0 & -1 & 0 & \gamma_{13} & \gamma_{23} & \gamma_{33} & \gamma_{43} & 0 & 0 & \psi_3 & \gamma_{53} & 0 & \beta_3 \\
0 & 0 & 0 & -1 & \gamma_{14} & \gamma_{24} & \gamma_{34} & \gamma_{44} & 0 & 0 & 0 & \gamma_{54} & \psi_4 & \beta_4 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & \alpha_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \alpha_{12} & \alpha_{22} & \alpha_{32} & \alpha_{42} & \alpha_{52} \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{43} & \alpha_{53} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} & \alpha_{54} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \alpha_{15} & \alpha_{25} & \alpha_{35} & \alpha_{45} & \alpha_{55}
\end{array} . \tag{12}$$

A necessary condition for identification of all the coefficients in the β and α matrices is that there are at least $r^2 = 81$ restrictions on the structural α matrix which contains 126 elements. Without any loss of generality we have normalised the first 4 equations as the budget share equations in (12). 9 restrictions have been placed on each of the remaining 5 equations so that the variables $w_{1t}, w_{2t}, w_{3t}, w_{4t}$,$\ln p_{1t}$,$\ln p_{2t}$,$\ln p_{3t}$,$\ln p_{4t}$ and $\widetilde{I}_{1t}$ are thought of (arbitrarily) as being driven by $\widetilde{I}_{2t}, \widetilde{I}_{3t}, \ln p_{5t}, \widetilde{I}_{4t}$ and z_t. As it stands there are only 7 restrictions on each of the first four rows in (12), the budget share equations, which are the normalisations of the coefficients on the budget shares and the exclusion restrictions on all the indices but the 'own' demographic index. These restrictions sum to 73 in all. Homogeneity, $\sum_{i=1}^{5} \gamma_{ij} = 0, j = 1, ..., 4$ adds another one restriction to each demand equation and the symmetry relations:

$$\begin{aligned}
\gamma_{21} &= \gamma_{12} \\
\gamma_{31} &= \gamma_{13} \\
\gamma_{41} &= \gamma_{14} \\
\gamma_{32} &= \gamma_{23} \\
\gamma_{42} &= \gamma_{24} \\
\gamma_{43} &= \gamma_{34}
\end{aligned}$$

add a further 6 restrictions giving 83 restrictions in all so that the necessary condition is satisfied. A necessary and sufficient condition for identification is that the Jacobian matrix for the relations $\pi' = \alpha\beta'$ has full column rank (cf., for example, Doornik [7, 1995]). That is:

$$\frac{\partial \text{vec}(\pi')}{\partial \text{vec}(\phi')'} = [\beta \otimes I_p] \frac{\partial \text{vec}(\alpha)}{\partial \text{vec}(\phi')'} = J'$$

where ϕ is the 43×1 vector of unknown coefficients in (12) after imposing homogeneity and symmetry. We used the rank procedure in GAUSS [8, 2002], with random values for the β matrix and ϕ vector, to verify that J' has full column rank.

Of course, with the exclusion restrictions in (12) plus homogeneity and symmetry, the elements of α and β are overidentified, in the sense that there are two overidentifying restrictions. An interesting aspect of this analysis is that the "unrestricted model" can be written as containing the homogeneity and 4 of the symmetry restrictions - the "restricted model" then restricts the remaining 2 sets of symmetric coefficients. Put another way, in the unrestricted model we are estimating a demand system with homogeneity and some symmetry already imposed by normalisation and which is perfectly consistent with the data in the sense that it will generate an identical likelihood to a completely unrestricted model with rank(π) = 9. It is the large number of cointegrating equations relative to the number of stochastic trends which enables sufficient normalisations to identify a complete demand system (less two symmetry conditions).

PcGive [15, 2001], which uses the switching algorithm, failed to converge when estimating the model. However, since all the coefficients are just identified, without the additional 2 symmetry restrictions, the matrix β is unrestricted and the restrictions on the matrix α in (12) are all linear, we can use the following iterative procedure to obtain maximum likelihood solutions for the structural coefficients.. Let:

$$\text{vec}(\alpha) = H_o + H_1\phi$$

with H_o and H_1 known matrices. Taking the rank r estimate of π from the ML procedure, say $\widetilde{\pi}$, and solving the following set of equations iteratively starting with random values for the coefficients in β gives us ML estimates of the restricted α.

$$\begin{aligned}
\phi^{(s)} &= \left(W^{(s)\prime}W^{(s)}\right)^{-1} W^{(s)\prime}\left(\text{vec}(\widetilde{\pi}') - \left(\beta^{(s)} \otimes I_p\right) H_o\right) \\
\text{vec}(\alpha^{(s)}) &= H_o + H_1\phi^{(s)} \\
\text{vec}(\beta^{(s+1)}) &= I_p \otimes \left(\alpha^{(s)\prime}\alpha^{(s)}\right)^{-1} \alpha^{(s)\prime}\text{vec}(\widetilde{\pi}')
\end{aligned}$$

where $W = (\beta \otimes I_p) H_1$ and (s) denotes the s^{th} iteration. The process was assumed to have converged when the differences between estimates were of the order $|0.00001|$ in successive iterations. The procedure produces α and β matrices such that α contains all the normalisations and the product $\beta\alpha'$ is identically equal to the rank r matrix $\widetilde{\pi}$.

Maximum likelihood estimates of the coefficients of the demand equations with normalisations for homogeneity and four symmetry conditions imposed are given in Table 1. A Wald test of the remaining symmetry relations (abitrarily chosen as $\gamma_{32} = \gamma_{23}$ and $\gamma_{24} = \gamma_{42}$) produced a statistic of 0.741 with 2 degrees of freedom so the null hypothesis of overall symmetry cannot be rejected.

Table 1 *Here*

The indices in the Food and Alcohol & Tobacco equations are not significantly different from zero in the cointegrating equations. This does not mean that these indices can be dropped from the analysis as they do have an impact in the dynamic part of the VECM. Because of the large number of coefficients we do not give them here but report that lagged changes in the alcohol and tobacco index do have a significant impact in all the other demand equations and lagged changes in the food index have a significant impact on demand for clothing and footwear and on fuel and housing.

Conventional demand elasticities for the AIDM model, calculated at the point of the sample mean of the variables, are given in Table 2.

Table 2 *Here*

The formula for the price elasticities was derived on the assumption of the true price index given by:

$$\ln P_t = const + \sum_k \alpha_k \ln p_{kt} + \frac{1}{2}\sum_j \sum_k \gamma_{kj} \ln p_{kt} \ln p_{jt}$$

with the α_k and γ_{kj} as given in (9).

The income elasticity of demand for commodity i at time t is given by:

$$\eta_{it} = \frac{\beta_i}{w_{it}} + 1$$

and own price elasticity[8] of demand by:

$$\eta_{iit} = \frac{\gamma_{ii}}{w_{it}} - \beta_i - 1 + \frac{\beta_i^2 z_t + \beta_i \psi_i \widetilde{I}_{it}}{w_{it}}.$$

All the own price elasticities for the full ML system have the correct negative sign. Food and Alcohol & Tobacco are close to being unit price elastic while Clothing & Footwear and Fuel & Housing are price inelastic. Income elasticities classify all goods as necessities ($0 < \eta_i < 1$).

Table 2 also gives point estimates of eigenvalues which imply that the substitution matrix is at least negative semi-definite. Finally, a Wald test for equality of the $\psi_J^{(g)}$ in (11) results in a test statistic of 22.8 with 64 degrees of freedom so that equality cannot be rejected and the set of demand equations satisfy the adding-up restriction.

3. The Translog Model

The starting point for the Translog model (see for example Lewbel & Ng [11, 2004]) is:

$$w_{hjt} = \frac{\alpha_{oj} + \sum_i \gamma_{ij} \ln p_{it} - c_j \ln(x_{ht})}{1 + \sum_i c_i \ln p_{it}} \tag{13}$$

[8] The term $\frac{\beta_i^2 z_t + \beta_i \psi_i \widetilde{I}_{it}}{w_{it}}$ is neglible in practice and has been omitted from the calculations as it makes little or no difference to the results quoted. The same applies in the symmetry condition in the substitution matrix below.

where, as in the previous section, w_{hjt} is the budget share for good j at time t for household h, x_{ht} is per-household total (nominal) income and p_{it} is the price of commodity i at time t. The coefficient c_j is assumed constant across all households. As in the previous section, assume that the constant α_{oj} subsumes a fixed effect for each age group in the population so that in (13):

$$\alpha_{oj} = \theta_{oj} + \theta_{gj}.$$

Aggregating over households in the gth age group and then across all age groups then proceeds in the same way as in the previous section and we obtain the structural demand model for each commodity group, j:

$$w_{jt} = \frac{\theta_{oj} + \psi_j \widetilde{I}_{jt} + \sum_i \gamma_{ij} \ln p_{it} - c_j \ln(x_t)}{1 + \sum_i c_i \ln p_{it}} + u_{jt} \tag{14}$$

where x_t is aggregate nominal income, u_{jt} is a random error and $\widetilde{I}_{jt}$ the estimated demographic index discussed above. Conditions on the c's for adding up are $\sum_i c_i = 0$, and for homogeneity $c_j = \sum_i \gamma_{ij}$.

The main estimation problem in (14) is that the equations are non-linear. Lewbel & Ng [11, 2004] overcome the problem by multiplying through by the denominator taking the term $w_{jt} \sum_i c_i \ln p_{it}$ to the right hand side and applying an instrumental variable technique - assuming resulting equation errors are white noise. In this paper we transform (14) to:

$$w^*_{jt}(\mathbf{c}) = \theta_{oj} + \psi_j \widetilde{I}_{jt} + \sum_i \gamma_{ij} \ln p_{it} + v_{jt},$$

where $w^*_{jt}(\mathbf{c}) = w_{jt}\left(1 + \sum_i c_i \ln p_{it}\right) + c_j \ln(x_t)$ so that, for given values of the c coefficients, $w^*_{jt}(c_j)$ is linear. A two step estimation procedure was then adopted. First, in an unrestricted VAR containing all the variables in levels with the transformation imposed[9], we searched over a grid for the c coefficients maximising, at each point, the concentrated log likelihood with the c_js are constrained to satisfy $\sum_j c_j = 0$. The values of the vector $\widetilde{\mathbf{c}}' = (0.0557\ 0.0239\ 0.0569\ 0.1227\ {-0.2593})$ which form the supremum for all the maximised values was then chosen for the second step in the estimation which consists of forming a VECM containing the variables $x_t = \left(w^*_{1t}(\widetilde{\mathbf{c}}), ..., w^*_{4t}(\widetilde{\mathbf{c}}), \widetilde{I}_{jt}, lnp_{1t}, ..., lnp_{5t}\right)$ and proceeding to estimate and test the model within the VECM framework.

[9] The alternative would be to work with the original untransformed w_{jt}s and use the Jacobian transformation in the likelihood. The method used in the text was applied to simulated data producing excellent results. The resulting estimates are consistent but not efficient.

In the time series data, we tested the composite variables $w_{jt}^{*}(\widetilde{\mathbf{c}})$ for a unit root using the procedures by Ng and Perron [13, 1997] and Perron and Ng [16, 1996] which optimally choose the lag length for the ADF test. Their DF-GLS test for a unit root does not reject unit roots for any of the $w_{jt}^{*}(\widetilde{\mathbf{c}})$ variables[10]. As in the AIDM the equation for Other Goods is dropped to avoid a singularity and means that the demographic index for this commodity group does not appear in the system we are estimating and testing. Sufficient conditions for adding up to be satisfied are then:

$$\sum_{j}^{J}\theta_{oj}=1;\ \sum_{j}^{J}\gamma_{ji}=0,\forall i;\ \sum_{j}^{J}c_{j}=0$$

and:

$$\sum_{j}^{J}\psi_{j}\widetilde{I}_{jt}=0. \tag{15}$$

As for the AIDM the G solutions to the equations:

$$\psi_{J}^{(g)}=-\frac{\sum_{j}^{J-1}\psi_{j}\widetilde{\theta}_{gj}}{\widetilde{\theta}_{gJ}} \tag{16}$$

must all be equal. With the estimates $\widetilde{\theta}_{gj}$ and estimates of ψ_{j}, $j=1,..,J-1$, for the NTLOG model, this equality hypothesis can be tested. We report the result of the test below.

As for the AIDM, we tested for cointegration in the demand system using Johansen's [9, 1995] likelihood ratio procedures. That is, we test for the rank of the matrix π in the vector error correction formulation:

$$\triangle x_{t}=\theta_{o}+\theta_{1}\triangle x_{t-1}+...+\theta_{s}\triangle x_{t-s}+\pi x_{t-1}+\zeta_{t},$$

where now $x_{t}=\left(w_{1t}^{*}(\widetilde{\mathbf{c}}),...,w_{4t}^{*}(\widetilde{\mathbf{c}}),\widetilde{I}_{jt},lnp_{1t},...,lnp_{5t}\right)$ contains the set of 13 variables, i.e., 4 budget shares, 5 prices and 4 demographic indices - nominal income being contained in the composite share variables. BIC, Hannan-Quinn and Akaike information tests (obtained with PcGive [15, 2001]) gave results for lag lengths of 1, 4, and 4 respectively in levels (0, 3 and 3 in first differences) so tests and subsequent estimation was carried out using 3 lags in first differences in the VECM. For the NTLOG model, the trace test statistic for the null of 7 cointegrating vectors is 138.72 with a 5% critical value of 94.15 so we can reject 7 cointegrating vectors in favour of 8 or more. The λ-*max* statistic for the null of 7 is 40.23 which is close to the 5% critical value of 39.37 so that, with this statistic, we can also reject 7 in favour of 8 cointegrating vectors. The omission of one non-stationary variable from the analysis, compared with the analysis for the AIDM, that is income is included in the composite share terms, appears to result in one less cointegrating relation, so we analyse the NTLOG with 8 cointegrating vectors.

[10] The 5% critical value for the test is -1.98 and the test statistics for all the $w_{jt}^{*}(\widetilde{\mathbf{c}})$, j = 1,...,4, variables in the time series data set were greater than -0.42.

With $p = 13$ in the model and $r = 8$ cointegrating equations it follows that the rank of the $p \times p$ matrix π is equal to r so that we can write:

$$\pi = \gamma\alpha'$$

where γ is $p \times r$ and α is $p \times r$ and is the matrix of cointegrating coefficients. To identify the coefficients of the demand equations we need at least 8 restrictions on each equation. Consider the following structural definition of the cointegrating vectors, α' :

$$\begin{array}{ccccccccccccc}
w_{1t}^* & w_{2t}^* & w_{3t}^* & w_{4t}^* & \ln p_{1t} & \ln p_{2t} & \ln p_{3t} & \ln p_{4t} & \widetilde{I}_{1t} & \widetilde{I}_{2t} & \widetilde{I}_{3t} & \ln p_{5t} & \widetilde{I}_{4t} \\
-1 & 0 & 0 & 0 & \gamma_{11} & \gamma_{21} & \gamma_{31} & \gamma_{41} & \psi_1 & 0 & 0 & \gamma_{51} & 0 \\
0 & -1 & 0 & 0 & \gamma_{12} & \gamma_{22} & \gamma_{32} & \gamma_{42} & 0 & \psi_2 & 0 & \gamma_{52} & 0 \\
0 & 0 & -1 & 0 & \gamma_{13} & \gamma_{23} & \gamma_{33} & \gamma_{43} & 0 & 0 & \psi_3 & \gamma_{53} & 0 \\
0 & 0 & 0 & -1 & \gamma_{14} & \gamma_{24} & \gamma_{34} & \gamma_{44} & 0 & 0 & 0 & \gamma_{54} & \psi_4 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \alpha_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \alpha_{12} & \alpha_{22} & \alpha_{32} & \alpha_{42} & \alpha_{52} \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{43} & \alpha_{53} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} & \alpha_{54}
\end{array}. \quad (17)$$

A necessary condition for identification of all the coefficients in the β and α matrices is that there are at least $r^2 = 64$ restrictions on the structural α matrix which contains 104 elements. Without any loss of generality we have normalised the first 4 equations as the budget share equations in (17). 8 restrictions have been placed on each of the remaining 4 equations so that the variables $w_{1t}^*, w_{2t}^*, w_{3t}^*, w_{4t}^*$,$\ln p_{1t}$,$\ln p_{2t}$,$\ln p_{3t}$ and $\ln p_{4t}$ are thought of (arbitrarily) as being driven by $\widetilde{I}_{1t}, \widetilde{I}_{2t}, \widetilde{I}_{3t}, \ln p_{5t}$, and $\widetilde{I}_{4t}$. As it stands there are 7 restrictions on each of the first four rows in (17), the budget share equations, which are the normalisations of the coefficients on the budget shares and the exclusion restrictions on all the indices but the 'own' demographic index. With the 32 restrictions on the remaining 4 equations the restrictions then sum to 60 in all. Homogeneity, $\sum_{i=1}^{5} \gamma_{ij} = c_j$, $j = 1, ..., 4$ adds another one restriction to each demand equation giving 64 so that the model is just identified with homogeneity included. The symmetry relations:

$$\begin{aligned}
\gamma_{21} &= \gamma_{12} \\
\gamma_{31} &= \gamma_{13} \\
\gamma_{41} &= \gamma_{14} \\
\gamma_{32} &= \gamma_{23} \\
\gamma_{42} &= \gamma_{24} \\
\gamma_{43} &= \gamma_{34}
\end{aligned}$$

add a further 6 restrictions. Of course, with the exclusion restrictions in (17) plus homogeneity and symmetry, the elements of α and β are overidentified, in the sense that there are 6 overidentifying restrictions.

We obtained estimates in the same way as detailed in the estimation of the AIDM coefficients and they are given in Table 3 for the just identified case - only homogeneity imposed. A Wald test of the 6 symmetry relations produced a statistic of 254.25 with 6 degrees of freedom so that symmetry is firmly rejected for the NTLOG model.

Table 3 *Here*

All the coefficients on the demographic indices are significantly different from zero so they are as important in the NTLOG as in the AIDM.

Conventional demand elasticities for the NTLOG model, calculated at the point of the sample mean of the variables, are given in Table 4.

Table 4 *Here*

The income elasticity of demand for commodity j at time t is given by:

$$\eta_{it} = 1 - \frac{c_i}{w_{it}\left(1 + \sum_j c_j \ln p_{jt}\right)}$$

and price elasticity of demand by:

$$\eta_{iit} = \frac{\frac{\gamma_{ii}}{w_{it}} - c_i}{\left(1 + \sum_j c_j \ln p_{jt}\right)} - 1.$$

All the own price elasticities for the full ML system for NTLOG have the correct negative sign. Demands for all commodity groups are price inelastic for the NTLOG, although Clothing & Footwear is close to being unit elastic. Income elasticities classify all goods as necessities ($0 < \eta_i < 1$).

Table 4 also gives point estimates of eigenvalues which imply that the substitution matrix is at least negative semi-definite. Finally, a Wald test for equality of the $\psi_J^{(g)}$ in (16) results in a test statistic of 22.35 with 64 degrees of freedom so that equality cannot be rejected and the set of demand equations satisfy the adding-up restriction.

3.1. Testing for Structural Breaks. We tested for structural breaks in the sample period for both models using the procedure developed by Bai, Lumsdaine & Stock [3, 1998] for multivariate time series. Their method assumes the system with given cointegrating vectors, estimates the corresponding VECM and computes two test statistics, Sup-W and lExp-w, for a shift in the mean of the VECM. For the Translog, the Sup-W statistic, with three lags, is 59.92 (73.34) and the lExp-w statistic is 25.55 (32.52). 5% critical values are given in parenthesis for the null hypothesis of no structural break. For the AIDM the Sup-W statistic, also with three lags, is 62.02 and the lExp-w statistic is 26.99. None of the test statistics therefore rejects the null hypothesis of no structural break in either of the models so the whole analysis was conducted without allowing for structural breaks in the sample period.

4. Comparison of Estimates of Demand Models

Before turning to the forecasting performance of the AIDM and NTLOG models we summarise the tests carried out so far. For the AIDM homogeneity, symmetry and negativity are not rejected while for the NTLOG homogeneity and negativity are not rejected but symmetry is rejected by a very large margin. With the Stone approximation for the price index the AIDM is linear and straightforward to estimate. By contrast, the NTLOG is non-linear and the transformation applied above requires a good deal of preprocessing for the first step of the two step procedure. Moreover, the two step procedure has the disadvantage that test statistics are conditioned on the estimated values of the c_j coefficients. For both models the demographic indices are highly significant - for the NTLOG in the cointegrating relations as well as the dynamic model - and estimated price and income elasticities are similar in both magnitude and sign. In the next section we compare the AIDM model without the demographic indices.

4.1. The Standard AIDM Model Without Demographic Variables. Without the demographic indices, estimation of the AIDM is a straightforward application of time series analysis for which we use PcGive [15, 2001] with the same data set. The VECM consists of the set of 10 variables, i.e., 4 budget shares, 5 prices and log real per capita. Testing for cointegration in the demand system using Johansen's [9, 1995] likelihood ratio procedures results in a model which contains just four cointegrating vectors. For this model, not surprisingly, the number of lags for the VECM was also determined as 4 in levels (3 in first differences). The trace test statistic does not reject the null of 4 cointegrating vectors at either the 1% or 5% level in PcGive whereas 5 or more is rejected at 1%. The λ-max test[11] is weaker with a value of 42.82 for the null of 3 cointegrating vectors which has a 5% critical value of 45.28. We need to have at least 4 cointegrating vectors to specify the AIDM correctly and so we accepted the null of 4 cointegrating vectors bearing in mind that with the omission of the significant demographic variables the standard model would be misspecified. With four cointegrating equations there are four normalisations required for each equation. The unit coefficient on one budget share and zero coefficients on the other three in all four equations use up all the normalisations so that homogeneity places one testable restriction on each demand equation while symmetry imposes a further six testable restrictions. PcGive gives a likelihood ratio (LR) test statistic of 12.67 for the null of homogeneity, the statistic is distributed as chi-square with 4 degrees of freedom. The p-value is 0.013 so that the hypothesis is rejected at the 5% but not the 1% level. This result is consistent with other papers which find some evidence for homogeneity in the AIDM when testing in a full time series context, e.g., Attfield ([1, 1997]), Ng ([12, 1995]). Extending the test to include the null of symmetry however results in a LR test statistic of 34.12 which is distributed as chi-square with 10 degrees of freedom. The p-value for the test is 0.0002 so that the joint null of homogeneity and symmetry is strongly rejected. Moreover, some price and income elasticities computed at the point of sample means make little economic sense, e.g., the price elasticity for Food is positive so that the substitution matrix is not nega-

[11] The λ-max is not available in PcGive so was computed from GAUSS [8, 2002] routines using the COINT [4, 1994] module.

tive semi-definite while the income elasticity for Clothing and Footwear is negative implying that these goods are inferior. Overall the AIDM without the demographic indices has to be rejected.

5. Comparison of Forecasting Performance

In this section we compare the forecasting performance of the two preferred models: (i) the AIDM with demographic indices and (ii) the NTLOG with demographic indices. For this exercise we estimated the models using all time periods up to and including 1997Q4. We then forecast the 15 observations from 1998Q1 to 2001Q3 using the VECM with the appropriate number of cointegrating vectors for the competing models, i.e., 9 for AIDM and 8 for NTLOG. Homogeneity and symmetry were not imposed on the models for forecasting. For the AIDM, as we have seen, the restricted model, with the imposition of homogeneity and symmetry, is not significantly different from the unrestricted model. Symmetry is rejected for NTLOG so imposing symmetry is likely to produce poorer forecasts. The forecasts used 3 lags of first differences and are dynamic in the sense that the forecasts for 1998Q1 use all the data up to 1997Q4. Forecasts for 1998Q2 use, as inputs, the forecasts for 1998Q1 and so on. Figure 1 plots the actual change in budget shares (demeaned and seasonally adjusted) with the forecasts from each of the models. For NTLOG the forecasts of the $w^*_{jt}(\widetilde{\mathbf{c}})$ were transformed back to forecasts of budget shares using:

$$\widehat{w}_{jt} = \frac{\widehat{w}^*_{jt}(\widetilde{\mathbf{c}})}{\left(1 + \sum_i \widetilde{c}_i \ln p_{it}\right)} - \widetilde{c}_j ln(x_t)$$

where the symbol $\widehat{}$ represents a forecast. Comparing forecasts by the root mean square error (RMSE) criterion, the AIDM outperforms the NTLOG for Food and for Fuel and Housing while the NTLOG outperforms the AIDM for Alcohol & Tobacco and for Clothing & Footwear. The latter is relatively poorly forecast by both models and there is some evidence of residual seasonality in the forecasts, particularly those for Alcohol & Tobacco. Towards the end of the out of sample period the AIDM forecasts are superior for all the commodities except Alcohol & Tobacco where the forecasts are almost identical. On balance the forecasts from AIDM outperform those from the NTLOG model. Figure 2 rolls forward the forecasts from the AIDM until the end of 2005 predicting that the change in the food share will continue to decline while the change in the share of clothing & footwear and of fuel & housing will be increasing while the change in the share of alcohol & tobacco will stay pretty much the same. The 95% confidence intervals for the forecasts were obtained from PcGive.

Figure 3 compares the forecasts from the AIDM with and without demographics. Forecasts from the standard AIDM, without demographic indices, and 95% confidence intervals for the forecasts are plotted together with forecasts from the AIDM with demographics. The forecasts are rolled forward to 2005 and show that by 2004 forecasts from the model with demographics are outside the confidence intervals for the standard model without demographics. Omitting the demographic indices, which track the changing structure of the population, could therefore produce misleading forecasts.

6. Demand for Health, Communications, Recreation and Education

In this section we apply the preferred AID model to time series data on expenditures on the services health, communications, recreation and education. The series are from the ONS databank for the same period 1971Q1 to 2001Q3. It is not possible to analyse these variables jointly with the four groups food, alcohol & tobacco, clothing & footwear and fuel & housing in the previous sections because of the very large dimension of the resulting VECM. Instead we analyse health, communications, recreation and education jointly confining all other goods and services to the Other Goods category. Of course, we don't have demographic indices for these new variables but we do have an index for all Other Goods from the previous analyses in the above sections where health, communications, recreation and education were included in Other Goods. In the absence of individual indices, we therefore take the Other Goods demographic index and include it in each of the share equations for health, communications, recreation and education. The share equations are then:

$$w_{jt} = \theta_{oj} + \psi_j \widetilde{I}_t + \sum_i \gamma_{ji} lnp_{it} + z_t \beta_j + u_{jt}$$

where shares, prices and real income are defined and constructed in the same way as in previous sections but now represent health, communications, recreation and education. Note that $\widetilde{I}_t$ has no j subscript so that it is the same demographic index variable for all the service budget shares. We analysed the model in the way described in previous sections. First, we tested all variables for stationarity using the procedures of Ng and Perron [13, 1997] and Perron and Ng [16, 1996] which optimally choose the lag length for the ADF test. Their DF-GLS test did not reject unit roots for any of the variables. The 5% critical is -1.98 and all the test statistics were greater than -0.82. Next we tested for the rank of the cointegrating space using the Johansen MLE method and found we couldn't reject 6 cointegrating vectors against 7 with the max-λ statistic so carried out the estimation with $r = 6$ cointegrating vectors[12]. There are 11 variables in the model so it is puzzling why there are 5 stochastic trends in this model, the same number as in the AIDM with 14 variables in section 2.1. There are 3 I(1) variables omitted from this model, 3 demographic indices associated with each of the variables. A combination of the omitted trends transmitting through prices and income and the approximation of using a composite demographic index in place of the individual trends probably accounts for the number of trends.

With 6 cointegrating equations there are 36 restrictions required. The 4 normalisations on the four share equations and the 6 normalisations on the remaining two equations gives 28 restrictions. Homogeneity restrictions, 4, and 4 of the 6 symmetry conditions gives us a just identified model and the two remaining symmetry conditions can be tested with a Wald test as in section 2.1.

Table 5 *Here*

[12] The max-λ statistic is 33.39 against a 5% critical value of 33.46. The trace statistic is 90.81 against a 5% critical value of 68.52 rejecting 6 against 7 or more cointegrating vectors. With 7 vectors the demand equations wouldn't be identified without further restrictions in addition to the homogeneity and symmetry conditions.

Table 5 gives the ML estimates for the model. Adding up is automatically satisfied in this model because the same index is common to all equations so the coefficient on the omitted Other Goods equation can be equated to the sum of the coefficients on the index variable. The two additional symmetry conditions are rejected by a Wald test statistic of 14.59 which has a pval of 0.001.

Table 6 *Here*

Table 6 gives own price and demand elasticities for the four services. Price elasticities have the correct sign, Health and Communication being close to unit elastic while Recreation and Education are price elastic - although the own price coefficients are poorly determined. Income elasticities classify Communications and Education as necessities, although the Communications elasticity is close to zero. Expenditure on Health services is a luxury but, given that the income coefficient is poorly determined, is closer to unit income elasticity. Recreation is very much a luxury.

Dynamic forecasts for the shares of the four services are given in Figure 4, rolled forward to 2013. There is evidence of residual seasonality in the series but recreation and education shares look relatively flat with a slight decline in education and slight increase in recreation over the forecast period. The change in the share of expenditure on health is forecast to increase steadily while communications, after peaking in 2006, are predicted to decline to the same rate of change as around 1997.

7. Conclusion

In this paper we have estimated and compared the almost ideal demand model and the non-stationary translog model using recent quarterly data series for the UK. In both models we include demographic variables in the form of the proportion of individuals in each of the age groups from 19 to 84 inclusive in the population. Indices are formed, using cross section data from the Family Expenditure Surveys, which combine the demographic age effect on each commodity group with a measure of income distribution among the age groups and the indices are incorporated into the demand models. The demographic indices are found to be significant in both models. For the AIDM homogeneity, symmetry and negativity are satisfied in the sense that the restrictions imposed by these properties of theoretical demand equations are not rejected against an unrestricted model - with the correct number of cointegrating equations. The NTLOG model is found to satisfy homogeneity and negativity but symmetry is rejected. Estimates of price and income elasticities are similar and have sensible values for both models. The "correct" number of cointegrating equations is found using maximum likelihood tests resulting in 9 for the AIDM and 8 for NTLOG. It is the resulting number of normalisations on each cointegrating equation, 9 for AIDM and 8 for NTLOG, which enable a number of the demand propositions to be imposed without restricting the models. Using multivariate techniques, both models are tested for structural breaks within the sample period but the null of "no structural break" cannot be rejected for either model.

On balance the AIDM with demographic indices is the preferred model as it is more straightforward to estimate than the NTLOG, which contains non-linearities, satisfies the propositions of demand theory and provides marginally superior out of sample forecasts. The demographic indices are important as without them the standard AIDM does not satisfy either symmetry or negativity and some estimates of price and income elasticities are bizarre.

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Table 1. ML Estimates of Full AID System

Dependent Variable	Explanatory Variables										Diagnostics	
	$\ln p_1$	$\ln p_2$	$\ln p_3$	$\ln p_4$	$\tilde{I}_1$	$\tilde{I}_2$	$\tilde{I}_3$	$\ln p_5$	$\tilde{I}_4$	z	R^2	Box-Ljung($\sqrt{T} = 10$ lags)
w_1	-0.0131 (0.0093)	0.0015 (0.0029)	0.0014 (0.0075)	-0.0177 (0.0051)	-0.0002 (0.0280)	-	-	0.0280 (0.0082)	-	-0.0135 (0.0107)	0.81	11.67 (*pval* = 0.31)
w_2	0.0015 (0.0029)	0.0007 (0.0021)	0.0005 (0.0026)	0.0046 (0.0023)	-	0.0270 (0.0378)	-	-0.0074 (0.0111)	-	-0.0232 (0.0030)	0.83	3.80 (*pval* = 0.96)
w_3	0.0014 (0.0075)	0.0024 (0.0070)	0.0003 (0.0088)	0.0047 (0.0057)	-	-	-0.6247 (0.0815)	-0.0087 (0.0117)	-	-0.0465 (0.0095)	0.72	8.36 (*pval* = 0.59)
w_4	-0.0177 (0.0051)	0.0095 (0.0055)	0.0047 (0.0057)	-0.0091 (0.0054)	-	-	-	0.0126 (0.0086)	2.495 (0.4133)	-0.0304 (0.0264)	0.83	7.02 (*pval* = 0.72)

Commodity groups are: 1. Food; 2. Alcohol & Tobacco; 3. Clothing & Footwear; 4. Fuel & Housing; 5. Other Goods

$\tilde{I}_j$ Demographic index for commodity group j

Estimated asymptotic standard errors in parenthesis

Table 2. AIDM Model. Estimated Demand Elasticities

	ML System Estimates			
			Calculated at Sample Means	
Commodity Group	Own price coefficient	Income coefficient	Own Price Elasticity	Income Elasticity
Food	-0.0131 (0.0093)	-0.0135 (0.0107)	-1.0841	0.9082
Alcohol & Tobacco	0.0007 (0.0021)	-0.0232 (0.0030)	-1.0423	0.5570
Clothing & Footwear	0.0003 (0.0088)	-0.0465 (0.0095)	-0.7309	0.3482
Fuel & Housing	-0.0091 (0.0054)	-0.0304 (0.0264)	-0.5988	0.8186

Estimated asymptotic standard errors in parenthesis

Mean eigenvalues of "Substitution Matrix": -0.164, -0.131, -0.066, -0.039
Eigenvalues of substitution matrix at mean data points: -0.155, -0.141, -0.066, -0.040.

Table 3. ML Estimates of Full NTLOG System

Dependent Variable	Explanatory Variables										Diagnostics	
	$\ln p_1$	$\ln p_2$	$\ln p_3$	$\ln p_4$	$\tilde{I}_1$	$\tilde{I}_2$	$\tilde{I}_3$	$\ln p_5$	$\tilde{I}_4$	$\tilde{c}_j$	R^2	Box-Ljung($\sqrt{T}=10$ lags)
w_1	0.0297	-0.0054	-0.0054	-0.0360	0.1154	-	-	0.0728	-	0.0557	0.76	5.79 (*pval* = 0.83)
	(0.0083)	(0.0050)	(0.0056)	(0.0062)	(0.0111)			(0.0146)		-		
w_2	0.0078	0.0073	-0.0067	-0.0636	-	-0.6026	-	0.0791	-	0.0239	0.86	6.50 (*pval* = 0.77)
	(0.0040)	(0.0025)	(0.0038)	(0.0026)		(0.0728)		(0.0092)		-		
w_3	-0.0248	-0.0042	0.0102	0.0907	-	-	-0.8273	-0.0150	-	0.0569	0.79	6.22 (*pval* = 0.80)
	(0.0106)	(0.0066)	(0.0077)	(0.0076)			(0.0806)	(0.0115)		-		
w_4	-0.0181	-0.0083	0.0094	0.0848	-	-	-	0.0549	-0.5627	0.1227	0.85	6.06 (*pval* = 0.81)
	(0.0073)	(0.0046)	(0.0052)	(0.0054)				(0.0109)	(0.0675)	-		

Commodity groups are: 1. Food; 2. Alcohol & Tobacco; 3. Clothing & Footwear; 4. Fuel & Housing; 5. Other Goods

$\tilde{I}_j$ Demographic index for commodity group j

Estimated asymptotic standard errors in parenthesis

Table 4. NTLOG Model. Estimated Demand Elasticities.

	ML System Estimates			
			Calculated at Sample Means	
Commodity Group	Own price coefficient g_{jj}	Income coefficient c_j	Own Price Elasticity	Income Elasticity
Food	0.0297 (0.0083)	0.0557 -	-0.8540	0.6204
Alcohol & Tobacco	0.0073 (0.0025)	0.0239 -	-0.8840	0.5390
Clothing & Footwear	0.0102 (0.0077)	0.0569 -	-0.9143	0.1988
Fuel & Housing	0.0848 (0.0054)	0.1227 -	-0.6142	0.2633

Estimated asymptotic standard errors in parenthesis

Mean eigenvalues of "Substitution Matrix": -0.138, -0.088, -0.056, -0.033.
Eigenvalues of substitution matrix at mean data points: -0.133, -0.092, -0.057, -0.035.

Table 5. ML Estimates of AIDM System. Health, Communications, Recreation, Education Share Equations

Dependent Variable	Explanatory Variables							Diagnostics	
	$\ln p_1$	$\ln p_2$	$\ln p_3$	$\ln p_4$	$\tilde{I}$	$\ln p_5$	z	R^2	Box-Ljung($\sqrt{T} = 10$ lags)
w_1	0.00083 (0.0012)	0.00002 (0.0010)	-0.00181 (0.0028)	-0.00337 (0.0014)	0.01731 (0.0100)	0.00433 (0.0039)	0.00220 (0.0024)	0.61	21.21 (*pval* = 0.02)
w_2	0.00002 (0.0010)	-0.00037 (0.0027)	-0.02518 (0.0074)	0.00373 (0.0051)	0.02275 (0.0223)	0.02181 (0.0072)	-0.01778 (0.0051)	0.60	11.47 (*pval* = 0.32)
w_3	-0.00181 (0.0028)	-0.00057 (0.0066)	0.00922 (0.0170)	-0.03565 (0.0035)	-0.59610 (0.0682)	0.02881 (0.0207)	0.13920 (0.0163)	0.78	11.74 (*pval* = 0.30)
w_4	-0.00337 (0.0014)	-0.00016 (0.0013)	-0.03565 (0.0035)	-0.00547 (0.0023)	-0.09751 (0.0109)	0.04466 (0.0079)	-0.00288 (0.0026)	0.61	3.67 (*pval* = 0.96)

Commodity groups are: 1. Health; 2. Communications; 3. Recreation; 4. Education; 5. Other Goods

$\tilde{I}$ Common demographic index for all share equations

Estimated asymptotic standard errors in parenthesis

Table 6. AIDM Model. Estimated Demand Elasticities. Health, Communications, Recreation, Education Share Equations

	ML System Estimates			
			Calculated at Sample Means	
Service Group	Own price coefficient	Income coefficient	Own Price Elasticity	Income Elasticity
Health	0.00083 (0.0012)	0.00220 (0.0024)	-0.9395	1.1823
Communications	-0.00037 (0.0027)	-0.01778 (0.0051)	-1.0939	0.0309
Recreation	0.0092 (0.0170)	0.1392 (0.0163)	-1.4483	2.3306
Education	-0.0055 (0.0023)	-0.0028 (0.0026)	-1.5904	0.7082

Estimated asymptotic standard errors in parenthesis

Mean eigenvalues of “Substitution Matrix”: -0.099, -0.018, -0.011, -0.00006
Eigenvalues of substitution matrix at mean data points: -0.099, -0.018, -0.012, -0.0002.

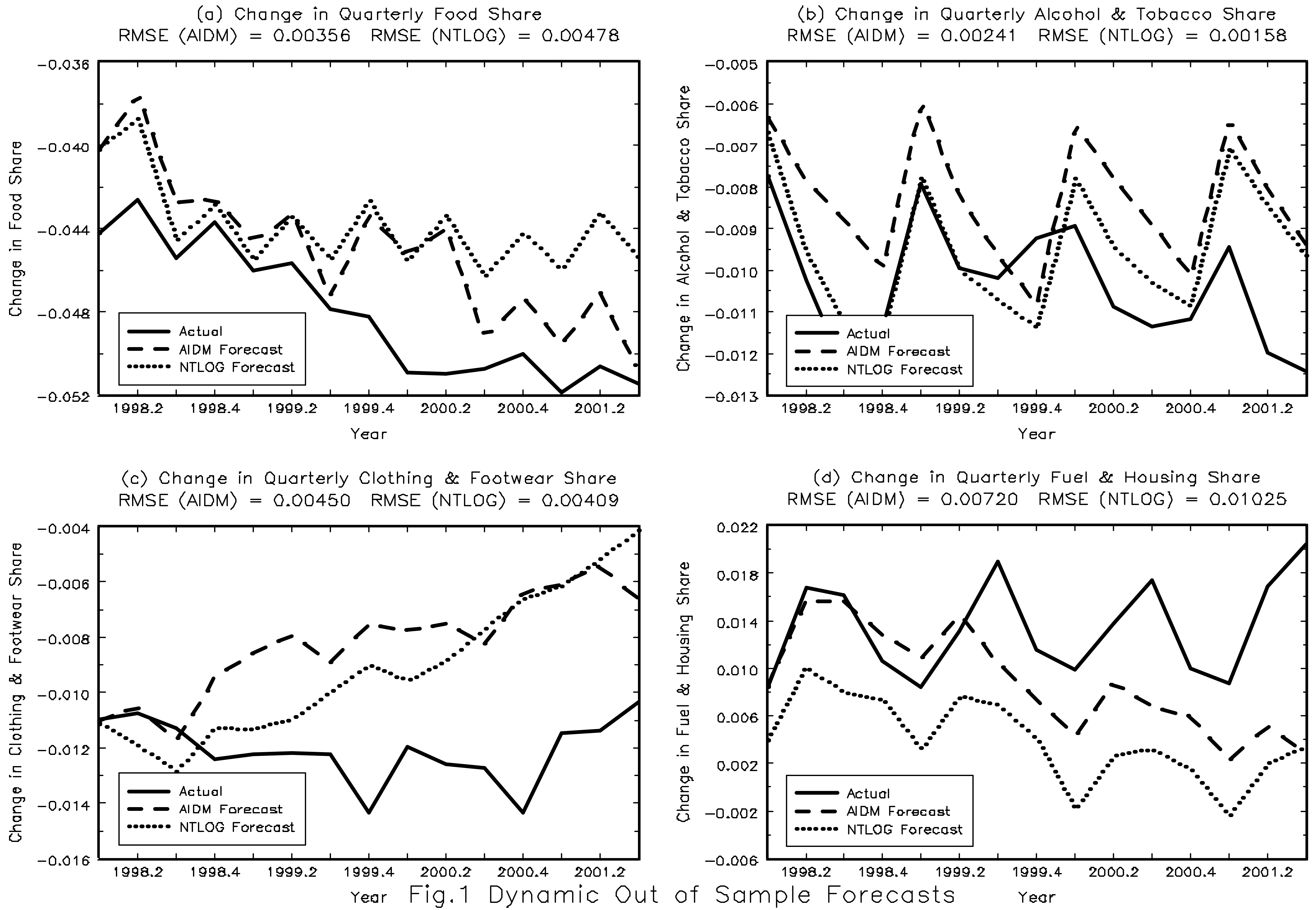

Fig.1 Dynamic Out of Sample Forecasts

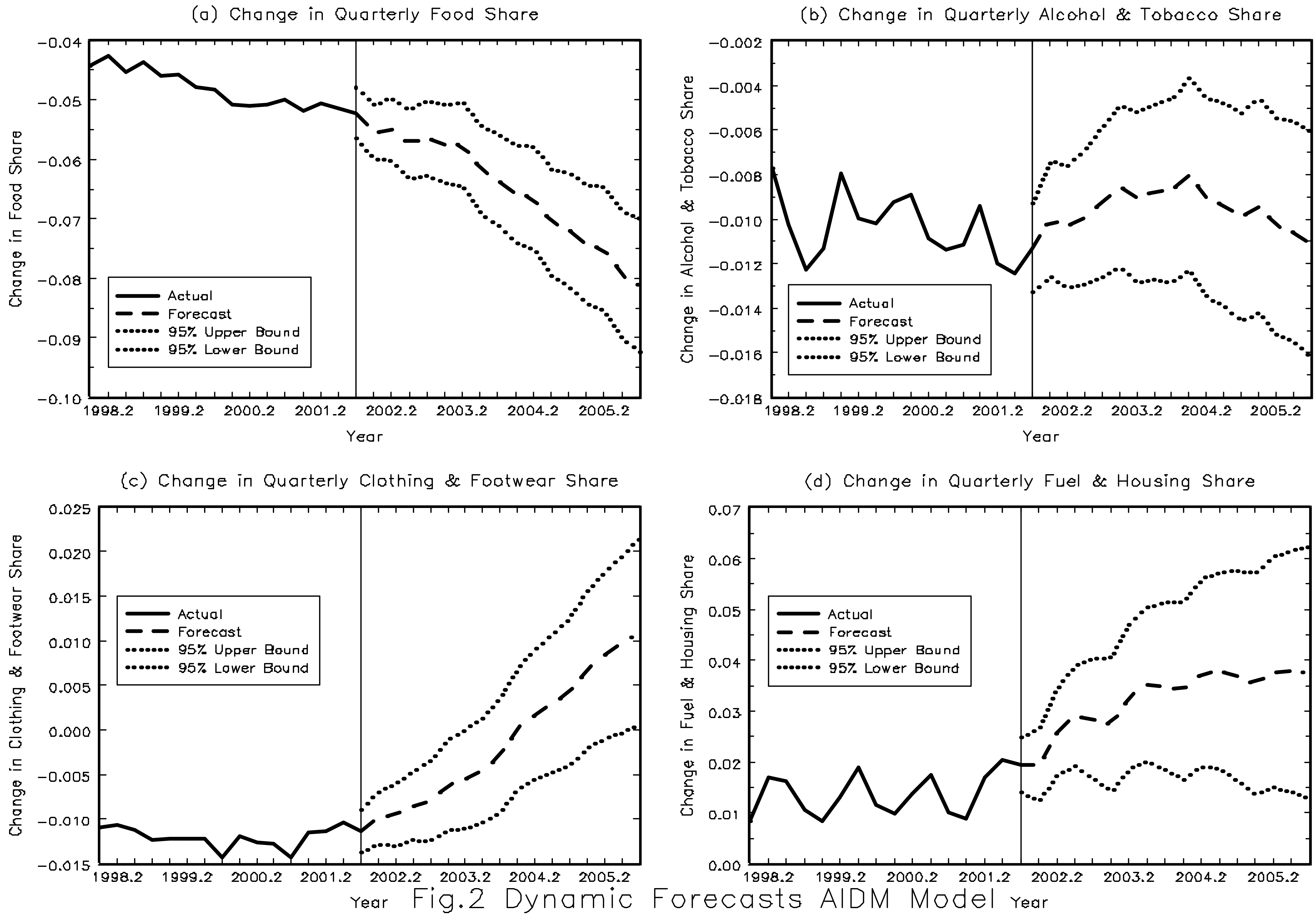

Fig.2 Dynamic Forecasts AIDM Model

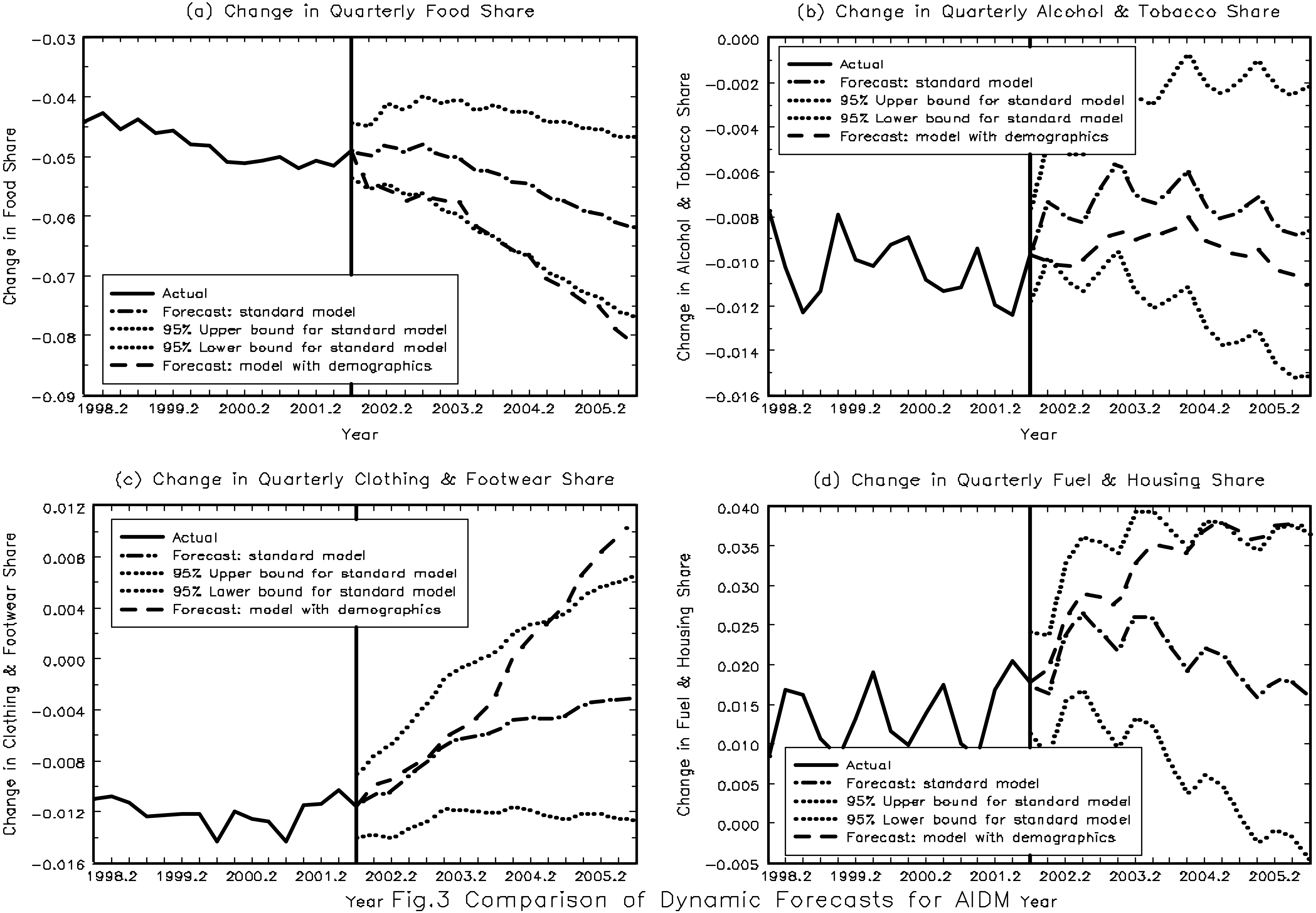

Fig.3 Comparison of Dynamic Forecasts for AIDM

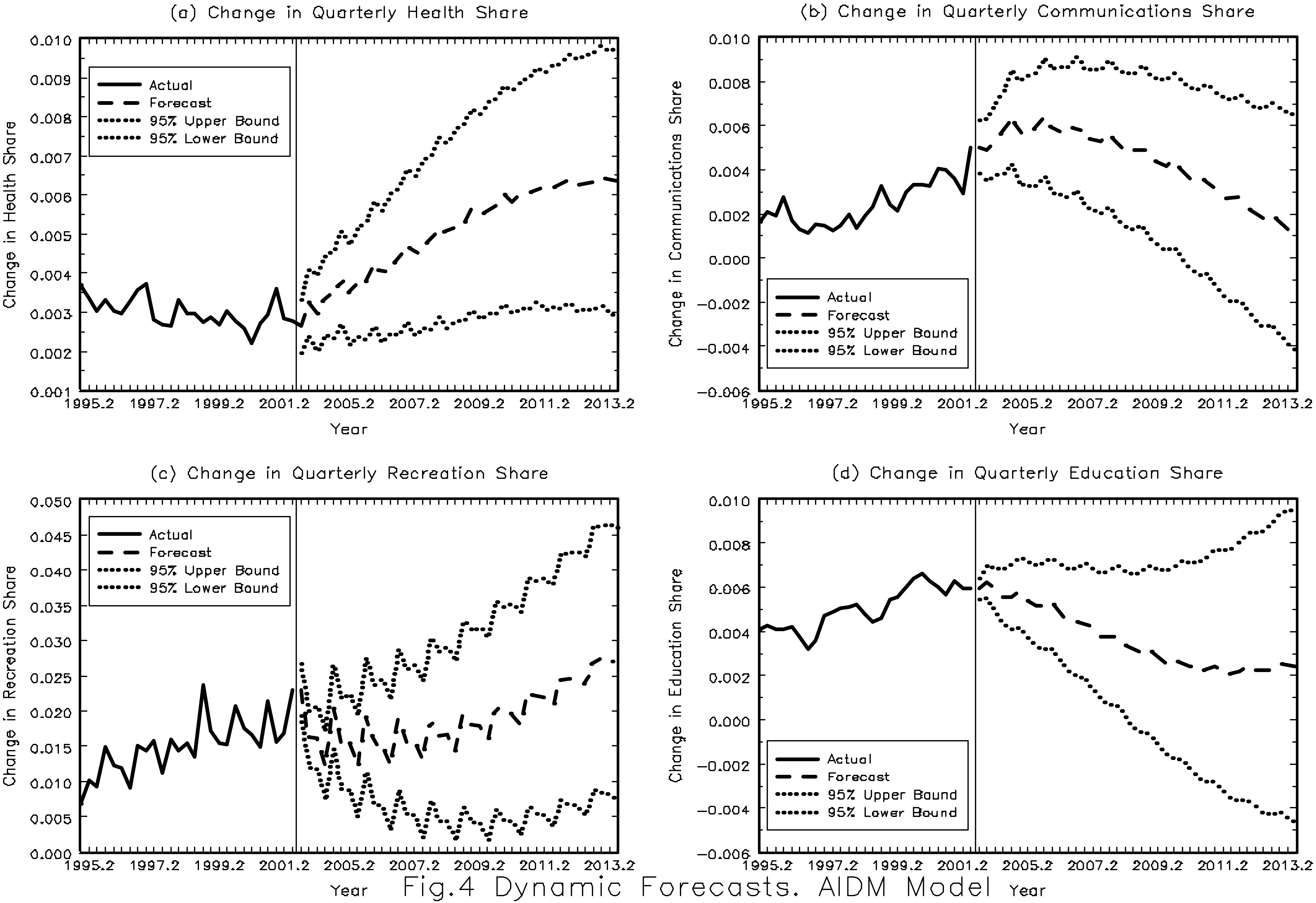

Fig.4 Dynamic Forecasts. AIDM Model