Balanced Growth and Output Convergence in Europe

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Abstract.

Using OECD quarterly data on consumption, output and investment from 1980, the balanced growth hypothesis is tested country by country for seven European economies, Belgium, Finland, France, Holland, Italy, Spain and the UK. Output series for each of the countries is then modelled as an output system and the hypothesis of convergence in trend output growth tested. Finally, the hypothesis of balanced growth *and* convergence, is tested in a system framework.

Key words: Convergence, Balanced Growth, Stochastic Trends,

JEL Classification: C32; C51

1. Introduction

The UK Government has specified five economic criteria which must be satisfied before the UK's entry into the European Monetary Union (EMU). One of the criteria is concerned with 'sustainable convergence' between the UK and European economies. In this paper we examine, for a number of EU economies, the evidence for the balanced growth model and, through this model, the evidence for sustainable convergence of output growth in Europe. Using data from the OECD, where available, we examine the evidence for stability (stationarity) of the consumption/income, C/Y, and investment/output, I/Y, ratios. These are the 'great ratios' that King et al [10, (1991)], hereafter KPSW, found stable in their balanced growth model for the USA. Using the results of Horvath and Watson [7, (1995)] for testing the null of zero cointegrating vectors against a *known* number with *known* coefficients, it is found that constancy of the great ratios is consistent with the OECD dataset i.e. that the hypothesis that the three variables lnC, lnI and lnY are cointegrated with unit coefficients can be accepted for each of the countries individually. Given this balanced growth within economies we next address the question of whether there is a single common trend driving output growth by testing the output series across the countries for common trends in a multivariate analysis similar to that of Bernard and Durlauf [2, (1995)]. Their definition of convergence requires the economies to have identical long run stochastic trends in output growth. Finding that there is evidence for a single common trend we combine all the variables for all countries to test the joint hypothesis of balanced growth and convergence. The underlying theoretical model is sketched in section 2, empirical results are analysed in section 3 and section 4 concludes.

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2. The Model

2.1. Balanced Growth. Let c_t, i_t and y_t be the natural logarithms of consumption, investment and income variables for a single economy and let $x_t' = (c_t, i_t, y_t)$. If x_t is $I(1)$ and the great ratios $c_t - y_t$ and $i_t - y_t$ are stationary then a VECM exists of the form:

$$\triangle x_t = \theta_o + \theta_1 \triangle x_{t-1} + ... + \theta_k \triangle x_{t-k} + \beta\alpha' x_{t-1} + \zeta_t \quad (1)$$

where $\triangle x_t = x_t - x_{t-1}, \zeta_t$ is a Gaussian error and:

$$\alpha' = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}. \quad (2)$$

The Wold representation of equation (1) is given by:

$$\triangle x_t = \mu + C(L)\zeta_t$$

where μ is the mean of $\triangle x_t$ and $C(L) = I + C_1 L + C_2 L^2 + ...$ with L the lag operator.

Using the Beveridge-Nelson [3, (1981)] decomposition into trend and transitory components, we define trend output as (cf. Cochran [4, (1981)]):

$$\begin{aligned} x_t^* &= \lim_{k\to\infty} E_t\left(x_{t+k} - k\mu\right) \quad (3) \\ &= x_t + \sum_{j=1}^{\infty} E_t\left(\triangle x_{t+j} - \mu\right). \end{aligned}$$

After some manipulation and differencing, trend growth for the vector of variables can be obtained as:

$$\triangle x_t^* = C(1)\theta_o + C(1)\zeta_t \quad (4)$$

where $\mu = C(1)\theta_o$. An expression for $C(1)$ is readily available, (cf. Johansen [8, (p.52, 1995)]):

$$C(1) = \alpha_\perp \left(\beta_\perp' \Theta(1)\alpha_\perp\right)^{-1} \beta_\perp'$$

where $\Theta(L) = I - \theta_1 L - \theta_2 L^2 - ... - \theta_k L^k$, and where $\alpha_\perp$ is the orthogonal complement of α such that $\alpha_\perp' \alpha = 0$ and likewise $\beta_\perp' \beta = 0$. Let p be the number of variables in x_t, for the case of a single country $p = 3$, and assume that the number of cointegrating vectors, r, is two, then $\alpha_\perp'$ is $(p - r \times p)$ and α is $(p \times r)$. Likewise, $\beta_\perp'$ is $(p - r \times p)$ and β is $(p \times r)$. Then, for a single country, with $r = 2$, we have:

$$C(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \left(\beta_\perp' \Theta(1)\alpha_\perp\right)^{-1} \beta_\perp'$$

since, for the α' defined in (2), we have $\alpha_\perp' = (1, 1, 1)$. It then follows that:

$$\triangle x_t^* = \begin{bmatrix} \triangle c_t^* \\ \triangle i_t^* \\ \triangle y_t^* \end{bmatrix} = C(1)\theta_o + C(1)\zeta_t = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \xi_t$$

where ξ_t is the scalar $\left(\beta_\perp' \Theta(1)\alpha_\perp\right)^{-1} \beta_\perp' (\theta_o + \zeta_t)$. The long run trend in growth is common to all variables which is the balanced growth result that KPSW found empirically for the economy of the USA. The matrix α' in (2) has two testable restrictions - the unity constraints in the elements in the last column. In the section 3 we test this model for seven European economies.

2.2. Convergence of Output Growth. Bernard and Durlauf [2, (1995)] define convergence in output in two economies, i and j, to be the equality of their long run output forecasts, i.e.:

$$\lim_{k\to\infty} E_t(y_{i,t+k} - y_{j,t+k}) = 0 \tag{5}$$

while the two economies will have a common trend in output if:

$$\lim_{k\to\infty} E_t(-y_{i,t+k} + \gamma y_{j,t+k}) = 0. \tag{6}$$

The definition in (6), they argue, is equivalent to the output series being cointegrated with cointegrating coefficient γ which measures the proportionate difference in the long run forecast. If the series are cointegrated with $\gamma = 1$, as in definition (5), then the countries have an identical common trend and long run output will have converged[1].

Suppose, in the case of two countries, that output does cointegrate with vector:

$$\alpha' = (-1, \gamma).$$

Then the left hand side of (6) becomes:

$$\lim_{k\to\infty} E_t(\alpha' w_{t+k}) \tag{7}$$

where $w_t' = (y_{1t}, y_{2t})$. Defining the long run trend generated by just the output variables as:

$$w_t^* = \lim_{k\to\infty} E_t\left(w_{t+k} - k\mu_{\triangle w}\right)$$

where $\mu_{\triangle w} = E(\triangle w)$ and premultiplying by α' yields:

$$\alpha' w_t^* = \lim_{k\to\infty} E_t\left(\alpha' w_{t+k} - k\alpha'\mu\right) \tag{8}$$

but, $\alpha'\mu = \alpha' C(1)\theta_o = 0$, since $\alpha' C(1) = 0$. It follows that the definitions in (7) and (8) are equivalent. So, premultiplying the trend output vector, or trend growth vector as in (4), by α' will result in zero when α' is a cointegrating vector. This follows from the Wold representation:

$$\triangle w_t = \mu_{\triangle w} + C(L)\eta_t$$

leading to:

$$\triangle w_t^* = \mu_{\triangle w} + C(1)\eta_t$$

and hence:

$$\alpha' \triangle w_t^* = \alpha' \mu_{\triangle w} + \alpha' C(1)\zeta_t = 0.$$

If $\alpha' = (-1, \gamma)$ with $\gamma \neq 1$ then, from this analysis the long run growth trends are proportional, which can be tested by testing for cointegration between y_{1t} and y_{2t}. If the outputs are cointegrated then if $\gamma = 1$, long run growth trends will be identical - growth will have converged, which can be tested by a LR test of the null that $\gamma = 1$. In a system of m countries a test for a common trend is a test that the cointegrating space is equal to $m - 1$ and a test of convergence that there are $m - 1$ restrictions of unity on the final column of α'.

[1]The Bernard-Durlauf definition requires that there are no constants or time trends in the cointegrating equations.

2.3. Balanced Growth and Convergence. Suppose now that we combine the balanced growth result with the convergence result for two countries. The vector of all variables becomes:

$$x_t' = (c_{1t}, i_{1t}, c_{2t}, i_{2t}, y_{1t}, y_{2t})$$

where c_{1t} is consumption for country 1 and c_{2t} is consumption for country 2 etc., and if the balanced growth hypothesis is valid for both, then $c_{1t} - y_{1t}$, $i_{1t} - y_{1t}$, $c_{2t} - y_{2t}$ and $i_{2t} - y_{2t}$ are all stationary. If, in addition, the output series for the two countries are cointegrated, so that $\gamma y_{2t} - y_{1t}$ is stationary, the two countries will have a common trend in output growth which is proportional, the coefficient γ, but growth will not have converged in the sense of an identical common trend, $\gamma \neq 1$. That is, for this case α' will be:

$$\alpha' = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & \gamma \end{bmatrix}$$

or, without any loss of generality since in the VECM of (1) we can always write $\beta\Xi^{-1}\Xi\alpha' = \beta_*\alpha_*'$, where Ξ is an elementary matrix such that:

$$\alpha_*' = \Xi\alpha' = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & \gamma \\ 0 & -1 & 0 & 0 & 0 & \gamma \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & \gamma \end{bmatrix}.$$

The orthogonal complement of α_* is $\alpha_{*\perp}' = (\gamma, \gamma, 1, 1, \gamma, 1)$ so that trend growth for each component of country 1 is proportional to the corresponding component of country 2 with the proportion being equal to the cointegrating coefficient between each country's outputs. Of course, if $\gamma = 1$ trend growth in the two economies would be identical and output growth would have converged.

In a system context with m countries and p variables in each country a test for convergence and balanced growth is a test that there are $mp - 1$ cointegrating equations and $mp - 1$ unit restrictions on the last column of α_*' so that we have the form:

$$\alpha_*' = (-I_{mp-1}|\iota_{mp-1}) \tag{9}$$

with I_{mp-1} the identity of order $mp-1$ and ι the vector of ones of order $mp-1 \times 1$. With seven countries and three variables if we can't reject the hypothesis that $\alpha_*' = (-I_{20}|\iota_{20})$, then we have evidence not only for convergence but that this convergence is sustainable in the long term via balanced growth in consumption, investment and output.

One of the problems, however, with carrying out such a test for anything above two or three economies is that in a multivariate setting the dimensions of the VECM, with just a few lags, can lead to catastrophic failure in some numerical routines. On the other hand, it is the dynamic interaction between variables and across countries that we wish to utilise in our tests. We compromise by testing trivariate models for stability of the great ratios and then for common trends in the output series alone before finally combining them in the full system.

3. Empirical Results

3.1. Data Series. In this section we model quarterly, seasonally adjusted real, per capita, consumption expenditure, C_t, output, Y_t, and investment, I_t for the EU members Belgium, Finland, France, Holland, Italy, Spain and the UK for the period 1980Q1 to 2001Q3. The economic series were obtained from the OECD Main Economic indicators data bank at MIMAS (Manchester Information Service at Manchester University) using the Timeweb Explorer interface. Consumption is defined as: private final consumption expenditure; output as gross domestic product minus government final consumption expenditure and investment as gross fixed capital formation. All series are expressed in logarithms, quarterly annualised, seasonally adjusted and in constant, 1995, prices. For comparison across members of the EU, all values were converted to U.S. dollars using the $/£ exchange rate end period. Annual population data was obtained from the U.S. Bureau of the Census, International Data Base and log-linearly interpolated to quarterly series. PPP adjusted quarterly series are not available from the OECD source. The EU countries excluded from the analysis are Greece, Ireland and Luxembourg because series on consumption and investment are not available from the OECD source, and Austria, Denmark, Germany, Portugal and Sweden because data on some of the series is not available prior to 1988Q1. The starting point of 1980Q1 was chosen as this included Belgium and Spain where series are not available until 1980Q1.

3.2. Results for the Great Ratios. For each country the logarithms of consumption, investment and output, c_t, i_t, y_t were tested for unit roots. Results are not given here but are available from the author as it is well known that the hypothesis of a unit root in macroeconomic time-series such as these is rarely rejected[2]. Table 1 gives the results for Johansen ML cointegration tests for the trivariate system for each country. The null of zero cointegrating equations is decisively rejected in favour of one while the null of one in favour of two is weaker, being around the 20% level for all countries except Belgium - which is less than 20%. It should be emphasised that some of these results can be 'improved' by increasing lag lengths for the Johansen test. Neither the AIC, BIC or LR testing down procedures are very useful for selecting the lag length for most of these series as the likelihood surface is very flat. Testing down from 8 lags usually selected 8 lags while AIC and BIC often selected zero. The lag lengths reported in Table 1 were obtained by testing down from 8 lags and choosing the smallest lag length consistent with a Johansen trace test statistic for two cointegrating relations significant at around the 20% level.

If we accept the above as evidence for two cointegrating equations, the next step is to test that the coefficient on y_t is unity in both the cointegrating investment and consumption equations for each country. The first two columns in Table 1 give the point estimates and they are all close to one with the exception of the investment coefficient for Finland which is negative but not significantly different from zero. LR tests of the null that both coefficients are equal to one are given in the last column but one in Table 1. The null is rejected at the 1% level for Belgium, Holland and Spain but can be accepted for the other countries.

Horvath and Watson [7, (1995)], hereafter H-W, have developed Wald type tests which can be applied in exactly the situation we have here, where the underlying theory predicts known coefficients in the cointegrating vectors - zeros and ones. They construct tables for a Wald statistic for the test of a null of zero cointegrating relations against a

[2]The null of a unit root cannot be rejected for any of the data series using an augmented Dickey-Fuller test with lags of any order up to 8.

KNOWN alternative and demonstrate considerable power advantages. Clearly, using the information of known relations we are more likely to reject the null of no cointegration in favour of the two specified relations. The statistic for testing the null of zero against the alternative given in equation (2) was calculated for each of the countries and is given in Table 1. The null can be rejected and the alternative accepted at around the 10% level or better for all countries except Holland[3]. Using this H-W test procedure then, the data is consistent with the great ratios being stationary for all countries for the sample period.

3.3. Results for Output Variables. Bernard and Durlauf [2, (1995)] using annual output data (1900 to 1987) for 15 international countries found evidence for "3 to 6" common trends driving the output series. Table 2 gives the multivariate test results for the output of the 7 European countries modelled as a system. The BIC selected a lag length of 1 while AIC and testing down as many as 8. The results are given for a lag length of 1, in first differences, where there are 5 cointegrating equations significant at the 5% level[4] and 6 at the 20% level. If we accept that there are 6 cointegrating relations between the 7 countries, we have one stochastic trend and this trend output growth is proportional across the countries. Table 2 gives a comparison of these proportions for the countries with the French coefficient normalised to unity. From the point estimates, long run log per capita dollar output for Belgium is 1.41 and for Holland is 1.54 times that of France while Finland is 0.22, Italy 0.25, Spain 0.55 and the UK 0.52 that of France. These coefficients should all be equal to unity if output has converged. Using conventional tests they all significantly different from unity (except France). A joint LR test rejects the null that they are jointly unity with a p-value of 0.0013. However, if we use the H-W statistic to test the null of zero cointegrating relations against the alternative of six known cointegrating relations of the form of equation (9), so that there is an identical stochastic trend for all the countries, i.e., the coefficients in Table 2 are restricted to unity, we obtain a test statistic of 170.06 which exceeds the 5% critical value in H-W tables of 103.02. On this evidence we can reject the null and accept the common-trend, convergence of output hypothesis.

3.4. The Complete System. Putting together the tests for balanced growth and convergence for all countries jointly, results in a vector of variables of dimension of 21 and a test for 20 cointegrating vectors. If the rank of the cointegrating space is 20 there is one common output trend driving all the variables in the system. If the cointegrating vectors take the form of $\alpha_*' = (-I_{20}|\iota_{20})$ in (9) then the hypothesis of both balanced growth and convergence cannot be rejected. With only one lagged first difference in the VECM there are 441 coefficients to be estimated in the θ_1 matrix alone which is approaching the limit for any sensible testing procedure (cf. Abadir et al [1, (1999)]). On the other hand, using information from all the economies allows interaction between them in the short run dynamics in the VECM, which has advantages as pointed out by Granger and Haldrup [6, (1997)]. In Table 3, with 1 lag in the full system, the hypothesis of 20 cointegrating vectors cannot be rejected at around the 20% level using the Johansen trace test statistic. Looking at individual coefficients in the last two columns of Table

[3] Increasing the lag length to 11 for Holland produces a Horwath-Watson test statistic of 27.57 which is significant at 1%.

[4] With two lags some instability in the estimation was evident from the deterioration in the estimates and large standard errors. The latter, errors for the cointegrating coefficients, were obtained from MICROFIT [14, (1996)]. All other results in the paper were obtained using PcGive10 [13, (2001)] and GAUSS [5, (2001)]

3, the only countries which really reject the balanced growth hypothesis are Finland and the UK. Likewise, in column one a test for convergence - all coefficients equal to 1 - is only rejected for Finland and the UK. A standard LR test for the null of balanced growth and convergence for all countries is easily rejected. Again though, using the H-W test the null of 20 zero cointegrating vectors against a known restricted matrix as in (9), $(-I_{20}|\iota_{20})$, can easily be rejected in favour of the alternative[5].

Finally, in Table 4, we give the results for the economies that look most likely to have converged in Table 3, Belgium, Holland, Italy, Spain and France, i.e., dropping the two countries UK and Finland. The results were obtained with two lagged differences in the VECM which was the maximum possible without instability problems arising in the estimation. The balanced growth results deteriorate in Table 4, particularly the coefficient on investment for both Italy and Spain while the coefficients for convergence are all insignificantly different from unity supporting convergence for these countries alone. A LR test rejects the joint balanced growth and convergence hypothesis while, once again, the H-W test of the null of 14 zero cointegrating vectors against a known restricted matrix as in (9), $(-I_{14}|\iota_{14})$, is easily be rejected in favour of the alternative[6].

4. Conclusion

Using quarterly, post 1980, OECD data for seven European economies, Belgium, Finland, France, Holland, Italy, Spain and the UK, this paper finds some evidence for the balanced growth hypothesis across the countries and for convergence in output growth if the Wald test of Horvath-Watson [7, (1995)] using a *known* alternative in the testing procedure is adopted. The data is then consistent with a single European balanced output growth trend driving the economies. These findings on convergence are not wholly consistent with those of Bernard and Durlauf [2, (1995)] but their analysis covered an earlier period, 1900-1987. The results in this paper are, however, consistent with Strazicich and Lee [16, (2001)] and Li and Papell [11, (1999)] who do find strong evidence for convergence when they incorporate endogenous breaks in trend in their tests for cointegration. The breaks occur around the first and second world wars and both these papers argue that they may be the reason for the Bernard and Durlauf result of non-convergence. This paper uses data from 1980 which avoids the breaks due to the world wars but the comparative weakness of some of the conventional cointegration tests does suggest that an analysis taking into account structural breaks, in a multivariate framework, may prove fruitful.

[5]The tables produced by H-W extend to 9 variable systems. Using the same procedures as detailed in their paper, we first replicated their results for 2, 3 and 7 variable systems and then obtained the 5% critical value for testing for 21 variables with 20 known cointegrating vectors on the alternative. The critical values were obtained with 10000 replications of samples of size 1000.

[6]Analysing the output series alone for these 5 countries results in 4 cointegrating vectors at 6% using the Johansen trace statistic. Coefficients are all insignificantly different from unity. The LR test still rejects the null of convergence (pval = 0.001) but the H-W test easily accepts the convergence hypothesis with test statistic of 60.1 (critical 5% value: 53.9).

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Table 1 Tests of European Great Ratios

Country	Estimate of α_1 in equation: $-c_t + \alpha_1 y_t$	Estimate of α_2 in equation: $-i_t + \alpha_2 y_t$	Number of lags of first differences	Johansen Trace Statistic[(a)]		LR test of joint null $\alpha_1 = 1$ and $\alpha_2 = 1$ [(b)]	Horvath-Watson Test Statistic[(c)]
				Ho: r=0, Ha: r 1	Ho: r≤1, Ha: r 2		
Belgium	0.913 (0.028)	1.329 (0.049)	3	28.97	9.48	10.24	18.82
Finland	0.870 (0.282)	-0.190 (1.352)	4	26.05	11.60	3.14	19.24
Holland	0.854 (0.015)	0.961 (0.052)	8	24.54	10.50	11.36	11.57
Italy	1.070 (0.074)	1.015 (0.259)	4	31.22	11.26	4.53	23.26
Spain	1.026 (0.028)	1.846 (0.421)	2	36.53	12.18	15.74	18.26
UK	1.052 (0.022)	1.089 (0.055)	7	37.67	12.04	3.13	35.87
France	0.895 (0.029)	1.001 (0.047)	7	28.85	10.74	7.69	21.85

Critical values:

			20%	10%	5%	1%
(a)	Johansen trace statistic:	Ho: r = 0 v Ha: r 1	23.64	26.79	29.68	35.65
		Ho: r ≤ 1 v Ha: r 2	11.07	13.33	15.41	20.04
	(From Osterwald-Lenum [18,(1992)], Table1, p. 468)					
(b)	LR statistic - chi-square(2)		-	4.61	5.99	9.21
(c)	Horvath-Watson [12, (1995)]	Ho: r = 0 v Ha: r = 2 with				
	cointegrating vectors of form (-1, 0, 1) and (0, -1, 1)		-	18.51	20.74	25.35

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Table 2 Tests of European Output Series

	Belgium	Finland	Holland	Italy	Spain	UK	France
Real $ output per capita as a proportion of output of France (γ_j)	1.408 (0.108)	0.223 (0.155)	1.541 (0.194)	0.258 (0.220)	0.554 (0.171)	0.522 (0.136)	1
	Ho: r≤4, Ha: r 5	Ho: r≤5, Ha: r 6					
Johansen Trace Statistic[(a)]	28.90	11.13					
LR test statistic[(b)]	21.79						
Horvath-Watson statistic[(c)]	170.06						

Critical values:

			20%	10%	5%	1%
(a)	Johansen trace statistic:	Ho: r ≤4 v Ha: r 5	23.64	26.79	29.68	35.65
		Ho: r ≤5 v Ha: r 6	11.07	13.33	15.41	20.04
	(From Osterwald-Lenum [12, (1992)], Table1, p. 468).					
(b)	LR statistic - chi-square(6)		-	10.60	12.60	16.80
(c)	Horvath-Watson [7, (1995)]	Ho: r = 0 v Ha: r = 6 with				
	cointegrating vectors of the form $\left(-I_6 \middle| \iota_6\right)$, where $\iota' = (1,1,1,1,1,1)$		-	98.48	103.02	112.77

Table 3 Tests of European System

Country	Real \$ output per capita as a proportion of output of France (γ_j)	Estimate of α_1 in the equation: $-c_t + \alpha_1 y_t$	Estimate of α_2 in equation: $-i_t + \alpha_2 y_t$
Belgium	1.121	1.183	1.322
	(0.070)	(0.045)	(0.129)
Finland	0.470	0.703	0.230
	(0.123)	(0.119)	(0.293)
Holland	1.073	1.040	1.003
	(0.122)	(0.082)	(0.130)
Italy	0.872	0.869	0.653
	(0.129)	(0.131)	(0.175)
Spain	0.879	0.918	0.748
	(0.107)	(0.113)	(0.195)
UK	0.281	0.207	0.028
	(0.147)	(0.175)	(0.239)
France	1	1.060	0.897
	-	(0.03)	(0.80)

	Ho: r≤18, Ha: r 19	Ho: r≤19, Ha: r 20
Johansen Trace Statistic[(a)]	24.75	10.59
LR test statistic[(b)]	132.5	
Horvath-Watson statistic[(c)]	3751.7	

Critical values:			20%	10%	5%	1%
(a)	Johansen trace statistic:	Ho: r ≤18 v Ha: r 19	23.64	26.79	29.68	35.65
		Ho: r ≤19 v Ha: r 20	11.07	13.33	15.41	20.04
	(From Osterwald-Lenum [12, (1992)], Table1, p. 468).					
(b)	LR statistic - chi-square(20)		-	28.40	31.40	37.60
(c)	Horvath-Watson [7, (1995)]	Ho: r = 0 v Ha: r = 20 with				
	cointegrating vectors of the form $\left(-I_{20} \middle| \iota_{20}\right)$		-	927.1	941.5	972.5

Table 4 Belgium, Holland, Italy, Spain & France

Country	Real \$ output per capita as a proportion of output of France (γ_j)	Estimate of α_1 in the equation: $-c_t + \alpha_1 y_t$	Estimate of α_2 in equation: $-i_t + \alpha_2 y_t$
Belgium	0.998	1.201	0.812
	(0.117)	(0.047)	(0.298)
Holland	0.836	0.923	0.667
	(0.206)	(0.131)	(0.262)
Italy	0.783	0.733	0.401
	(0.140)	(0.139)	(0.182)
Spain	0.703	0.781	0.026
	(0.112)	(0.115)	(0.403)
France	1	1.175	0.738
	-	(0.071)	(0.119)

	Ho: r≤12, Ha: r 13	Ho: r≤13, Ha: r 14
Johansen Trace Statistic[(a)]	24.23	13.18
LR test statistic[(b)]	79.68	
Horvath-Watson statistic[(c)]	1129.18	

Critical values:			20%	10%	5%	1%
(a)	Johansen trace statistic:	Ho: r ≤12 v Ha: r 13	23.64	26.79	29.68	35.65
		Ho: r ≤13 v Ha: r 14	11.07	13.33	15.41	20.04
	(From Osterwald-Lenum [12, (1992)], Table1, p. 468).					
(b)	LR statistic - chi-square(14)		-	21.10	23.70	29.10
(c)	Horvath-Watson [7, (1995)]	Ho: r = 0 v Ha: r = 14 with				
	cointegrating vectors of the form $\left(-I_{14} \middle| \iota_{14}\right)$		-	467.3	477.1	498.1