# Endogenous growth and demographics in a model with old age support

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**Abstract**

This paper examines the relationships between economic and demographic outcomes when children provide old age support to parents and longevity is endogenous. Reproductive agents in overlapping generations mature safely through two periods of life and face a probability of surviving for a third period. Given this probability, each agent maximises her expected lifetime utility by choosing consumption, the number of children and investment in each child. If life expectancy is low parents choose not to invest in their children's education and the economy stays in a development trap. A growth equilibrium occurs when life expectancy is high enough.

**Keywords:** Longevity; fertility; human capital; growth.

**JEL Classification:** J13; O11; O40.

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## 1. Introduction

Demographic transition refers to a shift in reproductive behaviour from a state of high birth and death rates to a state of low birth and death rates. Ever since Kuznets (1965), economists have noticed the synchronised pace of demographic transition and economic growth. Early-developed countries have all experienced a demographic transition, while slow-growing economies are still those that could not get out of the high-fertility trap. Hence, an important question to ask is whether there exists a theoretical relationship between demographic transition and development. In the literature there have been two main approaches to endogenising the fertility decision into an economic model. The first is to assume that children are a consumption good and as such appear in their parents' utility function (Barro and Becker, 1988; Becker and Barro, 1988; Galor and Weil, 1996). The second is to assume that children provide old age support (Ehrlich and Lui, 1991; Raut and Srinivasan, 1994; Chakrabarti, 1999). The latter approach assume that parents are selfish and raise children just to provide support for their own old-age expenditure, an argument more compatible with the literature in development economics[1] although it may still be regarded of some importance in developed countries[2].

The aim of this paper is to study the relationships between demographic and macroeconomic variables when children are valued as a source of old age support and the duration of life is uncertain. It distinguishes itself from the existing literature by suggesting a simple framework able to capture all the main aspects of the demographic transition which can be summarised as consisting of three stages: (1) Advances in medical care and

[1] Eswaran (1998) considers fertility choice in an OLG model of a developing country when old age security is the motive for having children. However the paper concentrates on child mortality and is not an endogenous growth model.
[2] See Ehrlich and Lui (1991)

sanitation increase life expectancy; (2) In the first stages of economic development population growth rises because of the decline in mortality associated with and increase in fertility rate; (3) The fertility rate declines with development while life expectancy increases. To the best of my knowledge, no model with a child-as-old-age-support argument has been studied to explain all these three stages at the same time. For example, Ehrlich and Lui (1991) have to include "companionship" as well as material support in order to account for the stylised facts of the demographic transition. On the other hand, in Chakrabarti (1999) development traps are characterised by low fertility, in contrast with much of the literature.

We consider an overlapping generations economy in which the life expectancy of agents extends probabilistically to three periods. Agents are bearers of children, investors in their children's education and producers and consumers of output. Educational investment is the means of accumulating human capital which raises the future productivity of labour. A small exogenous decrease in mortality increases agents' fertility and has no growth effect in the economy. However, if life expectancy rises above a certain threshold, by raising the future returns to human capital accumulation it increases the opportunity costs of current consumption. Under such circumstances, agents spend more on children's education and have fewer numbers of children, implying a higher growth rate of output and a lower growth rate of population. These results complement those obtained in certain other models of fertility and growth (see, e.g., Becker *et al*., 1990; Galor and Weil, 1990; Nelson, 1956; Pavilos, 1995; Raut and Srinivasan, 1994), as well as those in the broader literature on poverty traps and threshold externalities. They are also related to the growing body of work on the development of economies over the very long run and the transition from pre-industrial to post-industrial societies (see, e.g., Galor and Weil, 1998; Kremer, 1993; Jones, 1999; Tamura, 1996, 1999).

The model is set out in Section 2. In Section 3 we solve for the optimal decisions of individuals. Section 4 contains our analysis of growth and demographic transition. Concluding remarks appear in Section 5.

## 2. The model

Time is discrete and indexed by $t = 0,\ldots,\infty$. There is an endogenous population of reproductive agents belonging to overlapping generations with finite but uncertain lifetimes. Each agent matures safely through two periods of life and has a probability of surviving for a third period. After being raised and educated by her parent in the first period of life, an agent becomes active as a bearer of children, an investor in their education, and a producer and consumer of output. If the agent survives to the third period of life he becomes old and depends entirely on his children's support. All agents have identical preferences and technologies, and are aware of their life expectancies.

The expected lifetime utility of an agent of generation *t*, $U^t$, is given by

$$U^t = \Phi(c_y^t) + \pi^t \theta \Phi(c_o^t);\ \ \pi^t, \theta \in (0,1); \Phi'(\cdot) > 0; \Phi''(\cdot) < 0 \tag{1}$$

where $c_y^t$ and $c_o^t$ denote consumption when young (second period of life) and consumption when old respectively, and $\pi^t$ is the probability of surviving to the third period. In our analysis parents are non-altruistic in the sense that they do not derive utility from the welfare of offspring.

Each agent enters her second period of life with a given amount of human capital which is the result of his parent's education effort. In this paper, the concept of human capital need not be restricted to including just technical knowledge and skills, but may be broadened to encompass other personal attributes such as health. An agent born in period *t* consumes $c_y^t$ units of a homogeneous output when young, gives birth to $n^t$ children, supplies one unit of labour inelastically to the labour market, transfers a given fraction $\psi$ of his income as a gift to his parent[3] and a fraction $q$ for rearing each child. She also invests in children's education by spending a fraction $e^t$ of her income on each child. If an agent survives to the third period, then she will consume a fixed fraction $\psi$ of his children's income. The budget constraints are therefore:

$$c_y^t = w^t h^t \left(1 - qn^t - e^t n^t - \psi\right) \tag{2}$$

$$c_o^t = n^t w^{t+1} h^{t+1} \psi \tag{3}$$

where $w$ is the wage per efficiency unit of labour, which in the rest of the model is normalised to one for simplicity. An agent produces her children's human capital, $h^{t+1}$, in accordance with

$$h^{t+1} = h^t \Omega\left(e^t, \pi^t\right);\ \Omega_e(\cdot), \Omega_\pi(\cdot) > 0;\ \Omega_{ee}(\cdot), \Omega_{\pi\pi}(\cdot) < 0;\ \Omega_{e\pi} > 0 \tag{4}$$

This is a Lucas (1988) type of human capital production function where this generation's human capital is an increasing function of the previous generation's level of human capital, $h^t$, and on parental level of investment in the education of each child, $e^t$. The novelty in this equation is that we assume that good health status, here proxied by parental life expectancy[4], raises the return to education and training and is complementary to educational investment. In other words, what we assume is simply that a healthy parent is more productive in his educational efforts. Or one could think of it as assuming that parent's health have an important effect on child's health, and then consider the abundant evidence (for example WHO, 1999) that adult health depends in part on child health and itself

[3] In this model the fraction of income allocated to parental support is exogenous to simplify the analysis. One could endogenise it by assuming that children are altruistic towards their parents. For example, if parental consumption entered children's utility function linearly with $\beta$ coefficient, $\psi$ would be equal to $\beta/(1+\beta+\pi\theta)$.

[4] In fact, one may consider $\pi$ as an index of health status. In this interpretation, when $\pi=0$ the agent is not deriving any utility from consumption in old age because of his poor health.

directly influences labour productivity. Also a rapidly growing literature documents the effects of ill health on children's enrolment, learning and attendance rates in school. In our setting mortality and morbidity risks are viewed as monotonically related, an assumption which seems consistent with the empirical evidence.

The model is completed by specifying the survival probability, $\pi^t$. One may think of $\pi^t$ as being influenced by a number of factors, both internal and external to an agent. These factors might range from private expenditures of income, time and effort (e.g., on medical treatment, hygiene and exercise), to government provided services and the quality of the environment (such as the extent of public health care, sanitation and pollution). Changes in life expectancy also depend on changes in public awareness and personal lifestyles brought about unintentionally by changes in levels of education through which human capital is accumulated. Finally, there are important externalities affecting a country's life expectancy, stemming from technological change in public health and medicine developed elsewhere. This type of externality seems to be much more powerful than the standard spillovers in technology. In fact, the evidence points to an increasing convergence in life expectancy across the world, which stands in marked contrast to the continuing disparities in levels of economic development. This view of events invites a simple and tractable characterisation of $\pi^t$:

$$\pi^t = \pi\left(h^t, H^t\right) \tag{5}$$

where $H^t$ is the human capital externality corresponding to the world level of human capital and $\pi_h(\cdot), \pi_H > 0$, $\pi_{hh}(\cdot), \pi_{HH}(\cdot) < 0$, $\pi(0,0) = \underline{\pi}$ and $\lim_{h,H\to\infty} \pi(\cdot) = \overline{\pi} \leq 1$. Although to keep the analysis simple we chose to black-box all these mechanisms, one could model $\pi^t$ with more explicit consideration of the various determinants outlined above. For example, in the Appendix we attend to one of these considerations explicitly: the case in which $\pi^t$ is determined by private medical expenditures. Compared to our main body of analysis, this extension of the model provides additional microfoundations but has no bearing on the marginal decisions of individuals (and leads ultimately to the same type of reduced form expression for $\pi^t$).

## 3. Individual fertility and education choices

An agent is faced with the problem of maximising (1) subject to (2), (3) and (4). The first order conditions state that for an interior solution the marginal return on child quality, that is the rate of return on human capital investment, must be equal to the marginal return on quantity, that is:

$$\frac{\Omega(\pi^t, e^t)\psi}{q + e^t} = \Omega_e(\pi^t, e^t)\psi \tag{6}$$

However, given that $\pi^t$ and $e^t$ are complementary in the production of $h^{t+1}$, a corner solution with no investment in human capital may exists in this model when $\pi^t$ is so low that the return on human capital investment is below the return on fertility for any level of $e^t$. In this case parents concentrate on quantity of children only and children simply inherit the level of human capital of the older generation.

In order to get explicit solutions we now assume that consumption is substituted intertemporally with unit elasticity by considering $\Phi(\cdot) = \log(\cdot)$. This assumption makes the analysis more tractable than it would otherwise be without causing much loss of generality. We also assume that the $\Omega$ function is defined as:

$$\Omega(\cdot) = (1 + \pi^t \delta e^t)^\gamma, \ \ \gamma \in (0,1), \ \ \delta > 0 \tag{7}$$

Solving this problem, the interior optimal decision rules for the fertility rate and educational investment are:

$$n^t = \frac{\delta\theta\left(\pi^t\right)^2(1-\gamma)(1-\psi)}{\left(\delta\pi^t q - 1\right)\left(1+\pi^t\theta\right)}, \tag{8}$$

$$e^t = \frac{\delta\gamma q\pi^t - 1}{\delta\pi^t(1-\gamma)}. \tag{9}$$

In this case human capital evolves according to the following

$$h^{t+1} = h^t\left(\frac{\gamma\left(\delta\pi^t q - 1\right)}{1-\gamma}\right)^\gamma. \tag{10}$$

These decision rules depend on the parameters $q$ (the cost of child rearing), $\gamma$ and $\delta$ (the productivity of investment in human capital), and $\theta$ (the discount factor) in the ways that one would expect. They also depend on $\pi^t$, the probability of life extension. It is assumed that $q$ is small enough such that an increase in this probability reduces the fertility rate[5] and increases the amount spent on children's education. Intuitively, a higher life expectancy raises future return to educating children relative to the expected marginal rate of substitution in consumption.

However, given the form of the learning technology, there is a corner solution for education when parental human capital is below a certain threshold and the rate of return on child quality is less than the rate of return on quantity. A necessary condition for the optimal noncorner solution (8)-(10) is that $\pi^t > \hat{\pi}$, where $\hat{\pi} = 1/\delta q\gamma$. If longevity is below the threshold then an agent chooses not to invest in educating his children. Hence fertility is

$$n^t = \frac{\pi^t\theta(1-\psi)}{q\left(1+\pi^t\theta\right)} \tag{11}$$

and

$$h^{t+1} = h^t. \tag{12}$$

[5] The assumption is that $q<(\theta+2)/\delta$

In this case the relationship between $n^t$ and $\pi^t$ goes in the opposite direction.

## 4. Demographics and development

The expressions in (10) and (12) describe the equilibrium path of development of the economy. If life expectancy is below the threshold the economy is locked in a low development trap with no growth and high fertility. In this situation, a decrease in mortality which leaves $\pi^t$ below the threshold has the effect of increasing fertility. Such a state of affairs, which may be defined as Malthusian state, is an interesting result of this paper as it captures one of the phases of the demographic transition which is often ignored in theoretical papers. The economy switches from the Malthusian regime to the modern growth regime when life expectancy rises above the threshold. In this case parents find it convenient to invest in human capital and equation (8) becomes the key to growth in this economy. Along the balanced growth path fertility decreases and the amount of resources spent on education rises.

**(Figure 1 and Figure 2 about here)**

As human capital increases, assuming initial heterogeneity in human capital or longevity across countries, the mortality revolution spreads in other areas as well. Eventually there might well be convergence in life expectancy depending on the precise function for $\pi$. Some of the countries previously locked in a poverty trap may move on a balanced growth path if the longevity spillover is enough to push them above the threshold $\hat{\pi}$. In other countries however, significant improvements in life expectancy might occur without setting off a growth process if the productivity of investment in human capital, $\delta$, is low. The empirical evidence seems to offer some support to the results of this paper. An earlier phase of reduction in mortality has in fact been identified in some countries of north-western Europe from the late seventeenth to early nineteenth centuries, even though after that mortality conditions have not always improved (and have even worsened in some areas), due to the disease environment in the cities at the beginning of the industrial revolution. Also, the lengthening of life and associated improvement in health and economic growth had a similar geographic pattern of diffusion: the north-western Europe and its overseas descendent first, then the rest of Europe and Japan, then Latin America, Middle East and Asia and finally the sub-Saharan Africa.[6] In some of these regions, particularly in sub-Saharan Africa, this improvement in life expectancy has been occurring with no economic growth. This may be interpreted, in the light of the model, as being the effect of a threshold of $\hat{\pi}$ which keeps the economy in a low development trap.

Our analytical results are confirmed by numerical simulations of a calibrated version of the model under alternative specifications of $\pi^t$. One simple specification is the logistic function:

$$\pi(h^t) = \frac{\underline{\pi}\left[\exp(\Delta\varphi)+1\right]+\overline{\pi}\left[\exp\left(\Delta h^t\right)-1\right]}{\exp(\Delta\varphi)+\exp\left(\Delta h^t\right)} \tag{11}$$

The parameters $\varphi$ and $\Delta$ determine the turning point in $\pi$ and the speed of transition.

Treating each adult period as lasting 30 years and the first childhood period as lasting 10 years (with the additional assumption that agents have children at the age of 30), our baseline parameter values are: $\overline{\pi} = 0.9$, $\underline{\pi} = 0.1$, $\Delta = 0.02$, $\varphi = 0.3$, $\gamma = 0.9$, $\delta = 45$, $q$=0.05, $\psi = 0.25$, and $\theta = 0.63$. This economy is on a balanced growth path when $\pi = 0.5$, i.e. when life expectancy is 55 years. Along the growth path, life expectancy increases to 67 years of age, the total fertility rate per couple diminishes from 6.4 to 2.1 and human capital per person increases tenfold in the first 120 years of economic growth.

[6] For a full account of the mortality revolution see Easterlin (1998).

## 5. Conclusions

Theoretical models of fertility choice with old age support motive have ignored the role of lifetime extension and health improvements on education choices. The model developed in this paper is an attempt at filling in some of the gaps by allowing for endogenous lifetimes and two-way causality in the relationship between longevity and economic activity. By incorporating these features it yields additional insights into the process of demographic transition. As well as being able to explain the switch from a Malthusian regime to a modern growth regime, our analysis has important implications for the long-term development of an economy and the extent to which initial inequalities in life expectancy and economic standards between countries are likely to persist. These implications are complementary to those found in the existing literature on poverty traps, growth miracles and threshold externalities, but are derived from a different perspective that sheds new light on the issue of why some countries may undergo a “mortality revolution” and still lag behind others while other countries converge in both life expectancy and economic standards.

## Appendix: $\pi(\cdot)$ as function of health expenditure

Suppose that agents can undertake certain activities that increase their chances of survival. In particular, let us assume that each agent has at least some control over the probability of her survival through direct expenditures on improving her health. The probability of survival is specified as:

$$\pi^t = \pi(z^t); \pi'(z) > 0; \pi''(z) < 0; \pi(0) > 0; \lim_{z\to 1}\pi(z) = \overline{\pi} \tag{12}$$

The utility function can be rewritten as:

$$U^t = \log(c_y^t) + \pi(z^t)\theta\log(c_o^t), \quad \pi^t, \theta \in (0,1) \tag{13}$$

and the first period budget constraints becomes:

$$c_y^t = w^t h^t (1 - qn^t - e^t n^t - \psi)(1 - z^t) \tag{14}$$

where $(1\text{-}z^t)$ is the portion of disposable income spent on health. Under these circumstances and given our previous results we can write the following condition for the maximisation of utility with respect to $z^t$:

$$-\frac{1}{1-z^t} + \pi'(z^t)\theta\log(c_o^t) + \frac{\theta\pi'(z^t)(\gamma\delta q\pi(z^t)-1)}{\delta q\pi(z^t)-1} = 0 \tag{15}$$

This expression can be rewritten as:

$$\frac{1}{1-z^t} = \pi'(z^t)\theta v(h^t, z^t) \tag{16}$$

where

$$v(h^t, z^t) = \log(c_o^t) + \frac{\delta\gamma q\pi(z^t)-1}{\delta q\pi(z^t)-1}, \quad v_h(\cdot) > 0 \text{ and } v_z > 0 \text{ if } \pi > \frac{1}{\delta q\gamma} \tag{17}$$

This expression implies:

$$z'(h^t) = -\frac{\theta\pi'(z^t)v_h(\cdot)}{-\frac{1}{(1-z)^2} + \theta(v(\cdot)\pi''(z^t) + v_z(\cdot)\pi'(z^t))} > 0 \tag{18}$$

which gives

$$\pi^t = \pi\left(z(h^t)\right) = \hat{\pi}(h^t); \hat{\pi}'(h^t) = \pi'(z^t)z'(h^t) > 0 \quad (19)$$

$$\hat{\pi}''(h^t) = \pi''(z^t)z'(h^t)^2 + \pi'(z^t)z''(h^t) \quad (20)$$

An expression for $z''$ can be found from (17) as:

$$z''(h^t) = -\frac{\theta}{\left[-1\big/\left(1-z(h^t)\right)^2 + \theta\left(v(\cdot)\pi''(h^t) + v_z(\cdot)\pi'(h^t)\right)\right]^2} * \quad (21)$$
$$\Big\{\left[-1\big/\left(1-z(h^t)\right)^2 + \theta\left(v(\cdot)\pi''(h^t) + v_z(\cdot)\pi'(h^t)\right)\right]\left[v_h(\cdot)\pi''(z^t)z'(h^t) + \pi'(z^t)\left(v_{hh}(\cdot) + v_{hz}(\cdot)z'(h^t)\right)\right] -$$
$$\pi'(z^t)v_h(\cdot)\Big[-2z'(h^t)\big/(1-z)^3 + \theta\Big(v(\cdot)\pi'''(z^t)z'(h^t) + \pi''(z^t)\left(v_h(\cdot) + v_z(\cdot)z'(h^t)\right) +$$
$$v_z(\cdot)\pi''(z^t)z'(h^t) + \pi'(z^t)\left(v_{zh}(\cdot) + v_{zz}(\cdot)z'(h^t)\right)\Big)\Big]\Big\}$$

which is negative unless $\pi'''(z^t)$ is positive and very large. Assuming this is not the case, then $\pi''(h^t) < 0$ as in the main text.

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**Figure 1.** The effect of longevity on fertility

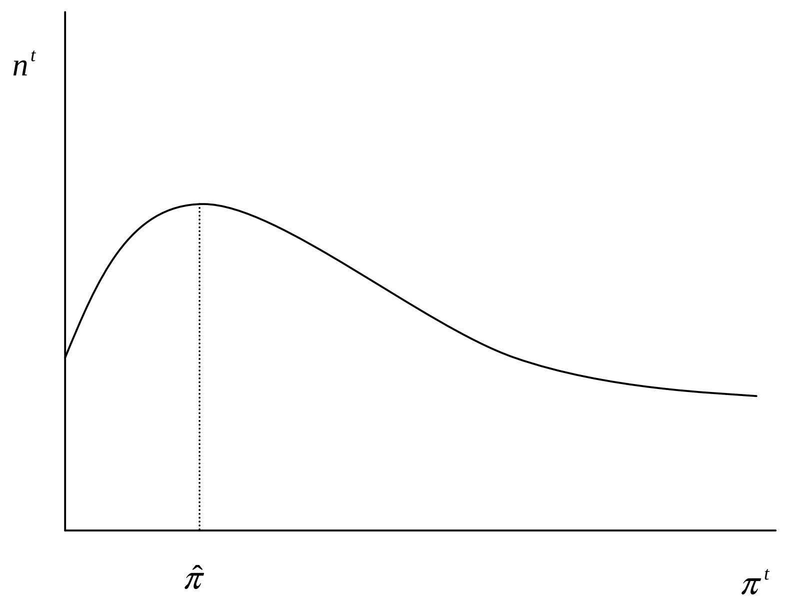


**Figure 2.** Dynamics of human capital