A Tournament Model of Fund Management

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Abstract

We develop a two-period model of portfolio management where managers aim for a higher-valued portfolio than their rivals whilst making a respectable return. It predicts that "losing" managers will show a greater tendency to adopt extreme portfolios; this will be more marked the further behind the fund is and the nearer the final ranking period; and, when taking extreme positions, losing managers will choose high /low market exposure depending upon whether they (a) expect a rising or falling market, and (b) have sufficient assets to take advantage of a rising market. Tests, using UK investment trusts data, broadly support these predictions.

Classification Code: G1, G10, G11
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1 Introduction

There is at present considerable interest in whether decisions taken by managers of pooled investment vehicles are optimal for the individuals on whose behalf those decisions are made. One area of research, such as Brown, Goetzmann and Park (2001) and Carpenter (2000), is the analysis of the impact on risk-taking of a convex managerial reward system. Other work investigates whether the ranking of a mutual fund has consequences for future inflows of investment funds (see, for example, Chevalier and Ellison (CE) (1997) and Goetzmann and Peles (1997)). Our paper concentrates on the tournament aspect of the fund management sector and its effect on portfolio choice, taking as given that ranking is an important consideration for fund managers. We develop a model of portfolio management in which fund managers aim to achieve a higher-valued portfolio than their rivals whilst making a 'respectable' return.[1] We draw out from the model a set of predictions which we test using monthly data on UK investment trusts covering the four years ending March 31, 2001.

Our model extends the literature in three main ways. First, it is a genuine two-period model. Existing work on tournaments and fund management is based on the intuition derived from theoretical models that essentially relate to one period only (see, for example Taylor (2000)). In such models, the portfolio values are inherited at the beginning of the period, and fund managers take *one* set of decisions to maximise the probability of winning at the end of that period. They predict that inheritors of an underperforming portfolio will take more risk. Our model explicitly allows for *two* sequential sets of decisions that are taken to maximise the probability of winning at some given future date. Implications can be loosely drawn from it about a more prolonged decision process. Secondly, our definition of risk-taking is different from the definition other models employ. Empirical papers, such as Brown, Harlow and Starks (BHS) (1996) and CE (1997), define risk primarily in terms of variability of portfolio value. For example, BHS compare portfolio standard deviations at year-ends with those at the mid-way stage. In contrast, we categorise portfolio positions by how 'extreme' they are: an 'extreme' portfolio consists mainly of cash or mainly of shares, that is, very low or very high market exposure. Which of these extreme positions is adopted depends primarily, but not entirely, on whether the fund manager believes the market will rise or fall. In other words, we incorporate the idea of taking bets, either with or against the market.[2]

[1] If we assume that fund managers are *exclusively* driven by the aim of beating their rivals, then the reward to a fund manager who made heavy losses would be exactly the same as if he made exceptional profits, provided all other fund managers made even worse losses, even if share prices were rising.

[2] CE decompose the portfolio variance into a specific risk component and a component reflecting the variance caused by moving the portfolio's beta away from 1. The latter

Finally, we set the problem within a signal-extraction framework which allows us to recognise the market timing aspect of fund managers' activities: they try to assess whether the market is falling or rising, and adopt appropriately defensive or aggressive investment strategies (see, for example, Pesaran and Timmermann (2000) for a recent UK study and Granger (1992) for a summary of related results for the US).

The testable hypotheses we derive from our model are:

- Managers of 'losing funds' will show a greater tendency than average to adopt extreme portfolios; this tendency will be more marked the further 'behind' the fund is.
- This tendency to adopt extreme portfolios is likely to increase as the final ranking period approaches.
- When taking an extreme position, the manager of a losing fund will choose between high market exposure or low market exposure according to (a) whether he expects the market to rise or fall;[3] and (b) whether he has sufficient assets to warrant adopting a high market exposure when he expects the market to rise. This last condition is one not found elsewhere in the literature, and arises because our model is based on absolute share price movements rather than percentage returns, as is more conventional in the finance literature. This unorthodox approach may help explain the 'anomaly' in Elton *et al.* (2001), who find that funds often adopt betas less than one when they might expect a higher rate of return from higher beta strategies.

Our empirical procedures differ from those generally used in US studies into tournament models of portfolio management. US studies tend to examine *changes* in the risk-taking strategy of loser funds between the mid-point and the end of the 'tournament' at 31 December, and have, on the whole, generated inconclusive results. But, in the UK at least, the date of the final ranking period may be less clear-cut. Furthermore, losers may have been losers for some time when they are sampled; and, finally, losers under one ranking criterion - for example, performance over the previous year - may not be losers under another - for example, performance over the last three years. For all these reasons, *changes* in strategy may well not be observed between one period and another. In this paper we examine instead *cross-sectional* differences between losers and winners, where losers and winners

enters their regressions as a dependent variable in the form $|\beta\text{-}1|$, so that no distinction is made between decisions to increase the portfolio beta above 1, or to reduce it below 1. CE describe such risk as "variance from implicit bets with or against the market as a whole" (p.1187). We distinguish between the two types of bet.

[3] We use the term 'he' throughout to denote the more cumbersome 'he or she'.

are defined in three different ways, following the ranking criteria of the Financial Times newspaper. We find strong support for the three hypotheses mentioned above.

The paper is in two broad sections. In the first we develop our model and draw out its main implications; and in the second we explain our data and the results of our tests. We end with a set of conclusions and a brief discussion of the implications of our findings.

2 A Model of Fund Management

2.1 Assumptions and Notation

We consider two representative, risk-neutral traders, denoted A and B. B manages the 'average' sector fund against which A's performance will be measured: specifically, A will 'win' if, in the final period, his portfolio is worth more than B's.[4] A's decisions are the focus of our model.

We assume three periods. At the beginning of the first period, $t-1$, the two traders are endowed with initial portfolios consisting of a combination of a risky asset (shares) and a risk-free asset (cash). Borrowing is precluded.[5] B's portfolio (the average portfolio) consists of N^B shares and cash, C^B. Cash gives a zero return. The number of shares in B's portfolio is assumed to be known to trader A and does not change over the three periods, although the value of B's portfolio changes as the share price changes. The share price, P, is assumed to follow a random walk, so that

$$P_{i+1} = P_i + \varepsilon_{i+1} \tag{1}$$

where ε_{i+1} is the 'shock' to the share price between periods i and $i+1$ ($i = t-1, t$), is i.i.d, and is known to be distributed as $N(0, \sigma_\varepsilon^2)$. We assume that trader A has no information about the size of the shock to the share price more than one period ahead. His expectation of ε_{i+1} formed in period $i-1$ will therefore be zero with an associated variance of σ_ε^2. But, importantly for what follows, we assume that trader A obtains 'news' during period i which allows him to make an inference about the shock to the share price in period $i+1$.This news can be viewed as information that is specific to A, or as A's interpretation of more widely available information. Formally, we decompose the shock to the share price into trader A's inference plus an error that is uncorrelated with that inference:

$$\varepsilon_{i+1} = \varepsilon_{i+1}^A + v_{i+1}^A \tag{2}$$

where ε_{i+1}^A is trader A's inference of the price shock in period $i+1$ *after* hearing this news in period i; and the v_{i+1}^A are i.i.d. and known to be

[4] We assume that A's portfolio is sufficiently small within the sector to allow the performance of the average fund to be exogenous.

[5] The predictions of the model hold as long as borrowing is restricted to some degree.

distributed as $N(0, \sigma^2_{v^A})$. The inverse of $\sigma^2_{v^A}$ can be seen as a measure of the precision of A's beliefs - the greater is $\sigma^2_{v^A}$, the less accurate are trader A's inferences about future share price movements. Note that $\sigma^2_{v^A} \leq \sigma^2_{\varepsilon}$ (that is, receipt of the news does not decrease the precision of A's inference about the shock to the share price[6]).

We adopt the following other notation:

N^A_i is the number of shares held by trader A *at the beginning of* period i, where $i = t-1, t, t+1$;

N^B is the constant number of shares held by trader B;

C^A_i is the amount of cash held by trader A *at the beginning of* period i;

C^B is the constant amount of cash held by trader B; and

PF^J_i is the value of trader J's portfolio in period i; $J = A, B$.

Figure 1 shows a timeline of events, together with the decisions that A must take.

FIGURE 1 ABOUT HERE

2.2 Trader A's Decision Process

The strategy underlying trader A's decision process is the same for both periods: his aim is to maximise the probability that the value of his portfolio in period $t+1$ will exceed that of B's portfolio. He attempts to achieve this aim by arranging his portfolio in period i (i.e. selecting the number of shares he will be holding at the beginning of period $i+1$) *after* hearing the news relating to the share price in period $i+1$. His portfolio choice depends on whether the news leads him to expect the price to rise or to fall.

The precise decisions A makes depend on the period in which he makes them. We first analyse A's decision in period t, the period immediately before the 'final period' when the tournament is decided. We then analyse his decision in period $t-1$.

2.2.1 A's decision in period t

The value of trader J's portfolio in period i can be written,

$$PF^J_i = P_i N^J_i + C^J_i \tag{3}$$

And each trader's budget constraint can be written

$$P_i N^J_{i+1} = P_i N^J_i + C^J_i - C^J_{i+1} \tag{4}$$

[6]This a commonly-used definition of information. A notable exception is Beaver (1968), who argues that "a decision maker may be more uncertain about a given event after receiving a message about the event than he was before he received the message. To use Theil's terminology, the entropy may increase as a result of a message, yet the message has information content." (footnote 8, citing Theil 1967 Ch.1)

Updating equation (3) by one period and using equations (1) and (4) we can write

$$PF^J_{i+1} = PF^J_i + \varepsilon_{i+1} N^J_{i+1} \tag{5}$$

This states that the value of trader J's portfolio in any period equals its value in the previous period, adjusted by the shock to the share price multiplied by the number of shares in the portfolio when the shock occurs. Three implications follow directly from this equation: (i) if A's portfolio value were *higher* than B's at period t then A would be certain to win the tournament if he held the *same* number of shares as B since any share price movement would then affect the two portfolios equally;[7] (ii) for the same reason, if A were *behind* at period t and held the *same* number of shares as B then he would be certain to lose; and (iii) there are two ways in which A might overtake B: by holding *more* shares than B when the share price *rises* sufficiently; or holding *fewer* shares when the share price *falls* sufficiently. The first two of these implications are straightforward. In the rest of this subsection we consider the more complex third implication and the decision which it leads A to make. We therefore impose the condition that A's portfolio value at the beginning of period t is less than B's, that is $PF^A_t < PF^B_t$.

Under this condition A's problem in period t, formally stated, is

$$\max_{N^A_{t+1}} \left[\Pr\left(PF^A_{t+1} > PF^B_{t+1} \right) \right] \tag{6}$$

Given that A is maximising this probability *after* hearing the news relating to the share-price movement between period t and $t+1$, his problem can be conveniently recast as that of selecting N^A_{t+1} to,

$$\max_{N^A_{t+1}} \int_{-\infty}^{-L} f(x)dx \equiv \max_{N^A_{t+1}} N(-L) \tag{7}$$

where $f(x)$ is the standard Normal distribution density function; $N(.)$ is the corresponding cumulative density function; and the limit L can be written,

$$L = \frac{1}{\sigma^A_v} \left[\frac{PF^B_t - PF^A_t - \varepsilon^A_{t+1}(N^A_{t+1} - N^B)}{|N^A_{t+1} - N^B|} \right] \tag{8}$$

From the properties of the Normal distribution it is clear that the probability that A wins is maximised when L is minimised. If A chooses $N^A_{t+1} > N^B$ then the limit L will be negatively related to N^A_{t+1}. This implies that if

[7] This is analogous to 'covering' in a two-boat sailing race, or to indexing in the financial markets.

A decides to hold *more* shares than B he should hold the *maximum* number of shares he can, which is PF_t^A/P_t , and the limit L becomes

$$L_S \equiv \frac{1}{\sigma_v^A}\left[\frac{PF_t^B - PF_t^A}{PF_t^A/P_t - N^B} - \varepsilon_{t+1}^A\right] \tag{9}$$

If $N_{t+1}^A < N^B$ then L is positively related to $N_{t+1}^A$and A will want to hold as few shares as possible, which is zero. In this case L becomes

$$L_C \equiv \frac{1}{\sigma_v^A}\left[\frac{PF_t^B - PF_t^A}{N^B} + \varepsilon_{t+1}^A\right] \tag{10}$$

The two limits, L_S and L_C, allow us to identify ε_{t+1}^*, the value of ε_{t+1}^A which will trigger the movement from an all-cash to an all-shares strategy. Formally, it will be where $L_S = L_C$ and so,

$$\varepsilon_{t+1}^* \equiv \left(\frac{PF_t^B - PF_t^A}{2}\right)\left(\frac{1}{PF_t^A/P_t - N^B} - \frac{1}{N^B}\right) \tag{11}$$

If ε_{t+1}^A exceeds ε_{t+1}^*, A's optimum strategy is to choose shares, hoping that the price rises sufficiently to close the gap between his portfolio and B's. If ε_{t+1}^A is less than ε_{t+1}^*, his optimum strategy is to choose cash and hope that the price falls sufficiently to eliminate B's advantage. Notice that ε_{t+1}^* will be zero if the maximum number of shares that A can hold $\left(PF_t^A/P_t\right)$equals twice the number that B holds. The reason for this is that a *rise* in price of \$$x$ when A is holding $2N^B$ shares, and a *fall* in price of \$$x$ when he is holding none, will cause exactly the same closure of the gap between A's and B's portfolios. The probability of winning with the all-shares strategy and the probability of winning with the all-cash strategy are, in this sense and under this special condition, symmetric around a zero value of ε_{t+1}^A. On the other hand, if A were holding more than $2N^B$ shares, then a price rise of \$$x$ would mean a larger closing of the gap than would a price fall of \$$x$ if he held no shares. Hence, if A has sufficient funds to hold more than $2N^B$ shares ε_{t+1}^* is negative and A's optimal strategy may be to hold all shares even though he expects a (slight) *fall* in the share price. The opposite is also true when A cannot hold more than $2N^B$ shares.

A's decision process in period t can now be summarised as follows:

1. If A is in front at t, he should hold N^B shares and will win at $t+1$ with probability 1.

2. If A is behind at t and his portfolio is not sufficiently valuable to enable him to hold more shares than B he should hold all cash and the probability that he wins will be $N(-L_C)$.

3. If A is behind at t, but his portfolio is sufficiently valuable to enable him to hold more shares than B, he will, depending upon the value of ε_{t+1}^{A}, either hold all shares (in which case the probability that he wins is $N(-L_S)$), or all cash (with a probability of winning equal to $N(-L_C)$) .

A's decision at period t clearly has an all-or-nothing quality: provided A inherits a portfolio in period t which is lower in value than B's A will hold either all cash or all shares; only if he inherits a portfolio which is higher in value than B's will A hold a diversified portfolio. As we shall now show, the same is not true of period $t-1$.

2.2.2 A's decision at $t-1$

A's problem at $t-1$ is to choose the number of shares to hold in period t, N_t^A, to maximise the probability that he will win in period $t+1$. This probability, π^w, can be written,

$$\pi^w \equiv \pi^f + (1-\pi^f)\pi^{wb} \tag{12}$$

where
π^f is the probability that A gets in front at t;[8] and
π^{wb} is the probability that A will win at $t+1$ even if he is behind at t.

If A's portfolio in period $t-1$ exceeds B's then his problem is trivial - he should hold N^B shares in periods t and $t+1$ and will then be certain to win. So we consider only the case where A is behind at $t-1$. In these circumstances, A can no longer act simply to maximise π^f, the probability of having a higher portfolio value than B in the next period. Were he to do so, he would, by the logic of the previous section, choose either an all-cash or an all-shares strategy in period $t-1$ and thereby run the risk of falling so much further behind (if the share price moved the 'wrong' way) that he would have no chance of making up the gap in the final period. In some circumstances an extreme strategy might still be optimal, for example if he were so far behind that only successive extreme strategies with two favourable share price movements offer him a chance of overtaking B. But since in this two-period model he will still have another chance to win, he may sometimes be willing at $t-1$ to sacrifice some probability of getting ahead immediately, in exchange for reducing the probability of being too far behind at t to win. More formally, to maximise π^w he must consider the effects of his decision on both π^f and π^{wb}, the probability that he will win at $t+1$ despite being behind at period t. To show that this may lead to A adopting neither an all-shares nor an all-cash strategy in period $t-1$ we need to work out the various probabilities in equation (12).

[8] Note that once A does get in front, he simply needs to hold N^B shares and will then be certain to win.

The calculation of π^f This is simply the probability derived in the previous section lagged one period, and it can be derived by rolling back equations (7) and (8) one period to give:

1. If $N_t^A > N^B$ $\pi^f = N(-L_1)$ where $L_1 \equiv \frac{1}{\sigma_v^A}\left[\frac{PF_{t-1}^B - PF_{t-1}^A}{N_t^A - N^B} - \varepsilon_t^A\right]$
2. If $N_t^A < N^B$ $\pi^f = N(L_1)$
3. If $N_t^A = N^B$ $\pi^f = 0$

The calculation of π^{wb} This probability depends on A's assessment at $t-1$ of his probable strategy at t. He knows that he will either choose to carry an all-shares portfolio into period $t+1$, in which case he will win if ε_{t+1} is sufficiently high; or he will choose all cash, in which case he will win if ε_{t+1} is sufficiently low. Calculating π^{wb} therefore involves the calculation of three separate probabilities: (i) the probability that A will choose an all-shares strategy at period t;[9] (ii) the probability that ε_{t+1} is sufficiently high to ensure that this strategy allows A to overtake B in period $t+1$; and (iii) the probability that ε_{t+1} is sufficiently low to ensure that the alternative all-cash strategy allows A to overtake B in period $t+1$. The detailed derivation of these probabilities, all of which are assessed in period $t-1$, is given in the appendix. Here we give the main results.

First, A's assessment (in period $t-1$) of the probability that he will choose an all-shares strategy for the final period will depend upon the relative shareholdings that A and B carry into period t. This is because these shareholdings will determine the relative portfolio values after the period t price change, and hence the strategies available at the end of that period. The precise relationships are as follows:

1. If A chooses $N_t^A > 2N^B$ the probability, assessed in period $t-1$, that he will choose an all-shares strategy for the final period equals $N(-L_2).N(-L_3)$
2. If A chooses $N^B < N_t^A \leq 2N^B$ it equals $N(L_2).N(-L_3)$
3. If A chooses $N_t^A \leq N^B$ it equals $N(L_2).N(L_3)$

where:

$$L_2 = \frac{1}{\sigma_v^A}\left[\frac{2N^B P_{t-1} - PF_{t-1}^A}{N_t^A - 2N^B} - \varepsilon_t^A\right]$$

$$L_3 = \frac{1}{\sigma_v^A}\left[\frac{N^B P_{t-1} - PF_{t-1}^A}{N_t^A - N^B} - \varepsilon_t^A\right]$$

[9]The probability that A will choose an all-cash strategy is, as the previous section implied, simply one minus the probability that A selects an all-shares strategy.

Secondly, the probability (evaluated in period $t-1$) that ε_{t+1} is sufficiently high to ensure that an all-shares strategy allows A to overtake B in period $t+1$ can be written as

$$\pi^H = \Pr\left\{ \begin{array}{c} \varepsilon_{t+1}\left(PF^A_{t-1} - N^B P_{t-1}\right) - \varepsilon_t\left(PF^B_{t-1} - PF^A_{t-1}\right) + \\ \left(N^A_t - N^B\right)\varepsilon_t(P_{t-1} + \varepsilon_t + \varepsilon_{t+1}) > P_{t-1}\left(PF^B_{t-1} - PF^A_{t-1)}\right) \end{array} \right\} \quad (13)$$

This probability cannot be expressed analytically and needs to be simulated. We return to this below.

Finally, the probability (evaluated in period $t-1$) that ε_{t+1} is sufficiently low to ensure that an all-cash strategy allows A to overtake B in period $t+1$ can be written as

$$\pi^L = N(L_4) \quad (14)$$

where $L_4 = \frac{PF^A_{t-1} - PF^B_{t-1} - \left(N^B - N^A_t\right)\varepsilon^A_t}{\sqrt{N^{B2}\sigma^2_\varepsilon + \left(N^B - N^A_t\right)^2 \sigma^2_{vA}}}$

2.2.3 A's decision strategy at $t-1$

The previous section shows that A's evaluation of π^w will vary according to the relative shareholdings of the two traders. Using conditions 1-3 in the calculations of π^f and π^{wb}, we can summarise as follows.

If A selects a value of N^A_t such that $2N^B < N^A_t$ then he will evaluate π^w as,

$$\pi^w_1 = N(-L_1) + N(L_1)\left\{ \begin{array}{c} \pi^H N(-L_2)\,N(-L_3) \\ +\left[1 - N(-L_2)\,N(-L_3)\right]N(L_4) \end{array} \right\} \quad (15)$$

If A selects a value of N^A_t such that $N^B < N^A_t \leq 2N^B$ then he will evaluate π^w as,

$$\pi^w_2 = N(-L_1) + N(L_1)\left\{ \begin{array}{c} \pi^H N(L_2)\,N(-L_3) \\ +\left[1 - N(L_2)\,N(-L_3)\right]N(L_4) \end{array} \right\} \quad (16)$$

And if he selects a value of N^A_t such that $0 < N^A_t \leq N^B$ then he will evaluate π^w as,

$$\pi^w_3 = N(L_1) + N(-L_1)\left\{ \begin{array}{c} \pi^H N(L_2)\,N(L_3) \\ +\left[1 - N(L_2)\,N(L_3)\right]N(L_4) \end{array} \right\} \quad (17)$$

A's decision strategy in period $t-1$ can therefore be seen as calculating the particular values of N^A_t that maximise each of π^w_1, π^w_2, and π^w_3, and from these to select the value of N^A_t which generates the maximum among π^w_1, π^w_2, and π^w_3. In following this strategy A is clearly constrained by the value of his portfolio in period $t-1$. If his portfolio allows him to hold more than $2N^B$ shares then all three of the probabilities above will enter into his decision. If the value of his portfolio is such that he can hold more than N^B shares, but not more than $2N^B$ shares, then only the second and third probabilities are relevant. And if he cannot hold N^B shares then only the third probability is relevant.

2.3 Simulations of A's decision strategy at time period $t-1$

Because π^H has a non-standard distribution, A's selection of an optimal value of N_t^A cannot be derived entirely analytically. To derive A's optimal decision - the optimal value of N_t^A - we therefore simulated the value of π^H for given values of the exogenous or pre-determined variables PF_{t-1}^B, PF_{t-1}^A, N^B, P_{t-1}, σ_ε^2, $\sigma_{\upsilon^A}^2$, and ε_t^A, and the choice variable, N_t^A. For these simulations we normalised $PF_{t-1}^B$and P_{t-1} to 1 and, for each combination of PF_{t-1}^A, N^B, σ_ε^2, and $\sigma_{\upsilon^A}^2$, we generated $10,000$ values of ε_t^A and ν_t^A to derive the value of π^H. From this and the resulting values of L_1, L_2 , L_3 and L_4, we derived the optimal value of N_t^A in each case.[10] Table 1 and panels A to E of figure 2 illustrate some of the key results that emerge from this procedure.

In table 1 we present the results of repeatedly using the procedure to derive the optimal value of N_t^A for selected values of PF_{t-1}^A, N^B, σ_ε^2, $\sigma_{\upsilon^A}^2$, and ε_t^A, and then calculating the proportion of times this optimal value was within one step of either of the two extreme strategies, *viz.* $N_t^A = 0$ and $N_t^A = PF_{t-1}^A$[11]. Each cell in table 1 therefore gives the proportion of times the optimal value of N_t^A was 'extreme' for the particular values of PF_{t-1}^A, N^B, $\sigma_{\upsilon^A}^2$, σ_ε^2 and ε_t^A indicated. Figure 2 presents, again for selected values of the relevant variables, the probability of A's portfolio being of higher value than B's in period $t+1$, as a function of A's decision in period $t-1$ about N_t^A.

TABLE 1 ABOUT HERE

FIGURE 2 ABOUT HERE

Both the table and figure 2 suggest the following key features of the relationship between A's optimal selection of N_t^A, and PF_{t-1}^A, N^B, ε_t^A and $\sigma_{\upsilon^A}^2$.

[10] The simulations were carried out using GAUSS (1992) to generate the required 10,000 values of ε_t^A and ν_t^A. In each case the optimal value of N_t^A was derived from a grid-search from 0 to PF_{t-1}^A in steps of 0.003. The ranges of values for σ_ε^2, $\sigma_{\nu^A}^2$ and ε_t^A used in the simulations were chosen with reference to 'normal' market behaviour, based on a normalised share price of 1. σ_ε^2, the variance of the share price, ranged between 0.04 and 0.01. $\sigma_{\nu^A}^2$ ranged from a minimum of 0.0025 to a maximum, governed by the value of σ_ε^2, such that $\sigma_{\nu^A} = \sigma_\varepsilon - 0.05$. The value of ε_t^A ranged between 0.1 and -0.1.

[11] In each case we repeated the procedure 100 times.

- First, there are certain values of these variables for which a diversified portfolio is optimal even though the motive is to win in period $t+1$. The thicker lines in panels A-D show cases where a mildly diversified portfolio is optimal. In each case the optimal portfolio - the peak in the line - consists of around 90% shares. The thinner lines show cases where the optimal portfolio is either all cash or all shares, i.e. they peak at 1 or 0.

- More specifically, if the value of A's inherited portfolio value falls sufficiently below B's, A will tend to adopt either the extreme all-cash or the extreme all-shares strategy depending upon his view of the likely change in the share price. The thinner line in panel B gives an example of an all-shares strategy being optimal because A's portfolio is well below B's *and* A expects the share price to rise.

- Even if A's and B's portfolios are close in period $t-1$ it will still be optimal for A to adopt an extreme strategy in period $t-1$ if the expected change in the share's price is itself sufficiently extreme: the safety afforded by a diversified portfolio is traded off against the anticipated gain from holding an extreme portfolio. For example, in panel C the thinner line indicates that if A expects a fall of 2% in the share price it will induce him to adopt an extreme all-cash strategy even though his portfolio is close in value to B's.

- If A inherits a portfolio which, whilst lower in value than B's in period $t-1$, is close to it, and if the expected change in the price of shares between periods $t-1$ and t is itself modest then it can be optimal for A to adopt the traditionally 'safer' strategy in period $t-1$ of selecting a diversified portfolio. A is, as it were, keeping his powder dry for the extreme strategy he knows he will have to adopt in period t. It is, in these special circumstances, optimal for him not to run the risk of putting himself even further behind in period t and thereby reducing the chances of success from whichever extreme strategy he adopts in period t. Such cases are illustrated in the thicker lines in panels A-D.[12]

[12] The sharp dip in A's estimated probability of winning when he selects a value of N_t^A close to N^B - most clearly apparent in panels D and E - arises because if A decides to carry into period t the same number of shares as B, then, *whatever happens to the share price between the two periods*, he cannot close the gap on B between period $t-1$ and t. Consequently, his chances of winning are governed solely by whichever extreme strategy he will adopt in period t and by the share price movement in the final period. Since, in the examples illustrated, A's and B's portfolio values are very close in period $t-1$, and since the expected value of the share price movement *two* periods ahead is zero, A's estimate in period $t-1$ of his probability of winning with whatever extreme strategy he adopts in period t is approximately 0.5. Should he choose any other value of N_t^A A will have some chance of closing the gap on B between periods $t-1$ and t, though of course he will also then run the risk of *widening* the gap and hence his estimated probability of winning may be greater or less than 0.5.

3 Empirical testing

The model developed in section 2 suggests the following testable predictions about the behaviour of fund managers who are subject to periodic ranking:

- Managers of losing funds, by the mere fact of being losers, will be driven to adopt extreme portfolios. Hence, in a sample of fund managers, the tendency of the losers to adopt extreme portfolios will be more marked than it is for winners.
- The incentive to adopt an extreme portfolio will be greater the worse the fund's performance has been. In a sample of losing funds the tendency of any fund to adopt an extreme portfolio will therefore be greater the worse the fund's performance.
- The tendency for losing funds to adopt extreme portfolios will be sensitive to time. As the date of the final ranking period approaches, all losers will be more likely to adopt an extreme portfolio but this tendency will become less dependent on the degree of under-performance.
- When choosing between two possible extreme portfolios, managers of losing funds are more likely to choose the all-shares portfolio when (a) they are anticipating a share price rise and (b) the maximum number of shares they can hold generates sufficient market exposure for them to win if the market does rise.

In this section we test all these predictions by estimating a series of probit models, but before doing so we clearly need data on the behaviour of investment funds that are subject to periodic ranking, and we need precise empirical counterparts to theoretical constructs employed above, *viz.* 'losing funds', 'extreme portfolios', the 'final period', 'anticipated market movements' and 'maximum shareholdings'.

The US literature in this area is concerned with mutual funds, which are open-ended pooled investment vehicles. The exact UK equivalent is the unit trust, but we will concentrate our tests on investment trusts, which are closed-ended companies offering opportunities for pooled investment.[13] Although there is no inflow and outflow of funds to provide an incentive for a high ranking, as there is with open-ended funds, investment trusts are ranked regularly and publicly both with respect to one another and in comparison with the industry average, so it is clear that rankings are of some considerable concern within the sector.

[13] We choose investment trusts rather than unit trusts because of the availability of a more comprehensive data set.

3.1 Losing funds

Our procedure for identifying losing funds is to define investment trust values according to a base date, 0, from which portfolio performance will be measured. The normalised fund value of investment trust j at some date τ, where $\tau > 0$, is defined as $IT_\tau^j \equiv P_\tau^j / P_0^j$, where P_τ^j is the share price of the investment trust at date τ. Similarly, the normalised value of the 'average' fund, IT_τ^{av}, is the relative price of the relevant sector index. Losing funds are then identified as funds for which $IT_\tau^{av} - IT_\tau^j$ is positive: the higher this measure is, the further behind is the fund.[14]

3.2 Extreme portfolios

In our model, managers have a choice between one risky and one risk-free asset. Clearly in reality there are many risky assets to choose from, but investment trusts tend to track a market index so we use an appropriate index to represent 'the' risky asset. Although the Association of Investment Trust Companies publish comprehensive UK monthly statistics, there is insufficient detailed data on fund composition, so we proxy the proportion of the fund held in the form of shares by the trust's market model beta.[15] Funds with extreme portfolios (i.e. extreme betas) in any period are then defined as those whose betas are, in that period, more than 1 standard deviation away from the average beta of their sector. We discuss below how we measure betas, and the way in which we classify funds and hence extreme betas.

3.3 The final period

The empirical counterpart to the model's 'final period' (period $t+1$) is less clear. US work on mutual funds, such as Busse (2001) and Brown *et al.* (1996), takes the final rankings to be at the end of the calendar year (31 December). UK investment trusts are ranked weekly in the FT and, furthermore, the rankings are based on one-, three- and five-year performance so each fund has three different rankings in the FT alone. (There are also other sources of rankings, but the FT is the most widely read financial paper.) Consequently there are many 'intermediate' ranking days and no obvious final ranking day, though one might conjecture that there are periods in the UK when rankings may be more important than at other

[14] An alternative measure would base the fund value on the value of the investments in the fund, the 'net asset value' (NAV). However, it is well-established that an investment trust's shares trade at a discount to its NAV and we are interested in the value of the portfolio from the point of view of an investor in the fund, not an investor in the underlying assets of the fund.

[15] Using a fund's beta as our risk measure addresses the problem noted in Busse (2001), who stresses the importance of distinguishing between risk "changes that are due to changes in common factors and changes attributable to the fund managers." (p.72)

times. The most likely of these are the end of December, when there are many reviews assessing fund managers' performance, and the month or so leading up to the end of the tax year (31 March for companies and 5 April for individuals) when investors are disposing of surplus funds and may well be looking at rankings to help them decide where to invest.

3.4 Maximum shareholdings

While it is more usual to discuss portfolio composition in terms of proportions rather than absolute values, our model shows that this obscures an issue that is important when borrowing is precluded or restricted[16]. While the loser can always hold the same *proportion* of shares as the average fund, he cannot always expose the same *total asset value* to the market. It may be not be possible in a rising market for a loser fund to stake a sufficiently large bet to win.[17] Indeed, this may explain the anomaly in Elton *et al.* (2001, Abstract): "Surprisingly, funds on average have a beta less than one when a beta greater than one would have provided a higher expected return with potentially the same tracking error."

To proxy a fund's maximum possible shareholding we use the NAV of a fund - a measure of the total assets available for exposure to the market. As with the normalised value of investment trust j, we can define a normalised NAV: $NAV_\tau^j \equiv NAV_\tau^{Uj}/NAV_0^{Uj}$, where NAV^{Uj} is the unadjusted net asset value of investment trust j. As discussed above, β is the proxy for the *proportion* of assets held in the form of shares, so the *value* of these assets at time τ is $\beta_\tau^j \times NAV_\tau^j$. The maximum number of shares that fund J can hold relative to the average fund, $MAXN$, is therefore $MAXN_\tau^j \equiv (NAV_\tau^j)/(\beta_\tau^{av} NAV_\tau^{av})$.

3.5 Anticipated market movements

Observations on each fund manager's news about future share price movements are not available. To capture any common element in the news in any

[16] Investment trusts can borrow within the limits laid down by their Articles of Association. As with any company there are other limits to their borrowing that are a function of the value of their assets. It is interesting to note that the 'split-capital' investment trust (a form of fund which was introduced into the UK relatively recently and is often designed to increase volatility and expected returns by gearing up) has, at the time of writing, been subject to much adverse comment in the financial press: "Analysts are concerned that many of these new trusts ... have heavy borrowings [and] are now close to breaching their asset-to-loan agreements..." (Financial Times, 11 August, 2001).

[17] Taylor (2000), which most closely resembles our paper, avoids this issue by implicitly assuming in his equation (1) that the final portfolio value is the sum of the performances in the first and the second halves of the year, rather than the result of a compounded return on the portfolio value at the end of the first half. This is equivalent to assuming that there are two independent tournaments, the winner being the fund with the best performance overall.

particular month we have made use of the monthly summaries produced by the UK Treasury of the forecasts of the rate of growth of UK GDP over the forthcoming year made by (generally) 35 independent bodies such as Barclays Bank, Goldman Sachs and Merill Lynch. We take the average of these forecasts for month i as a indicator of each fund's ε_{i+1}^{A}; specifically we assume that a high value for this average suggests that each fund is more likely to expect a rise in share prices; a low value suggests that each fund is more likely to expect a price fall.

3.6 Data

We used the categories of investment trusts designated by the Association of Investment Trust Companies as at May 31, 2001[18] and, since this makes the estimation of expected market movements easier, we chose those who specialise in UK companies. Trusts are either grouped by 'style' such as high income or growth, or by sector specialism such as biotechnology and property. There were 47 sector specialist funds in total, specialising in 11 different sectors, giving sample sizes that were too small. The four style categories were high income (31 companies), high growth (30), growth and income (45) and smaller companies (35). To carry out our tests we need to estimate betas for each fund and this of course requires some appropriate market index. It was difficult to construct a meaningful market index for the first two categories, so our final sample consisted of the 80 companies in the last two categories. Of these, 19 had insufficiently long price histories, so the sample was reduced to 61 companies, 33 in growth and income and 28 in smaller companies.[19]

Other than the UK Treasury data, all data were collected from Datastream, which provides prices and NAVs for investment trusts within each sector and for the sector index (the 'average' fund). The sample period is the 48 months ended February 28, 2001. Treasury forecasts were available only from September 1997, so tests involving estimates of future market movements were limited to 42 months of observations.

At each month end the sample of investment trusts was divided into those who performed worse than the sector index, the losers, and those who performed better than the sector index. The 'winner' category excluded the top four funds in each sector at each month end, as they are likely to be

[18]Since the tests use historic data, there is a risk of survivorship bias. Past loser funds may have been wound up, resulting in a sample that is disproportionately weighted in favour of successful funds. However, the performance index of the 'average' fund, which is used to categorise sampled funds as winners or losers, is a historic sector index that takes account of all funds within the sector at the time the index is calculated.

[19]Not all companies sampled had sufficient data to be included in the earlier months, particularly when tests involved 5-year performance. The maximum final sample size was 2,486 firm-months.

competing for top ranking and therefore may be operating a strategy that is different from the other funds, as suggested in CE (1997).[20]

As the FT publishes rankings based on one-, three- and five-year fund performance, we constructed separate groups based on performance over each of these periods. All tests were carried out separately under each performance measure.

3.7 Betas

We measured betas using the market model (the FTSE All-Share index proxied the market for the growth-and-income funds, the FTSE Small-Cap for the smaller companies), based on log changes in NAV rather than changes in price, since the latter will be affected by variations in discount. (Using NAVs also removed the estimation problems caused by thin trading.)

Since we are effectively treating each month's observations as independent (see below for further discussion of this point), we would be dealing with overlapping data were we to use historic monthly returns to estimate the betas, as is standard. We therefore based our estimates on daily NAV returns over the month starting on the observation day. This clearly reduces the number of data points used in the beta estimation. One compromise would be to halve the sample size, taking observations only every two months and measuring the betas over two-month intervals. Although this would improve the reliability of the betas, it would increase the standard errors in the tests of the model.

We used both approaches. The betas based on one-month returns did throw up a number of negative betas, which one would not expect to find, while such 'anomalies' were considerably fewer with the two-month betas. On the other hand, the t-statistics of both sets of betas were, in general, very high, with an average value of 8.7 for one-monthly betas and 12.3 for two-monthly ones. Furthermore the patterns of test results were robust to the choice of beta, except for the last set of tests (see below), for which the sample sizes were extremely small when using two-month betas. We therefore report results only for tests based on one-month betas.

To define extreme betas we treated the two sectors in our sample separately, as the small-company sector had a higher mean beta with a larger standard deviation than the income-and-growth sector. We measured the standard deviation of each sector's betas around the mean beta of the sector index over the sample period (the standard deviations were 0.39 for the growth-and-income funds, 0.63 for small-company funds). We defined an extreme beta as one lying more than one standard deviation away from the relevant sector index beta in the month of observation. This classified 22%

[20] All the tests reported below were repeated using the whole sample but with appropriate zero/one dummies for these top funds. The results of the tests were not qualitatively altered.

of the growth-and-income funds and 21% of the small-company funds as extreme.

3.8 Descriptive statistics and tests

Descriptive statistics are given in table 2. The mean and median betas are slightly lower than 1, as found in Elton et al (2001), and losers appear to have lower betas than winners. Returns are positive overall and the difference between the performance of the sector index and that of an individual fund is greater when the fund is a loser than when it is a winner, as one would expect when the top funds are omitted.

TABLE 2 ABOUT HERE

Our tests consisted of a series of probit equations. We first tested the hypothesis that losers are more likely to have extreme betas than are winners by estimating the following probit equations:

$$\Pr\{EXTBETA_{j,\tau}\} = a + bLOSEB_{j,\tau} + \mu_{j,\tau} \tag{18}$$

$$\Pr\{EXTBETA_{j,\tau}\} = a' + b'WINB_{j,\tau} + \mu'_{j,\tau} \tag{19}$$

where:

$EXTBETA_{j,\tau}$ takes the value 1 if fund j has an extreme beta in month τ, and 0 otherwise;

$LOSEB_{j,\tau} = BEHIND_{j,\tau}$ if observation j is a losing fund in month τ, and 0 otherwise, where $BEHIND_{j,\tau} = IT_{\tau}^{av} - IT_{\tau}^{j}$ (positive for losers and negative for winners); and

$WINB_{j,\tau} = BEHIND_{j,\tau}$ if observation j is a winning fund in month τ.

We predict that losers are more likely to have extreme betas than are winners, and that this tendency will be more marked the further behind they are. We would therefore expect the coefficient on $LOSEB_{j,\tau}$ in the first equation to be significantly positive. Conversely, the coefficient on $WINB_{j,\tau}$ in the second equation should not be significantly different from zero. To take account of the fact that we are repeatedly sampling the same funds in consecutive months we use a Huber and White robust variance estimator adjusted for clustering of observations within company. We also report the results of adding a lagged dependent variable, $\Pr\{EXTBETA_{j,\tau-1}\}$, to allow for any inertia in the movement of a fund's beta.

Tables 3 and 4 report the results for losers and winners respectively, with and without the lagged dependent variable.

TABLES 3 and 4 ABOUT HERE

The tables show that in all cases the coefficient on $LOSEB_{j,\tau}$ is highly positively significant, as expected, while the coefficient on $WINB_{j,\tau}$ is not significantly different from zero. Panel B of each table shows that there is significant inertia in the beta chosen each period, but the results are robust to inclusion of the lagged dependent variable.

Having established that losing funds are indeed associated with more extreme betas, we proceed to investigate in more detail the effect of time and the loser's performance on the choice of beta. We estimate equation (20) for losers only (under each of the three performance criteria).

$$\begin{aligned}\Pr\{EXTBETA_{j,\tau}\} &= a'' + b''BEHIND_{j,\tau} + \sum_T c''_T Q^T_{j,\tau} \\ &\quad + \sum_T d''_T Q^T BEH_{j,\tau} + \mu''_{j,\tau}\end{aligned} \tag{20}$$

where:
$Q^T_{j,\tau}$ is a quarterly dummy, taking the value 1 if $\tau = T$ and 0 otherwise ($T = \{1, 2, 4\}$); and
$Q^T BEH_{j,\tau}$ are slope dummies equal to $Q^T_{j,\tau} \times BEHIND_{j,\tau}$.

Our model suggests that the incentive for losers to choose high betas is greater as the ranking period approaches. We chose quarter 3 (July-September) as the 'base' period, as this is the least likely to be a final ranking period. We therefore expect at least one of the quarter dummies to be significantly positive. Positive coefficients on Q^1 (January-March) or Q^4 (October-December) would support our earlier argument that the end of the tax year or the end of the calendar year respectively are candidates for final ranking periods, . The interaction terms between $BEHIND_{j,\tau}$ and the quarter dummies test the prediction of our model, that the degree of under-performance becomes less important as the ranking period approaches: in the period just before the end of the game, simply being a loser is sufficient to ensure that an extreme portfolio is chosen. We would therefore expect that when the coefficient on the quarterly dummy is positively significant, suggesting that the final ranking day is close, the coefficient on the interaction term should be significantly negative, indicating that the choice of an extreme beta is less sensitive to the degree of under-performance.

The results are given in tables 5 and 6, with and without the lagged dependent variable respectively.

TABLE 5 and 6 ABOUT HERE

As expected, the coefficient on $BEHIND_{j,\tau}$ is strongly significantly positive in all cases: the more behind the fund is, the more likely is the manager to choose an extreme beta. In table 6 the lagged dependent variable is again highly significantly positive and the remaining coefficients are not significantly affected by its inclusion.

All the quarterly intercept dummies are positive and all the interaction slope dummies are negative. The results suggest that quarter 1 is the most likely candidate as the final ranking period, although quarter 2 is also a contender. In both tables these quarters have significantly positive coefficients under all three performance measures, and quarter 1 has particularly strong negative coefficients on the interaction term, indicating that for this quarter the decision to adopt an extreme beta is more influenced by the date than the degree of the fund's under-performance. This supports the hypothesis that March, the end of the tax year, is likely to be seen as a 'final' ranking period.

Our predictions apply to losers only, so we repeated the tests on winner funds, to confirm that the same results do not apply to them. The results are given in tables 7 and 8, with and without the lagged dependent variable respectively. There are very few significant coefficients at all in these results, other than the lagged dependent variable and the constant. In particular, performance ($BEHIND_{j,\tau}$) is still not significant. The quarterly intercept and slope coefficients tend to have the same sign on them as in the loser regressions, but only the 5-year winners have any significant coefficients.

TABLE 7 and 8 ABOUT HERE

For funds that have adopted an extreme position, we now investigate the choice between a high and a low beta. As discussed above, the variables that are important here are the fund's maximum possible market exposure compared with average, $MAXN$, and anticipated market movements. According to our model, a losing fund that is expecting the market to fall will go into cash (adopt a low beta). Conversely, a losing fund that is expecting the market to rise will only adopt a high beta if it can expose a sufficiently high asset value to the market compared with the average (in fact, if it has a $MAXN$ above 1). A losing fund that has a $MAXN$ below 1 may choose a low beta in the hope that the market might fall, even if it is expected to rise. Consequently, our probit equation must reflect the fact that a negative ε^A will always decrease the probability of choosing a high beta, but a positive ε^A will only increase that probability if $MAXN$ is also above 1. The independent variable in the probit equation is therefore an interaction term between ε^A and a high $MAXN$.

We estimate equation (21) for losing funds with extreme betas.

$$\Pr\{HIBETA_{j,\tau}\} = \gamma + \delta HIMAXN\varepsilon_{j,\tau} + \zeta_{j,\tau} \tag{21}$$

where:

$HIBETA_{j,\tau}$ takes the value 1 if observation j has an extreme beta that is higher than the sector index beta in month τ, and 0 otherwise; and

$HIMAXN\varepsilon_{j,\tau} = HIMAXN_{j,\tau} \times \varepsilon^A_{j,\tau}$, where $MAXN_{j,\tau}$ is a proxy for fund j's maximum possible market exposure in month τ compared with average, $HIMAXN_{j,\tau}$ is a dummy that equals 1 if $MAXN_{j,\tau}$ exceeds 1, and $\varepsilon^A_{j,\tau}$ is a proxy for the expected market change between months τ and $\tau + 1$ (the proxies are discussed in detail above).

$HIMAXN\varepsilon_{j,\tau}$ will be positive if ε^A is positive and $MAXN$ exceeds 1, negative if ε^A is negative and $MAXN$ exceeds 1, and zero otherwise. Our model predicts that for losing funds, the coefficient on $HIMAXN\varepsilon_{j,\tau}$ will be positive. As our model suggests that only losing funds are constrained in this way, we also estimated it separately for winning funds, to confirm that the coefficient for these funds is not significantly different from zero.

The results excluding the lagged dependent variable are presented in table 9. Panel A applies to losers only and panel B to winners. For losers, all coefficients on $HIMAXN\varepsilon$ are positive, but only the one-year loser coefficient is significant. The constant term is significantly negative, showing that losers adopting extreme positions are more likely to choose low betas than high. For winners there is no evidence that $HIMAXN\varepsilon$ plays any role in the choice of a high beta. The constant is also not significant, indicating that winners are equally likely to choose high and low betas when adopting extreme positions.

TABLE 9 ABOUT HERE

Inclusion of a lagged dependent variable is more problematic here. We cannot include only $HIBETA_{j,\tau-1}$, since this will assign a zero both to those funds with extremely low betas in the previous period and to those with middling, but not extremely high, betas. To capture these differences we included both $HIBETA_{j,\tau-1}$ and $MIDBETA_{j,\tau-1}$, which takes the value 1 if observation j had a non-extreme beta in month $\tau - 1$, and 0 otherwise. It turned out that $HIBETA_{j,\tau-1}$ was a perfect predictor of $HIBETA_{j,\tau}$ in four out of the six regressions, and was extremely highly significant in the other two. This is not surprising, given the inertia in beta choice which was identified earlier, and tells us that firms choosing a high beta in one month are unlikely to change that decision in the following month, regardless of forecasts of market movements. We therefore dropped the observations for which $HIBETA_{j,\tau-1}$ was a perfect predictor and re-estimated the equation with only $MIDBETA_{j,\tau-1}$ as the lagged dependent variable. Results are presented in table 10.

TABLE 10 ABOUT HERE

For these observations, $HIMAXN\varepsilon$ becomes more significant for all three types of loser, but remains not significantly different from zero for winners. For both losers and winners $MIDBETA_{j,\tau-1}$ is significantly positive as one might expect.

We can conclude that loser funds adopting extreme positions are unlikely to change from a high beta to a low one, or vice versa, between one period and the next. However, if a firm adopts an extreme position for the first time, a loser will be strongly influenced by the expected market movement and the maximum exposure it can achieve relative to the average, while these will not be important considerations for winners.

4 Conclusions

We have developed a model of fund management in which fund managers see themselves as, to some extent, engaged in a tournament, but also as required to make a reasonable return. A number of predictions follow from our model, notably that managers of losing funds will increasingly adopt extreme market positions as the final date of the tournament approaches. The particular position adopted will generally be related to the bullishness or bearishness of the market, but fund managers of very low-value funds may elect to bet against the market (adopt low betas when the market is expected to rise) if they cannot expose sufficient assets to the market to allow their relative position to benefit from a rise in share prices. Our tests of these predictions, using recent UK data and cross-sectional beta distributions, are generally very supportive of the model. Funds worth less than average are more likely to select extreme betas and this tendency seems to be more pronounced around the end of the tax year. Losing funds generally increase market exposure by adopting high betas when the market is expected to rise, but are also subject to constraints imposed by the low value of their total assets. They may even adopt low betas when the market is expected to rise, if they cannot improve their position sufficiently to 'beat' the average in a rising market.

The additional risk-taking caused by the tournament aspect of fund management has been identified by other writers as a possible source of distortion in decision-making in this market. However, we show that the 'unwarranted' adoption of extreme positions by loser funds is likely to be a significant problem only at certain times of the year. Conversely, our model suggests an additional distortion: where it may be optimal for loser funds to choose high market exposure there can be a disincentive for them to do so if their fund value is too low.

5 Appendix

5.0.1 The probability that A chooses shares at t

From the analysis in section 2.2.1, A will choose shares at t if:

- his portfolio is valuable enough to allow him to hold more than N^B shares; and

- he believes that ε_{t+1} will exceed ε^*_{t+1} in equation (11)

In period $t-1$, A has no information about ε_{t+1} and $\varepsilon_{t+1} \sim N(0, \sigma^2_\varepsilon)$. The required probability is therefore $\Pr\left\{0 > z \text{ and } \frac{PF^A_t}{P_t} > N^B\right\}$, where $z = \left(\frac{PF^B_t - PF^A_t}{2}\right)\left(\frac{1}{PF^A_t/P_t - N^B} - \frac{1}{N^B}\right)$.

This can be written as $\Pr\left\{0 > z \quad | \frac{PF^A_t}{P_t} > N^B\right\} \times \Pr\left\{\frac{PF^A_t}{P_t} > N^B\right\}$. We take each part in turn.

- $\Pr\left\{0 > z \quad | \frac{PF^A_t}{P_t} > N^B\right\}$

By assumption $PF^A_t < PF^B_t$, so the required probability equals $\Pr\left\{\frac{PF^A_t}{P_t} - 2N^B > 0\right\}$, which, from equations (1) and (5) of the main text, can be recast as:

$\Pr\left\{\varepsilon_t\left(N^A_t - 2N^B\right) > 2N^B P_{t-1} - PF^A_{t-1}\right\}$.

Since this probability is being assessed at $t-1$, *after* news about the price shock in period t, and since $\frac{\varepsilon_t - \varepsilon^A_t}{\sigma^A_v} \sim N(0,1)$, then if $N^A_t > 2N^B$ this probability will be $N(-L_2)$ where $L_2 = \frac{1}{\sigma^A_v}\left[\frac{2N^B P_{t-1} - PF^A_{t-1}}{N^A_t - 2N^B} - \varepsilon^A_t\right]$.

If $N^A_t \leq 2N^B$ this probability will be $N(L_2)$.

- $\Pr\left\{\frac{PF^A_t}{P_t} > N^B\right\}$

This is equivalent to $\Pr\left\{\frac{PF^A_t}{P_t} - 2N^B > 0\right\}$ with $2N^B$ replaced by N^B. Hence, when $N^A_t > N^B$ this probability will be $N(-L_3)$ where $L_3 = \frac{1}{\sigma^A_v}\left[\frac{N^B P_{t-1} - PF^A_{t-1}}{N^A_t - N^B} - \varepsilon^A_t\right]$. And if $N^A_t < N^B$ the probability will be $N(L_3)$.

Therefore at $t-1$ the probability that A will choose shares at t, if his portfolio value is less than B's in period t, is given by:

1. $N(-L_2)N(-L_3)$ when $N^A_t > 2N^B$;
2. $N(L_2)N(-L_3)$ when $N^B < N^A_t \leq 2N^B$;
3. $N(L_2)N(L_3)$ when $N^A_t \leq N^B$

5.0.2 The probability that A chooses cash at t

This probability equals (1− the probability that A chooses shares at t) and hence can be derived directly from the previous analysis.

5.0.3 Probability that ε_{t+1} will be sufficiently high for A to win if he adopts an all-shares strategy for the final period

If A chooses all shares at t, his portfolio at $t+1$ will be worth $N_{t+1}^{A}P_{t+1}$, where $N_{t+1}^{A} = PF_{t}^{A}/P_{t}$. Using equations (1) and (5) again to substitute for P_t, P_{t+1} and PF_{t}^{A}, we can establish that:

$$PF_{t+1}^{A} = \tfrac{PF_{t-1}^{A} + \varepsilon_t N_t^A}{P_{t-1}+\varepsilon_t} \times (P_{t-1} + \varepsilon_t + \varepsilon_{t+1})$$

Similarly, $PF_{t+1}^{B} = PF_{t-1}^{B} + N^{B}(\varepsilon_t + \varepsilon_{t+1})$. The required probability is therefore

$$\Pr\left\{\tfrac{PF_{t-1}^{A} + \varepsilon_t N_t^A}{P_{t-1}+\varepsilon_t} \times (P_{t-1} + \varepsilon_t + \varepsilon_{t+1}) > PF_{t-1}^{B} + N^{B}(\varepsilon_t + \varepsilon_{t+1})\right\}$$

Collecting terms, we have,

$$\Pr\pi^{H} = \Pr\left\{\begin{array}{c} \varepsilon_{t+1}\left(PF_{t-1}^{A} - N^{B}P_{t-1}\right) - \varepsilon_t\left(PF_{t-1}^{B} - PF_{t-1}^{A}\right) + \\ \left(N_t^A - N^B\right)\varepsilon_t(P_{t-1} + \varepsilon_t + \varepsilon_{t+1}) > P_{t-1}\left(PF_{t-1}^{B} - PF_{t-1)}^{A}\right) \end{array}\right\}$$

5.0.4 Probability that ε_{t+1} will be sufficiently low for A to win if he adopts an all-cash strategy for the final period

If A chooses cash at t his portfolio at $t+1$ will be worth the same as it was in t, PF_t^A. Using the same relationships as in the previous section, the required probability is $\Pr\left\{PF_{t-1}^{A} + \varepsilon_t N_t^A > PF_{t-1}^{B} + N^{B}(\varepsilon_t + \varepsilon_{t+1})\right\}$ which re-arranges to $\Pr\left\{N^B\varepsilon_{t+1} + \varepsilon_t\left(N^B - N_t^A\right) < PF_{t-1}^{A} - PF_{t-1}^{B}\right\}$. Since A is assessing this probability at $t-1$, after receiving information about the next period's price shock, and since ε_t and ε_{t+1} are independent, with $N^B\varepsilon_{t+1} \sim N(0, N^B\sigma_\varepsilon)$ and $\varepsilon_t\left(N^B - N_t^A\right) \sim N\left(\left(N^B - N_t^A\right)\varepsilon_t^A, \left(N^B - N_t^A\right)\sigma_V^A\right)$ the required probability is $N(L_4)$ where $L_4 = \frac{PF_{t-1}^{A} - PF_{t-1}^{B} - \left(N^B - N_t^A\right)\varepsilon_t^A}{\sqrt{N^{B^2}\sigma_\varepsilon^2 + \left(N^B - N_t^A\right)^2\sigma_{vA}^2}}$.

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FIGURES

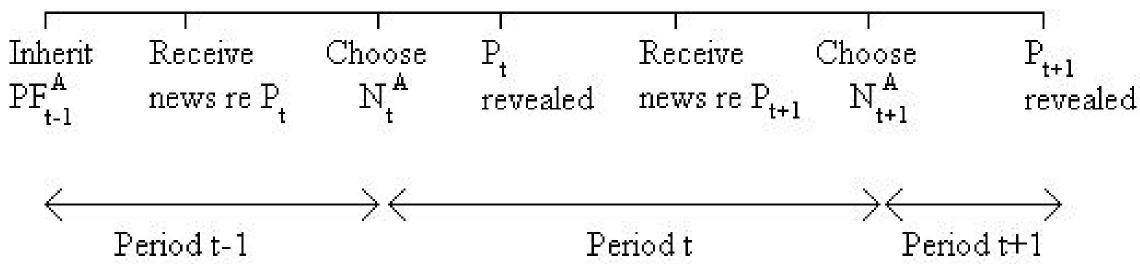
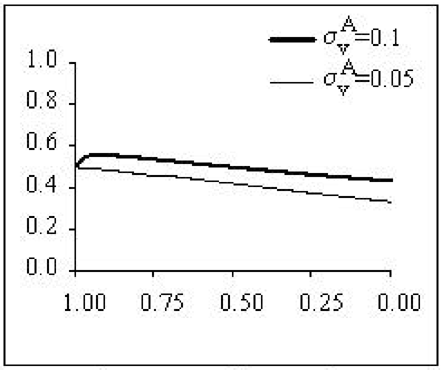
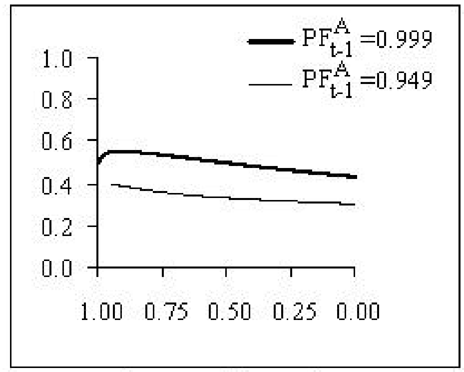


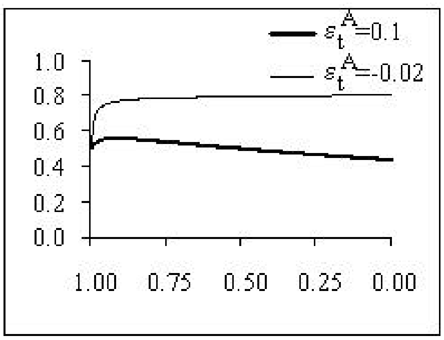
Figure 1 ***A*****'s tournament timeline**



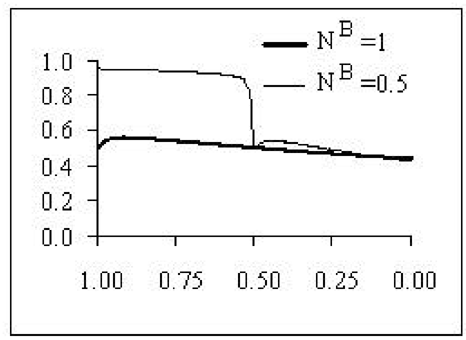
Panel A PF^A_{t-1}=0.999; N^B=1; ε^A_t=0.1; σ_ε=0.2



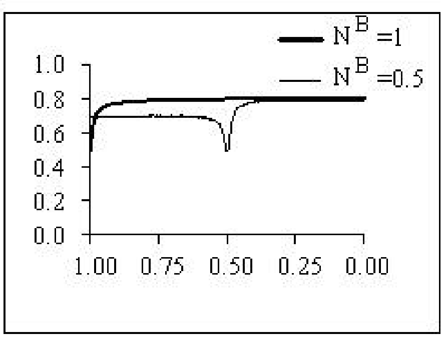
Panel B σ^A_v=0.1; N^B=1; ε^A_t=0.1; σ_ε=0.2



Panel C PF^A_{t-1}=0.999; N^B=1; σ^A_v=0.1; σ_ε=0.2



Panel D PF^A_{t-1}=0.999; ε^A_t=0.1; σ^A_v=0.1; σ_ε=0.2



Panel E PF^A_{t-1}=0.999; ε^A_t=-0.02; σ^A_v=0.1; σ_ε=0.2

Figure 2 ***A*'s decision at $t-1$.** In all panels, the x-axis shows the number of shares A holds in period t and the y-axis shows the associated expected probability of winning.

TABLES

PF^A_{t-1}	0.999	0.949	0.899	0.849	0.799
$\sigma_{v^A} = 0.15$					
$\varepsilon^A_t = 0.1$	0	1*	1*	1*	1*
$\varepsilon^A_t = 0.0$	1	1	1	1	1
$\varepsilon^A_t = -0.02$	1	1	1	1	1
$\sigma_{v^A} = 0.1$					
$\varepsilon^A_t = 0.1$	0	1*	1*	1*	1*
$\varepsilon^A_t = 0.0$	1	1	1	1	1
$\varepsilon^A_t = -0.02$	1	1	1	1	1
$\sigma_{v^A} = 0.05$					
$\varepsilon^A_t = 0.1$	1*	1*	1*	1*	1*
$\varepsilon^A_t = 0.0$	1	1	1	1	1
$\varepsilon^A_t = -0.02$	1	1	1	1	1

Panel A: $N^B = 1$

PF^A_{t-1}	0.999	0.949	0.899	0.849	0.799
$\sigma_{v^A} = 0.15$					
$\varepsilon^A_t = 0.1$	1*	1*	1*	1*	1*
$\varepsilon^A_t = 0.0$	0	0.6	0.9	1	1
$\varepsilon^A_t = -0.02$	0.1	0.7	0.9	1	1
$\sigma_{v^A} = 0.1$					
$\varepsilon^A_t = 0.1$	1*	1*	1*	1*	1*
$\varepsilon^A_t = 0.0$	0	0.8	0.9	1	1
$\varepsilon^A_t = -0.02$	0.1	0.7	0.9	1	1
$\sigma_{v^A} = 0.05$					
$\varepsilon^A_t = 0.1$	1*	1*	1*	1*	1*
$\varepsilon^A_t = 0.0$	0	0.6	0.8	1	1
$\varepsilon^A_t = -0.02$	0.2	0.7	0.9	1	1

Panel B: $N^B = 0.5$

Notes

1. * Indicates that the 'extreme' portfolio was an all-shares portfolio
2. For all simulations reported above, $\sigma_\varepsilon = 0.2$, $PF^B_{t-1} = P_{t-1} = 1$

Table 1: **Proportion of times that A's portfolio choice is 'extreme'**

	No	Mean	Median	Std.dev
Betas				
All	2,486	0.90	0.86	0.46
Losers	1,487	0.87	0.84	0.43
Winners	999	0.96	0.93	0.50
Log Return				
All	2,486	0.07	0.04	0.19
Losers	1,487	0.01	-0.00	0.17
Winners	999	0.14	0.10	0.20
$\mathbf{IT}_{\tau}^{av}-\mathbf{IT}_{\tau}^{j}$				
Losers	1,487	0.09	0.06	0.09
Winners	999	-0.05	-0.04	0.05
MAXN				
All	2,486	1.07	1.07	0.23
Losers	1,487	1.04	1.05	0.23
Winners	999	1.10	1.10	0.22

Notes

1. All data, including the categorisation of losers and winners, have a base date of -1 year. Losers(winners) are those funds with a 1-year return $\left(IT_{\tau}^{j}\right)$ below(above) that of the sector index $\left(IT_{\tau}^{av}\right)$.
2. 'Winners' excludes the top four funds in each sector each month.
3. MAXN is a proxy for the fund's maximum possible market exposure compared with average.

Table 2: **Descriptive statistics**

	1-year	3-year	5-year
LOSEB	2.315	1.842	1.171
	$(3.68)^{**}$	$(4.05)^{**}$	$(5.49)^{**}$
CONSTANT	-0.951	-1.121	-1.061
	$(-9.18)^{**}$	$(-11.47)^{**}$	$(-9.09)^{**}$
Pseudo R-squared	0.022	0.035	0.045
No. of observations	$2,486$	$2,190$	$1,720$

Panel A: No lag effects

	1-year	3-year	5-year
LOSEB	1.407	1.319	0.794
	$(3.38)^{**}$	$(4.47)^{**}$	$(6.06)^{**}$
$EXTBETA_{j,\tau-1}$	1.325	1.211	1.185
	$(8.49)^{**}$	$(8.80)^{**}$	$(6.86)^{**}$
CONSTANT	-1.269	-1.368	-1.315
	$(-17.27)^{**}$	$(-17.11)^{**}$	$(-14.65)^{**}$
Pseudo R-squared	0.178	0.159	0.163
No. of observations	$2,437$	$2,154$	$1,693$

Panel B: Lagged dependent variable

Notes
1. The equation tested was $\Pr\{EXTBETA_{j,\tau}\} = a + bLOSEB_{j,\tau} + \mu_{j,\tau}$.
In Panel B a lagged dependent variable was added.
2. ** = significant at 1%; * = significant at 5% (t-statistics in parentheses)

Table 3: **Effect of being a loser fund on probability of choosing an extreme beta**

	1-year	3-year	5-year
WINB	0.094	0.322	1.164
	(0.10)	(0.27)	(0.93)
CONSTANT	-0.813	-0.945	-0.878
	$(-7.34)^{**}$	$(-7.88)^{**}$	$(-6.03)^{**}$
Pseudo R-squared	0.000	0.000	0.002
No. of observations	2,486	2,190	1,720

Panel A: No lag effects

	1-year	3-year	5-year
WINB	-0.268	0.358	0.604
	(-0.39)	(0.41)	(0.72)
$EXTBETA_{j,\tau-1}$	1.366	1.285	1.273
	$(8.88)^{**}$	$(8.98)^{**}$	$(7.98)^{**}$
CONSTANT	-1.203	-1.257	-1.217
	$(-16.17)^{**}$	$(-14.32)^{**}$	$(-11.34)^{**}$
Pseudo R-squared	0.170	0.144	0.145
No. of observations	2,437	2,154	1,693

Panel B: Lagged dependent variable

Notes
1. The equation tested was $\Pr\{EXTBETA_{j,\tau}\} = a + bWINB_{j,\tau} + \mu_{j,\tau}$
In Panel B a lagged dependent variable was added.
2. ** = significant at 1%; * = significant at 5% (t-statistics in parentheses)

Table 4: **Effect of being a winner fund on probability of choosing an extreme beta**

	1-year losers	3-year losers	5-year losers
BEHIND	4.926	4.153	4.409
	(6.12)**	(3.40)**	(3.08)**
Q^1 (Jan-Mar)	0.433	0.426	0.396
	(3.06)**	(1.75)	(1.73)
Q^2 (Apr-June)	0.374	0.495	0.719
	(2.11)*	(2.03)*	(3.34)**
Q^4 (Oct-Dec)	0.277	0.303	0.309
	(1.76)	(1.23)	(1.24)
Q^1BEH	−3.786	−2.968	−3.874
	(−4.42)**	(−2.46)*	(−2.78)**
Q^2BEH	−1.206	−2.521	−3.844
	(−1.06)	(−1.76)	(−2.78)**
Q^4BEH	−0.781	−1.267	−2.900
	(−0.95)	(−1.14)	(−2.15)*
Constant	−1.415	−1.564	−1.457
	(−8.08)**	(−7.19)**	(−6.27)**
Pseudo R squared	0.064	0.070	0.090
No. of observations	1,487	1,308	1,004

Notes

1.The equation tested was

$$\Pr\{EXTBETA_{j,\tau}\} = a'' + b''BEHIND_{j,\tau} + \sum_T c''_T Q^T_{j,\tau} + \sum_T d''_T Q^T BEH_{j,\tau} + \mu''_{j,\tau}$$

2. ** = significant at 1%; * = significant at 5% (t-statistics in parentheses)

Table 5: **Effect of performance and time-to-ranking on probability of choosing an extreme beta (loser funds only)**

	1-year losers	3-year losers	5-year losers
BEHIND	3.595	3.041	2.990
	(5.06)**	(3.26)**	(3.58)**
$EXTBETA_{j,\tau-1}$	1.314	1.101	1.225
	(6.37)**	(6.91)**	(5.46)**
Q^1 (Jan-Mar)	0.419	0.412	0.386
	(3.20)**	(1.92)*	(2.01)*
Q^2 (Apr-June)	0.451	0.524	0.727
	(2.61)**	(2.38)*	(3.92)**
Q^4 (Oct-Dec)	0.312	0.361	0.341
	(1.97)*	(1.60)	(1.74)
Q^1BEH	−2.986	−2.037	−2.766
	(−3.60)**	(−2.17)*	(−3.42)**
Q^2BEH	−1.055	−1.846	−2.706
	(−0.85)	(−1.42)	(−3.30)**
Q^4BEH	−0.563	−1.114	−2.045
	(−0.61)	(−1.06)	(−2.56)**
Constant	−1.733	−1.774	−1.713
	(−12.70)**	(−9.44)**	(−9.05)**
Pseudo R squared	0.214	0.170	0.211
No. of observations	1,459	1,285	988

Notes

1. The equation tested was

$$\Pr\{EXTBETA_{j,\tau}\} = a'' + b''BEHIND_{j,\tau} + \gamma''\{EXTBETA_{j,\tau-1}\} + \sum_T c''_T Q^T_{j,\tau} + \sum_T d''_T Q^T BEH_{j,\tau} + \mu''_{j,\tau}$$

2. * = significant at 10%; ** = significant at 5% (t-statistics in parentheses)

Table 6: **Effect of performance and time-to-ranking on probability of choosing an extreme beta (loser funds only); including lagged dependent variable**

	1-year winners	3-year winners	5-year winners
BEHIND	0.032	−0.408	−3.323
	(0.01)	(−0.17)	(−1.40)
Q^1 (Jan-Mar)	0.017	−0.005	0.727
	(0.08)	(−0.02)	(2.47)*
Q^2 (Apr-June)	0.158	0.272	0.765
	(0.69)	(1.05)	(2.22)*
Q^4 (Oct-Dec)	0.017	−0.166	0.565
	(0.08)	(−0.61)	(1.58)
Q^1BEH	0.683	0.511	3.577
	(0.20)	(0.17)	(1.34)
Q^2BEH	−1.682	−0.631	1.684
	(−0.42)	(−0.21)	(0.74)
Q^4BEH	−0.487	−3.397	0.215
	(−0.15)	(−1.15)	(0.08)
Constant	−0.890	−1.171	−1.847
	(−4.41)**	(−5.10)**	(−5.15)**
Pseudo R squared	0.007	0.017	0.034
No. of observations	999	882	716

Notes

1.The equation tested was

$$\Pr\{EXTBETA_{j,\tau}\} = a'' + b''BEHIND_{j,\tau} + \sum_T c''_T Q^T_{j,\tau} + \sum_T d''_T Q^T BEH_{j,\tau} + \mu''_{j,\tau}$$

2. The top four funds in each sector each month were excluded

3. ** = significant at 1%; * = significant at 5% (t-statistics in parentheses)

Table 7: **Effect of performance and time to ranking day on probability of choosing an extreme beta (winner funds only)**

	1-year winners	3-year winners	5-year winners
BEHIND	−0.733	0.695	−2.880
	(−0.23)	(0.34)	(−1.33)
$EXTBETA_{j,\tau-1}$	1.355	1.422	1.121
	(8.62)**	(7.96)**	(6.19)**
Q^1 (Jan-Mar)	0.238	0.119	0.828
	(1.02)	(0.45)	(2.94)**
Q^2 (Apr-June)	0.322	0.370	0.756
	(1.25)	(1.55)	(2.07)*
Q^4 (Oct-Dec)	0.230	0.034	0.662
	(0.95)	(0.13)	(1.91)
Q^1BEH	1.265	−0.006	3.423
	(0.33)	(−0.00)	(1.36)
Q^2BEH	−1.095	−1.605	0.614
	(−0.24)	(−0.57)	(0.26)
Q^4BEH	2.187	−2.148	0.403
	(0.57)	(−0.82)	(0.15)
Constant	−1.379	−1.514	−2.110
	(−6.74)**	(−6.80)**	(−6.66)**
Pseudo R squared	0.162	0.175	0.132
No. of observations	978	869	705

Notes

1. The equation tested was

$$\Pr\{EXTBETA_{j,\tau}\} = a'' + b''BEHIND_{j,\tau} + \gamma''\{EXTBETA_{j,\tau-1}\} + \sum_T c''_T Q^T_{j,\tau} + \sum_T d''_T Q^T BEH_{j,\tau} + \mu''_{j,\tau}$$

2. The top four funds in each sector each month were excluded
3. ** = significant at 1%; * = significant at 5% (t-statistics in parentheses)

Table 8: **Effect of performance and time to ranking day on probability of choosing an extreme beta (winner funds only); including lagged dependent variable**

	1-year	3-year	5-year
HIMAXNε	0.421	0.113	0.232
	$(3.74)^{**}$	(0.77)	(1.16)
CONSTANT	-0.977	-0.801	-0.974
	$(-3.84)^{**}$	$(-3.39)^{**}$	$(-3.21)^{**}$
Pseudo R-squared	0.108	0.009	0.034
No. of observations	282	212	174

Panel A: Losers

	1-year	3-year	5-year
HIMAXNε	-0.066	0.109	0.061
	(-0.39)	(0.56)	(0.32)
CONSTANT	-0.019	-0.136	-0.156
	(-0.06)	(-0.41)	(-0.42)
Pseudo R-squared	0.002	0.007	0.002
No. of observations	175	122	90

Panel B: Winners (excluding top four funds)

Notes
1. The equation tested was $\Pr\{HIBETA_{j,\tau}\} = \gamma + \delta HIMAXN\varepsilon_{j,\tau} + \zeta_{j,\tau}$
2. ** = significant at 1%; * = significant at 5% (t-statistics in parentheses)

Table 9: **Probability of choosing a high beta, conditional on choosing an extreme beta**

	1-year	3-year	5-year
HIMAXNε	0.381	0.141	0.366
	(3.05)**	(1.09)	(2.17)**
MIDBETA$_{\tau-1}$	0.959	1.690	1.885
	(2.48)**	(3.76)**	(3.35)**
CONSTANT	−2.376	−2.456	−2.915
	(−6.20)**	(−5.50)**	(−5.25)**
Pseudo R-squared	0.185	0.194	0.274
No. of observations	208	187	152

Panel A: Losers

	1-year	3-year	5-year
HIMAXNε	−0.093	0.116	0.118
	(−0.65)	(0.59)	(0.63)
MIDBETA$_{\tau-1}$	1.621	1.696	1.750
	(4.34)**	(3.17)**	(3.49)**
CONSTANT	−1.605	−1.986	−1.871
	(−4.04)**	(−3.80)**	(−2.75)**
Pseudo R-squared	0.192	0.178	0.190
No. of observations	136	88	76

Panel B: Winners (excluding top four funds)

Notes

1. The equation tested was $\Pr\{HIBETA_{j,\tau}\} = \gamma + \delta HIMAXN\varepsilon_{j,\tau} + \lambda MIDBETA_{j,\tau-1} + \zeta_{j,\tau}$

   The regression was run on observations which did not have a high beta in period $\tau - 1$.
2. ** = significant at 1%; * = significant at 5% (t-statistics in parentheses)

Table 10: **Probability of choosing a high beta, conditional on choosing an extreme beta; including lagged dependent variable**