Stochastic Trends, Demographics and Demand Systems

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Abstract. Techniques for determining the number of stochastic trends generating a set of non-stationary panel data are applied to budget shares for a number of commodity groups from the family expenditure survey (FES) for the UK for the years 1973-2001. It is argued that some stochastic trends in macro data are generated by the aggregation of fixed demographic effects in the micro data. From cross section data, fixed effect coefficients are estimated which incorporate both age and income distribution effects. The estimated coefficients are combined with age proportion variables to form a set of I(1) indices for broad commodity groups which are then incorporated into a system of aggregate demand equations. The equations are estimated and tested in a non-stationary time series setting.

Keywords: Demand Equations, Age Demographics, Stochastic Trends.

JEL Classification: C1, C3, D1.

1. Introduction

In recent years, although empirical demand systems have been estimated in a time series setting, e.g., Lewbel and Ng [18, 2004], Ng [20, 1995], Attfield [1, 1997], central theoretical propositions such as homogeneity and symmetry are generally still not found to be satisfied. One of the arguments for this failure, and one recently analysed by Lewbel and Ng [18, 2004], is the omission of demographic effects in most empirical analyses. In this paper we also incorporate demographic effects into the demand analysis but take a different approach from Lewbel and Ng. It is argued that as the proportion in each age group in the population is shifting over time - exemplified by the "ageing population" - these changing proportions generate stochastic trends, I(1) variables, which impact on aggregate budget shares. The mechanism generating the trends lies in the aggregation across households when there are fixed age cohort effects. The aggregation of the micro data naturally leads to the stochastic proportions appearing in the aggregate demand equations.

Recent research, Bai [2, 2002], and Bai and Ng [3, 2002] and [4, 2004], has shown that it is possible to obtain the number of stochastic trends generating a set of I(1) variables by application of factor model techniques to a set of panel data. In section 2 we adopt these techniques to obtain the number of stochastic trends generating a panel of budget shares using the Family Expenditure Surveys (FES) from 1973

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to 2001. Having obtained the number of non-stationary factors (stochastic trends) driving budget shares over time, in section 3 we argue that demographic trends, formed from the aggregation of micro-data, play an important role in aggregate demand systems. We demonstrate that fixed age group effects at the household level - which also contain income distributional effects - when aggregated, lead to the inclusion of variables which are functions of the proportion of each age group in the total population. These proportions are significant I(1) variables and their exclusion from previous analyses would lead to misspecification problems. In section 4 we estimate the fixed effect parameters and use them to construct the functions of the age proportion variables, thereby forming demographic indices for commodity groups. Section 5 of the paper incorporates the demographic indices into Deaton and Muellbauer's [9, 1980] "Almost Ideal Demand Model", (AIDM), in a pure non-stationary time series setting. The number of stochastic trends - estimated from the Johansen maximum likelihood procedure - are found to be consistent with the number of trends in the panel data. In the aggregate time series data, with the inclusion of the demographic indices, we find that the relatively small number of stochastic trends (relatively large number of cointegrating equations) enables us to specify a demand system which includes homogeneity and some symmetry as simple normalisations and that the null of the remaining symmetry of the price coefficients can be tested and is not rejected by the data. Moreover, "adding up" is satisfied and the substitution matrix is negative semi-definite. The empirical demand system therefore satisfies all the theoretical properties of demand models. Section 6 concludes the paper.

2. Analysis of Aggregate Budget Shares

To analyse the number of stochastic trends generating budget shares we constructed budget shares for all households for 53 commodities from the FES for each of the years 1973 to 2001. The commodities are 25 components of "Food", 7 components of "Alcohol & Tobacco", 12 components of "Clothing & Footwear", 3 components of "Fuels", "Housing" goods, "Durable" goods, "Miscellaneous" goods, "Other" goods, "Service" goods and "Transport" goods. Further details of the groups are given in Table 1.

Table 1 *Here*

The aggregation results in a matrix of budget shares, W, with w_{jt}, for $j = 1, ..., J$ and $t = 1, ..., T$ and $J = 53, T = 29$. In a series of papers Bai [2, 2002] and Bai and Ng [3, 2002] and [4, 2004] show that the method of principal components can be used to obtain the number of unknown factors, both stationary and non-stationary, generating a set of data.

Assume that there are a set of k "significant" unknown factors F_t and factor loadings λ_j such that there exists a relationship of the form:

$$w_{jt} = c_j + \lambda_j' F_t + \varepsilon_{jt} \tag{1}$$

where the ε_{jt}s are stochastic errors and the F_ts and ε_{jt}s may, or may not be non-stationary. Of course, if either or both are non-stationary then the shares w_{jt} may be non-stationary. Bai [2, 2002] shows that as $(J, T) \to \infty$ the following information criteria converge on the correct number of non-stationary factors in the vector F_t:

$$
\begin{aligned}
IPC_1(k) &= \widetilde{V}(k) + k\widetilde{\sigma}^2\alpha_T\left(\frac{J+T}{JT}\right)\ln\left(\frac{JT}{J+T}\right) \\
IPC_2(k) &= \widetilde{V}(k) + k\widetilde{\sigma}^2\alpha_T\left(\frac{J+T}{JT}\right)\ln\left(\min\{J,T\}\right) \qquad (2)\\
IPC_3(k) &= \widetilde{V}(k) + k\widetilde{\sigma}^2\alpha_T\left(\frac{J+T-k}{JT}\right)\ln(JT)
\end{aligned}
$$

where $\alpha_T = T/[4\ln(\ln(T))]$, $\widetilde{V}(k) = (JT)^{-1}\sum_j^J\sum_t^T \widetilde{\varepsilon}_{jt}^2$ and $\widetilde{\sigma}^2 = \widetilde{V}(kmax)$, where $kmax$ is the maximum value for k. To find the value of k which minimises the criterion, Bai suggests setting $kmax$ to $8[(T/100)^{1/4}]$ and finding the minimum value for k, using this value of k for $kmax$ and continuing to iterate until k no longer changes.

Figures 1& 2 *Here*

This procedure results in Figure 1 for the budget shares data where $IPC_1(k)$ is minimised for all criteria at $k = 5$ implying five stochastic trends are responsible for generating the matrix of budget shares.

We can test whether the budget shares matrix, W, and the errors in equation (1) are non-stationary by applying a pooled augmented Dickey-Fuller test for a unit root. Assuming that the ε_{jt} are independent across j, to test the null that $\rho_j = 1$ against the null that $\rho_j < 1$ in the model:

$$\triangle\widetilde{\varepsilon}_{jt} = \alpha_o + \rho_j\widetilde{\varepsilon}_{jt-1} + \theta_1\triangle\widetilde{\varepsilon}_{jt-1} + ... + \theta_p\triangle\widetilde{\varepsilon}_{jt-p} + error$$

calculate the p-value for each commodity group j, $pval_j$; the $pval$s are distributed as uniform variates over the interval [0,1] and therefore $-2\ln(pval_j)$ is a chi square variate with two degrees of freedom. It follows that the statistic:

$$P_{\widetilde{\varepsilon}} = \frac{-2\sum_j^J \ln(pval_j) - 2J}{\sqrt{4J}}$$

is asymptotically distributed as a standard normal variate (cf. Choi [7, 2001], and Bai and Ng [4, 2004, p.13]) and we reject the null of a unit root if the test statistic exceeds the critical value in the upper tail of the standard normal, i.e., 1.65 at the 5% level.

With k set to 5 and the number of lags in the ADF test set to:

$$p = 4ceil\{(min(J,T)/100)^{1/4}\},$$

i.e., $p = 4$, the pooled $P_{\widetilde{\varepsilon}}$ test statistic for the null of non-stationarity of the equation errors is 18.83 so we can reject non-stationarity in the equation errors and assume that all the non-stationarity in the model comes from the stochastic trends, F, and not from the errors. A similar test on the raw data matrix, W, resulted in a test statistic of -0.48, so that we cannot reject the hypothesis that the budget shares are non-stationary, as might be expected since the shares are a linear combination of the 5 non-stationary factors[1].

This analysis suggests that across the years there are at least five stochastic trends generating aggregate demand for commodities. The theory of demand and empirical demand studies generally have budget shares as functions of prices and income (total expenditure). There is a good deal of empirical evidence that income and prices are $I(1)$ variates [cf. Lewbel and Ng [18, 2004], Ng [20, 1995], Attfield [1, 1997]], so we would anticipate trends associated with income and with prices. Some studies include higher order functions of income so other trends could be associated with such functions, although Gorman [13, 1981] has shown that for utility maximising consumers, exactly aggregable budget share Engel curves can have a maximum of two functions of income. Lewbel and Ng [18, 2004] show that an "evolving" population could be an important factor. They derive a model in which aggregation with a slowly changing population results in a non-stationary error process in the demand equations.

In recent history in the UK the combined effects of birth control and lower mortality, due to advances in health care, have led to proportions in the lower age groups declining while proportions in the upper age groups have been increasing. The Office for National Statistics (ONS) calculates the percentage of the population of 25 years of age and under in 1971 to be 38% falling to 30% by 2001, while the proportion of those aged 50 and over rose from 32% to 34% over the same period. Figure 3 graphs these movements.

Figure 3 *Here*

The Lewbel and Ng model is not appropriate for proportions but in the next section we show how aggregation over a fixed effect model can lead to a demand structure which reflects these changes in the demographic structure of the economy and we show formally that we cannot reject the null hypothesis that the age proportion variables behave like unit root processes.

3. Aggregation and Demographics

In figure 2 are the results of applying the analysis in the previous section to the 25 items in the food group alone. The results are very similar to those reported for all commodities together, the information criteria being minimised at 4, 5 and 6 stochastic trends. What is striking is figure 4 where we show the results of aggregating within commodity groups but disaggregating across age cohorts.

Figure 4 *Here*

That is, the top left panel in figure 4 gives the result of applying the information criteria to a matrix of budget shares for aggregate food, for age cohorts 19 to 84 across the years 1973-2001. The remaining panels give the results for the other

[1] The analysis by Bai & Ng assumes large T and large J but they report good results for simulations with smaller samples in the case of determining both stationary and non-stationary factors. When min{J,T} $\geqq$ 40, for example, they find their criteria give precise estimates of the number of factors [3, 2002, p.203].

groups of commodities where we have combined the 25 food components into a Food group; the 7 alcohol and tobacco components into an Alcohol & Tobacco group; the 12 components of clothing and footwear into a Clothing & Footwear group; the 3 components of fuel and the housing goods into a Fuel & Housing group; and combined the durable, miscellaneous, other, service and transport goods into an Other Goods group. For food (and all other commodity groups) the number of stochastic trends drops dramatically to only one trend in most cases. An interpretation of this result is that aggregating across age groups increases the number of trends when there are fixed age cohort effects. We show below that if the shares at age cohort level are functions of a shift demographic parameter, of real income and of prices, aggregation across age cohorts leads to the introduction of non-stationary demographic variables. Figure 4 implies that at age cohort level there is only one stochastic trend whereas we would expect at least two due to prices and income. However, we are dealing with small samples which may lack sensitivity and if the value of $kmax$ is increased to fifteen for a number of the commodity groups the value of k rises to two - but to no more than two.

To see how aggregating across age cohorts can lead to the introduction of a number of stochastic trends suppose all households at a particular time are grouped into those with heads the same age and that there are $\mathbf{G}$ such age groups denoted by $\mathcal{G}_{gt}$, $g = 1, ..., \mathbf{G}$. Let $\xi_{gt} = n_{gt}/N_t$ be the proportion of households in age group $\mathcal{G}_{gt}$, n_{gt}, in the total number of households, N_t. We formally tested these proportions for a unit root for the age groups from 19 to 84 across all the 29 years of the survey using the pooled ADF test statistic, P_ξ. The test statistic, with 4 lags[2], is $P_\xi = 0.46$ so the null of a unit root in these proportion series cannot be rejected under the standard normal distribution.

To incorporate these demographic variables into the analysis we need to specify a formal demand system. Lewbel and Ng [18, 2004] point out that demand systems that have Diewert [8, 1974] flexibility, i.e., do not impose unlikely constraints on demand elasticities, are the AIDM of Deaton and Muellbauer [9, 1980] and the Translog of Jorgenson, Lau and Stoker [15, 1982]. Both these models include, as explanatory variables, the log of real income and logged prices. Moreover, Lewbel and Ng [18, 2004] show for data for the USA when the square of log real income is included as an additional explanatory variable in the AIDM (as in the QUAIDS models of Blundell, Pashardes and Weber [5, 1993] and Banks, Blundell and Lewbel [6, 1997]) the demand equations are not cointegrated. This implies that the square of log real income cannot be a candidate for a stochastic trend in a cointegrated demand system. As mentioned in the previous section however, they do find some empirical evidence to support the argument that aggregation over heterogenous consumers in a slowly changing population can lead to the non-stationarity of equation errors.

To incorporate the age proportion variables directly into the demand system suppose the budget share for good j at time t for household h is given by the same

[2] In a later section we test for unit roots in age proportions in a pure time series setting using the procedures of Ng & Perron (1997) which automatically select lag length.

functional form as in the AIDM, that is:

$$w_{hjt} = \alpha_{oj} + \sum_i \gamma_{ij} \ln p_{it} + \ln(x_{ht}/P_t^*)\beta_j \tag{3}$$

where x_{ht} is per-household total income, p_{it} is the price of commodity i at time t, and $\ln P_t^*$ is Stone's price index[3] which linearises the theoretical AIDM model, Deaton and Muellbauer [9, 1980, p.316], and the coefficient β_j is constant across all households. We assume that the constant α_{oj} subsumes a fixed effect for each age group in the population, which can be thought of as a taste parameter in the utility function, so that the intercept in (3) is given by:

$$\alpha_{oj} = \theta_{oj} + \theta_{gj}.$$

Then, budget shares of good j for household h are:

$$w_{hjt} = \frac{x_{hjt}}{x_{ht}}$$

where $x_{hjt} = p_{jt}q_{hjt}$ is expenditure on good j by household h, and $x_{ht} = \sum_j x_{hjt}$ is total expenditure on all goods by household h. Aggregate budget shares for all households in group g are then:

$$w_{gjt} = \frac{x_{gjt}}{x_{gt}} = \frac{\sum_{h \in \mathcal{G}_{gt}} x_{hjt}}{\sum_{h \in \mathcal{G}_{gt}} x_{ht}} = \frac{\sum_{h \in \mathcal{G}_{gt}} x_{ht} w_{hjt}}{\sum_{h \in \mathcal{G}_{gt}} x_{ht}} = \theta_{oj} + \theta_{gj} + \sum_i \gamma_{ij} \ln p_{it} + z_{gt}\beta_j, \tag{4}$$

where z_{gt} is log real income per capita for age group g. Aggregation within an age group is along the same lines as the overall aggregation in Deaton and Muellbauer [9, 1980, p.314]. That is, we can assume that there is a component, say $\ln k_{gt}$, which reconciles the aggregation over levels with the aggregation over logarithms such that:

$$\ln k_{gt} = -\sum_h \left(\frac{x_{ht}}{x_{gt}}\right) \ln \left(\frac{x_{ht}}{x_{gt}}\right).$$

Deaton and Muellbauer [9, 1980, p.315] refer to $\ln k_{gt}$ as the log of Theil's [25, 1972] entropy measure of equality. Testing $\ln k_{gt}$ for a unit root, using the pooled test, we found the null of a unit root could be rejected for all lags up to 3 in the ADF test with statistics 37.15, 19.59, 8.91, 4.01 for lags 0, 1, 2 and 3. With 4 lags the test statistic is 1.33. The critical 5% value under the standard normal is 1.65 so it is safe to assume that $\ln k_{gt}$ is stationary. If income were equal within the group, $\ln k_{gt}$ would be a constant but would not be the same constant across groups. We tested for equality of group means of $\ln k_{gt}$ - over time - using a Wald test. The result, 5783 with 65 degrees of freedom, rejects the null of equality at any conventional significance level.

[3] Stone's price index is defined as $\ln P_t^* = \sum_j w_{jt} \ln(p_{jt})$, where w_{jt} is the budget share for the jth commodity at time t aggregated across all households.

Since the $\ln k_{gt}$ are stationary we assume that each is equal to a constant (its mean) plus a random error. The constant is absorbed into θ_{gj} and the random component into the equation error. This means that the estimates of each θ_{gj} contain a fixed age effect plus a measure of the inequality of the income distribution for that age cohort.

Now, aggregating over all G age groups gives:

$$w_{jt} = \frac{x_{jt}}{x_t} = \frac{\sum_g x_{gjt}}{\sum_g x_{gt}} \equiv \frac{\sum_g x_{gt} w_{gjt}}{\sum_g x_{gt}} = \theta_{oj} + \frac{\sum_g x_{gt}\theta_{gj}}{\sum_g x_{gt}} + \sum_i \gamma_{ij} \ln p_{it} + z_t \beta_j \tag{5}$$

where z_t is the log of total real income per capita. The aggregation procedure is similar to that outlined above but now the discrepancy index is given by:

$$\ln k_t = -\sum_g \left(\frac{x_{gt}}{x_t}\right) \ln \left(\frac{x_{gt}}{x_t}\right).$$

We assume $\ln k_t$ is stationary[4] with mean 4.1104 and estimated standard error 0.0022. As in the case of age groups, we assume that $\ln k_t$ is a constant (its mean) plus a random error so that the constant is absorbed into the equation intercept and the error into the equation disturbance.

The second term in the final expression in (5) can be written:

$$\frac{\sum_g x_{gt}\theta_{gj}}{\sum_g x_{gt}} = \frac{\sum_g \frac{x_{gt}}{n_{gt}} \theta_{gj} \frac{n_{gt}}{N_t}}{\frac{\sum_g x_{gt}}{N_t}} = \sum_g \frac{\overline{x}_{gt}}{\overline{x}_t} \theta_{gj} \xi_{gt} \tag{6}$$

where $\overline{x}_{gt} = x_{gt}/n_{gt}$ is average total expenditure per household in age group g, $\overline{x}_t = x_t/N_t$ is average total expenditure across all households and $\xi_{gt} = n_{gt}/N_t$. The ratio of group means to overall means, $\overline{x}_{gt}/\ \overline{x}_t$, turns out to be stationary as ratios often are, e.g., the "great" ratios consumption/income and investment/output[5].

In its present form (6) is difficult to construct for researchers working with aggregate time series as although the population proportion variable is readily available the parameter θ_{gj} has to be estimated, and the variable x_{gt} obtained, from cross section sources. Since the ratio of means, $\overline{x}_{gt}/\ \overline{x}_t$, is stationary, we assume:

$$\frac{\overline{x}_{gt}}{\overline{x}_t} = \delta_g + v_{gt} \tag{7}$$

where v_{gt} is a random error. The parameter δ_g for each age group is assumed constant over time and can be directly estimated by least squares from the cross section data

[4] The Ng & Perron [21, 1997] test stistic is -1.96 with a 5% critical value of -1.98 so it is on the borderline of being non-stationary.

[5] The pooled test statistic is greater than 3.4 for all lags less than and including 4, so the null of a unit root in $\overline{x}_{gt}/\ \overline{x}_t$ is rejected.

to give $\widehat{\delta}_g$ which is, of course, the sample mean of the ratio for the gth group. The null hypothesis that the mean of the ratio $\overline{x}_{gt}/\overline{x}_t$ is the same across all g, i.e., $\delta_g = c$, for all g, is comprehensively rejected by a Wald test with statistic 69540 with 65 degrees of freedom. Substituting (7) into (6) yields:

$$\frac{\widetilde{\sum_g x_{gt}\theta_{gj}}}{\sum_g x_{gt}} = \sum_g \widehat{\delta}_g \theta_{gj} \xi_{gt}. \tag{8}$$

Substituting the estimate in (8) into (5) results in:

$$w_{jt} = \theta_{oj} + \sum_g \widehat{\delta}_g \theta_{gj} \xi_{gt} + \sum_i \gamma_{ij} \ln p_{it} + z_t \beta_j \tag{9}$$

which contains stochastic trends associated with the demographic, income and price variables. The omission of the demographic variables could explain the "no cointegration" result of many demand studies. Lewbel and Ng [18, 2004], for example, show that for data for the USA, budget share demand systems which include z_t and logged prices do not cointegrate, i.e., have a non-stationary equation error.

To make (8) operational, in the next section, we estimate θ_{gj} using pooled cross section/time series data, to form a set of demographic indices for each commodity group, $\widetilde{I}_{jt}$, of the form:

$$\widetilde{I}_{jt} = \sum_g \widehat{\delta}_g \widetilde{\theta}_{gj} \xi_{gt}. \tag{10}$$

The construction of an index for $\ln k_{gt}$, the income distributional part of θ_{gj}, from cross section data is a suggestion of Deaton and Muellbauer [9, 1980, pp.314-315].

4. Estimating the Demographic Indices

In this section we use datasets created by forming variables for age cohorts over time to estimate a demand system in the form of equation (4) for each commodity group, and use the estimates of the coefficients on the age proportion variables, θ_{gj}, to form the demographic index which measures the impact of a changing population structure on that commodity.

For age cohorts we treat the budget share equations as a set of share Engel curves. That is, we suppress the price variables and treat the relationships as a system of cross section Engel curve equations of the form:

$$w_{gjt} = \alpha_{oj} + \theta_{gj} + z_{gt}\beta_j + u_{gjt}.$$

where u_{gjt} is a random error term. To estimate the model we need series on commodity prices over time to construct the real expenditure variable. From published sources it isn't possible to obtain price series for each of the 53 commodities we analysed in section 2, but only for broad groups. In addition, when we turn to aggregate time series data and methods in the next section it isn't feasible to estimate a model with a large number of commodities. We therefore aggregated within the commodity groupings, as in the previous section, to form five major groups, viz., Food, Alcohol & Tobacco, Clothing & Footwear, Fuel & Housing and Other Goods. We obtained annual price indices for these five groups over the time period of the analysis from the ONS data bank.

We estimated the coefficients for each commodity group by the least squares dummy variable (LSDV) method by defining a set of dummy variables for each age group and dropping the last group - aged 84 - to avoid collinearity. When the coefficient for the dropped group is required in constructing the demographic indices below, we assume that the coefficient for the age 84 group is the same as for the group aged 83.

The estimation procedure ignores any non-stationarity in the group shares and income but the point estimates are still consistent and we employed the HAC procedure of Newey and West [19, 1987] to allow for any serial correlation caused by non-stationarity. Also, we tested for stationarity of the equation errors using the ADF method proposed for panel data by Kao [16, 1999]. Two lags were used in the ADF model - the maximum in the Kao procedure - and we obtained test statistics of -6.43, -8.52, -13.92, -3.86 and -2.15 for the groups Food, Alcohol & Tobacco, Clothing & Footwear, Fuel & Housing and Other Groups respectively. Kao shows that as $T \rightarrow \infty$ and $J \rightarrow \infty$ sequentially, the test statistic tends to a standard normal variate so that in all cases the null of no cointegration can be rejected. This confirms that the budget share equation errors are stationary before aggregating over all households.

Because of the large number of coefficients estimated we do not tabulate them in detail but give plots in figure 5. Since we are using budget shares as the dependent variable, the coefficients θ_{gj} sum to zero across commodities so that some have to be negative. If the demographic variables had no impact on budget shares the graphs in figure 5 would all be horizontal. The coefficients for Fuel and Housing and for Other Goods form a U-shape while those for Alcohol & Tobacco, Clothing & Footwear and Food are inverted U. Food appears to be a reflection of Other Goods. This latter contains expenditures on house purchase and other durables. "Housing" in Fuel & Housing contains only expenditure on household goods such as white goods and small electrical items, floor coverings and furnishings, kitchen utensils and cleaners. The impact of the age coefficients is to cause the share of these Other Goods to fall between the ages of 19 and 30, flatten out and then increase from age 70. The share of Food increases (relatively - the coefficients are in the negative quadrant) from age 19 to 30 and then levels out until age 70 when it starts to decline. The Fuel & Housing share declines very slightly from age 19 to 50 but then increases to age 80 and over.

Using the HAC estimated variance-covariance matrix we used the Wald statistic to test the null hypothesis that the age effect coefficients within each commodity group are: (i) equal - which would imply constancy across all age groups thereby forming a constant in the commodity equation (the proportions sum to unity) which would be of little interest, (ii) jointly zero and thus irrelevant. For both (i) and (ii) the null hypothesis could easily be rejected at the 5% level so the age proportion variables do make a significant contribution.[6]

5. Time Series Demand Systems with Demographic Indices

For most practical uses of demand systems researchers employ time series data where it isn't possible to calculate the demographic variables. In this section we use the demographic indices calculated above with quarterly time series data from the ONS data bank[7] for the period 1971Q1 to 2001Q1 firstly, to test for the cointegrating rank of an AIDM model, and secondly, to estimate the parameters of the demand model. Annual series on age proportions were obtained from the Government Actuarial service but are only available from 1971 for each age group in the population[8].

The starting point is the structural demand model of equation (9) for each commodity group, j:

$$w_{jt} = \theta_{oj} + \psi_j \widetilde{I}_{jt} + \sum_i \gamma_{ji} \ln p_{it} + z_t \beta_j + u_{jt} \tag{11}$$

where u_{jt} is a random error and we have aggregated over all G age groups and include a parameter, ψ_j, on the estimated demographic index $\widetilde{I}_{jt}$, firstly, to allow for any differences in magnitude between the cross section and time series data, and secondly, to allow a fixed linear relationship between the proportion of households in each age group, in the FES samples, and the proportion of each age group in the population in the ONS series. The budget shares and prices are ordered $j = 1, ..., 5$ for Food, Alcohol & Tobacco, Clothing & Footwear, Fuel & Housing and Other Goods.

In the time series data, we tested all variables for unit roots using the procedures by Ng and Perron [21, 1997] and Perron and Ng [23, 1996] which optimally choose the lag length for the ADF test. Their DF-GLS test for a unit root did not reject unit roots for any of the variables, including the demographic indices[9]. In the estimation procedures and system tests which follow, one equation has to be dropped because on the null hypothesis of a demand system, the "adding up" restriction leads to a singularity if all equations are used. Dropping the equation for Other Goods means that the demographic index for this commodity group does not appear in the system we are estimating and testing. Sufficient conditions for adding up to be satisfied are:

$$\sum_j^J \theta_{oj} = 1; \ \sum_j^J \gamma_{ji} = 0, \forall i; \ \sum_j^J \beta_j = 0$$

[6] The Wald statistics for Food, Alcohol & Tobacco, Clothing & Footwear, Fuel & Housing and Other Goods were 761, 2677, 1651, 492, 298 with 65 degress of freedom for the hypothesis in (i) and 687, 2365, 1499, 467, 263 with 64 degrees of freedom for the hypothesis in (ii).

[7] Quaterly, seasonally unadjusted, series on real and nominal expenditures for all categories of goods were obtained and aggregated into the five main groups described in the main text. Commodity price indices and total expenditures (income) were derived from these data sets. Prior to analysis, seasonal components were removed using seasonal dummies.

[8] Prior to 1971 population statistics are available for 5-yearly age groupings only. The annual population series were converted to quarterly using the logarithmic interpolation procedure.

[9] The 5% critical value for the test is -1.98 and the test statistics for all the variables in the time series data set were greater than -1.58.

and:

$$\sum_{j}^{J} \psi_j \widetilde{I}_{jt} = 0. \tag{12}$$

Substituting the formulae for the indices in equation (10) into (12) yields:

$$\sum_{g} \left(\sum_{j}^{J} \psi_j \widetilde{\theta}_{gj} \right) \widehat{\delta}_g \xi_{gt} = 0$$

so that adding up is satisfied if $\left(\sum_{j}^{J} \psi_j \widetilde{\theta}_{gj} \right) = 0, \forall g$. We can therefore write ψ_J - the coefficient on the Other Goods demographic index which isn't estimated - as:

$$\psi_J^{(g)} = -\frac{\sum_{j}^{J-1} \psi_j \widetilde{\theta}_{gj}}{\widetilde{\theta}_{gJ}} \tag{13}$$

where the superscript (g) denotes that there will be G "solutions" to the equations. The solutions, $\psi_J^{(g)}$, must all be equal. With the estimates $\widetilde{\theta}_{gj}$ and estimates of ψ_j, $j = 1, .., J-1$, this equality hypothesis can be tested. We report the result of the test below.

We tested for cointegration in the demand system using Johansen's [14, 1995] likelihood ratio procedures. That is, we test for the rank of the matrix π in the vector error correction formulation:

$$\triangle x_t = \theta_o + \theta_1 \triangle x_{t-1} + ... + \theta_s \triangle x_{t-s} + \pi x_{t-1} + \zeta_t,$$

where x_t contains the set of 14 variables, i.e., 4 budget shares, 5 prices, 4 demographic indices and log real per capita. To find the number of lags in first differences, s, we estimated an unrestricted equation in levels with lags 1 to 4. The BIC, Hannan-Quinn and Akaike information tests (obtained with PcGive [11, 2001]) gave results for lag lengths of 1, 4, and 4 respectively (in levels [0, 3 and 3 in first differences]). We therefore carried out the tests and subsequent estimation using 3 lags in first differences. The trace test statistic for the null of 8 cointegrating vectors is 136.21 with a 5% critical value of 94.15 so we can reject 8 cointegrating vectors in favour of 9 or more. The λ-*max* statistic for the null of 8 is 39.84 with 5% critical value of 39.37 so that, with this statistic, we can also reject 8 in favour of 9 cointegrating vectors. The trace test statistic for the null of 9 cointegrating vectors is 96.36 with a 5% critical value of 68.52 so we can reject 9 cointegrating vectors in favour of 10 or more but the λ-*max* statistic for the null of 9 is 30.39 with 5% critical value of 33.46 so that we cannot reject 9 in favour of 10[10]. Taking evidence from both test statistics together we accept the null of 9 cointegrating vectors.

The result of 9 cointegrating vectors ties in extremely well with the finding, in section 2, of 5 stochastic trends generating the budget shares. The relation between

[10] Critical values were obtained from Osterwald-Lenum [22, 1992]

stochastic trends and cointegrating equations is outlined in Stock and Watson [24, 1988] and can be summarised as follows. Suppose x_t is a $p \times 1$ vector of $I(1)$ variables and there are $r < p$ cointegrating relations, then from the Wold theorem the moving average representation of the VECM model is:

$$\triangle x_t = \mu_o + C(L)\zeta_t,$$

where $C(L) = I + C_1 L + C_2 L^2 + \ldots$. Substituting for $C(L) = C(1) + (1-L)C^*(L)$ we obtain:

$$\triangle x_t = \mu_o + C(1)\zeta_t + C^*(L)\triangle\zeta_t$$

and therefore:

$$x_t = const + \mu_o t + C(1)\sum_i^t \zeta_i + C^*(L)\zeta_t.$$

In the term $C(1)\sum_i^t \zeta_i$ it can be shown that $C(1)$ has rank $p-r$ so we can always define a non-singular elementary matrix A such that $C(1)A^{-1} = [C_1, 0]$ where C_1 is $p \times p - r$. It follows that:

$$\begin{aligned} C(1)\sum_i^t \zeta_i &= C(1)A^{-1}A\sum_i^t \zeta_i \\ &= [C_1, 0]\, A\sum_i^t \zeta_i. \end{aligned} \tag{14}$$

Let $\tau_t = A\sum_i^t \zeta_i$, then τ_t is a vector of p random walks, i.e.

$$\tau_t = \tau_{t-1} + A\zeta_t = A\sum_i^t \zeta_i.$$

Partition τ_t into:

$$\tau_t = \begin{bmatrix} \tau_{1t} \\ \tau_{2t} \end{bmatrix}$$

where τ_{1t} is $p - r \times p$ and then the expression in (14) becomes:

$$C(1)\sum_i^t \zeta_i = C_1\tau_{1t}$$

and is a function of just $p - r$ random walks, or common stochastic trends, and therefore the model can be written:

$$x_t = const + \mu_o t + C_1\tau_{1t} + C^*(L)\zeta_t$$

and the vector of observed variables x_t are generated by a constant, a time trend, a set of $p - r = k$ stochastic trends and a stationary term.

With $p = 14$ in the model and $r = 9$ cointegrating equations there will be $p - r = k = 5$ stochastic trends so the results from the panel of budget shares and the results from the time series analysis complement each other.

It follows then that the rank of the $p \times p$ matrix π is equal to r so that we can write:

$$\pi = \gamma\alpha'$$

where γ is $p \times r$ and α is $p \times r$ and is the matrix of cointegrating coefficients.

To identify and estimate the cointegrating equations we need some structure on the relations. Since there are $r = 9$ cointegrating relations we can always write:

$$\pi = \beta\alpha' = \beta G^{-1} G\alpha'$$

where G is any $r \times r$ nonsingular matrix. Therefore, to identify the coefficients of the demand equations we need at least 9 restrictions on each equation. Consider the following structural definition of the cointegrating vectors, α' :

$$\begin{array}{cccccccccccccc}
w_{1t} & w_{2t} & w_{3t} & w_{4t} & \ln p_{1t} & \ln p_{2t} & \ln p_{3t} & \ln p_{4t} & \widetilde{I}_{1t} & \widetilde{I}_{2t} & \widetilde{I}_{3t} & \ln p_{5t} & \widetilde{I}_{4t} & z_t \\
-1 & 0 & 0 & 0 & \gamma_{11} & \gamma_{21} & \gamma_{31} & \gamma_{41} & \psi_1 & 0 & 0 & \gamma_{51} & 0 & \beta_1 \\
0 & -1 & 0 & 0 & \gamma_{12} & \gamma_{22} & \gamma_{32} & \gamma_{42} & 0 & \psi_2 & 0 & \gamma_{52} & 0 & \beta_2 \\
0 & 0 & -1 & 0 & \gamma_{13} & \gamma_{23} & \gamma_{33} & \gamma_{43} & 0 & 0 & \psi_3 & \gamma_{53} & 0 & \beta_3 \\
0 & 0 & 0 & -1 & \gamma_{14} & \gamma_{24} & \gamma_{34} & \gamma_{44} & 0 & 0 & 0 & \gamma_{54} & \psi_4 & \beta_4 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & \alpha_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \alpha_{12} & \alpha_{22} & \alpha_{32} & \alpha_{42} & \alpha_{52} \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{43} & \alpha_{53} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} & \alpha_{54} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \alpha_{15} & \alpha_{25} & \alpha_{35} & \alpha_{45} & \alpha_{55}
\end{array} . \quad (15)$$

A necessary condition for identification of all the coefficients in the β and α matrices is that there are at least $r^2 = 81$ restrictions on the structural α matrix which contains 126 elements. Without any loss of generality we have normalised the first 4 equations as the budget share equations in (15). 9 restrictions have been placed on each of the remaining 5 equations so that the variables $w_{1t}, w_{2t}, w_{3t}, w_{4t}$,$\ln p_{1t}$,$\ln p_{2t}$,$\ln p_{3t}$,$\ln p_{4t}$ and $\widetilde{I}_{1t}$ are thought of (arbitrarily) as being driven by $\widetilde{I}_{2t}, \widetilde{I}_{3t}, \ln p_{5t}, \widetilde{I}_{4t}$ and z_t. As it stands there are only 7 restrictions on each of the first four rows in (15), the budget share equations, which are the normalisations of the coefficients on the budget shares and the exclusion restrictions on all the indices but the 'own' demographic index. These restrictions sum to 73 in all. Homogeneity, $\sum_{i=1}^{5} \gamma_{ij} = 0$, $j = 1, ..., 4$ adds another one restriction to each demand equation and the symmetry relations:

$$\begin{aligned}
\gamma_{21} &= \gamma_{12} \\
\gamma_{31} &= \gamma_{13} \\
\gamma_{41} &= \gamma_{14} \\
\gamma_{32} &= \gamma_{23} \\
\gamma_{42} &= \gamma_{24} \\
\gamma_{43} &= \gamma_{34}
\end{aligned}$$

add a further 6 restrictions giving 83 restrictions in all so that the necessary condition is satisfied. A necessary and sufficient condition for identification is that the Jacobian matrix for the relations $\pi' = \alpha\beta'$ has full column rank (cf., for example, Doornik [10, 1995]. That is:

$$\frac{\partial \text{vec}(\pi')}{\partial \text{vec}(\phi')'} = [\beta \otimes I_p] \frac{\partial \text{vec}(\alpha)}{\partial \text{vec}(\phi')'} = J'$$

where ϕ is the 43×1 vector of unknown coefficients in (15) after imposing homogeneity and symmetry. We used the rank procedure in GAUSS [12, 2002], with random values for the β matrix and ϕ vector, to verify that J' has full column rank.

Of course, with the exclusion restrictions in (15) plus homogeneity and symmetry, the elements of α and β are overidentified, in the sense that there are two overidentifying restrictions. An interesting aspect of this analysis is that the "unrestricted model" can be written as containing the homogeneity and 4 of the symmetry restrictions - the "restricted model" then restricts the remaining 2 sets of symmetry coefficients. Put another way, in the unrestricted model we are estimating a demand system with homogeneity and some symmetry already imposed by normalisation and which is perfectly consistent with the data in the sense that it will generate an identical likelihood to a completely unrestricted model with rank$(\pi) = 9$. It is the large number of cointegrating equations relative to the number of stochastic trends which enables sufficient normalisations to identify a complete demand system (less two symmetry conditions).

To obtain estimates operationally, since the restrictions on the matrix α in (15) are all linear, we can write:

$$\text{vec}(\alpha) = H_o + H_1\phi$$

with H_o and H_1 known matrices. Taking the rank r estimate of π from the ML procedure, say $\widetilde{\pi}$, and solving the following set of equations iteratively starting with random values for the coefficients in β gives us ML estimates of the restricted α.

$$\begin{aligned}
\phi^{(s)} &= \left(W^{(s)\prime}W^{(s)}\right)^{-1} W^{(s)\prime} \left(\text{vec}(\widetilde{\pi}') - \left(\beta^{(s)} \otimes I_p\right) H_o\right) \\
\text{vec}(\alpha^{(s)}) &= H_o + H_1\phi^{(s)} \\
\text{vec}(\beta^{(s+1)}) &= I_p \otimes \left(\alpha^{(s)\prime}\alpha^{(s)}\right)^{-1} \alpha^{(s)\prime}\text{vec}(\widetilde{\pi}')
\end{aligned}$$

where $W = (\beta \otimes I_p) H_1$ and (s) denotes the s^{th} iteration. The process was assumed to have converged when the differences between estimates were of the order $|0.00001|$ in successive iterations. The procedure produces α and β matrices such that α contains all the normalisations and the product $\beta\alpha'$ is identically equal to the rank r matrix $\widetilde{\pi}$.

Maximum likelihood estimates of the coefficients of the demand equations with normalisations for homogeneity and four symmetry conditions imposed are given in Table 2. A Wald test of the remaining symmetry relations (abitrarily chosen as $\gamma_{32} = \gamma_{23}$ and $\gamma_{24} = \gamma_{42}$) produced a statistic of 0.741 with 2 degrees of freedom so the null hypothesis of overall symmetry cannot be rejected.

Table 2 Here

The indices in the Food and Alcohol & Tobacco equations are not significantly different from zero in the cointegrating equations. This does not mean that these indices can be dropped from the analysis as they do have an impact in the dynamic part of the VECM. Because of the large number of coefficients we do not give them here but report that lagged changes in the alcohol and tobacco index do have a significant impact in all the other demand equations and lagged changes in the food index have a significant impact on demand for clothing and footwear and on fuel and housing.

Conventional demand elasticities for the AIDM model, calculated at the point of the sample mean of the variables, are given in Table 3.

Table 3 Here

The formula for the price elasticities was derived on the assumption of the true price index given by:

$$\ln P_t = const + \sum_k \alpha_k \ln p_{kt} + \frac{1}{2}\sum_j \sum_k \gamma_{kj} \ln p_{kt} \ln p_{jt}$$

with the α_k and γ_{kj} as given in (11).

The income elasticity of demand for commodity i at time t is given by:

$$\eta_{it} = \frac{\beta_i}{w_{it}} + 1$$

and price elasticity[11] of demand by:

$$\eta_{iit} = \frac{\gamma_{ii}}{w_{it}} - \beta_i - 1 + \frac{\beta_i^2 z_t + \beta_i \psi_i \widetilde{I}_{it}}{w_{it}}.$$

All the own price elasticities for the full ML system have the correct negative sign. Food and Alcohol & Tobacco are close to being unit price elastic while Clothing & Footwear and Fuel & Housing are price inealstic. Income elasticities classify all goods as necessities ($0 < \eta_i < 1$).

Table 3 also gives point estimates of eigenvalues which imply that the substitution matrix is at least negative semi-definite. Finally, a Wald test for equality of the $\psi_J^{(g)}$ in (13) results in a test statistic of 22.8 with 64 degrees of freedom so that equality cannot be rejected and the set of demand equations satisfy the adding-up restriction.

6. CONCLUSION

In this paper we have found agreement between the number of stochastic trends generating budget shares in a set of panel data derived from the Family Expenditure survey and aggregate time series data obtained from the ONS. It is argued that at least some of the stochastic trends are generated by demographic shifts in non stationary age-group proportion variables. The proportion variables enter into the

[11] The term $\frac{\beta_i^2 z_t + \beta_i \psi_i \widetilde{I}_{it}}{w_{it}}$ is neglible in practice and has been omitted from the calculations as it makes little or no difference to the results quoted. The same applies to the symmetry condition in the substitution matrix below.

demand equations in aggregating over fixed age group effects. The age group fixed effects are found to be highly significant in cross section data and indices formed using these effects plus the effects of aggregation from household to population level are significant in the aggregate time series equations. The omission of the demographic variables could account for the finding of "no cointegration" in some previous demand studies.

The slow-moving shifts in population age group proportions observed in comparatively recent years are likely to have dramatic effects on the economy and we would expect them to have an impact on demand for commodities. The demographic indices constructed in this paper should enable researchers and forecasters to increase the precision of their results by incorporating these indices into demand systems thereby allowing for such demographic effects.

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Table 1. Commodity Group Composition Definitions

Food Group		Clothing & Footwear Group	
1.	Bread, milk loaves, rolls	33.	Men's outerwear
2.	Flour, biscuits,cakes, composite, cereals	34.	Men's underwear & hosiery
3.	Beef and veal, including minced meat	35.	Women's outerwear
4.	Mutton and lamb	36.	Women's underwear & hosiery
5.	Pork	37.	Boy's outerwear
6.	Bacon and ham, uncooked	38.	Boy's and girl's underwear
7.	Offal, sausages uncooked,sausage meat, poultry, other meat, meat products	39.	Girl's outerwear
8.	Fish – fresh, canned, frozen, fish and chips	40.	Men's, women's and children's accessories: headgear, belts, ties, gloves, scarves, haberdashery
9.	Eggs, fresh and dried	41.	Men's footwear
10.	Butter	42.	Women's footwear
11.	Margarine	43.	Children's and infant's footwear
12.	Fresh milk, cream, yoghurt, fresh cream, skimmed milk, canned and dried milk and cream, yoghurt, other milk products, baby milk foods	44.	All other clothing and clothing charges
13.	Cheese, including processed		**Fuel, Power & Housing Group**
14.	Lard, cooking and other oils and fats	45.	Anthracite, coal, coke, smokeless fuels, concessionary coal & coke
15.	Vegetables, tomatoes - fresh, canned, frozen, dried	46.	Gas, electricity
16.	Raw and all potato products	47.	Paraffin, fuel oil & other fuel
17.	Fruit- fresh, canned, bottled, frozen, dried; juices - fresh, canned, bottled	48.	Housing
18.	Tea		**Other goods**
19.	Coffee, coffee essence	49.	Durables
20.	Food drinks not baby milk foods	50.	Services
21.	Sugar	51.	Miscellaneous
22.	Syrup, lemon curd, honey, jam, marmalade	52.	Transport
23.	Ice cream	53.	Other
24.	Sweets and chocolates		
25.	All other food including school meals & meals out		
	Alcohol & Tobacco Group		
26.	Beer, stout, shandy, cider, home & away		
27.	All wines, fortified and unfortified, home & away		
28.	All spirits and liqueurs, home & away		
29.	All other alcohol, home & away		
30.	Cigarettes, cigarette tobaco and papers		
31.	Pipe tobacco		
32.	Cigars, snuff		

Table 2. ML Estimates of Full System

Dependent Variable	Explanatory Variables										Diagnostics	
	$\ln p_1$	$\ln p_2$	$\ln p_3$	$\ln p_4$	$\tilde{I}_1$	$\tilde{I}_2$	$\tilde{I}_3$	$\ln p_5$	$\tilde{I}_4$	z	R^2	Box-Ljung($\sqrt{T} = 10$ lags)
w_1	-0.0131 (0.0093)	0.0015 (0.0029)	0.0014 (0.0075)	-0.0177 (0.0051)	-0.0002 (0.0280)	-	-	0.0280 (0.0082)	-	-0.0135 (0.0107)	0.81	11.67 (*pval* = 0.31)
w_2	0.0015 (0.0029)	0.0007 (0.0021)	0.0005 (0.0026)	0.0046 (0.0023)	-	0.0270 (0.0378)	-	-0.0074 (0.0111)	-	-0.0232 (0.0030)	0.83	3.80 (*pval* = 0.96)
w_3	0.0014 (0.0075)	0.0024 (0.0070)	0.0003 (0.0088)	0.0047 (0.0057)	-	-	-0.6247 (0.0815)	-0.0087 (0.0117)	-	-0.0465 (0.0095)	0.72	8.36 (*pval* = 0.59)
w_4	-0.0177 (0.0051)	0.0095 (0.0055)	0.0047 (0.0057)	-0.0091 (0.0054)	-	-	-	0.0126 (0.0086)	2.495 (0.4133)	-0.0304 (0.0264)	0.83	7.02 (*pval* = 0.72)

Commodity groups are: 1. Food; 2. Alcohol & Tobacco; 3. Clothing & Footwear; 4. Fuel & Housing; 5. Other Goods

$\tilde{I}_j$ Demographic index for commodity group j

Estimated asymptotic standard errors in parenthesis

Table 3. Estimated Demand Elasticities

	ML System Estimates			
			Calculated at Sample Means	
Commodity Group	Own price coefficient	Income coefficient	Own Price Elasticity	Income Elasticity
Food	-0.0131 (0.0093)	-0.0135 (0.0107)	-1.0841	0.9082
Alcohol & Tobacco	0.0007 (0.0021)	-0.0232 (0.0030)	-1.0302	0.5570
Clothing & Footwear	0.0003 (0.0088)	-0.0465 (0.0095)	-1.1507	0.3482
Fuel & Housing	-0.0091 (0.0054)	-0.0304 (0.0264)	-1.0597	0.8186

Estimated asymptotic standard errors in parenthesis

Mean eigenvalues of Substitution Matrix: -0.166, -0.132, -0.078, -0.049
Eigenvalues of substitution matrix at mean data points: -0.157, -0.141, -0.078, -0.050.

Fig. 1. Plot of LEVELS Criteria $IPC_1(k)$, $IPC_2(k)$ and $IPC_3(k)$
All Commodities by All Years
kmax = 10

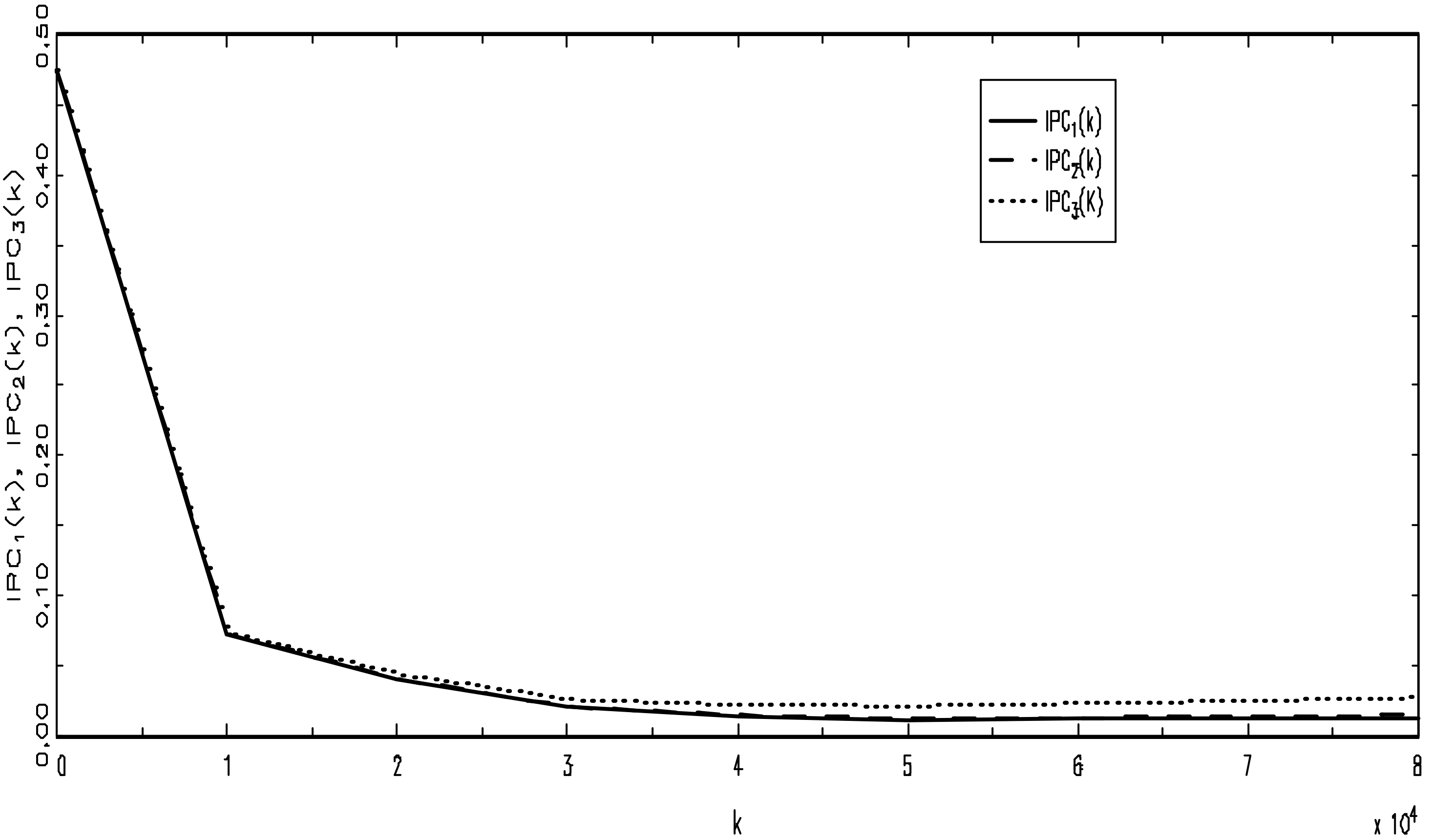
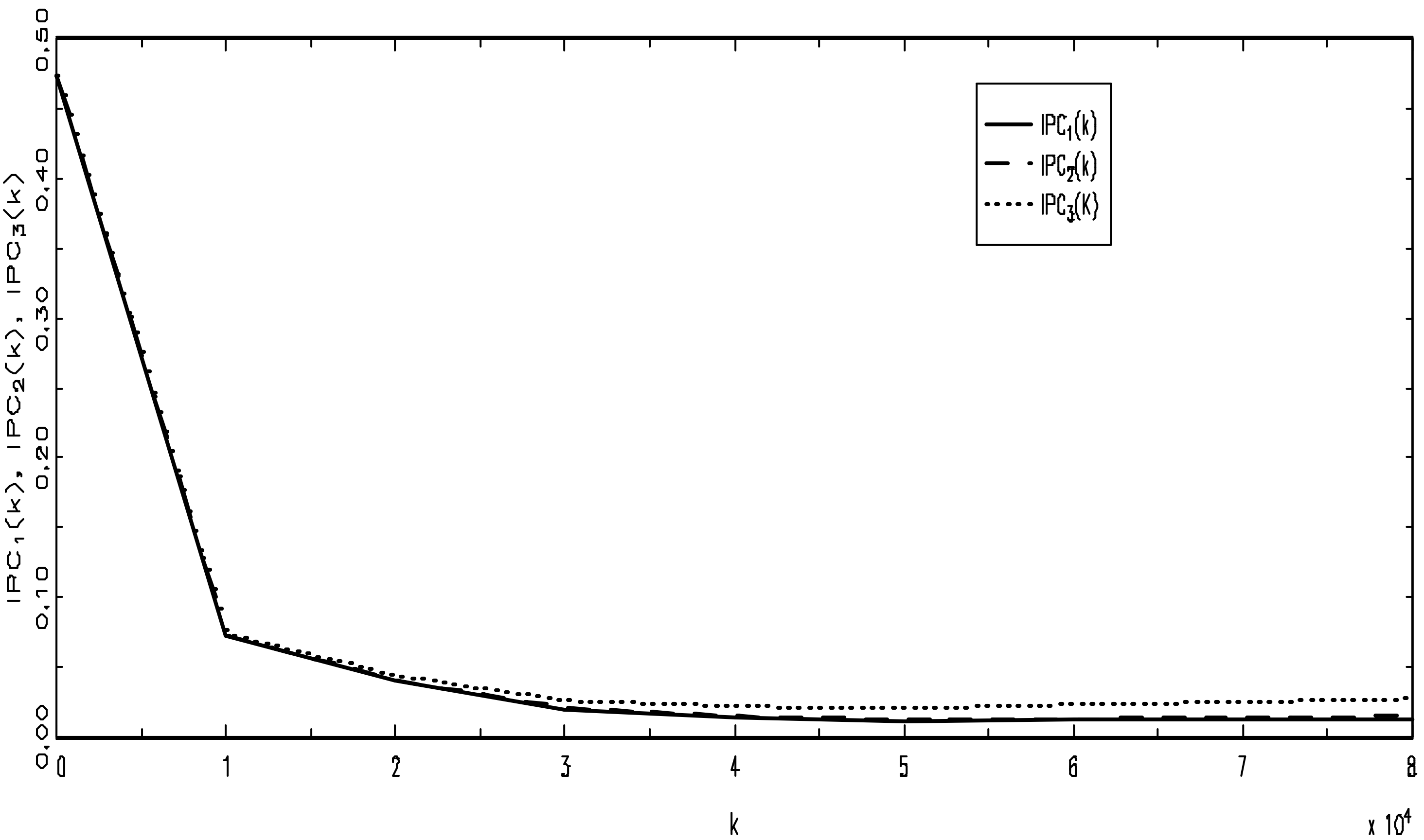


Fig. 2. Plot of LEVELS Criteria $IPC_1(k)$, $IPC_2(k)$ and $IPC_3(k)$
All Commodities by All Years
kmax = 3



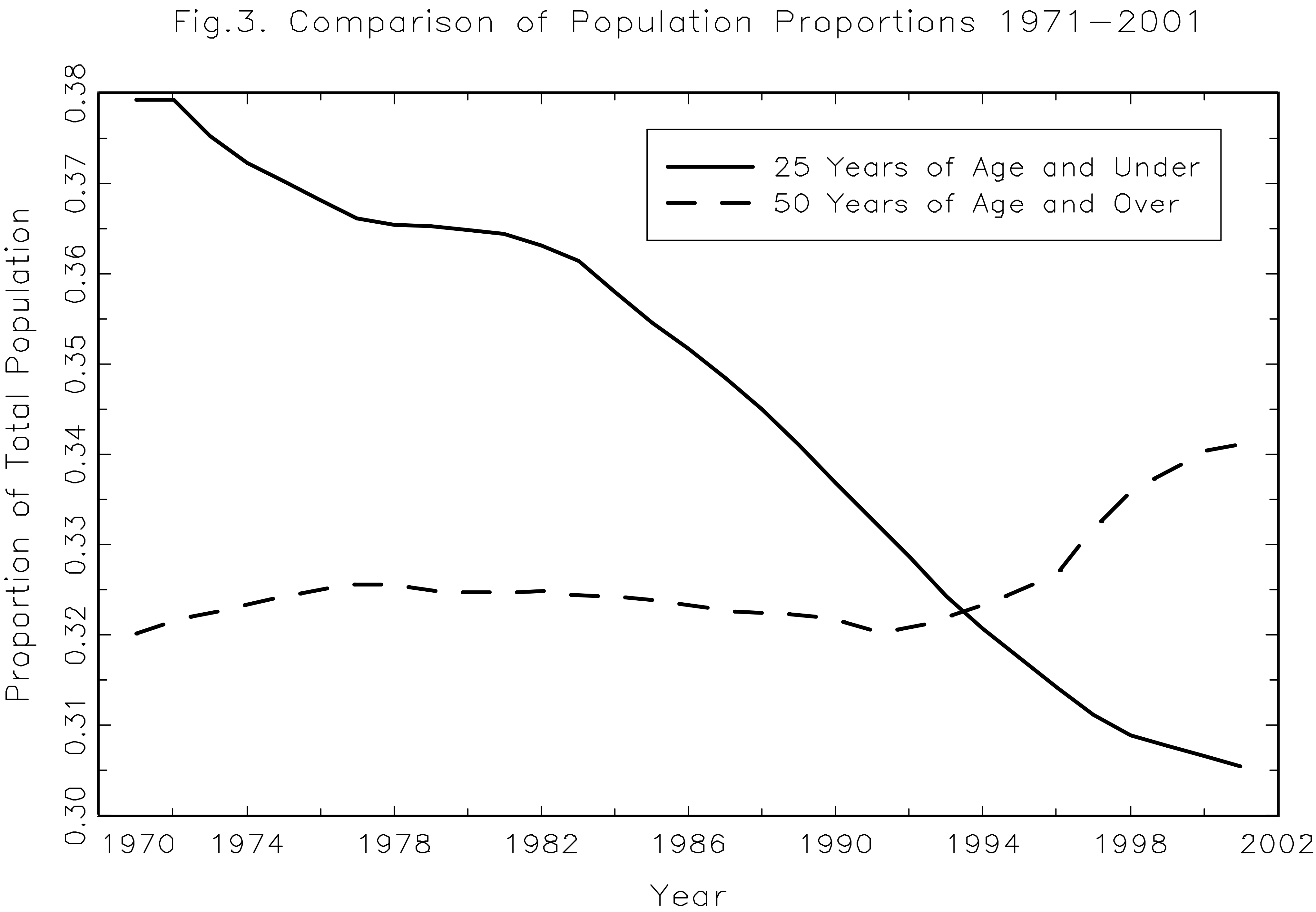


Fig.3. Comparison of Population Proportions 1971-2001
25 Years of Age and Under
50 Years of Age and Over
Proportion of Total Population
0.30
0.31
0.32
0.33
0.34
0.35
0.36
0.37
0.38
1970
1974
1978
1982
1986
1990
1994
1998
2002
Year

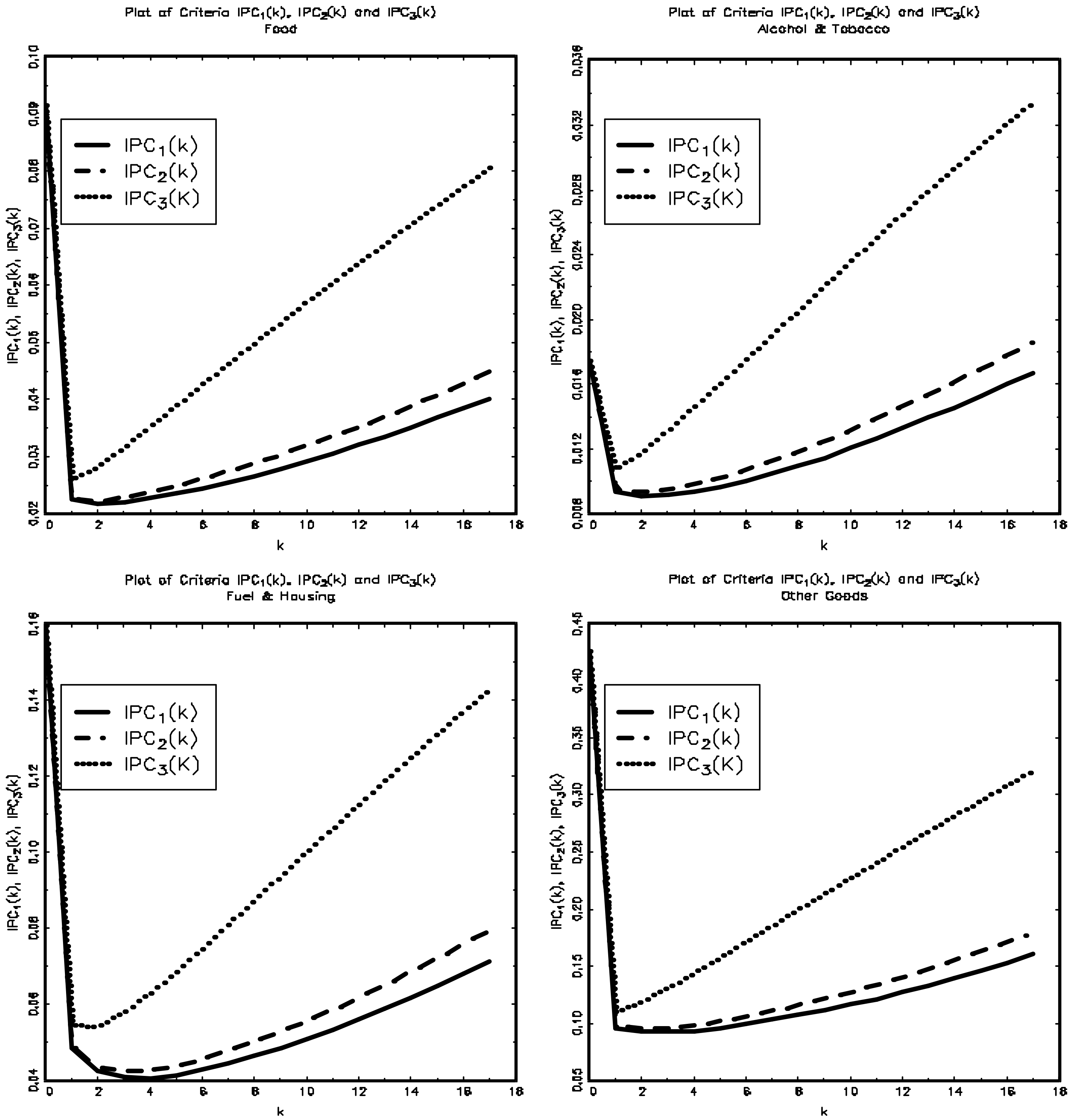


Fig 4

Information Criteria for Shares

All Age Groups by All Years

Fig.5. Age Cohort Coefficients, Θ_{gj}, for Commodity Groups