The New Keynesian Phillips Curve and Imperfect Information

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We develop and test a New Keynesian model of inflation dynamics based on the presence of idiosyncratic shocks to firms' marginal costs. We show that if firms ignore economy-wide influences on their marginal costs, inflation will exhibit a greater degree of inertia than the conventional New Keynesian model suggests. Credible disinflationary policies are likely to be accompanied by recessions rather than booms and the economy can, in principle, respond to monetary shocks in a way that is consistent with the stylised facts. The model imposes testable restrictions which cannot be rejected using quarterly data for the UK and the US. Finally we demonstrate, through a series of simulation experiments, the plausibility of the model's central assumption - that firms will rationally choose to ignore economy-wide influences on their costs.

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1 Introduction

The New Keynesian Phillips Curve (NKPC) has recently been described by Mankiw (2001) as 'appealing from a theoretical standpoint, but ... ultimately a failure'. From a theoretical perspective it is attractive because it can be derived from the dynamically optimising behaviour of monopolistically competitive firms faced with significant costs to adjusting prices. Its failure is empirical: it simply cannot explain the dynamics of inflation without the introduction of *ad hoc* and rather contrived assumptions such as the existence of a sub-set of firms who set prices by a rule of thumb[1] or who form non-rational expectations.[2] Its specific empirical failures are that it cannot explain the degree of persistence in inflation, it makes the implausible prediction that a credible deflationary policy produces a boom, and, more generally, it cannot account for the stylised facts about the response of the economy to monetary policy shocks.

In this paper we first show that a simple and theoretically appealing modification to the basic NKPC can in principle overcome all of these empirical failings. We go on to show that this modified NKPC imposes testable restrictions which cannot be rejected using UK or US data. Our modification is to assume that firms experience idiosyncratic stochastic technology shocks which cause differences across firms in their costs. As a consequence, information about the behaviour of *economy-wide* costs may be of little value to a firm when predicting *its own* future costs, and, as a consequence, they might rationally ignore it.[3] We show that, if it is ignored, each firm can rationally confuse movements in costs that are specific to itself with those that are common to all firms; and if the processes driving the aggregate and idiosyncratic movements are different, the firm can rationally make forecasting errors that would not be made by a firm, or indeed any outside observer, equipped with full information. We show that, under these assumptions,

[1] For examples see Fuhrer and Moore (1995), Galí and Gertler (1999) and Galí *et al.* (2001). For similar models applied to the UK case see Balakrishnan and López-Salido (2000, 2001).

[2] For examples see Roberts (1997, 1998) who measures expectations from survey data; Mankiw (2001) who incorporates adaptive expectations into a NKPC to explain the stylised facts of the dynamic response of inflation and unemployment to monetary shocks; and Ball (2000), who assumed that firms use only lagged inflation when forming expectations, a feature he labels 'near-rationality'. In models closer to the one presented in this paper, Mankiw and Reis (2001) assume information disperses slowly across the economy and Woodford (2001) considers the effects of noisy information in a Phelps island model context.

[3] The problem is analogous to that analysed by Pischke (1995), Demery and Duck (1999, 2000) and Goodfriend (1992) in the context of the permanent income hypothesis. In those studies the process driving each agent's labour income has an aggregate or common component and an idiosyncratic one. The processes driving the two components are assumed to be dynamically different, and agents are assumed either not to be able to distinguish the two or not to consider it worth their while to distinguish them.

whilst *its* forecasting errors will appear as white noise to each firm, *in aggregate* such forecasting errors will be serially correlated and that this serial correlation in *average* forecasting errors introduces an additional source of inertia in aggregate price behaviour. So, inertia in inflation arises naturally from stochastic productivity differences across firms and from the resulting different information sets firms choose to use when forming expectations. The resulting model suggests that credible disinflationary policies are likely to be accompanied by recessions rather than booms, and that the economy can, in principle, respond to monetary shocks in a way that is consistent with the stylised facts reported in Mankiw (2001).

There are four further sections to the paper. In the first we develop our version of the NKPC and show how it can in principle account for the major failings of the simple NKPC; in the second we describe our data, explain our testing procedures and report our results; and in the third we use our results to assess the plausibility of our assumption that firms will ignore available aggregate information when forming expectations about their own future costs. In a final section we present a brief set of conclusions.

2 The Model

The assumptions underlying our version of the NKPC are initially the same as those presented in Galí *et al.* (2001) with the simplification that the production function is linear in the single factor, labour. This implies that a firm's marginal costs are independent of *its own* level of output and hence that the firm's expectations of future marginal costs do not require it to form expectations of its own future output. This assumption is quite consistent with a change in *aggregate* output causing a change in the firm's costs through its effects on the wage rate.

We assume a continuum of firms indexed by $j \in [0, 1]$. Each firm is a monopolistic competitor and produces a differentiated good $Y_t(j)$, which it sells at the nominal price $P_t(j)$. Each firm faces an iso-elastic demand curve given by $Y_t(j) = (P_t(j)/P_t)^{-\phi} Y_t$ where Y_t and P_t are aggregate output and the aggregate price level respectively. The production function for firm j is $Y_t(j) = A_t(j) N_t(j)$, where $N_t(j)$ is the quantity of labour employed by firm j in period t and $A_t(j)$ is a technological factor affecting firm j.[4] We assume that $A_t(j)$ varies stochastically across firms and hence marginal costs differ stochastically across firms.

Firms are assumed to set nominal prices as suggested in Calvo (1983): each firm resets its price with probability $1 - \theta$ each period, where θ is independent of the time elapsed since the last adjustment. So, each period,

[4] This technological factor is normally assumed to be common to all firms (see, for example, in Galí *et al.* (2001)). Our alternative assumption - that it may have a firm-specific component - is the reason why information can be considered firm-specific.

$1-\theta$ of firms reset their prices. θ is therefore a measure of price rigidity. Those firms which do reset are assumed to do so with the aim of maximising their expected discounted profits subject to the constraints imposed by technology, the wage rate and the possibility (defined by θ) that they may reset price at some future date. The resulting optimal price-setting rule is that each firm should set its price as a markup over a discounted stream of expected future nominal marginal costs, where, if the firm faces a low probability of being able to reset its price, i.e. a high value of θ, the firm places more weight on expected future marginal costs. Formally, a logarithmic approximation to the optimizing rule is:[5]

$$p_t^*(j) = \log(\varphi) + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t(j) MC_{t+k}(j) \tag{1}$$

which, since we assume that $MC_t(j)$ is part of the jth firm's information set at date t, can be re-written more conveniently as:

$$p_t^*(j) = \log(\varphi) + MC_t(j) + \sum_{k=1}^{\infty}(\beta\theta)^k E_t(j)(\Delta MC_{t+k}(j)) \tag{2}$$

where $p_t^*(j)$ is the log of the newly-set price of firm j; φ ($\equiv \phi/(\phi-1)$) is the firm's desired gross markup; $MC_{t+k}(j)$ is the logarithm of the nominal marginal cost in period $t+k$ of a firm which last reset its price in period t; β is a subjective discount factor; and $E_t(j)$ is the expectation operator conditional on information available to firm j at date t. Notice that the marginal cost terms in equations (1) and (2) are indexed on j, i.e. we are allowing the marginal costs of firms who reset their prices to differ amongst themselves in any period. The source of these differences is the stochastic differences among firms in the values of the technological term $A_t(j)$: firms with greater than average $A_t(j)$ will have lower than average $MC_t(j)$.[6]

On the assumption that ΔMC_t, the change in the of log of the *economy-wide* average nominal marginal cost, follows a stationary process, it has, by the Wold representation theorem, the following invertible moving-average form:

$$\Delta MC_t - \mu = \sum_{i=0}^{\infty}\alpha_i \varepsilon_{t-i} \equiv \alpha(L)\varepsilon_t \tag{3}$$

where $\alpha_0 = 1$; μ is the mean of ΔMC_t; and ε_t is white noise.

We further assume that the deviation of $\Delta MC_t(j)$ from this average is itself a stationary invertible moving average error process such that:

$$\Delta MC_t(j) = \Delta MC_t + \sum_{i=0}^{\infty}\gamma_i u_{t-i}(j) \equiv \Delta MC_t + \gamma(L) u_t(j) \tag{4}$$

[5] See Galí *et al.* (2001), p.1244.

[6] We maintain the assumption that firms hire labour at a common wage rate.

where $\gamma_0 = 1$; and $u_t(j)$ is also white noise.

Combining equations (3) and (4) and reparamaterising gives the following invertible moving-average process:

$$\Delta MC_t(j) = \sum_{i=0}^{\infty} \rho_i \eta_{t-i}(j) \equiv \rho(L)\eta_t(j) \tag{5}$$

where $\rho_0 = 1$; $\eta_t(j)$ is a white noise error; and the values of the ρs are functions of the αs, the γs, and the relative variances of ε, and u.[7]

We interpret equations (3) - (5) in the following way. Suppose that firm j were to observe the aggregate and specific shocks *separately.* The expectation of, for example, $\Delta MC_{t+k}(j)$, which the firm must form in period t to solve the dynamic optimisation problem expressed in equation (2), would then be given by $\sum_{i=k}^{\infty} \alpha_i \varepsilon_{t-i+k} + \sum_{i=k}^{\infty} \gamma_i u_{t-i+k}$.[8] If, for some reason, it cannot, or chooses not to, observe the two shocks *separately,* but just observes its own costs, it will form its expectation of $\Delta MC_{t+k}(j)$ from equation (5) as $\sum_{i=k}^{\infty} \rho_i \eta_{t-i+k}(j)$. We shall assume this to be the case, i.e. we shall assume that whilst each firm observes its own particular $\Delta MC_t(j)$ series (and hence $\eta_t(j)$), no firm ever observes the current or past realisations of ε or $u(j)$ *separately.*

Our rationale for this assumption is that each firm will acquire information about its own costs, $MC_t(j)$, as part of its normal operations; there are no additional resource costs involved in acquiring it. Each firm, therefore can be thought of as observing the time series $MC_t(j)$, and hence the time series $\rho(L)\eta_t(j)$, costlessly. In contrast, information about *aggregate* or *average* costs, and therefore information about the precise composition of $MC_t(j)$, requires additional resource costs. The higher these additional resource costs are, the more likely it is that each firm will be unwilling to incur them, especially if the extra information obtained provides only a modest improvement in its ability to forecast its own future marginal cost and hence only a modest increase in its profits - something we assess later in the paper. Essentially we are applying to information gathering, and hence expectation formation, the basic economic argument that agents will carry out any activity up to the point where its marginal benefits equal its marginal costs: information about its own costs will be used because it is acquired virtually costlessly; the more expensive information about economy-wide costs may not be.

So, in our version of the NKPC, each firm can be seen as observing $\eta_t(j)$ but not ε_t and $u_t(j)$, and so each firm will form its expectation about *its own* future marginal costs using equation (5) and not equation (4). Note

[7] See, for example, Hamilton (1994, pp. 102-107). Note that the ρs will be common to all firms provided γs and the variance of $u_t(j)$ are common to all firms, which we shall assume they are.

[8] We assume that shocks dated t are part of the information set in period t.

that since $\eta_t(j)$ is white noise it follows that *from each firm's perspective* there is no pattern to its one-period-ahead forecasting error; *given its chosen information set* it is forming expectations rationally.

We can now write $p_t^*(j)$ as:

$$p_t^*(j) = \log(\varphi) + MC_t(j) + \sum_{i=0}^{\infty} k_i \eta_{t-i}(j) \tag{6}$$

where $k_0 = \sum_{i=1}^{\infty}(\beta\theta)^i \rho_i$; $k_1 = \sum_{i=1}^{\infty}(\beta\theta)^i \rho_{i+1}$; .....; $k_n = \sum_{i=1}^{\infty}(\beta\theta)^i \rho_{i+n}$.

The *average* price set by those firms who are resetting can therefore be written as:[9]

$$p_t^* = \log(\varphi) + MC_t + \sum_{i=0}^{\infty} k_i \eta_{t-i} \tag{7}$$

where η_{t-i} is the average value of $\eta_{t-i}(j)$ over the j firms resetting in period t. We assume that the average marginal cost over those firms re-setting their prices is equal to the average marginal cost of all firms.

We shall assume that a sufficiently large number of firms reset their prices each period to justify the assumption that the average value of $u_{t-i}(j)$ across those firms will be zero.[10] With this assumption, a comparison of equation (3) with an aggregation of equation (5) implies:

$$\alpha(L)\varepsilon_t = \rho(L)\eta_t \tag{8}$$

Before considering a key implication of this equation, we define the current price level as a weighted average of the prices of those firms who are resetting and those that are not. Since all previous prices have the same probability of being reset, the current price level can therefore be seen as a weighted sum of the average prices of those resetting and the average price level in the previous period. Formally:

$$p_t = (1-\theta)p_t^* + \theta p_{t-1} \tag{9}$$

From equations (7) and (9) we derive:[11]

$$\pi_t = \theta\pi_{t-1} + (1-\theta)\left[\Delta MC_t + k_0\eta_t + \sum_{i=1}^{\infty}((k_i - k_{i-1})\eta_{t-i})\right] \tag{10}$$

and hence:

$$\pi_t = \theta\pi_{t-1} + (1-\theta)\sum_{i=0}^{\infty} k_i^* \eta_{t-i} \tag{11}$$

[9] This follows from the fact the the k_is are common to all firms.

[10] We are also assuming that $u_t(j)$ is independent across firms and has a finite variance.

[11] For convenience, constants are supressed. See Appendix A for a more detailed derivation of this equation.

where $k_0^* = \rho_0 + k_0$; $k_1^* = \rho_1 + k_1 - k_0$; $k_2^* = \rho_2 + k_2 - k_1$; ....; $k_n^* = \rho_n + k_n - k_{n-1}$.

This is one way of representing our version of the NKPC, but its distinguishing features - especially their dependence on a key characteristic of η_t - can best be seen by first re-writing equation (8) as:

$$\eta_t = \frac{\alpha(L)}{\rho(L)}\varepsilon_t \tag{12}$$

As this equation makes clear, although $\eta_t(j)$ is, by construction, white noise for each firm, the *average* value of it across firms, η_t, is serially correlated.[12] This implies that, although each firm's one-period-ahead forecasting error will exhibit no pattern, its aggregate equivalent will exhibit serial correlation. This characteristic of η_t is the reason why our version of the NKPC predicts inertia in the inflation rate, as we now show.

Substituting equation (12) into equation (11) we obtain:

$$\pi_t = \theta\pi_{t-1} + (1-\theta)\sum_{i=0}^{\infty} k_i^* \frac{\alpha(L)}{\rho(L)}\varepsilon_{t-i} \tag{13}$$

which, for a finite lag length n, we can re-write as:[13]

$$\begin{aligned} \pi_t \quad = \quad & [\theta - \rho_1]\pi_{t-1} + [\theta\rho_1 - \rho_2]\pi_{t-2} + [\theta\rho_2 - \rho_3]\pi_{t-3} + ... \\ & + \theta\rho_n\pi_{t-(n+1)} + (1-\theta)\sum_{i=0}^{n} k_i^* \Delta MC_{t-i} \end{aligned} \tag{14}$$

If we had made the normal rational expectations assumption - that each firm is fully informed about the separate realisations of ε_t and $u_t(j)$ - then $\frac{\alpha(L)}{\rho(L)}$ would equal 1, η_t would equal ε_t and equation (11) could be written as:

$$\pi_t = \theta\pi_{t-1} + (1-\theta)\sum_{i=0}^{\infty} k_i^* \varepsilon_{t-i} \tag{15}$$

A comparison of equations (13) or (14) with (15) shows that a major implication of our version of the NKPC is that inflation will exhibit a greater degree of inertia than is implied by the conventional fully-rational expectations model. This, and other implications, can be seen more sharply if we write the conventional NKPC - equation (15) - in the following form:[14]

$$\pi_t = \beta E_t\pi_{t+1} + \lambda(MC_t - p_t) \tag{16}$$

[12] The reason why η_t is serially correlated although each $\eta_t(j)$ is not is that $\eta_t(j)$ is white noise by construction: the presence of the terms in $u_t(j)$ forces the selection of the particular values of the reparameterisation coefficients - the ρs - which guarantee that, for each j, $\eta_t(j)$ is white noise. On aggregation, the terms in $u_t(j)$ disappear whilst the ρs are the same, and there is no reason why the terms in ρ when applied solely to the εs will necessarily produce a white noise error.

[13] Note that in the special case of price flexibility, where $\theta = 0$, k_i^* will equal ρ_i and equation (14) collapses to $\pi_t = \Delta MC_t$.

[14] See appendix B for a detailed derivation of this result.

where λ ($= \frac{(1-\theta)(1-\beta\theta)}{\theta}$) is positive.

Assume, as does Mankiw (2001), an economy in long-run equilibrium with an inflation rate of zero, where a change in monetary policy takes place in period t which will credibly involve no change in inflation in period t but a drop in inflation in period $t+1$. What must happen to the log of real marginal cost in period t, $MC_t - p_t$, if equation (16) is to hold? Clearly it must rise. But a *rise* in economy-wide real marginal cost will most plausibly be associated with a *rise* in aggregate real output above its natural level.[15] Hence the conventional NKPC implies that (credible) disinflations will be associated with booms, or, to put it another way, credible monetary contractions cause booms, an implication that Mankiw (2001) describes as 'strange' and as suggesting something fundamentally wrong with the model. Our modified NKPC does not have this implication. To see this most simply, assume a value for n in equation (14) of 1 to give:

$$\pi_t = [\theta - \rho_1]\pi_{t-1} + \theta\rho_1\pi_{t-2} + (1-\theta)k_0^*\Delta MC_t + (1-\theta)k_1^*\Delta MC_{t-1} \quad (17)$$

Rewrite this as:

$$\begin{aligned}\pi_t \quad = \quad & [\theta - \rho_1]\pi_{t-1} + \theta\rho_1\pi_{t-2} + (1-\theta)(k_0^*(MC_t - p_t) \\ & + (k_1^* - k_0^*)(MC_{t-1} - p_{t-1}) - k_1^*(MC_{t-2} - p_{t-2})) \\ & + (1-\theta)k_0^*\pi_t + (1-\theta)k_1^*\pi_{t-1}\end{aligned}$$

or:

$$\begin{aligned}\pi_t \quad = \quad & \frac{1}{(1-(1-\theta)k_0^*)}\{[\theta - \rho_1 + (1-\theta)k_1^*]\pi_{t-1} + \theta\rho_1\pi_{t-2} \\ & + (1-\theta)(k_0^*(MC_t - p_t) + (k_1^* - k_0^*)(MC_{t-1} - p_{t-1}) \\ & - k_1^*(MC_{t-2} - p_{t-2}))\}\end{aligned} \quad (18)$$

Starting from a long-run equilibrium inflation rate of zero, assume a credible monetary policy which reduces inflation in period $t+1$ onwards to some constant negative value. What will be the resulting time path of aggregate real marginal cost and by implication, the aggregate output gap? For equation (18) to hold as inflation moves from zero to a negative value, aggregate real marginal cost must initially *fall* provided:

$$0 < (1-\theta)k_0^* < 1$$

By implication the aggregate output gap must also fall. Hence in our version of the model, a credible inflation-reducing policy is consistent with a recession not a boom. More generally, this version of the NKPC is quite capable,

[15] Note that although we have assumed that each firm's marginal costs are constant over their own output levels, aggregate unit labour costs may change as *aggregate* output changes, in part because wage-setting behaviour by workers may be influenced by the aggregate level of activity.

in principle, of explaining the stylised facts of the reaction of economies to restrictive monetary policy: a drawn-out fall in inflation accompanied by a recession.

These distinguishing characteristics of the model all result from our assumption about expectations formation. Firms have *different* information sets because of stochastic differences in technology across firms which in turn cause stochastic differences in costs across firms. Information about its own costs is automatically acquired as part of a firm's day-to-day operations and therefore forms part of each firm's information set. Information about average costs cannot be so readily acquired and therefore may not form part of each firm's information set. Inflation inertia can therefore, in principle at least, be explained within the NKPC framework without destroying the theoretical attraction of that framework by introducing irrationality in expectations formation or other *ad hoc* non-optimising behaviour by some or all firms.

From an empirical perspective equation (14) has two important features. First, it allows us to identify the key structural parameters (θ, β and the ρs) whereas these parameters are only identified in equation (16) after imposing prior values to other parameters.[16] Secondly there are overidentifying restrictions in equation (14) which will provide a basis for a formal test of the model. In the next section we consider in more detail how our model of the NKPC can be tested and report the results of testing it using UK and US data.

3 The Data and Empirical Results

Our data, full details of which are given in Appendix D, are from the UK and the US and cover the private (non-government) sector.[17] We adopt (for both countries) data definitions similar to those employed by Batini, Jackson and Nickell (2000).[18] The data are quarterly and seasonally adjusted covering 1963Q2-2000Q4 in the UK and 1960Q1-2000Q4 in the US. In the UK case the inflation rate (π) is defined as the quarterly change in the log of

[16] Galí *et al.* (2001) identify θ when estimating equation (16) by imposing prior values for the curvature of the production function and the mark-up parameter. We also place restrictions on the production function but do not require a prior value for φ (and therefore ϕ) to identify θ.

[17] Galí and Gertler's (1999) study of US inflation measured the share of labour in the non-farm business sector, though their measure of inflation was based on the overall GDP deflator. Tests of our model using unit labour costs for the non-farm business sector are very similar to those we report below. We prefer the wider coverage to enable direct comparison with the UK.

[18] Batini, Jackson and Nickell (2000) examine the relationship between inflation and labour share in the UK. They find that 'in general the hypothesis that the labour share does not Granger cause inflation is rejected at standard levels of significance. This tentative evidence seems to favour the view that the share of labour may contain corroborative or incremental information for predicting inflation' (p12).

the overall GDP price deflator[19] and in the US the inflation rate is based on the non-government GDP deflator. A series for unit labour costs was constructed for both countries by taking the ratio of nominal non-government compensation of employees to real non-government GDP. The log of this ratio defines our variable MC. In both cases we adjust the published compensation estimates to include a labour income component of the income of self-employed (UK) or proprietors (US).[20] Real marginal cost (or equivalently the share of labour) is defined as the ratio of the adjusted series of labour income to nominal non-government GDP. The log of this ratio defines our variable $MC_t - p_t$.

The adjustment we make to employee compensation implies that the average return to labour of the self-employed/proprietors is equal to the average remuneration of employees in employment. Self-employment income is not separately identified in the UK accounts[21] so we follow the procedure used by Batini, Jackson and Nickell (2000), who adjust compensation by the ratio of total employment to the number of employees. The imputation of labour income of proprietors/self employed is particularly important given the growing importance of these sectors, especially in the UK, where the proportion of self-employment to total employment rose from around 8% in 1960 to 13% in 2000.

In the case of the UK, the three series - π, ΔMC and $MC_t - p_t$ (equivalently the log of labour share) - show clear evidence of stationarity: the ADF test statistics of -2.921, -4.018 and -3.815 (respectively) being significant at the 5% level (for preferred lags 1, 1 and 6 respectively)[22]. The US evidence is less clear-cut. ΔMC is clearly $I(0)$ with an ADF test statistic of -4.964 with one lag. We obtain ADF test statistics of -2.007 (no lags required) and -2.360 (4 lags) for $MC_t - p_t$ and π respectively, compared with an asymptotic critical value of 2.58 at the 10% significance level.[23] The KPSS statistics[24] for π and $MC_t - p_t$ (in which the null hypothesis is that inflation or real marginal cost is stationary about a constant level) were 0.347 and 0.809 (assuming a lag truncation parameter of 8), which compare with critical values of 0.347 (10%), 0.463 (5%) and 0.739 (1%). The null hypothesis

[19] The prices are basic prices for the UK and market prices for the US. As Batini *et al* point out, the use of basic prices means that value added is measured *net of indirect taxes*, which is theoretically more appropriate than measures in market prices. It was not possible to construct the non-government GDP deflator in the UK case due to the lack of a constant price government value added series.

[20] This procedure is adopted by Batini, Jackson and Nickell (2000). It has been used in other contexts when calculating aggregate labour income (see for example Blinder and Deaton (1985)).

[21] The income of the self-employed is now consolidated with other incomes in an 'Other Income' category.

[22] The lag length was determined by truncating at the last significant t-statistic.

[23] A measure of US inflation based on the overall GDP deflator has similar properties.

[24] See Kwiatkowski, Phillips, Schmidt and Shin (1992).

of stationarity is not rejected for US inflation, though the test is marginal at the 10% significance level. However we can reject the stationarity of the share of labour in the US case. More recent research[25] has found that US inflation is a fractionally-integrated (long-memory) stationary process. Since this inflation series is widely used in the empirical New Keynesian literature and given recent evidence in support of its stationarity, we proceed on the assumption that inflation and the growth rate in unit labour costs are both stationary processes.

We test our version of the NKPC by estimating equation (14) and testing the overidentifying restrictions it implies. To see the nature of these restrictions, assume that we have determined the appropriate value of n in equation (14) and hence the order of the lag on π_t, and, by implication, on ΔMC_t. The number of reduced-form (unrestricted) coefficients to be estimated is then $2(n+1)$. Because the k_i^*s in (14) are themselves functions of β, θ, and the ρs, there are $n+2$ *structural* parameters. The model therefore has n overidentifying restrictions.

Prior to estimating and testing equation (14), we report, for comparative purposes, our own estimates of the conventional NKPC model, making use of the approach adopted by Galí *et al.* (1999). They re-cast the NKPC in the form of equation (16), define z_t to be a vector of instruments observed at time t, and argue that equation (16) defines a set of orthogonality conditions:

$$E_t\{(\pi_t - \beta\pi_{t+1} - \lambda(MC_t - p_t))z_t\} = 0$$

Given these conditions, the model can be estimated using generalized method of moments (GMM).[26] Applying this procedure to our data we obtain the following results:

$$\text{UK} \quad : \quad \pi_t = \underset{(0.042)}{1.047}\, \pi_{t+1} + \underset{(0.015)}{0.011}\, (MC_t - p_t)$$

$$\text{US} \quad : \quad \pi_t = \underset{(0.031)}{0.973}\, \pi_{t+1} + \underset{(0.008)}{0.007}\, (MC_t - p_t)$$

Our instrument set includes one to six lags in π and 1 and 2 lags in $MC-p$, the output gap[27] and wage inflation. We report standard errors (in parenthesis) with a Newey-West correction. The results for the two countries are broadly similar. The NKPC is unsatisfactory on a number of counts: in

[25] See for example Bekdashe and Baum (2000), Hassler and Wolters (1994, 1995) and Baillie, Chung and Tieslau (1996).

[26] Our revised version of the model can also be expressed in the form shown in equation (16). However in our case, $E_t\pi_{t+1}$ is more correctly written as $\overline{E_t\pi_{t+1}}$ - the mean of firms' expectations given their (different) information sets. The procedure employed by Galí and Gertler (1999) is no longer appropriate since the instruments they use are not necessarily part of each firm's information set.

[27] The output gap is formed as the residual in a regression of the log of real GDP on the trend, the trend squared (in the case of the US) and with the additon of the trend cubed for the UK.

both countries the estimated value of λ is not significantly different from zero, the subjective discount rate is estimated to be negative in the case of the UK and the residuals show strong evidence of serial correlation.[28] Our data confirm the findings of others: the pure NKPC clearly fails to provide a satisfactory explanation of the data.

In Table 1 we present GMM estimates[29] of the structural coefficients of our version of the NKPC, as expressed in equation (14), for both the US and UK.[30] We initially estimated the model for higher value of n and reduced n if the last ρ coefficients were non-significant. The results shown in Table 1 are for the value of n thus selected, 5 for the UK and 3 for the US.

The results support the model, strongly so in the UK case. The estimated equations show no evidence of residual serial correlation and the Hansen tests of the instrument over-identifying restrictions (J) are satisfactory. For both countries the proportion of firms estimated *not* to reset their prices each quarter is around 0.8. The implied average duration of prices, T ($\equiv \frac{1}{1-\theta}$), indicates that prices were typically reset after around 6-7 quarters in the US and after just over 4 in the UK. For both countries the estimated values of the ρs are all positive and generally decline gradually, though the estimated ρs for the US are all lower than their UK equivalents and the shocks are less persistent in the US case. The point estimates of β suggests higher subjective rates of discount than is generally assumed, especially in the US case where β is both low and imprecisely estimated. In both countries it is not possible to reject a discount rate of 4% p.a. ($\beta = 0.99$). In Table 1 we report parameter estimates based on the restriction $\beta = 0.99$. Our conclusions are unaffected.

We also computed the Newey-West (1987) 'D' test statistic of the model's overidentifying restrictions, a test which is analogous to the likelihood ratio test.[31] The statistic is distributed as chi–square with degrees of freedom equal to the number of restrictions imposed. The p-value of this test is reported as $p(D)$ in Table 1. In both countries we fail to reject the model's restrictions at the conventional 5% and 10% significance levels: in the UK case the failure to reject the model's restrictions is particularly emphatic. The model's restrictions are also not rejected when we impose the restriction $\beta = 0.99$. All in all, our modified version of the NKPC appears to present

[28] The Ljung-Box portmanteau test for fourth-order residual serial correlation, based on the unweighted residuals, gives test statistics of 33.30 for the UK and 17.7 for the US, both of which imply a strong rejection of white noise errors.

[29] The structural models are estimated using weights based on a consistent estimator of the asymptotic covariance matrix of the unrestricted model. This permits tests of the overidentifying restrictions we discuss below.

[30] The instruments used were as follows: lags 1 to 6 in π, lags 1 to 5 in ΔMC, and lags 1 to 2 in both the output gap and wage inflation.

[31] See Newey and West (1987) p780, equation (2.9). The test statistic D requires that the same estimate of the covariance matrix is used in both the restricted and unrestricted models as this ensures that $D > 0$.

an empirically plausible account of UK and US inflation.

4 Imperfect Information and the Simulated Loss of Profits

The key assumption in our version of the NKPC is that, when forming expectations about future changes in its own costs, a firm either cannot or does not make use of information about changes in the costs of other firms. Specifically, we assume that it does not *separately* observe the shock to its costs that is common to other firms and the shock that is specific to itself. Instead it observes only the changes to its own costs which are the composite effect of the two shocks.

Is this assumption plausible? One rationale for it is that, whilst in principle information about aggregate behaviour is relatively cheaply available in official or non-official sources, in practice there are the usual publication delays and revisions which make current and recent aggregate data unavailable or unreliable. However, in our version of the NKPC a firm *never* identifies the aggregate and idiosyncratic shocks to its own costs regardless of how far back in time those shocks occurred, even though that information might by then be both reliable and cheap. To judge whether this can be considered plausible, we carried out a series of simulation experiments to gauge the loss of profits a firm would incur by basing its expectations on limited rather than full information. That is, we compared the firm's simulated, discounted, expected profits when it is assumed to observe the aggregate and idiosyncratic shocks to its costs *separately* with those obtained when it is assumed to observe only their *composite* effect. If this difference were large, our assumption that firms will typically not observe the two effects separately would be less convincing.

We made two sets of comparisons. In the first, we assume that ε_t and $u_t(j)$ have very simple but quite different effects on $\Delta MC_t(j)$. From the assumed values of the αs and γs, we then calculated the implied values for the ρs and σ_η^2 for different values of σ_ε^2 and σ_u^2. We then generated simulated values for ε_t and $u_t(j)$. On the assumption that all *previous* realisations of the shocks were zero, we then, for assumed values of θ, ϕ and β,[32] calculated two optimal prices for a firm which was resetting its price. The first assumed that the firm observes the true values of ε_t and $u_t(j)$ *separately* and applies the α and γ coefficients to them to form its expectations of its future marginal costs. The other assumed that the firm observes only the composite shock, $\eta_t(j) = \varepsilon_t + u_t(j)$, and applies the ρs to it to form its expectations of future marginal costs.

[32] In the simulations reported in Table 2, we assume $\phi = 6$, which implies a mark-up parameter (φ) of 1.2. This is the value assumed by Sbordone (2000) and is within the range (1.1 to 1.4) suggested as plausible by Galí *et al* (2001). We report later the effects of assuming different values for ϕ.

The two prices give rise to two streams of future profits for a particular stream of *future* realisations of ε and $u(j)$. We assumed that these future realisations were all zero and took the ratio of the two streams of profits as our indicator of the firm's *expected* loss of profits from forming expectations from the composite shock rather than the individual values of the two shocks. We express this ratio as DPF/DPF^* where DPF^* denotes the discounted value of future profits from the optimal price when both shocks are observed, and DPF denotes the discounted value of future profits from the optimal price when only the composite shock is observed. The closer is this ratio to 1 (it will always be less than 1), the smaller the loss of profits from not observing the shocks separately.[33] We simulated 1000 values of DPF/DPF^* and report their means in Table 2, Panel A.

The results shown in Table 2 assume $\alpha_0 = \gamma_0 = 1$; $\alpha_1 = -\gamma_1 = 0.9$; all higher-order αs and γs are assumed to be zero. The common shock therefore exerts a persistent and large effect on the level of a firm's marginal cost, whereas the firm-specific component has a less persistent effect. The results indicate that whilst the loss of profits from *not* observing the shocks separately can be very large, this is only when the variances of the two shocks are *both* 'high'. For example, if σ_ε^2 were 0.00001, then even if σ_u^2 were 100 times greater, the loss of profits would be minuscule - less than 0.04%; but if *both* variances are 0.001 then the potential loss of profits is in the region of 4%. Intuitively, as the ratio $\sigma_u^2/\sigma_\varepsilon^2$ rises, the value of ρ moves closer to the value of γ, and so the mistakes and loss of profits that arise from not separately observing potentially high (absolute) values of $u_t(j)$ are kept low. If σ_ε^2 were low too then the overall loss of profits would be low. But if *both* σ_ε^2 and σ_u^2 were high then, with the αs and γs different from each other, since the value of ρ cannot be close to both, it follows that potentially large errors can be made.

The *absolute* values of σ_ε^2 and σ_u^2 are therefore crucial. σ_ε^2 is the variance of the quarterly proportionate rate of change of the firm's nominal marginal costs. If σ_ε^2 were 0.0001, there would be a 95% chance of the annual growth rate of marginal costs being within ± 8 percentage points of its mean. If it were 0.00001, the equivalent figure would be ± 1.3 percentage points; if it were 0.001, the figure would be ± 13 percentage points. It seems therefore

[33] Some of the equations in the text are log-approximations and so the optimal price as expressed in equation (1) may at times not generate the optimum stream of profits. In the simulations, the optimal price for each assumption about expectations was defined as: $P_t^* = \frac{(1-\beta\theta)\phi}{(\phi-1)} \sum_{i=0}^{\infty} (\beta\theta)^i \widetilde{MC}_{t+i}$ where P_t^* is the level (not the log) of the optimal price and $\widetilde{MC}_{t+i}$ the level (not the log) of expected nominal marginal cost in period $t+i$. Profits in period $t+i$ were defined as $[P_t^* - \widehat{MC}_{t+i}].Y_{t+i}$. The value of Y_{t+i} was derived from the demand function. This required assumptions about the future course of the average level of prices and aggregate income. In the simulations, we allowed for a constant long-run inflation rate and long-run rate of growth of output. See Appendix C for a fuller explanation.

quite reasonable to expect the ratio DPF/DPF^* to be quite close to 1 for the UK and US.

To explore this further, we carried out a second set of simulations using values of the relevant parameters drawn from the actual data sets. First, we used the estimates of the ρs and of β and θ presented in Table 1. Then, for both countries we obtained estimates of the αs and σ_ε^2 from an $MA(q)$ empirical model for ΔMC_t. Using these as our parameter values, and for different assumed values of σ_u^2, we then solved for the values of the γs which are consistent both with a q-order invertible MA process driving $u_t(j)$ and with our estimated values of the ρs.[34] We then calculated the profit ratio (DPF/DPF^*) for each case assumed. The value of q was determined by the number of significant ρs in each country. For the UK we assumed an $MA(5)$ process for ΔMC_t; for the US we assumed an $MA(3)$. As the results in the table indicate,[35] the values of the parameters strongly suggest that both countries are in the parameter region where the loss of profits from not observing the two shocks separately is very low, however high the ratio $\sigma_u^2/\sigma_\varepsilon^2$.

So our results suggest that whilst in principle the failure to observe the two shocks separately *could* lead to a serious loss of profits, in practice the loss of profits in the UK and US economies over our data period is likely to be very small. Firms therefore have little incentive to observe the shocks separately.[36]

5 Conclusions

Our modification to the conventional NKPC appears both in principle and in practice to be capable of eliminating some of the failings of the conventional NKPC. Furthermore, it does so in a way which is theoretically natural. Its key assumption is that, faced with stochastic differences in their marginal costs due to stochastic technology differences, firms will largely ignore information on economy-wide components of their costs when forming expectations. This, we show, can explain a number of the stylised facts about

[34] We used numerical techniques to derive the required values of the γs.

[35] The results reported in Table 2 are based on zero long-term output growth and inflation. When the sample mean inflation and growth rates were used instead, the results were little changed.

[36] We also carried out simulations for values of ϕ of 4, 5 and 11 which roughly correspond to the markup parameters of 1.1 to 1.4 suggested in Galí *et al.* (2001). For these values, the loss of profits simulated using the parameter estimates for the US and the UK was still very small. The loss of profits simulated using the parameters of the illustrative model showed the same pattern as those presented in Panel A, but where $\phi = 11$ the loss of profits increased more quickly as σ_ε^2 increased so that when $\sigma_\varepsilon^2 = 0.001$, the loss of profits ranged from 7%-20% compared with the range 2%-5% shown in Panel A. In general, as one would expect, the simulations showed that a wrong price has a more dramatic effect on profits the higher the elasticity of demand.

inflationary dynamics: in particular the degree of inertia and the tendency for inflation to be positively correlated with economic activity.

Table 1: Imperfect Information Model
Equation (14)

	UK: 1963Q2-2000Q4, n=5		US: 1960Q1-2000Q4, n=3	
	β estimated	$\beta = 0.99$	β estimated	$\beta = 0.99$
Constant	0.000	0.000	0.000	0.000
	(0.001)	(0.001)	(0.000)	(0.000)
θ	0.774	0.771	0.840	0.863
	(0.028)	(0.027)	(0.037)	(0.032)
ρ_1	0.794	0.807	0.337	0.294
	(0.089)	(0.084)	(0.068)	(0.075)
ρ_2	0.508	0.520	0.185	0.183
	(0.089)	(0.087)	(0.070)	(0.067)
ρ_3	0.388	0.401	0.192	0.150
	(0.111)	(0.110)	(0.078)	(0.084)
ρ_4	0.226	0.228	-	-
	(0.097)	(0.098)	-	-
ρ_5	0.171	0.176	-	-
	(0.083)	(0.080)	-	-
β	0.926	-	0.617	-
	(0.165)	-	(0.304)	-
$Q(4)$	0.789	0.694	0.961	0.827
$Q(8)$	0.550	0.544	0.701	0.593
$p(J)$	0.852	0.891	0.479	0.393
$p(D)$	0.876	0.916	0.150	0.118
T	4.415	4.361	6.251	7.297

Notes: Estimated standard errors in (.) with a Newey-West correction.
$Q(n)$ is the p-value of the Ljung-Box test for nth order serial correlation.
$p(J)$ is the p-value of the Hansen test of over-identifying restrictions.
$p(D)$ is the p-value of the Newey-West test of the model's restrictions.
$T \equiv \frac{1}{1-\theta}$ is the expected duration of prices.

Table 2 Profit Loss (DPF/DPF^*) under Imperfect Information

Panel A: Illustrative Case

$\Delta MC_t = \varepsilon_t + 0.9\varepsilon_{t-1}; \Delta MC_t(j) = \Delta MC_t + u_t - 0.9u_{t-1}$; $\beta = 0.99; \theta = 0.8; \phi = 6$	$\sigma_\epsilon^2 = 0.00001$	$\sigma_\epsilon^2 = 0.0001$	$\sigma_\epsilon^2 = 0.001$	$\sigma_\epsilon^2 = 0.01$
$\sigma_u^2/\sigma_\epsilon^2 = 1$; $\rho = -0.000025$	0.999854	0.998377	0.983627	0.806845
$\sigma_u^2/\sigma_\epsilon^2 = 2$; $\rho = -0.170568$	0.999794	0.997953	0.978257	0.703445
$\sigma_u^2/\sigma_\epsilon^2 = 10$; $\rho = -0.514540$	0.999646	0.996646	0.963692	0.339306
$\sigma_u^2/\sigma_\epsilon^2 = 100$; $\rho = -0.796940$	0.999620	0.996242	0.958907	0.268164

Panel B: UK Case

α_{1-5}: $0.3258; 0.3757; 0.3212; 0.1007; 0.1480.$ $\phi = 6$	$\beta = 0.926; \theta = 0.774; x = 0.7167^*$; ρ_{1-2}: $0.794; 0.508;$ ρ_{3-5}: $0.388; 0.226; 0.171$; $\sigma_\epsilon^2 = 0.000266$	$\beta = 0.99; \theta = 0.771; x = 0.7634^*$; ρ_{1-2}: $0.8070; 0.5204;$ ρ_{3-5}: $0.4013; 0.2278; 0.1763$; $\sigma_\epsilon^2 = 0.000266$
$\sigma_u^2/\sigma_\epsilon^2 = 2$	0.997942	0.997109
$\sigma_u^2/\sigma_\epsilon^2 = 10$	0.998990	0.998629
$\sigma_u^2/\sigma_\epsilon^2 = 100$	0.999102	0.998832

Panel C: US Case

α_{1-3}: $0.2696; 0.3199; 0.0825.$ $\phi = 6$	$\beta = 0.617; \theta = 0.840; x = 0.5183^*$; ρ_{1-3}: $0.3368; 0.1853; 0.1924$; $\sigma_\epsilon^2 = 0.000072$	$\beta = 0.99; \theta = 0.863; x = 0.8545^*$; ρ_{1-3}: $0.2940; 0.1829; 0.1499$; $\sigma_\epsilon^2 = 0.000072$
$\sigma_u^2/\sigma_\epsilon^2 = 2$	0.999998	0.999999
$\sigma_u^2/\sigma_\epsilon^2 = 10$	1.00000	0.999999
$\sigma_u^2/\sigma_\epsilon^2 = 100$	1.00000	0.999998

Notes: The simulations were based on 1000 replications.

These results are based on zero long-term growth (g) and inflation rates ($\overline{\pi}$).

$$* \; x = (\beta\theta)\left[\frac{1+g}{(1+\overline{\pi})^{-\phi}}\right]$$

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6 Appendices

6.1 Appendix A: the derivation of equation (10).

From equation (5) we have:

$$\Delta MC_t(j) = \sum_{i=0}^{\infty} \rho_i \eta_{t-i}(j)$$

It follows that we can write:

$$E_t(j)\Delta MC_{t+k}(j) = \sum_{i=k}^{\infty} \rho_i \eta_{t-i+k}(j) \tag{A1}$$

Using equation (A1) and equation (2), the optimal price for a firm which is resetting its price will be:

$$\begin{aligned} p_t^*(j) &= \log(\varphi) + MC_t(j) + \beta\theta[\rho_1 \eta_t(j) + \rho_2 \eta_{t-1}(j) + ...] \\ &\quad + (\beta\theta)^2[\rho_2 \eta_t(j) + \rho_3 \eta_{t-1}(j) + ...] + ... \end{aligned}$$

or:

$$p_t^*(j) = \log(\varphi) + MC_t(j) + \sum_{i=0}^{\infty} k_i \eta_{t-i}(j)$$

where:

$$\begin{aligned} k_0 &= \sum_{i=1}^{\infty} (\beta\theta)^i \rho_i \\ k_1 &= \sum_{i=1}^{\infty} (\beta\theta)^i \rho_{i+1} \end{aligned}$$

or, in general:

$$k_n = \sum_{i=1}^{\infty} (\beta\theta)^i \rho_{i+n}$$

The average price of those firms who are resetting is therefore:

$$p_t^* = \log(\varphi) + MC_t + \sum_{i=0}^{\infty} k_i \eta_{t-i}$$

Given that $p_t = (1-\theta)p_t^* + \theta p_{t-1}$ and hence that $\pi_t = (1-\theta)\Delta p_t^* + \theta \pi_{t-1}$ it follows that:

$$\pi_t = \theta\pi_{t-1} + (1-\theta)[\Delta MC_t + k_0 \eta_t + \sum_{i=1}^{\infty}((k_i - k_{i-1})\eta_{t-i})] \tag{A2}$$

which is equation (10).

6.2 Appendix B: proof of equation (16) under fully rational expectations

We have from equation (15) in the main text:

$$\pi_t = \theta\pi_{t-1} + (1-\theta)\sum_{i=0}^{\infty} k_i^* \varepsilon_{t-i} \tag{B1}$$

It follows that we can write:

$$E_t\pi_{t+1} = \theta\pi_t + (1-\theta)\sum_{i=1}^{\infty} k_i^* \varepsilon_{t-i+1} \tag{B2}$$

And so:

$$\begin{aligned} \pi_t &= \beta E_t\pi_{t+1} + \lambda(MC_t - p_t) \\ &= \beta\theta\pi_t + \beta(1-\theta)\sum_{i=1}^{\infty} k_i^* \varepsilon_{t-i+1} + \lambda(MC_t - p_t) \end{aligned}$$

Since $\pi_t \equiv p_t - p_{t-1}$ it follows that we can write:

$$p_t = \left[\frac{1}{1-\beta\theta+\lambda}\right]\left[\begin{array}{c} (1-\beta\theta)p_{t-1} + \beta(1-\theta)\sum_1^{\infty} k_i^* \varepsilon_{t+1-i} \\ +\lambda MC_t \end{array}\right] \tag{B3}$$

First differencing equation (B3) and recognising the following:

$$\begin{aligned} \frac{1}{1-\beta\theta+\lambda} &= \frac{\theta}{1-\beta\theta} \\ \frac{\lambda}{1-\beta\theta+\lambda} &= 1-\theta \\ \frac{\beta(1-\theta)}{1-\beta\theta+\lambda} &= \frac{\beta\theta(1-\theta)}{1-\beta\theta} \end{aligned}$$

we can write:

$$\begin{aligned} \pi_t &= \theta\pi_{t-1} + (1-\theta)\sum_{i=0}^{\infty}\alpha_i\varepsilon_{t-i} + \left[\frac{\beta\theta(1-\theta)}{1-\beta\theta}\right] \\ &\qquad \left[\sum_1^{\infty} k_i^* \varepsilon_{t+1-i} - \sum_1^{\infty} k_i^* \varepsilon_{t-i}\right] \end{aligned} \tag{B4}$$

Equations (B4) and (B1) are identical if the coefficients attached to ε_{t-i} are identical for all i. These conditions are in fact met, as we now show.

- $\varepsilon_t : (1-\theta)k_0^* = (1-\theta)\alpha_0 + \left[\frac{\beta\theta(1-\theta)}{1-\beta\theta}\right] k_1^*$ since, under fully rational expectations, $\rho_i = \alpha_i$ and hence $k_0^* = \alpha_0 + k_0$; $k_1^* = \alpha_1 + k_1 - k_0$; and $k_j = \beta\theta(k_{j+1} + \alpha_{j+1})$
- $\varepsilon_{t-1} : (1-\theta)k_1^* = (1-\theta)\alpha_1 + \left[\frac{\beta\theta(1-\theta)}{1-\beta\theta}\right](k_2^* - k_1^*)$ since $k_1^* = \alpha_1 + k_1 - k_0$; and $k_2^* = \alpha_2 + k_2 - k_1$
- and so on for all ε_{t-i}

6.3 Appendix C: The derivation of the loss of profits ratio

Dropping the firm subscript in the interests of notational simplicity, we write the discounted value of the firm's expected future profits from setting the price P_1^* as:

$$E_t DPF = \sum_0^\infty (\beta\theta)^i E_t \left[P_1^* - \widetilde{MC}_{t+i}\right] \left[\frac{P_1^*}{P_{t+i}}\right]^{-\phi} Y_{t+i}$$

where $\widetilde{MC}$ is the level (not the log) of the firm's marginal cost. Therefore:

$$E_t DPF = \sum_0^\infty (\beta\theta)^i E_t \left[P_1^{*(1-\phi)} - \widetilde{MC}_{t+i} P_1^{*-\phi}\right] \left[\frac{Y_{t+i}}{P_{t+i}^{-\phi}}\right]$$

Assume that $Y_{t+i} = [1+g]^i Y_0$ and $P_{t+i} = [1+\overline{\pi}]^i P_0$; and normalise Y_0 so that $\frac{Y_0}{P_0^{-\varphi}} = 1$. Then maximising $E_t DPF$ with respect to P_1^* yields[37]

$$P_1^* = \left[\frac{\phi}{\phi-1}\right] \left[1 - \beta\theta \left(\frac{1+g}{(1+\overline{\pi})^{-\phi}}\right)\right] \sum_0^\infty (\beta\theta)^i \left[\frac{1+g}{(1+\overline{\pi})^{-\phi}}\right]^i E_t \widetilde{MC}_{t+i}$$

So, writing $(\beta\theta)\left[\frac{1+g}{(1+\overline{\pi})^{-\phi}}\right]$ as x, we can write that for any expected stream of marginal costs $E_1 MC_{t+i}$ the optimal price is:

$$P_1^* = \left[\frac{\phi}{\phi-1}\right] [1-x] \sum_0^\infty x^i E_1 \widetilde{MC}_{t+i}$$

and the associated expected discounted stream of profits from this price is

$$\begin{aligned} E_1 DPF_1 &= \left[\left[\frac{\phi}{\phi-1}\right] [1-x] \sum_0^\infty x^i E_1 \widetilde{MC}_{t+i}\right]^{1-\phi} \cdot \frac{1}{1-x} \\ &\quad - \left[\left[\frac{\phi}{\phi-1}\right] [1-x] \sum_0^\infty x^i E_1 MC_{t+i}\right]^{-\phi} \sum_0^\infty x^i E_1 \widetilde{MC}_{t+i} \end{aligned}$$

If a firm forms the expectation of this particular stream of marginal costs as $E_2 MC_{t+i}$ and sets the price accordingly, it will set the price:

$$P_2^* = \left[\frac{\phi}{\phi-1}\right] [1-x] \sum_0^\infty x^i E_2 \widetilde{MC}_{t+i}$$

A firm which expects the stream of marginal costs $E_1 MC_{t+i}$ will anticipate that the discounted stream of profits from this price (i.e. P_2^*) will be:

$$\begin{aligned} E_1 DPF_2 &= \left[\left[\frac{\phi}{\phi-1}\right] [1-x] \sum_0^\infty x^i E_2 \widetilde{MC}_{t+i}\right]^{1-\phi} \cdot \frac{1}{1-x} \\ &\quad - \left[\left[\frac{\phi}{\phi-1}\right] [1-x] \sum_0^\infty x^i E_2 MC_{t+i}\right]^{-\phi} \sum_0^\infty x^i E_1 \widetilde{MC}_{t+i} \end{aligned}$$

[37] Assuming that $0 < (\beta\theta)\left[\frac{1+g}{(1+\overline{\pi})^{-\phi}}\right] < 1$

Write:

$$\sum_0^\infty x^i E_2 \widetilde{MC}_{t+i} = \zeta \sum_0^\infty x^i E_1 \widetilde{MC}_{t+i}$$

Then we can write:

$$\begin{aligned} E_1 DPF_2 &= \zeta^{1-\phi} \left[\left[\frac{\phi}{\phi - 1} \right] [1 - x] \sum_0^\infty x^i E_1 \widetilde{MC}_{t+i} \right]^{1-\phi} \cdot \frac{1}{1-x} \\ &\quad - \zeta^{-\phi} \left[\left[\frac{\phi}{\phi - 1} \right] [1 - x] \sum_0^\infty x^i E_1 \widetilde{MC}_{t+i} \right]^{-\phi} \sum_0^\infty x^i E_1 \widetilde{MC}_{t+i} \end{aligned}$$

The ratio of the expected discounted profits with P_2^* to the expected discounted profits with P_1^* for a firm which expects the stream of marginal costs given by $E_1 \widetilde{MC}_{t+i}$ will therefore be:

$$\frac{E_1 DPF_2}{E_1 DPF_1} = \frac{\zeta^{(1-\phi)} \left(\frac{\phi}{\phi-1}\right)^{(1-\phi)} - \left(\zeta^{-\phi}\right) \left(\frac{\phi}{\phi-1}\right)^{-\phi}}{\left(\frac{\phi}{\phi-1}\right)^{(1-\phi)} - \left(\frac{\phi}{\phi-1}\right)^{-\phi}}$$

which has a maximum of 1 when $\zeta = 1$.

6.4 Appendix D

The raw data used in this paper can be downloaded from the following University of Bristol web site:

http://www.ecn.bris.ac.uk/www/ecdd/newk/newk.htm

6.4.1 Data Definitions for the UK

UK data were retrieved from the National Statistics DataBank Online at http://www.data-archive.ac.uk/. The four-digit codes are the relevant National Statistics codes for the series used.

π is the inflation rate defined as the first difference in the logarithm of the GDP deflator: $\pi_t = \log(DEF_t) - \log(DEF_{t-1})$, where $DEF = \frac{\text{ABML}}{\text{ABMM}}$, ABML is Gross Value Added (average) in current basic prices, seasonally adjusted; and ABMM is Gross Value Added in 1995 basic prices, seasonally adjusted.

The log of real marginal costs $(MC_t - p_t)$ or equivalently the log of the share of labour is defined as:

$$MC_t - p_t = \log \left[\frac{(\text{DTWM–NMXS}^a) \left(\frac{\text{DYZN+BCAJ}}{\text{BCAJ}}\right)}{\text{ABML-NMXV}^a\text{-NMXS}^a} \right]$$

where DTWM is total compensation of employees (£m) seasonally adjusted; NMXS^a is the variable NMXS seasonally-adjusted (X11), where NMXS is compensation of employees in government seasonally unadjusted; similarly NMXV^a is the variable NMXV seasonally-adjusted, where NMXV is general government gross operating surplus; DYZN is the number of self-employed workforce jobs (000, seasonally adjusted); and BCAJ is the number of employee workforce jobs (000, seasonally adjusted). Prior to 1978, the two employment series were available for the second quarter in each year only, so for these years observations for other quarters were derived by linear interpolation. This definition of labour share follows the preferred definition adopted by Batini, Jackson and Nickell (2000).

The logarithm of nominal marginal cost (MC) is defined as:

$$MC = \log\left[\frac{(\text{DTWM–NMXS}^a)\left(\frac{\text{DYZN+BCAJ}}{\text{BCAJ}}\right)}{\text{ABMM-}\left(\frac{\text{NMXV}^a\text{+NMXS}^a}{DEF}\right)}\right]$$

where, in the absence of a constant price series for government value added, we have assumed that the government value added deflator is the same as that for Gross Value Added. The growth in nominal marginal costs is defined as: $\Delta MC_t \equiv MC_t - MC_{t-1}$.

Real output (y) is ABMM, gross value added in 1995 basic prices, seasonally adjusted. The wage rate is defined as:

$$W = \frac{\text{DTWM}\left(\frac{\text{DYZN+BCAJ}}{\text{BCAJ}}\right)}{\text{DYZN+BCAJ}}$$

and wage inflation is defined as $\Delta w_t = \log(W_t) - \log(W_{t-1})$.

6.4.2 Data Definitions for the US

US data were obtained from:

- the US Bureau of Labor Statistics web site: http://stats.bls.gov/
- the Bureau of Economic Analysis web site: http://www.bea.doc.gov/.

All variables are seasonally adjusted and (where appropriate) at annual rates.

π is the inflation rate defined as the first difference in the logarithm of the non-government GDP deflator: $\pi_t = \log(DEF_t) - \log(DEF_{t-1})$, where $DEF = \frac{PY-PYG}{Y-YG}$, PY is GDP in current prices (\$b), PYG is General Government GDP (\$b), Y is GDP in billions of chained (1996) dollars (\$b) and YG is General Government GDP in billions of chained (1996) dollars (\$b).

The log of real marginal costs ($MC_t - p_t$) or equivalently the log of the share of labour is defined as:

$$MC_t - p_t = \log\left[\frac{COMP - COMPG}{PY - PYG - PROP}\right]$$

where: $COMP$ is total compensation of employees (\$b); $COMPG$ is government compensation of employees (\$b) and $PROP$ is proprietor's income (with inventory valuation and capital consumption adjustments (\$b).

The logarithm of nominal marginal cost (MC) is defined as:

$$MC = \log\left[\frac{COMP - COMPG + \left(\frac{COMP-COMPG}{PY-PYG-PROP}\right)PROP}{Y - YG}\right]$$

The growth in nominal marginal costs is defined as: $\Delta MC_t \equiv MC_t - MC_{t-1}$.

Real output (y) is $Y - YG$. The wage rate is defined as:

$$W = \frac{COMP - COMPG + \left(\frac{COMP-COMPG}{PY-PYG-PROP}\right)PROP}{L - LG}$$

where L and LG are (respectively) total and government employment (000) and wage inflation is defined as $\Delta w_t = \log(W_t) - \log(W_{t-1})$.