The Impact of Age Distribution Variables on the Long Run Consumption Function

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Abstract

Modigliani's Life Cycle Hypothesis (LCH) predicts that demographic variables should play a significant role in our understanding of the relationship between consumption and income. Understanding this relationship is particularly important given the demographic changes expected in the next few decades. Unfortunately, evidence for the importance of demographic variables is mixed: unsurprisingly since such variables change relatively slowly and most analysis is confined to post war data. In this paper we use a much longer time series of aggregate variables (1856-1996) which models consumption, income and demographic effects in a vector error correction framework allowing for structural breaks. Our analysis shows that demographic effects have an important effect in the manner predicted by the LCH.

Keywords: Consumption, ageing population, breaks in trend.

JEL Classification: C51, C53

1 Introduction

One of the most significant demographic changes predicted for the 21st century is in the age distribution of the population. The combined effects of control of family size and health care in developed countries are predicted to result in an "ageing population". The Office for National Statistics (ONS) calculates, for the UK, the percentage of the population between the ages of 20 and 64 years of

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age as 84% in 1955 and over 64 years of age as 16%; while for 2020 it predicts the 20 to 64 age group will have fallen to 75% and the over 64 years of age group increased to 25%. While the adult population (20 years of age and over) is predicted to continue to grow, the annual rate of growth for 2000-2020 is lower than the rate of growth for the 1980s and early 1990s. This raises interesting questions about the impact of such demographic changes on total per capita, real consumers' expenditure and on individual categories of consumption. Because demographic effects tend to impact on the economy gradually over long periods of time, we use long run annual data from 1856 to 1996 allowing for structural breaks due to the two world wars. We assume that the key parameters in the consumption function, the MPC and coefficients on the age distribution variables, are constant over the whole sample, but that there are differing deterministic time trends in the pre first world war, the inter-war and the post second world war periods. Fair and Dominguez [6, 1991] include age-distribution data for the United States in a consumption-income relationship. In section 2 we extend the Fair and Dominguez model to a time series setting and demonstrate that the variables under consideration are I(1). In section 3 we present appropriate maximum likelihood estimates of the model and section 4 concludes.

2 A Model of Consumption, Income and Age Distribution

One of the puzzles in much time series analysis of the consumption function is the lack of a long run equilibrium relationship between consumption and income. For the data in this paper[1] the Johansen maximal eigenvalue test statistic for no cointegration between per capita real consumption and per capita real personal disposable income is 9.6 (10% critical value 12.98) while the trace statistic is 13.3 (10% critical value 15.75) so that the null of no cointegration has to be accepted. It follows from this result that the equation error in this simple consumption income relationship is non-stationary. Suppose there is an omitted variable such that the true specification for the aggregate per capita relationship is:

$$c_t = \gamma_o + \gamma_1 y_t + \omega_t + u_t, \quad t = 1, ..., T$$

where c_t, y_t are *per capita* real consumption and disposable income respectively and ω_t is an omitted non-stationary variable. One explanation for ω_t might be in terms of aggregation; moving away from the representative agent model in favour of a model which allows for aged based heterogeneity, Fair and Dominguez [6, 1991] assume:

$$C_{it} = \gamma_o + \gamma_1 Y_{it} + \phi_1 D1_{it} + ... + \phi_J DJ_{it} + U_{it}, \quad i = 1, ..., N_t; \quad t = 1, ..., T$$

[1]All the data series used were obtained from Duck [4, (2002)] which contains full details of the construction of the series.

where C_{it} and Y_{it} are consumption and income of the ith individual in period t. There are assumed to be J age groups in the population of N_t and $\mathrm{D}j_{it}$ is a (0,1) dummy so that if the ith individual is in age group j at time t then $\mathrm{D}j_{it} = 1$, else $\mathrm{D}j_{it} = 0$. Suppose, for example that the ith individual is in the kth age group, then, in period t, their consumption will be:

$$C_{it} = \gamma_o + \phi_k + \gamma_1 Y_{it} + U_{it}, \quad i = 1, ..., N_t; \quad t = 1, ..., T.$$

The specification captures differences in age distribution in the population by allowing for differing intercepts. Aggregating across all individuals and dividing by population N_t results in a *per capita* relationship of the form:

$$c_t = \gamma_o + \gamma_1 y_t + \phi_1 p_{1t} + ... + \phi_J p_{Jt} + u_t, \quad t = 1, ..., T \tag{1}$$

with c_t and y_t *per capita* consumption and income respectively and p_{jt} is the *proportion* of individuals in age group j in the total population at time t.

In this model then, we would have:

$$\omega_t = \phi_1 p_{1t} + ... + \phi_J p_{Jt}.$$

If the population proportions are stationary, or if they are I(1) and the coefficients $(\phi_1, ..., \phi_J)$ form a cointegrating vector, then ω_t will be stationary. Otherwise ω_t will be non-stationary and, with c_t and y_t, may form a cointegrating relationship, which is the argument in this paper.

To reduce the number of parameters to estimate in (1) we impose the same restrictions on the ϕ coefficients as suggested by Fair and Dominguez [6, (1991, p 1280)], namely that:

$$\phi_j = \delta_o + \delta_1 j + \delta_2 j^2 \text{ and } \sum_{j=1}^{j=J} \phi_j = 0.$$

Without the restriction on the sum of the ϕ coefficients, equation (1) is not identified as the proportion variables sum to unity. The imposition of the quadratic form on the ϕ coefficients captures the “U-shape” prior of consumption behaviour in the life-cycle model. Combining the restrictions gives:

$$\phi = AH\delta$$

where $\phi = [\phi, ..., \phi_J]'$, $A = [I_J - J^{-1}\iota\iota']$, with ι the vector of ones, $\delta = [\delta_1, \delta_2]'$ and:

$$H = \begin{bmatrix} 1 & 1 \\ 2 & 2^2 \\ 3 & 3^2 \\ \vdots & \vdots \\ J & J^2 \end{bmatrix}. \tag{2}$$

The constant, δ_o, can then be obtained from the relation:

$$\delta_o = -J^{-1}\iota' H\delta.$$

Substituting the result in (2) into equation (1) results in:

$$c = \gamma_o + y\gamma_1 + z_1\delta_1 + z_2\delta_2 + u \tag{3}$$

where u is an error vector, $c = [c_1, ..., c_T]'$, $y = [y_1, ..., y_T]'$ and:

$$[z_1, z_2] = PAH$$

with:

$$P = \begin{bmatrix} p_{11} \cdots p_{J1} \\ \vdots \ \ddots \ \vdots \\ p_{1T} \cdots p_{JT} \end{bmatrix}.$$

Fair and Dominguez [6, (1991, p 1280)] estimate a model such as (3) assuming all variables are stationary. There is a great deal of evidence now, of course, that such time series variables as consumption and income are I(1) and the constructed variables $[z_1, z_2]$ are likely to behave like non-stationary variables as they are functions of the population proportions. We tested for a unit root using the sequential Dickey-Fuller minimum t-procedures suggested by Banerjee *et al* [2, (1992)] which allow for broken means and trends. In levels, the null of a unit root could not be rejected at the 5% level for any of the variables c_t, y_t, z_{1t}, z_{2t} using BIC to select the lag length allowing for either a mean break or break in trend. In first differences the null of a unit root can be rejected at the 5% level for all the variables again using BIC to select the lag length. Since we exclude a quadratic trend in levels in the sequel, in first differences we tested for a unit root with breaks in mean only[2]. This suggests that (3) should be modelled in a vector error correction framework - if the variables are cointegrated. We test for cointegration in the next section.

3 Maximum Likelihood Estimates

Given that we are using annual data from 1856 we have to assume structural breaks have occurred over the sample period. The technique we use to test the model, outlined below, restricts us to two break points[3] and so we chose the

[2]For levels for c_t, y_t, z_{1t}, z_{2t}, test statistics allowing for mean breaks were -0.75, -0.19, -3.22,-3.13 with critical 5% value -4.8 and allowing for trend breaks -4.22, -3.46, -2.61,-2.53 with critical 5% value -4.48. For first differences, test statistics allowing for mean breaks were -8.20, -10.35, -4.84, -5.25 with critical 5% value -4.8. Critical values are given in Banerjee et al [2, (1992, Table2, p.278)].

[3]It could be pointed out that the imposition of more than two structural breaks reduces the distinction between trend and difference stationary models (cf. Hansen [8, (2001)]).

arguably most natural breaks of the years immediately after the two World Wars. The first break point is then 1919, with T_1 observations in the first period, and the second break is 1946, with $T_2 - T_1$ observations in the second period. There are T observations altogether so that there are $T - T_2$ observations in the third period. Johansen *et al* [10, (2000)] derive a likelihood ratio test for cointegration in the presence of breaks in trend and mean at known points. We allow for broken means and trends in cointegrating relationships and broken means in first differences so that the VECM can be written:

$$\triangle x_t = \theta_o E_t + \sum_{i=1}^{k} \theta_j \triangle x_{t-j} + \beta(\alpha', \gamma') \begin{pmatrix} x_{t-1} \\ tE_t \end{pmatrix} + \sum_{i=1}^{k+1} \sum_{j=2}^{3} \kappa_{ji} D_{jt-i} + \zeta_t \qquad (4)$$

where $x_t = (c_t, y_t, z_{1t}, z_{2t})', D_{jt} = 1$ for $t = T_{j-1}$, with $T_o = 0$, and $D_{jt} = 0$ otherwise and $E_t' = (E_{1t}', E_{2t}', E_{3t}')$ with $E_{jt} = 1$ for $T_{j-1} + k + 2 \leqq t \leqq T_j$ and zero otherwise and where $T_o = 0$. Both the AIC and BIC select one lag, $k = 1$, for the VECM. The $E_{jt}s$ are dummies for the effective sample period for each sub-period and the $D_{jt-i}s$ have the effect of eliminating the first $k+1$ residuals of each period from the likelihood thereby producing the conditional likelihood function given the initial values in each period.

Johansen *et al* [10, (2000)] derive the distribution of the trace test statistic for testing for the rank of the cointegrating space in a model such as equation (4) and calculate the weights for the estimated response surface to enable critical values to be easily calculated from a $\Gamma-$distribution. Applying these methods[4] to the model in (4) we obtained the results in Table 1.

Table 1. Tests of Rank

Hypothesis	Test Statistic	p-value
$r = 0$	173.317	0.000
$r \leq 1$	86.324	0.003
$r \leq 2$	42.354	0.135
$r \leq 3$	10.749	0.787

Allowing for the two break points the likelihood rank test statistic rejects one cointegrating vector in favour of two but can't reject the null of two against three. We conclude that the rank of the cointegrating space is two and, without any loss of generality we can interpret the first of these as the consumption function in (3). We might expect there to be some impact of demographic effects on per capita income but normalising the coefficient on consumption to zero and the coefficient on the z_{1t} variable to 1 in the second equation and testing the restriction of 0 for

[4]All the results in this paper were obtained using the GAUSS programming language [7, (2001)].

the coefficient on income resulted in a LR test statistic of 2.7, with one degree of freedom, so that the null hypothesis that the coefficient on income is zero cannot be rejected. It follows that there is no interrelationship between income and the demographic variables and we interpret the second equation as a "spurious small sample" relationship between the z_{1t} and z_{2t} variables. Other variables, which we have not included, probably have a much larger impact on income and swamp any demographic effects.

With rank two, we can normalise two coefficients in the consumption equation. The first candidate is obviously the coefficient on c_t and we could also normalise the MPC to unity. This though would mean we would have no standard error for the MPC and it couldn't be tested[5]. In preference, we normalise the coefficient on z_{2t} to unity as the crucial estimate - the minimum point of the age quadratic - is determined by the ratio ($-\delta_1/2\delta_2$), so fixing $\delta_2 = 1$ ensures a minimum whilst allowing the actual minimum point to be determined by the estimate of δ_1 which can then be tested. We therefore set the coefficient on c_t to -1 and the coefficient on z_{1t} to 1 in the consumption function. The estimates of this unrestricted model are displayed in Table 2.

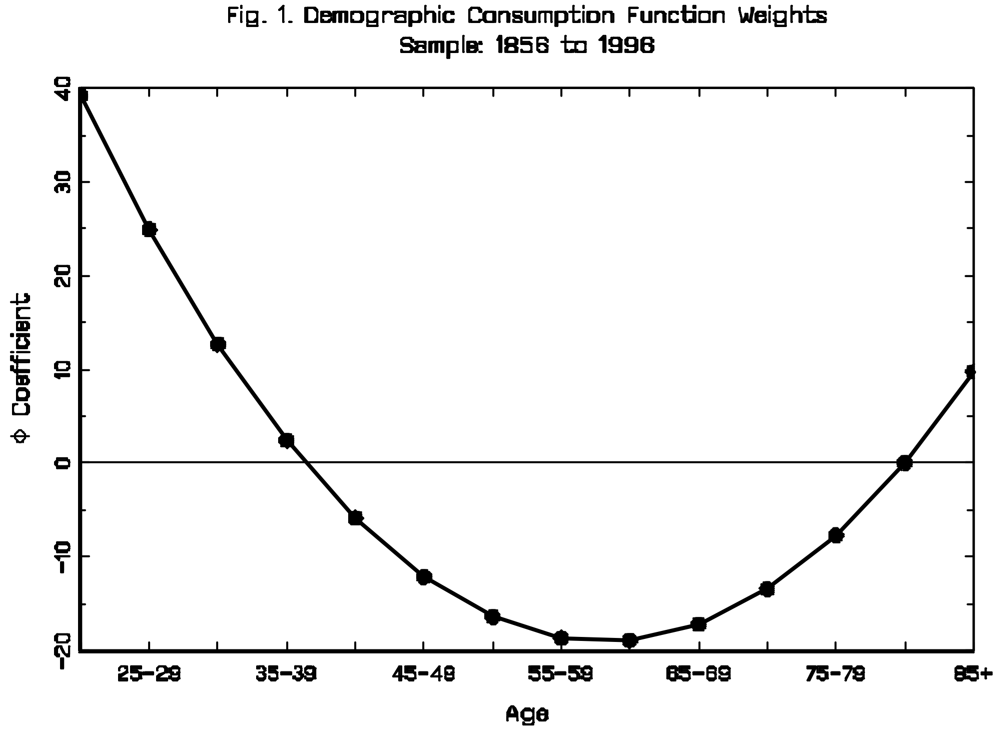
Table 2. Consumption Function Estimates

Variable	c_t	y_t	z_{1t}	z_{2t}	$trend_1$	$trend_2$	$trend_3$
Cointegrating Coefficients	-1	0.971	-17.265	1	-11.046	-21.095	-85.561
Estimated Standard Errors	-	(0.12)	(6.80)	-	(7.03)	(30.18)	(38.13)
Box-Ljung(11) = 9.23, pval=0.6							

The unrestricted estimate of the MPC, 0.971, is not significantly different from 1 and there is evidence of a trend in the first and last sub-periods which although not strong is certainly significant. The null of a white noise error in the corresponding equation in the VECM cannot be rejected with the Box-Ljung

[5] Standard errors were calculated using the switching technique of Doornik [3, (1995)].

statistic.



The turning point for the quadratic in age is given by the ratio ($-\delta_1/2\delta_2$) of the coefficients in equation (4)which is estimated as ($-\widehat{\delta}_1/2\widehat{\delta}_2$) $=(17.265/2) \approx$ 8.6 and is a minimum point giving a"U-shaped" relationship between age and consumption. Figure 1 depicts the implied quadratic obtained from these results[6]. Group 8 corresponds to age group 55-59 and group 9 to 60-64 so that the age distribution curve in the consumption function is the imposed U-shaped curve with a minimum point at around age 60. This is much higher than the findings of Fair and Dominguez [6, (1991, p 1280)] for the USA of an age, at the minimum point, of 41-42 but almost identical to the findings of Attfield [1, (2002)] using quarterly data for the post second world war period.

[6] The inclusion of a 'z_3' variable so that the age relationship is a cubic is not valid in the setup in this paper as z_3 apears to be integrated of order greater than unity. That is, the null of a unit root, with a mean shift, in the FIRST DIFFERENCE of z_3 cannot be rejected at the 5% level (test statistic: -4.46, critical value: -4.8).

4 Conclusion

In this paper we have estimated a consumption function over the period 1856 to 1956 allowing for structural breaks for the two world wars and which includes demographic variables in a vector error correction setting. The coefficients on the demographic age variables have an important impact on consumption in the way predicted by the LCH. The asymmetry between these coefficients on younger age groups, below 30 years of age, and older age groups, 75 and over, apparent in the curve in figure 1, implies that the "ageing population" effect of an increase in the proportion of older citizens matched with a decline in the proportion of the youngest age groups, will lead to a decline in overall per capita consumption for equivalent income levels.

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