Reputation and the Allocation of Ownership*

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Abstract

We show that the allocation of ownership matters even in long-term relationships where problems of opportunism are less severe unless the agents are very patient. Ownership structure is chosen to give agents the best incentives to cooperate. The optimal ownership structure of the static game restricts the gain from deviation to be the lowest but also the punishment will be minimal. The worst ownership structure of the one-shot game is good in the repeated setting because it provides the highest punishment but bad because the gain from deviation is also the highest. We show that when investment is inelastic to surplus share joint ownership is optimal. While for elastic investment the results of the one-shot game apply. Allowing for renegotiation of ownership softens the punishment under joint ownership. Joint ownership is then optimal if the agents are indispensable enough to each other.

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1 Introduction

The property rights theory of the firm (Grossman and Hart (1986) and Hart and Moore (1990)) is based on agents' opportunistic behaviour - self-interest seeking with guile in Williamson's (1985) terminology. Ownership gives power and allows the owner to expropriate value from the employees in bargaining. This gives the owner good incentives to invest in firm-specific human capital while the employees are subject to a greater holdup problem.[1] However, the behaviour we observe in the real world is not always opportunistic: workers are loyal to their employers, firms offer good quality products to their customers etc. Macaulay (1963) finds in his survey on contractual relations in business that:

> *Businessmen often prefer to rely on 'a man's word' in a brief letter, a handshake, or 'common honesty and decency' - even when the transaction involves exposure to serious risks.*

This kind of behaviour can be explained by reputation concerns. When the one-shot gain from opportunistic behaviour is outweighed by the loss of trust in the future holdup problems should not arise. The situations that involve firm-specific investments are exactly the ones where we would expect long term relationships to predominate and where reputation effects should matter. Can we avoid holdup problems under any ownership structure? Is there any scope for allocation of ownership in a repeated relationship? These are the issues raised in this paper. A repeated game is a natural way to analyse reputation effects.

We show that the allocation of ownership indeed matters even in a repeated relationship unless the agents are very patient. We have two types of results: one where joint ownership is optimal and second where the firm has a single owner.

Joint ownership is suboptimal in the basic property rights model (Hart and Moore (1990)) while in our model it can be optimal. In the real world we observe joint ownership, particularly in the form of joint ventures, which are quite common. Joint ownership can be optimal if the parties make investments in physical capital (Hart (1995)) or if it is important to ensure that

[1] See Holmstrom and Roberts (1998) and Holmstrom (1999) for a useful discussion on the property rights theory.

parties invest inside the relationship rather than outside (Rajan and Zingales (1998)). Joint ownership can also arise if a different bargaining solution is applied (de Meza and Lockwood (1998)). Reputation effects analysed in this paper can further contribute to our understanding of joint ownership.

The property rights theory of the firm analyses how holdup problems in firm-specific human capital can be minimized by appropriate allocation of ownership. Ownership gives power to an agent in the sense that the outside option is increased. If an agent owns an additional asset she can generate a higher value on her own. This higher outside option means that she can secure a higher share of the surplus in bargaining and therefore she has improved incentives to invest. Giving power to an agent means that we are taking power away from someone else - and that someone else has lower incentives. What the property rights theory is about is allocating power so that joint surplus is maximized.

Joint ownership does not give power to anybody. If two agents jointly own an asset, they have to reach a unanimous decision on the use of the asset. If they fail to reach an agreement, they cannot use the asset. Therefore the outside options are zero and the holdup problems are maximal in the static Hart and Moore model. It is better to give power to somebody rather than to give power to nobody. Therefore having a single owner dominates joint ownership in the static model.

In the repeated game the ownership structure is chosen to give the agents best incentives to cooperate. The best ownership structure is such that the gain from cheating is lowest relative to the punishment. The worst ownership structure of the one-shot game (joint ownership) has the advantage in the repeated game that it provides the highest punishment; the joint surplus is the lowest in the punishment path. However, the highest punishment does not imply that cooperation would be most sustainable. It is also true that when the punishment is highest so is the gain from deviation. When there are no outside options the bargaining will result in an even split of the surplus; the deviant gets half of the surplus generated by the opponent's first-best investment and gains a lot from deviation. While when the agents have outside options then the deviant cannot extract as much as half of the value of the efficient investment in bargaining and therefore gains less from cheating.

The trade-off present in the repeated game is the following. Joint ownership is good because it provides the highest punishment but bad because the gain from cheating is also the highest. Single ownership restricts the gain from deviation to be the lowest but also the punishment will be minimal.

When the investment is inelastic to surplus share it is optimal to ensure that punishment is maximal: joint ownership arises in equilibrium. While for elastic investment emphasis is on minimizing the gain from deviation; it is optimal for the firm to have a single owner. When we allow for renegotiation of ownership structure the punishment is softer under joint ownership. Then in addition to inelastic investment the agents have to be indispensable enough to each other for joint ownership to be optimal.

In related papers Klein and Leffler (1981), Telser (1981) and Bull (1987) show how cooperative agreements can be self-enforcing in a variety of situations. Their analysis relies on implicit contracts while in our model the interaction of implicit and explicit contracts is crucial. In Garvey (1995) reputation effects lead to a more equal sharing rule and the sharing rule is interpreted as the ownership structure. In our paper a range of sharing rules, equal and unequal ones, can implement first best while the definition of ownership is who has the residual control rights over the assets. Baker et al. (2000), subsequent to this paper, also analyse reputation effects in the theory of the firm. Our papers are complementary as we analyse a different environment. Their environment is asymmetric in the sense that only one party has an investment. They do not analyse joint ownership which is more natural in our symmetric environment.

The rest of the paper is organized as follows. A numerical example is presented in Section 2. Section 3 introduces the model. Section 4 briefly discusses the results of the one-shot game. The repeated game without renegotiation is analysed in Section 5. In Section 6 we allow for renegotiation of ownership structure. Section 7 concludes.

2 An example

We start with a simple numerical example. There is one asset, a, and two agents, 1 and 2. Ex ante agent 1 makes an investment in specific human capital. The investment can take three values: 0, 150, or 300. The cost of the investment is 0, 90, or 40γ respectively where $5 < \gamma < 6$. Accordingly the highest investment, 300, maximizes the joint surplus.[2] Asset a is essential to agent 1: her investment has no value unless she has access to the asset. Agent 2 does not have an investment but he is important as a trading partner: without his contribution agent 1 can realize only $\frac{1}{3}$ of the value of her investment (that is, either 0, 50, or 100). Due to high transaction costs ex ante contracts can be written only on the allocation of ownership. We compare two ownership structures: joint ownership and agent 1 control. In ex post bargaining the agents split the difference.

Under joint ownership the agents have to reach a unanimous agreement on the use of the asset. Therefore in the one-shot game agent 1 receives only half of the value of her investment which does not cover its cost (both $\frac{1}{2}150 < 90$ and $\frac{1}{2}300 < 40\gamma$). 1 will not invest and the joint surplus is equal to zero. When agent 1 owns the asset she receives $\frac{2}{3}$ of the value ($\frac{2}{3} = \frac{1}{2}(1+\frac{1}{3})$). Then 1 will choose the medium investment, 150, since $\frac{2}{3}150 > 90$ and $\frac{2}{3}300 < 40\gamma$. The joint surplus is equal to 60 and 1's share of it is equal to 10. The prediction of the one-shot game is that the only investing agent should own the asset.

When the trading relationship is repeated rather than one-shot the agents may support first best by the following trigger strategy. Agent 1 implicitly agrees to make the first-best investment and agent 2 in turn agrees to pay a transfer T to agent 1. Any deviations will trigger the outcome of the one-shot game as punishment: splitting the difference in bargaining and underinvestment by agent 1. Under joint ownership it may also be necessary to renegotiate the ownership structure.

When *agent 1 owns the asset* she does not cheat if and only if the discounted payoff stream from the efficient behaviour exceeds the payoff stream from the deviation path:

$$\frac{1}{1-\delta}(T - 40\gamma) \geq 10 + \frac{\delta}{1-\delta}10 \tag{1}$$

In the efficient path agent 1 receives T and pays her investment cost 40γ. The only way agent 2 can punish is by not paying the promised transfer but instead split the difference. If 1 cheats in investment, 2 observes it and starts the punishment already in the second half of the same period. Agent

[2] The assumption $\gamma < 6$ guarantees that the highest investment is efficient and $\gamma > 5$ is required for the costs to be convex.

1 chooses the deviation investment taking into account that the surplus will be divided by the split-the-difference rule. Therefore the cheating investment and the payoff from cheating equal their levels in the one-shot game. That is, by cheating agent 1 obtains 10 in every period. This explains agent 1's incentive compatibility constraint (1) which simplifies to:

$$T - 40\gamma \geq 10 \tag{2}$$

While agent 2 will not cheat if and only if:

$$\frac{1}{1-\delta}(300 - T) \geq 100 + \frac{\delta}{1-\delta}50 \tag{3}$$

In the efficient path agent 2 pays T and receives the rest of the surplus. However, by refusing to pay the promised transfer to agent 1 he can extract more surplus. In fact, he can obtain $\frac{1}{3}$ of the value of 1's investment, that is 100. But if 2 cheats in this period, from next period on 1 will underinvest and 2 receives a payoff of 50 $\left(= \frac{1}{3}150\right)$. The higher is T the less likely it is that (3) holds. Therefore to guarantee the best incentives for cooperation we choose T such that (2) just holds, that is $T = 40\gamma + 10$. Inserting this in (3) and simplifying we obtain:

$$\delta \geq \frac{40\gamma - 190}{50}. \tag{4}$$

Agent 1 ownership implements first best if and only if (4) holds.

Joint ownership is a dominated structure in the static game and therefore in analysing the repeated game we have to take into account that the ownership structure can be renegotiated after deviation. We assume that in bargaining they split the difference and that there is a renegotiation cost σ (which is for simplicity expressed as a cost per period). Renegotiation raises the joint surplus in the punishment path from zero to 60 and accordingly renegotiation pays for $\sigma < 60$. The agents will cooperate if and only if:

$$\frac{1}{1-\delta}(T - 40\gamma) \geq 0 + \frac{\delta}{1-\delta}\frac{1}{2}(60 - \sigma) \tag{5}$$

$$\frac{1}{1-\delta}(300 - T) \geq 150 + \frac{\delta}{1-\delta}\frac{1}{2}(60 - \sigma) \tag{6}$$

Again agent 1's deviation payoff is equal to her payoff in the one-shot game, that is zero. While agent 2 can extract half of 1's first best investment by cheating, that is 150 $\left(=\frac{1}{2}300\right)$. In the punishment path without renegotiation each agent would get a zero payoff. Splitting the gains from renegotiating to agent 1 ownership gives per period payoff $\frac{1}{2}(60-\sigma)$ to each agent.

We choose T such that (5) just binds, that is:

$$T = 40\gamma + \frac{1}{2}\delta(60-\sigma) \tag{7}$$

Inserting (7) in (6) and simplifying gives:

$$\delta \geq \frac{40\gamma - 150}{90+\sigma}. \tag{8}$$

If ownership structure is not renegotiated (which is optimal when $\sigma \geq 60$) the incentive compatibility constraints are like (5) and (6) except that the last term in both equations is replaced by zero. Then we choose $T = 40\gamma$ so that agent 1's IC just binds. Inserting this transfer in agent 2's IC and simplifying gives:

$$\delta \geq \frac{40\gamma - 150}{150} \tag{9}$$

We are now ready to determine the *optimal ownership structure* which is the structure that implements first best for the greatest range of discount factors. For $\sigma < 60$ joint ownership is optimal if and only if the right-hand-side of (8) is smaller than the right-hand-side of (4), that is if and only if:

$$\gamma > \frac{19\sigma + 960}{4\sigma + 160}. \tag{10}$$

When $\sigma \geq 60$ joint ownership is optimal if and only if the right-hand-side of (9) is smaller than the right-hand-side of (4), that is if and only if $\gamma > 5\frac{1}{4}$. Accordingly, joint ownership - the worst structure of the static game - implements first best for a greater range of discount factors than agent 1 control iff γ is high enough.

We assumed that $5 < \gamma < 6$. We now have to check that the right-hand-side of (10) is within this range. For $\sigma = 0$ it is equal to 6, for $\sigma = 60$ it is $5\frac{1}{4}$,[3] and it is monotonically decreasing in σ. Therefore for σ and γ high enough joint ownership is optimal in the repeated game.

To understand this result we have constructed Table 1 which gives the gain (G) and the loss (L) from deviation under the two structures. The gain shows how much more agent 2 can obtain by cheating than by cooperating[4] while the loss describes how much lower the payoff is in the punishment path relative to cooperation.

	joint ownership	*agent* 1 *control*
G	$40\gamma - 150$	$40\gamma - 190$
L, $\sigma < 60$	$240 - 40\gamma + \sigma$	$240 - 40\gamma$
L, $\sigma \geq 60$	$300 - 40\gamma$	$240 - 40\gamma$

Table 1

The best ownership structure is such that the gain from cheating is lowest relative to the loss. We can see from Table 1 that joint ownership implements a higher loss for the cheater but the gain from deviation is higher too. The loss is greater since the investment drops to zero rather than 150 in the no renegotiation case. When joint ownership is renegotiated in the punishment path the joint surplus drops to 60 as under agent 1 ownership but it is the renegotiation cost that makes the loss higher under joint ownership. The gain is greater under joint ownership since agent 2 can extract half rather than $\frac{1}{3}$ of the value of 1's efficient investment.

How does γ affect the incentives to cooperate? It is easy to see from Table 1 that the gain from deviation is increasing in γ. When the first-best investment becomes more expensive (γ increases) agent 2 has to pay a higher transfer to agent 1 to implement efficient investment. Since the value of the investment has not changed 2's payoff is now lower under cooperation. On the other hand 2's deviation payoff is unchanged since it is not related to investment costs. Therefore the gain from deviation is higher. Table 1 also shows that the loss from deviation is decreasing in γ. The drop in surplus

[3]Note that there is no jump in the critical value for γ when we move from renegotiation range to the range where renegotiation does not take place. When there is no renegotiation the critical value is $5\frac{1}{4}$ and with renegotiation and $\sigma = 60$ it is $5\frac{1}{4}$.

[4]Remember T was chosen so that agent 1's IC binds. Therefore in discussion we can concentrate on agent 2's incentives.

after deviation is smaller when the efficient surplus is not very high in the first place.

Since the best ownership structure is such that the gain from cheating is lowest relative to the loss joint ownership is optimal if moving from agent 1 control to joint ownership increases the punishment relatively more than the gain from deviation. In this example the relative increase in the punishment is 60/(240-40γ) or σ/(240-40γ) while the relative increase in the gain is 40/(40γ -190). When γ is high the gain is high and the punishment is low. Therefore it is easier to obtain a higher relative change in the punishment. (Note that the absolute changes do not in fact depend on γ.) Then joint ownership which maximizes punishment is optimal. When γ is low the opposite is true: the gain is low and the punishment is high. Then it is optimal to put all the weight in minimizing the gain since higher relative changes are easier to obtain there and agent 1 control is optimal.

In the rest of the paper we verify the result of this example in a model where both agents have a continuous investment.

3 The model

Our stage game is a simplified version of Hart and Moore (1990). We analyse a setup where agents 1 and 2 use asset a to supply consumers. Ex ante each agent makes an investment in human capital which is specific to asset a. We model the investment as agent i directly choosing the value of the investment, v_i. The investment makes the agent more productive in using the asset. The agent for example learns to know better the properties of the asset or the environment the firm operates and can therefore generate more surplus. The cost of the investment to agent i is $c(v_i)$. We make the following assumptions about the cost of investment:

Assumption 1. *$v_i \in [0, V]$ where $V > 0$. $c(v_i) \geq 0$ and $c(0) = 0$. c is twice differentiable. $c'(v_i) > 0$ and $c"(v_i) > 0$ for $v_i \in (0, V)$, with $\lim_{v_i \to 0} c'(v_i) = 0$ and $\lim_{v_i \to V} c'(v_i) = \infty$.* .

Investment in human capital is assumed to be too complex to be described adequately in a contract. It is observable to both agents but not verifiable to third parties like the court. Therefore the agents choose the investments noncooperatively. We also rule out profit-sharing agreements.[5] Ex ante contracts can only be written on the allocation of ownership. The possible ownership structures are *agent 1 ownership, agent 2 ownership* and *joint ownership.*

Outside options play a key role in the analysis. Under agent i ownership i can work alone with the asset after the investments are sunk and sell the final good to the customers. The value of the trade without agent j's contribution is λv_i where $0 \leq \lambda \leq 1$. The value of λ depends on the importance of agent j as a trading partner. If agent j is indispensable to asset a so that giving the control of a to agent i does not enhance the surplus he can generate on his own, then $\lambda = 0$. If agent j is dispensable so that agent i could replace him by an outsider without loss of value, then $\lambda = 1$. We assume that the agents are equally important as trading partners (have identical λ's).

When an agent does not control any asset on her own she has an outside option to work for another firm. We assume that asset a is essential to the agents so that the outside wage does not depend on their investment. Without loss of generality we normalize this fixed wage to zero.

Ex post the uncertainty is resolved and the agents negotiate a spot contract. The investments are observable to both agents at the time of bargaining and therefore the efficient bargaining solution will be reached. The only source of inefficiency in this model arises from the possible underinvestment. Consistent with Hart and Moore (1990) we assume that in bargaining the agents split the difference. Finally, production occurs and the final good is sold to the customers.

In our dynamic model the stage game described above is repeated infinitely. At date 0 the agents write a contract on the allocation of ownership to maximize the joint surplus. The contract can give the ownership of the asset to the same agent(s) for all the game or induce changes in ownership. Given our assumptions about contractibility the only event this contract can be contingent on is time. Skills depreciate and the environment changes and further investments can be made in the beginning of each period. We make the extreme assumption that the investment depreciates fully before the next period begins. In the second half of the period the gains from trade are realized and the spot contract on the division of surplus is written.

[5]See Hart and Moore (1990) for the justification of these assumptions.

The agents can renegotiate the date 0 contract on ownership structure. This becomes relevant after deviation from first best. We assume that there is a renegotiation cost σ. Transaction costs from ownership changes are not negligible, especially when we refer to renegotiation after trust has been breached.

4 One-shot game

In this section we briefly examine the static game. Equation (11) gives the first best investments, v_i^*:

$$1 - c'(v_i^*) = 0 \qquad i = 1, 2 \tag{11}$$

We simplify notation by writing $v_i^* \equiv v^*$ and denote the first best joint surplus by S^* where $S^* = 2\left[v^* - c\left(v^*\right)\right]$. Since ex ante contracts on trade cannot be written, the bargaining takes place after the investments are sunk. Agent i foresees that part of the surplus she generates by her investment is expropriated in ex post bargaining while she pays the full cost of investment. Therefore underinvestment (holdup) typically arises. Ownership is allocated to induce the highest investment.

Under *joint ownership* agent i can realize the value of her investment only by reaching an agreement with agent j; her investment has no value if she does not have access to her essential asset. The agents have to reach a unanimous agreement to use the asset. Since both agents have zero outside options they split the surplus 50:50 and the payoffs for the agents are:

$$P_i = \frac{1}{2}\left(v_1 + v_2\right) - c\left(v_i\right) \qquad i = 1, 2 \tag{12}$$

Therefore agent i receives only half of the value of her investment at the margin and the investments, v_1^J and v_2^J, are given by:

$$\frac{1}{2} - c'(v_i^J) = 0 \qquad i = 1, 2 \tag{13}$$

We simplify notation by writing $v_i^J \equiv v^J$ and denote the joint surplus by S^J.

With a *single owner*, say agent 1,[6] the payoffs for the agents are:

$$P_1 = \lambda v_1 + \frac{1}{2}\left[(1-\lambda)v_1 + v_2\right] - c(v_1) \tag{14}$$

$$P_2 = \frac{1}{2}\left[(1-\lambda)v_1 + v_2\right] - c(v_2) \tag{15}$$

The investments, v_1^1 and v_2^1, are given by the following first-order conditions:

$$\frac{1}{2}(1+\lambda) - c'(v_1^1) = 0 \tag{16}$$

$$\frac{1}{2} - c'(v_2^1) = 0 \tag{17}$$

The owner's investment is the greater the more dispensable the worker is (the higher is λ) while the worker's investment does not depend on λ. Denote the joint surplus by S^1.

In this setup the ownership decision is very simple. It is easy to see from (13) , (16) and (17) that joint ownership is strictly dominated for any $\lambda > 0$. Under joint ownership no agent has an outside option, while when there is a single owner, the owner has a positive outside option and therefore improved incentives to invest. Hart and Moore (1990) obtain the same result in a more general setup.

5 Repeated game without renegotiation

When the agents are in a long term relationship and care about the future, the holdup problems described in the previous section should not be so severe. In this section we analyse when the efficient investments can be supported using the trigger strategy and reversion to the Nash equilibrium of the static game as punishment. Obviously if the agents are very patient (the discount factor is close to one) the first best can be supported under any ownership structure. We are interested in situations when the agents are not completely patient and our aim is to find an ownership structure that guarantees first best for the greatest range of discount factors. The focus of this section is on

[6]Since the agents are identical agent 2 ownership would give the same joint surplus as agent 1 ownership.

high renegotiation costs ($\sigma >> 0$) so that the ownership structure will not be renegotiated. Section 6 examines lower values for σ.

The agents implicitly agree to make the efficient investments and share the surplus according to (P_1^*, P_2^*). (The sharing rule will be determined later.) Deviation from either investment or sharing rule will trigger punishment from the opponent for the rest of the game. In particular, if agent i cheats in investment the cooperation breaks down already in the second half of the day[7]: the surplus will be divided as in the static game, not according to the efficient sharing rule. Also if there is no deviation in investment but an agent does not agree to follow the sharing rule (P_1^*, P_2^*), then bargaining will result in the split-the-difference rule.

The trigger strategy for agent i is:

- in period 1 choose v^* and follow (P_1^*, P_2^*)
- if (P_1^*, P_2^*) and $v_j = v^*$ in 1,2,...,t-1, then choose v^* and follow (P_1^*, P_2^*) in t
- if either $v_j \neq v^*$ in t or not (P_1^*, P_2^*) in t-1 or t, then apply (P_1^N, P_2^N) in t,t+1,... and choose v^N in $t+1, t+2, ...$ where superscript N refers to the Nash equilibrium of the static game

Note that the only relevant information about the previous period when a new period begins is whether there was or was not deviation. Whether the deviation was in the investment or the sharing rule does not matter. This also means that the extensive form and the outcome of the bargaining game for the static model (as proposed in Sutton (1986)) is appropriate also here for the punishment phase. Whether the agents reach an agreement or fail to do so and have to take the outside option this period does not change the rest of the game. The next period starts from the same node.

It is easy to see that cheating in the investment dominates cheating in the sharing rule. When agent i deviates in the investment, she chooses her investment taking into account that the surplus will be divided with the split-the-difference rule. (The deviation investment is thus equal to the investment

[7]Note that by cooperation we refer to efficient behaviour: first-best investments and sharing rule. Of course this is a noncooperative game. Note also that even during punishment the agents get together and make the deal but the investments are lower and the division of surplus is different.

in the one-shot game.) By definition this is more than making the first-best investment and then switching to the split-the-difference rule. Only when agent i does not have an incentive to cheat in the investment (she has first-best incentives even in the one-shot game) might she choose to deviate in the sharing rule.

First best will be supported in equilibrium if and only if the discounted payoff stream from efficient behaviour exceeds the payoff stream from the deviation path for both agents. Equations (18) and (19) give the incentive compatibility constraints for the agents.

$$\frac{1}{1-\delta}\left[T - c(v^*)\right] \geq P_1^d + \frac{\delta}{1-\delta} P_1^p \tag{18}$$

$$\frac{1}{1-\delta}\left[2v^* - T - c(v^*)\right] \geq P_2^d + \frac{\delta}{1-\delta} P_2^p \tag{19}$$

where δ is the discount factor, T is the transfer agent 1 receives from 2 under cooperation, P_i^d is i's one-shot deviation payoff and P_i^p is i's payoff in the punishment path. If agent 1 deviates in investment, agent 2 observes it already in the same period and he will not pay T to agent 1. Agent 1 saves in investment costs but receives now a share of the surplus that is determined by Nash bargaining. The deviation payoffs with the single owner are:

$$P_1^d = \frac{1}{2}\left(1+\lambda\right) v_1^1 + \frac{1}{2} v^* - c\left(v_1^1\right) \tag{20}$$

$$P_2^d = \frac{1}{2}\left(1-\lambda\right) v^* + \frac{1}{2} v_2^1 - c\left(v_2^1\right) \tag{21}$$

while under joint ownership the deviation payoffs are:

$$P_i^d = \frac{1}{2} v^* + \frac{1}{2} v^J - c\left(v^J\right) \qquad i = 1, 2 \tag{22}$$

Proposition 1 designs a sharing rule such that both agents have best incentives to cooperate.

Proposition 1 *The optimal sharing rule is:*
$P_1^* = sP_1^d + (1-s)(S^* - P_2^d)$
$P_2^* = (1-s)P_2^d + s(S^* - P_1^d)$
where $s = (P_2^d - P_2^p)/(P_1^d + P_2^d - P_1^p - P_2^p)$.

Proof. When agent 2 pays a transfer T to agent 1, the payoffs are:

$$P_1^* = T - c(v^*) \tag{23}$$

$$P_2^* = 2v^* - T - c(v^*) \tag{24}$$

Then agent 1 will cooperate if and only if:

$$\delta \geq \frac{P_1^d - T + c(v^*)}{P_1^d - P_1^p} \tag{25}$$

Likewise agent 2 cooperates if and only if:

$$\delta \geq \frac{P_2^d - 2v^* + T + c(v^*)}{P_2^d - P_2^p} \tag{26}$$

Because agent 2's incentive to cooperate is decreasing in T while 1's incentive is increasing in T, the optimal T gives the agents balanced incentives to cooperate. Setting the right-hand-sides of equations (25) and (26) equal we can solve for T^*:

$$T^* = \frac{(P_2^d - P_2^p)\left[P_1^d + c(v^*)\right] + (P_1^d - P_1^p)\left[2v^* - P_2^d - c(v^*)\right]}{(P_1^d - P_1^p) + (P_2^d - P_2^p)} \tag{27}$$

Inserting T^* in equations (23) and (24) gives the expressions in the Proposition. ■

Neither agent would have an incentive to deviate if they could get their deviation payoff even under cooperation. Since this is not feasible the best we can do is to give each agent a certain proportion of her deviation payoff. It is like agent 1 gets her deviation payoff with probability s and agent 2 gets his deviation payoff with probability $(1 - s)$ leaving the rest of the surplus, $(S^* - P_2^d)$, to agent 1. The proportion s is chosen to balance the agents' incentives to cooperate.

Inserting the optimal sharing rule of Proposition 1 in (18) or (19) gives the range of discount factors for which first best can be supported:

$$\delta \geq \frac{P_1^d + P_2^d - S^*}{P_1^d + P_2^d - P_1^p - P_2^p} \tag{28}$$

To understand the incentives to cooperate we firstly examine the gain and the loss from deviation. The gain under agent 1 ownership is:

$$\begin{aligned} G^1 &= \left[\frac{1}{2}(1+\lambda)v_1^1 - c(v_1^1)\right] - \left[\frac{1}{2}(1+\lambda)v^* - c(v^*)\right] \\ &\quad + \left[\frac{1}{2}v_2^1 - c(v_2^1)\right] - \left[\frac{1}{2}v^* - c(v^*)\right] \end{aligned} \tag{29}$$

where $G \equiv \left(P_1^d + P_2^d - S^*\right)$. This expression is strictly positive since v_1^1 is chosen to maximize the first term is square brackets and v_2^1 is chosen to maximize the third term in square brackets. The same is true for joint ownership[8] and therefore the agents can gain by cheating; they can extract more of the value of the other party's efficient investment by deviating.

If agent i cheats in investment she gains in this period but from the next period on the payoff will be lower because agent j punishes by choosing lower investment. With a single owner the loss from deviation is equal to:

$$L^1 = 2\left[v^* - c\left(v^*\right)\right] - \left[v_1^1 - c(v_1^1)\right] - \left[v_2^1 - c(v_2^1)\right] \tag{30}$$

where $L \equiv (S^* - P_1^p - P_2^p)$. The loss is strictly positive since v^* maximizes the first term in square brackets. L shows how much lower the joint surplus will be in the punishment path. If the agents are patient enough the one-shot gain from cheating is outweighed by lower payoff in the future. Equation (28) simplifies to:

$$\delta \geq \frac{G}{G+L}. \tag{31}$$

The main focus of this paper is on equation (31). The gain and loss from deviation will differ in general for different ownership structures. Define $\underline{\delta} \equiv G/(G+L)$. In what follows we concentrate on finding the control structure that guarantees first best for the greatest range of discount factors, that is gives the lowest $\underline{\delta}$. The best ownership structure is such that the gain from deviation is lowest relative to the loss. Now it becomes clear that the optimal allocation gives the ownership to the same agent(s) for all the game. For example giving ownership of the asset to agent 1 for the first t periods

[8] For joint ownership subsitute $\lambda = 0$ and $v_1^1 = v_2^1 = v^J$.

and then making agent 2 the owner for the rest of the game does not improve the incentives to cooperate in any way (it may do no harm either if $\underline{\delta}$ is equal under both control structures).

Therefore we are left with the question: is $\underline{\delta}$ minimized by removing agent 1's outside option (joint ownership) or by giving her an outside option (single owner)? We can derive the optimal control structure by examining how the lowerbound for the discount factor with a single owner, $\underline{\delta}^1(\lambda)$, moves with λ. Examining how λ affects $\underline{\delta}^1$ is like comparing different ownership structures. This is because for $\lambda = 0$ all ownership structures are equivalent, which is stated in the following Lemma.

Lemma 1 $\underline{\delta}^1(0) = \underline{\delta}^J$.

When the agents are indispensable ($\lambda = 0$) joint ownership and agent 1 ownership are equivalent since owning the asset does not improve 1's incentives to invest in the punishment path. Agent 2 has the maximal holdup power: agent 1 cannot do anything without agent 2. Then all the ownership structures are equivalent.

We start by analysing the gain and loss from cheating.

Proposition 2 *With a single owner both the gain and loss from deviation are decreasing in λ.*

Proof. Equation (29) gives the gain from deviation under agent 1 ownership. Total differentiation gives:

$$dG^1/d\lambda = \left[\frac{1}{2}(1+\lambda) - c'(v_1^1)\right] \partial v_1^1/\partial\lambda + \frac{1}{2}(v_1^1 - v^*) = \frac{1}{2}(v_1^1 - v^*) < 0 \quad (32)$$

The investment effect is negligible and therefore we can ignore the first term in (32). Accordingly, the gain is decreasing in λ. Equation (30) gives the loss from deviation under agent 1 ownership. By total differentiation we obtain:

$$dL^1/d\lambda = -\left[1 - c'(v_1^1)\right] \partial v_1^1/\partial\lambda = -\frac{1}{2}(1-\lambda)\partial v_1^1/\partial\lambda < 0 \quad (33)$$

The first order condition (16) helps us to determine the sign of this expression and to simplify it. It is easy to see from (16) that $\partial v_1^1/\partial\lambda$ is positive. Therefore (33) is unambiguously negative. ■

A high loss and low gain from deviation would guarantee good incentives to cooperate. Proposition 2 tells that removing agent 1's outside option ($\lambda = 0$) provides the highest loss. In the punishment path agent 1 receives only half of the value of her investment at the margin and therefore the punishment investment and the joint surplus are the lowest possible. Single owner with $\lambda = 0$ is like joint ownership which is the worst structure in the one-shot game. In the repeated game this structure has the advantage that it provides the highest punishment.

However, the highest punishment does not imply that cooperation would be most sustainable. Proposition 2 shows that when the punishment is highest so is the gain from deviation. When agent 2 deviates in investment the spot contract will be written with the split-the-difference rule. When there are no outside options the agents simply split the gross surplus 50:50; the deviant gets half of the surplus generated by agent 1's first-best investment and therefore gains a lot from deviation. While when agent 1 has an outside option ($\lambda > 0$) agent 2 can extract less than half of the value of the efficient investment and therefore gains less from deviation. The optimal ownership structure of the one-shot game gives the highest possible outside option and consequently the highest share of the value of her investment to agent 1 and therefore best restricts the gain from deviation for agent 2. In the same time punishment will be minimal because 1's incentive to invest in the punishment path is maximized. The worst structure of the static game is good in the repeated game because it provides the highest punishment but bad because the gain from deviation is also the highest.

Proposition 2 tells that the gain and loss from deviation move to the same direction as we change λ and it is not immediately clear what is the effect on $\underline{\delta}^1$. The change in $\underline{\delta}^1$ is given by:

$$\frac{\partial \underline{\delta}^1}{\partial \lambda} \stackrel{s}{=} \frac{\mid \partial L^1 / \partial \lambda \mid}{L^1} - \frac{\mid \partial G^1 / \partial \lambda \mid}{G^1} \tag{34}$$

where $\stackrel{s}{=}$ denotes that the expressions have the same sign. The sign of (34) depends on the difference between the *relative* changes in the gain and loss. Both changes are negative and to ease the discussion we have chosen to use their absolute values. The first term gives the relative change in the loss from cheating and the second term is the relative change in the gain. If the gain decreases relatively more than the punishment then $\underline{\delta}^1$ is decreasing in λ while when the punishment effect is greater than the gain effect then $\underline{\delta}^1$

is increasing in λ. Analysing such a difference is very subtle. Therefore we introduce an explicit functional form for the investment cost.

Assumption 1'. *$c(v) = v^{\gamma}$ where $\gamma > 1$.* .

Lemma 2 examines the properties of $\underline{\delta}^1(\lambda)$.

Lemma 2 *(i) $\underline{\delta}^1(0) = \underline{\delta}^1(1) = \underline{\delta}^J$.*

(ii) For $0 < \lambda < 1$ $\underline{\delta}^1$ is $\left\{\begin{array}{c} < \\ = \\ > \end{array}\right\}$ $\underline{\delta}^J$ if and only if $\gamma \left\{\begin{array}{c} < \\ = \\ > \end{array}\right\} 2$.

Proof. In the Appendix.

Lemma 2 helps us to construct Figure 1. The Figure compares the lower-bounds for the discount factor under different ownership structures for various values of the outside option parameter λ.

Lemma 2 shows that $\underline{\delta}^1$ is nonmonotonic in λ. $\underline{\delta}^1(0) = \underline{\delta}^J$ because the ownership structures are equivalent even in the one-shot game. $\underline{\delta}^1(1) = \underline{\delta}^J$ because for $\lambda = 1$ the gain and loss from deviation are affected by only agent 2's investment under single ownership. This investment is equal to his investment under joint ownership $\left(v_2^1 = v^J\right)$. Agent 1's investment is not included because agent 2 cannot extract any of agent 1's investment in bargaining as he is dispensable. Neither does agent 1's investment provide any punishment because she has first best incentives even in the one-shot game. This explains the nonmonotonicity.

We can determine the optimal ownership structure by examining Figure 1.

Proposition 3 *For $\sigma >> 0$ joint ownership is optimal if and only if $\gamma > 2$.*

Proof. It is immediately clear from Figure 1(a) that for $\gamma > 2$ $\underline{\delta}^J \leq \underline{\delta}^1$. ■

When γ is large it becomes important to ensure that the punishment is maximal. Then joint ownership is optimal. The agents have to reach a unanimous agreement to use the asset. Otherwise they can work for another firm at a zero wage. Therefore the joint surplus is the lowest possible in the punishment path and cheating would lead to a very bad equilibrium.

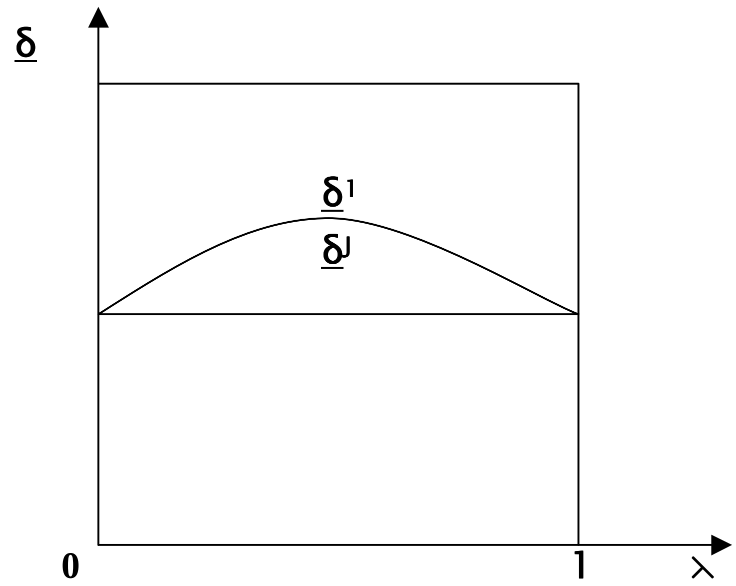

Figure 1(a) $\underline{\delta}$ for $\gamma > 2$

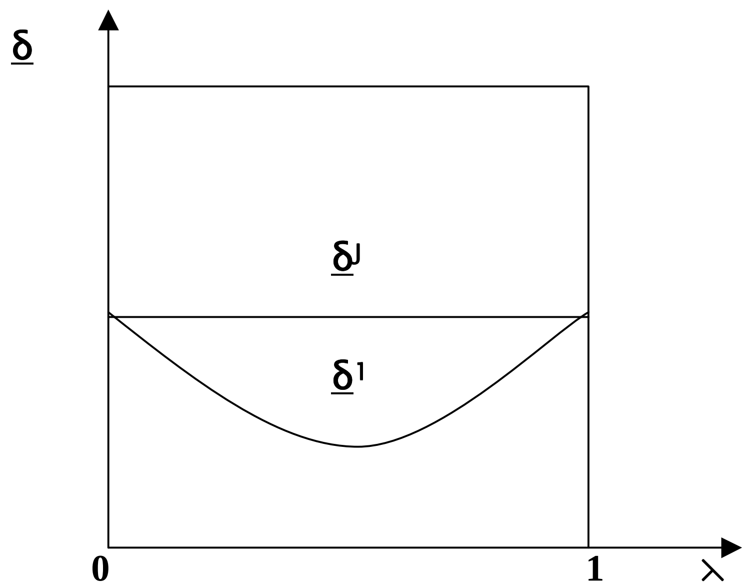

Figure 1(b) $\underline{\delta}$ for $\gamma < 2$

Proposition 4 *For $\sigma >> 0$ it is optimal to have a single owner if and only if $\gamma < 2$.*

Proof. Now Figure 1(b) where $\gamma < 2$ is appropriate. $\underline{\delta}^1 \leq \underline{\delta}^J$as the Figure illustrates. ■

Proposition 4 tells that when γ is small, the prediction of the one-shot game holds. In this parameter range it is more important to ensure that the gain from deviation is the smallest possible although then also the punishment is minimal. This is guaranteed when the asset has a single owner.

Proposition 5 *For $\sigma >> 0$ ownership does not matter if (i) $\lambda = 0$ or (ii) $\gamma = 2$.*

Proof. (i) Follows straightforward from Lemma 1. (ii) If $\gamma = 2$ $\underline{\delta}$ is equal for all ownership structures as Lemma 2 shows. ■

Proposition 5 gives the only two cases when the ownership structure does not matter in the repeated game. First, if all ownership structures are equivalent in the static game they will be equivalent in the dynamic game as well. This is the case when no allocation gives an outside option to agent 1; agent 2 is indispensable. Second, we have a more interesting equivalence result when the ownership structures differ in the one-shot game but the punishment and gain effect exactly offset each other. On the knife-edge ownership does not matter.

Our results depend on the parameter γ. To provide intuition for that remember that there are two ways to encourage cooperation: restricting the *share* of the first best investment a cheater can get or minimizing the *size* of the surplus (or maximizing the drop in surplus) after deviation. Let us concentrate on the size effect as the share does not depend on γ. The fall in surplus along the punishment path depends on the elasticity of investment (v_1^1) to surplus share $\left(\frac{1}{2}(1+\lambda)\right)$. This elasticity is equal to $\frac{1}{1-\gamma}$, i.e. for $\gamma > 2$ $(\gamma < 2)$ investment is inelastic (elastic) to surplus share. Joint ownership is optimal if without this ownership structure a deviation would only have little impact on the surplus generated. For inelastic investment joint ownership is needed to trigger a big fall in surplus along the punishment path. While for elastic investment even single ownership provides large enough punishment which combined with minimal gain from deviation results in best incentives.

6 Repeated game with renegotiation

In this section we allow σ to take any non-negative value and accordingly allow for the possibility of renegotiation of ownership structure in the punishment path. We firstly determine the parameter values for which renegotiation will take place. Under joint ownership there is renegotiation in the punishment path if and only if:

$$S^1(\lambda) - \sigma \geq S^J. \tag{35}$$

Equation (35) is satisfied for small enough σ and large enough λ. For large λ the joint surplus is much higher under agent 1 ownership and therefore the increase in joint surplus is less likely to be outweighed by the renegotiation cost σ. Define $\underline{\lambda}$ so that (35) just binds, that is $S^1(\underline{\lambda}) = S^J + \sigma$.

Lemma 3 *Ownership structure is renegotiated in the punishment path under joint ownership if and only if* $\lambda \geq \underline{\lambda}$. $\underline{\lambda}'(\sigma) > 0$ *and* $\underline{\lambda}(0) = 0$.

Proof. The first part of the Lemma follows directly from the definition of $\underline{\lambda}$ and from $\frac{\partial S^1}{\partial \lambda} > 0$. For $\sigma = 0$ we have $S^1(\underline{\lambda}) = S^J$ which holds for $\underline{\lambda} = 0$. The higher is σ, the higher $\underline{\lambda}$ has to be for $S^1(\underline{\lambda}) = S^J + \sigma$ to hold. ■

Lemma 3 states that when renegotiation is costless ($\sigma = 0$) renegotiation occurs for all λ. The higher is σ, the smaller is the renegotiation range (the higher is $\underline{\lambda}$) as the joint surplus has to be high enough with a single owner for renegotiation to pay.

How do the incentives to cooperate change when we allow for renegotiation? We start by assuming that we are in the renegotiation range and denote the critical discount factor under joint ownership by $\widetilde{\underline{\delta}}^J$.[9] After having examined the properties of $\widetilde{\underline{\delta}}^J$ we will take into account the range where renegotiation does not occur and analyse the optimal ownership structure.

[9] The analysis with a single owner does not change.

The agents have no incentive to deviate from first best if and only if:

$$\frac{1}{1-\delta}\left[T-c\left(v^{*}\right)\right] \geq P_{1}^{d}+\frac{\delta}{1-\delta}\left[P_{1}^{p}+\frac{1}{2}\left(S^{1}-P_{1}^{p}-P_{2}^{p}-\sigma\right)\right] \tag{36}$$

$$\frac{1}{1-\delta}\left[2v^{*}-T-c\left(v^{*}\right)\right] \geq P_{2}^{d}+\frac{\delta}{1-\delta}\left[P_{2}^{p}+\frac{1}{2}\left(S^{1}-P_{1}^{p}-P_{2}^{p}-\sigma\right)\right] \tag{37}$$

These incentive compatibility constraints differ from (18) and (19) with respect to the last term in square brackets. In the punishment path the ownership structure is renegotiated and the agents split the gains from renegotiation. Adding up the incentive compatibility constrains[10] and simplifying gives the lowerbound for the discount factor:

$$\delta \geq \frac{P_{1}^{d}+P_{2}^{d}-S^{*}}{P_{1}^{d}+P_{2}^{d}-S^{1}+\sigma} \equiv \underline{\widetilde{\delta}}^{J} \tag{38}$$

The gain and loss from deviation under joint ownership are now:

$$\widetilde{G}^{J}=2\left[\frac{1}{2}v^{J}-c\left(v^{J}\right)\right]-2\left[\frac{1}{2}v^{*}-c\left(v^{*}\right)\right] \tag{39}$$

$$\widetilde{L}^{J}=S^{*}-S^{1}+\sigma \tag{40}$$

Renegotiation has not changed the gain from deviation because the gain is realized before the renegotiation occurs. However, the loss is different. Joint surplus drops from S^{*} to $(S^{1}-\sigma)$ in the punishment path compared to a drop from S^{*} to S^{J} without renegotiation. Accordingly renegotiation of ownership structure softens the punishment. In the previous section the strength of joint ownership was that it maximizes punishment. Now that the punishment is softened[11] can joint ownership ever be optimal? Lemma 4 is the first step in providing an answer to that question.

[10] The agents can find a sharing rule such that neither agent has an incentive to cheat if and only if this combined incentive compatibility constraint holds. A sharing rule $\left(\frac{1}{2}S^{*},\frac{1}{2}S^{*}\right)$ gives the agents balanced incentives.

[11] Note that the loss from deviation is still higher under joint ownership even with renegotiation: $\left(S^{*}-S^{1}+\sigma\right)$ compared to $\left(S^{*}-S^{1}\right)$ with a single owner.

Lemma 4 *(i)* $\frac{\partial \widetilde{\underline{\delta}}^J}{\partial \sigma} < 0$ *and* $\frac{\partial \widetilde{\underline{\delta}}^J}{\partial \lambda} > 0$.

(ii) $\widetilde{\underline{\delta}}^J \geq \underline{\delta}^1$ *for* $\sigma = 0$.

(iii) $\widetilde{\underline{\delta}}^J = \underline{\delta}^J$ *for* $\lambda = \sigma = 0$.

(iv) $\widetilde{\underline{\delta}}^J = \underline{\delta}^J$ *for* $\lambda = \underline{\lambda}$.

Proof. (i) σ only appears in the denominator of (38), thus $\frac{\partial \widetilde{\underline{\delta}}^J}{\partial \sigma} < 0$. $\frac{\partial \widetilde{\underline{\delta}}^J}{\partial \lambda} > 0$ is implied by $\frac{\partial S^1}{\partial \lambda} > 0$.

(ii) For $\sigma = 0$ the loss is equal under joint ownership and agent 1 ownership and from Proposition 2 we know that the gain is (weakly) higher under joint ownership. Therefore $\widetilde{\underline{\delta}}^J \geq \underline{\delta}^1$.

(iii) For $\lambda = 0$ $S^1 = S^J$. Substituting this and $\sigma = 0$ in (38) gives:

$$\widetilde{\underline{\delta}}^J = \frac{P_1^d + P_2^d - S^*}{P_1^d + P_2^d - S^J}$$

which is equal to $\underline{\delta}^J$ from equation (28).

(iv) By definition $S^1(\underline{\lambda}) = S^J + \sigma$. Substituting this in (38) gives:

$$\widetilde{\underline{\delta}}^J = \frac{P_1^d + P_2^d - S^*}{P_1^d + P_2^d - S^J}$$

which is equal to $\underline{\delta}^J$ from (28). ■

Lemma 4 states that the higher is the renegotiation cost the lower is the critical discount factor under joint ownership. This is because the punishment is the stronger the higher is σ. Lemma 4 also finds that $\widetilde{\underline{\delta}}^J$ is increasing in λ (regardless of γ). Higher λ increases the surplus in the punishment path (as ownership is renegotiated to agent 1 ownership) but does not have an effect on the gain from deviation. Therefore $\widetilde{\underline{\delta}}^J$ unambiguously increases in λ.

Lemma 4 does not give the optimal ownership structure directly as we have to take into account the range where renegotiation does not occur. Lemma 4 gives the critical discount factor in the renegotiation range while when renegotiation does not occur Lemma 2 is relevant. Combining the two gives the relevant lowerbound for the discount factor. It is to that we proceed next and construct Figure 2.

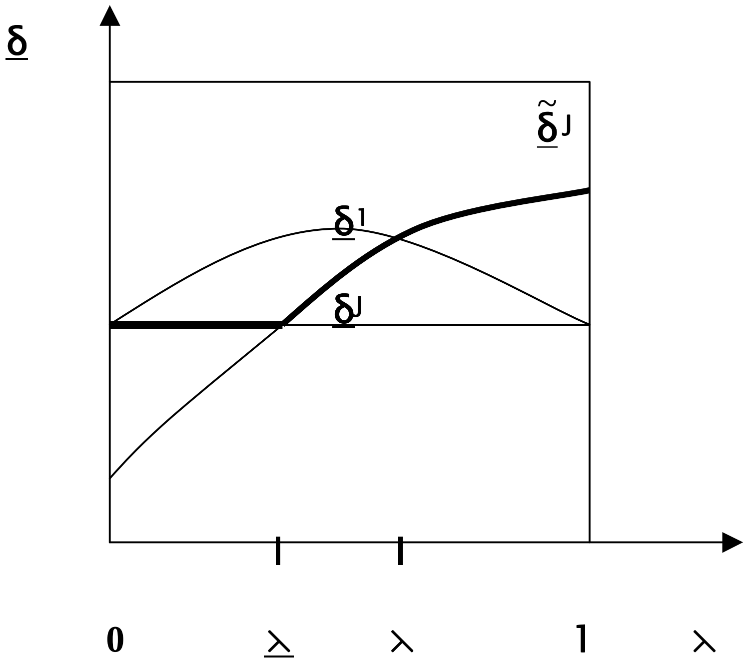

Figure 2(a) $\underline{\delta}$ for $\gamma > 2$

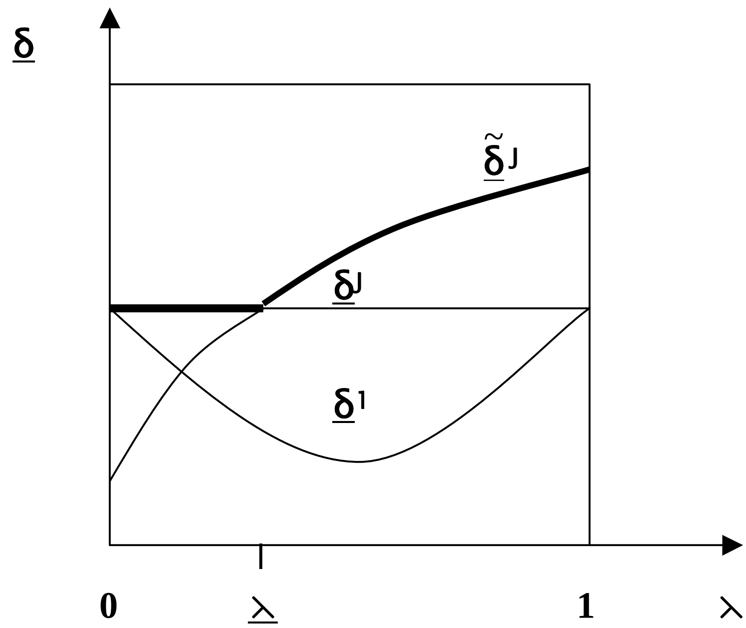

Figure 2(b) $\underline{\delta}$ for $\gamma < 2$

In Figure 2 $\underline{\delta}^J$ and $\underline{\delta}^1$ are as in Figure 1. In addition we have drawn $\widetilde{\underline{\delta}}^J$ for $\sigma > 0$. Part (iii) of Lemma 4 says that $\widetilde{\underline{\delta}}^J$ for $\sigma = 0$ and $\underline{\delta}^J$ start from the same point and according to part (i) $\widetilde{\underline{\delta}}^J$ is decreasing in σ. Therefore for $\sigma > 0$ $\widetilde{\underline{\delta}}^J$ at $\lambda = 0$ is below $\underline{\delta}^J$. Furthermore part (i) states that $\widetilde{\underline{\delta}}^J$ is increasing in λ. There facts help us to draw $\widetilde{\underline{\delta}}^J$.

Part (iv) obtains that when we move from no renegotiation range to the range where renegotiation occurs, the relevant critical discount factor is continuous. That is, we know that renegotiation range starts where $\widetilde{\underline{\delta}}^J$ cuts the horizontal $\underline{\delta}^J$ schedule. Therefore the relevant critical discount factor is given by the bold line.

The effect of the softer punishment due to renegotiation is clearly seen in Figure 2: the critical discount factor for joint ownership is (weakly) higher. We can now analyse the optimal ownership structure.

Proposition 6 *Joint ownership is optimal*
(i) *for all* $\lambda > 0$ *if and only if* $\gamma > 2$ *and* $\sigma \geq \overline{\sigma}$,
(ii) *for* $\lambda \leq \overline{\lambda}$ *if* $\gamma > 2$ *and* $\sigma < \overline{\sigma}$.

Proof. (i) Define $\overline{\sigma}$ for which $\underline{\lambda}(\overline{\sigma}) = 1$. (See Lemma 3.) For $\sigma \geq \overline{\sigma}$ renegotiation does not occur for any λ and therefore we repeat the result of Proposition 3.

(ii) See Figure 2(a). Define $\overline{\lambda}$ such that $\underline{\delta}^1\left(\overline{\lambda}\right) = \widetilde{\underline{\delta}}^J\left(\overline{\lambda}\right)$. It is clear from the figure that the critical discount factor is lowest under joint ownership for $\lambda \leq \overline{\lambda}$. ■

Proposition 7 *It is optimal to have a single owner if* (i) $\gamma < 2$, (ii) $\gamma > 2, \sigma < \overline{\sigma}$ *and* $\lambda > \overline{\lambda}$, *or* (iii) $\gamma = 2$ *and* $\lambda > \underline{\lambda}$.

Proof. (i) is obvious from Figure 2(b) and (ii) follows from the proof of Proposition 6. (iii) Lemma 4(i) proves that $\widetilde{\underline{\delta}}^J$ is increasing in λ for all γ while Lemma 2 obtains that $\underline{\delta}^1 = \underline{\delta}^J$ for $\gamma = 2$. Therefore whenever we are in the renegotiation range $(\lambda > \underline{\lambda})$ $\underline{\delta}^1 < \widetilde{\underline{\delta}}^J$ if $\gamma = 2$. ■

Comparing Propositions 6 and 7 to Propositions 3 to 5 in the previous section we can conclude that taking into account renegotiation of ownership structure makes a stronger case for the asset to have a single owner - that is, for the results of the static game to hold. In the previous section it was optimal to have a single owner if and only if investment is elastic to surplus share. In this section we additionally have single owner for inelastic investment if σ is low and λ is high and for unit elastic investment if λ is high. Renegotiation softens punishment under joint ownership and therefore the parameter range for which it is optimal is smaller.

However, even with softer punishment joint ownership is optimal for a wide parameter range. In particular, what is required for joint ownership to be optimal is that λ is not too high. In other words the agents have to be indispensable enough to each other for joint ownership to be optimal. Note that joint ownership can be optimal even when renegotiation would occur in the punishment path. In Figure 2(a) for $\underline{\lambda} \leq \lambda \leq \overline{\lambda}$ we are in the renegotiation range and $\widetilde{\underline{\delta}}^J < \underline{\delta}^1$.

Additionally we get a stronger result that ownership *does* matter in the repeated game. In the previous section ownership did not matter for unit elastic investment while now it does matter as long as λ is high as Proposition 7 states. This is also due the softer punishment under joint ownership.

7 Conclusions

In this paper we show that joint ownership is optimal (*i*) when the agents' investment has low value without the other party even if they have access to the asset and (*ii*) when investment is inelastic to surplus share (i.e. the marginal cost of investment is convex). When the parties are very important to each other mutual dependency through ownership structure supports cooperation. The majority of all joint ventures are between domestic partners and foreign business firms. These international joint ventures often combine the technological knowledge of the foreign firm with the local knowledge of the domestic firm - neither has much value without the other. Marriage is another form of joint ownership. Time and effort invested in building and maintaining a good relationship have indeed very little value without the spouse even with access to the material possessions. Therefore the first condition holds in these forms of joint ownership. The second condition, convex marginal cost, can result from time constraints. Marginal cost increases at an increasing rate as time for other activities is crowded out.

If at least one of the conditions does not hold then it is optimal to have a single owner. When parties are relatively unimportant to each other we should observe a hierarchical firm. In a typical hierarchical firm the value of the owner's specific knowledge about the firm's products and customers is not greatly diminished when a worker leaves the firm. This observation matches with our result.

Appendix

Proof of Lemma 2: **Step 1.** We first derive $\underline{\delta}^1$.

Under Assumption 1' the marginal cost is $\gamma v^{\gamma-1}$. Inserting this is (11) we get the first-best investment:

$$v^* = (1/\gamma)^{\frac{1}{(\gamma-1)}} \tag{A.1}$$

From equations (16) and (17) the punishment investments with the single owner are:

$$v_1^1 = [(1+\lambda)/2\gamma]^{\frac{1}{(\gamma-1)}} \tag{A.2}$$

$$v_2^1 = (1/2\gamma)^{\frac{1}{(\gamma-1)}} \tag{A.3}$$

We aim to derive:

$$\underline{\delta}^1 = \frac{G^1}{G^1 + L^1} \tag{A.4}$$

Using equation (29) we obtain:

$$\begin{aligned} G^1 &= \left[c(v^*) - c\left(v_1^1\right)\right] + \left[c(v^*) - c\left(v_2^1\right)\right] - \frac{1}{2}(1+\lambda)\left(v^* - v_1^1\right) - \frac{1}{2}\left(v^* - v_2^1\right) \\ &= (1/\gamma)(1/\gamma)^{\frac{1}{(\gamma-1)}} \left[\begin{array}{c} \left[1 - [(1+\lambda)/2]^{\frac{\gamma}{(\gamma-1)}}\right] + \left[1 - [1/2]^{\frac{\gamma}{(\gamma-1)}}\right] \\ -\frac{1}{2}(1+\lambda)\gamma\left(1 - [(1+\lambda)/2]^{\frac{1}{(\gamma-1)}}\right) - \frac{1}{2}\gamma\left(1 - (1/2)^{\frac{1}{(\gamma-1)}}\right) \end{array} \right] \end{aligned}$$

And from (29) and (30) we get:

$$\begin{aligned} G_1 + L_1 &= \frac{1}{2}(1-\lambda)\left(v^* - v_1^1\right) + \frac{1}{2}\left(v^* - v_2^1\right) \\ &= (1/\gamma)(1/\gamma)^{\frac{1}{(\gamma-1)}} \left[\frac{1}{2}(1-\lambda)\gamma\left(1 - [(1+\lambda)/2]^{\frac{1}{(\gamma-1)}}\right) + \frac{1}{2}\gamma\left(1 - (1/2)^{\frac{1}{(\gamma-1)}}\right)\right] \end{aligned}$$

Now we obtain the lowerbound for the discount factor under agent 1 control:

$$\underline{\delta}^1(\lambda) = \begin{array}{c} \{ \left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{\gamma}{\gamma-1}}\right]+\left[1-\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right]- \\ \frac{1+\lambda}{2}\gamma\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{1}{\gamma-1}}\right]-\frac{1}{2}\gamma\left[1-\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]\}/ \\ \left\{\frac{1-\lambda}{2}\gamma\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{1}{\gamma-1}}\right]+\frac{1}{2}\gamma\left[1-\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]\right\} \end{array} \tag{A.5}$$

We simplify notation by defining $\varepsilon \equiv (\gamma-2) > -1$ and $\eta \equiv (1+\lambda)/2$. Since $0 \leq \lambda \leq 1$, then $1/2 \leq \eta \leq 1$.

$$\begin{aligned} \underline{\delta}^1(\lambda) &= \left\{ \begin{array}{c} \left[1-\eta^{\frac{2+\varepsilon}{1+\varepsilon}}\right]+\left[1-\left(\frac{1}{2}\right)^{\frac{2+\varepsilon}{1+\varepsilon}}\right]-\eta(2+\varepsilon)\left[1-\eta^{\frac{1}{1+\varepsilon}}\right] \\ -\frac{1}{2}(2+\varepsilon)\left[1-\left(\frac{1}{2}\right)^{\frac{1}{1+\varepsilon}}\right] \end{array} \right\} / \\ &\quad \left\{(1-\eta)(2+\varepsilon)\left[1-\eta^{\frac{1}{1+\varepsilon}}\right]+\frac{1}{2}(2+\varepsilon)\left[1-\left(\frac{1}{2}\right)^{\frac{1}{1+\varepsilon}}\right]\right\} \end{aligned} \tag{A.6}$$

Next define $\nu \equiv \eta^{1/(1+\varepsilon)}$ and $\phi \equiv 2^{-1/(1+\varepsilon)}$

$$\begin{aligned} \underline{\delta}^1(\lambda) &= \left[\begin{array}{c} (1-\eta\nu)+(1-\phi/2)-(2+\varepsilon)(1-\nu)\eta \\ -(2+\varepsilon)(1-\phi)/2 \end{array} \right] / \\ &\quad [(2+\varepsilon)(1-\nu)(1-\eta)+(2+\varepsilon)(1-\phi)/2] \end{aligned} \tag{A.7}$$

Step 2. Now we prove that $\underline{\delta}^1(0) = \underline{\delta}^1(1)$.

For $\lambda = 0$ we have $\eta = 1/2$ and $\nu = \phi$. Substituting these into (A.7) we obtain:

$$\underline{\delta}^1(0) = [2(1-\phi/2)-(2+\varepsilon)(1-\phi)]/[(2+\varepsilon)(1-\phi)] \tag{A.8}$$

For $\lambda = 1$ we obtain $\eta = 1$ and $\nu = 1$. Substituting these into (A.24) we have:

$$\begin{aligned} \underline{\delta}^1(1) &= [(1-\phi/2)-(2+\varepsilon)(1-\phi)/2]/[(2+\varepsilon)(1-\phi)/2] \\ &= [2(1-\phi/2)-(2+\varepsilon)(1-\phi)]/[(2+\varepsilon)(1-\phi)] \end{aligned} \tag{A.9}$$

This proves that $\underline{\delta}^1(0) = \underline{\delta}^1(1)$.

Step 3. Next we derive $\underline{\delta}^J$. From Lemma 1 we know that $\underline{\delta}^J = \underline{\delta}^1(0)$. Therefore we can use equation (A.8).

$$\underline{\delta}^J = \left[(1-\phi/2) - (2+\varepsilon)(1-\phi)/2\right] / \left[(2+\varepsilon)(1-\phi)/2\right] \tag{A.10}$$

Step 4. Now we derive $\underline{\delta}$ in the case where both agent's punishment investment is equal to

$$\widehat{v} = \left[(1+\lambda)/2\gamma\right]^{\frac{1}{(\gamma-1)}} \tag{A.11}$$

Denote it by $\widehat{\underline{\delta}}$. This does not correspond to any ownership structure but the symmetry simplifies the proof greatly and we will use $\widehat{\underline{\delta}}$ in the next step of the proof.

The gain and the loss from deviation are:

$$\widehat{G} = \gamma^{\frac{-\gamma}{(1-\gamma)}}\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{\gamma}{\gamma-1}}\right] - \frac{(1+\lambda)}{2}\gamma^{\frac{-1}{(\gamma-1)}}\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{1}{\gamma-1}}\right] \tag{A.12}$$

$$\widehat{L} = \gamma^{\frac{-1}{(1-\gamma)}}\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{1}{\gamma-1}}\right] - \gamma^{\frac{-\gamma}{(\gamma-1)}}\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{\gamma}{\gamma-1}}\right] \tag{A.13}$$

Therefore the lowerbound for the discount factor is:

$$\begin{aligned}\widehat{\underline{\delta}} &= \left\{\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{\gamma}{\gamma-1}}\right] - \frac{(1+\lambda)}{2}\gamma\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{1}{\gamma-1}}\right]\right\} / \\ &\quad \frac{(1-\lambda)}{2}\gamma\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{1}{\gamma-1}}\right]\end{aligned} \tag{A.14}$$

Differentiating (A.14) with respect to λ we obtain:

$$\begin{aligned}\frac{\partial\widehat{\underline{\delta}}}{\partial\lambda} &\overset{s}{=} \left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{\gamma}{\gamma-1}}\right]\left\{(\gamma-1) - \left(\frac{1+\lambda}{2}\right)^{\frac{1}{\gamma-1}}\left[(\gamma-1) - \frac{(1-\lambda)}{(1+\lambda)}\right]\right\} \\ &-\gamma\left[1-\left(\frac{1+\lambda}{2}\right)^{\frac{1}{\gamma-1}}\right]\left\{(\gamma-1) - \left(\frac{1+\lambda}{2}\right)^{\frac{1}{\gamma-1}}\left[(\gamma-1) - \frac{(1-\lambda)}{2}\right]\right\}\end{aligned}$$

We aim to prove that $\frac{\partial \widehat{\delta}}{\partial \lambda} \overset{s}{=} (\gamma - 2)$. To simplify notation define $\varepsilon \equiv (\gamma - 2)$ and $\eta \equiv (1+\lambda)/2$. Then the previous equation simplifies to:

$$\begin{aligned} F_\eta(\varepsilon) &= \left[1 - \eta^{\frac{2+\varepsilon}{1+\varepsilon}}\right] \left\{(1+\varepsilon) - \eta^{\frac{1}{1+\varepsilon}} \left[(1+\varepsilon) - \frac{(1-\eta)}{\eta}\right]\right\} \\ &\quad - (2+\varepsilon)\left[1 - \eta^{\frac{1}{1+\varepsilon}}\right] \left\{(1+\varepsilon) - \eta^{\frac{1}{1+\varepsilon}}(1+\varepsilon) - (1-\eta)\right\} \end{aligned} \tag{A.15}$$

Next define $\nu \equiv \eta^{\frac{1}{1+\varepsilon}}$. Substituting ν in (A.15) and simplifying we obtain:

$$\begin{aligned} F_\eta(\varepsilon) &= (1-\nu\eta)\left[(1+\varepsilon)(1-\nu) + \nu(1-\eta)/\eta\right] \\ &\quad -(2+\varepsilon)(1-\nu)\left[(1+\varepsilon)(1-\nu) + \nu(1-\eta)\right] \\ &= (1-\nu\eta)(\nu-\eta)/\eta - \varepsilon(2+\varepsilon)(1-\nu)^2 \end{aligned} \tag{A.16}$$

From ν's definition we have $\varepsilon = \{[\ln(\eta)/\ln(\nu)] - 1\}$. Substituting this in (A.16) gives:

$$\begin{aligned} F_\eta(\varepsilon) &= (1-\nu\eta)(\nu-\eta)/\eta - \left\{[\ln(\eta)/\ln(\nu)]^2 - 1\right\}(1-\nu)^2 \\ &= \left\{\nu(1-\eta)^2 - \eta(1-\nu)^2 [\ln(\eta)/\ln(\nu)]^2\right\}/\eta \\ &= \frac{(1-\eta)^2(1-\nu)^2}{\eta[\ln(\nu)]^2}[f(\nu) - f(\eta)] \end{aligned} \tag{A.17}$$

where $f(\nu) = \frac{\nu[\ln(\nu)]^2}{(1-\nu)^2}$. It is straightforward to show that:

$$\begin{aligned} -1 &< \varepsilon < 0 \Leftrightarrow 0 < \nu < \eta < 1 \\ \varepsilon &= 0 \Leftrightarrow 1/2 \le \nu = \eta < 1 \\ \varepsilon &> 0 \Leftrightarrow 1/2 \le \eta < \nu < 1 \end{aligned}$$

We aim to prove that $F_\eta(\varepsilon) \overset{s}{=} \varepsilon$. It is easy to verify from (A.17) that $F_\eta(0) = 0$. Furthermore $F_\eta(\varepsilon) \overset{s}{=} [f(\nu) - f(\eta)]$. Therefore $f'(\nu) > 0$ for $\nu \in (0,1)$ implies that $F_\eta(\varepsilon) \overset{s}{=} \varepsilon$.

$$f'(\nu) = \frac{(1+\nu)[\ln(\nu)]^2}{(1-\nu)^3} + \frac{2\ln(\nu)}{(1-\nu)^2} = k(\nu)h(\nu) \tag{A.18}$$

where $k(\nu) = [2(1-\nu)/(1+\nu) + \ln(\nu)]$ and $h(\nu) = (1+\nu)\ln(\nu)/(1-\nu)^3$. $h(\nu) < 0$ for $\nu \in(0,1)$. $k(1) = 0$ and $k'(\nu) = (1-\nu)^2/\nu(1+\nu)^2 > 0$ and therefore $k(\nu) < 0$ for $\nu \in (0,1)$. Accordingly $f'(\nu) = h(\nu)k(\nu) > 0$ for $\nu \in(0,1)$. This completes the proof that $\frac{\partial\widehat{\delta}}{\partial\lambda} \overset{s}{=} (\gamma-2)$.

Step 5. Finally we prove that $\underline{\delta}^1(\lambda) - \underline{\delta}^J \overset{s}{=} \varepsilon$ for $0 < \lambda < 1$. We start by solving:

$$\begin{aligned}\underline{\delta}^1(\lambda) - \underline{\delta}^J &= \frac{(1-\eta\nu) + (1-\phi/2) - \eta(2+\varepsilon)(1-\nu) - (2+\varepsilon)(1-\phi)/2}{(1-\eta)(2+\varepsilon)(1-\nu) + (2+\varepsilon)(1-\phi)/2} \\ &- \frac{(1-\phi/2) - (2+\varepsilon)(1-\phi)/2}{(2+\varepsilon)(1-\phi)/2} \\ &\overset{s}{=} (1-\eta\nu)(1-\phi) - 2(1-\eta)(1-\nu)(1-\phi/2) \\ &+ (1-2\eta)(2+\varepsilon)(1-\nu)(1-\phi) \qquad \text{(A.19)}\end{aligned}$$

Next we obtain:

$$\begin{aligned}\widehat{\underline{\delta}} - \underline{\delta}^1(\lambda) &= \frac{(1-\eta\nu) - \eta(2+\varepsilon)(1-\nu)}{(1-\eta)(2+\varepsilon)(1-\nu)} \\ &- \frac{(1-\eta\nu) + (1-\phi/2) - \eta(2+\varepsilon)(1-\nu)}{(1-\eta)(2+\varepsilon)(1-\nu) + (2+\varepsilon)(1-\phi)/2} \\ &+ \frac{(2+\varepsilon)(1-\phi)/2}{(1-\eta)(2+\varepsilon)(1-\nu) + (2+\varepsilon)(1-\phi)/2} \\ &\overset{s}{=} (1-\eta\nu)(1-\phi) - 2(1-\eta)(1-\nu)(1-\phi/2) \\ &+ (1-2\eta)(2+\varepsilon)(1-\nu)(1-\phi) \qquad \text{(A.20)}\end{aligned}$$

This proves that $\widehat{\underline{\delta}} - \underline{\delta}^1(\lambda) \overset{s}{=} \underline{\delta}^1(\lambda) - \underline{\delta}^J$. Therefore one of the following has to be true:

(*i*) $\widehat{\underline{\delta}} < \underline{\delta}^1(\lambda) < \underline{\delta}^J$

(*ii*) $\widehat{\underline{\delta}} = \underline{\delta}^1(\lambda) = \underline{\delta}^J$

(*iii*) $\widehat{\underline{\delta}} > \underline{\delta}^1(\lambda) > \underline{\delta}^J$

$\widehat{\underline{\delta}}(0) = \underline{\delta}^J$ and from step 4 of the proof we know that $\partial\widehat{\underline{\delta}}/\partial\lambda \overset{s}{=} \varepsilon$. Therefore for $\lambda = 0$ and/or $\varepsilon = 0$ $\widehat{\underline{\delta}} = \underline{\delta}^J$ and (*ii*) holds. For $\lambda \in (0,1)$ and $\varepsilon < 0$ $\widehat{\underline{\delta}} < \underline{\delta}^J$ and therefore (*i*) holds. Respectively for $\lambda \in (0,1)$ and $\varepsilon > 0$ $\widehat{\underline{\delta}} > \underline{\delta}^J$ and (*iii*) holds. ■

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