PARAMETER IDENTIFICATION BY THE VIRTUAL FIELDS METHOD: APPLICATION TO THE STIFFNESS RADIAL VARIABILITY OF P. PINASTER WITHIN THE STEM

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OUTLINE

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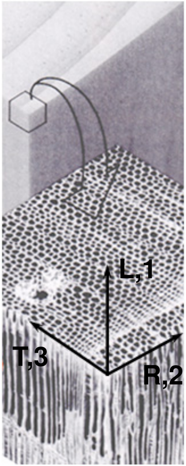
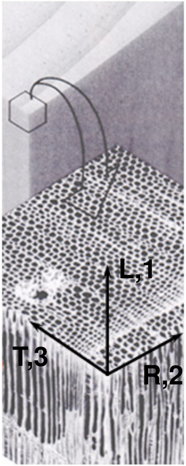
1. INTRODUCTION
2. IDENTIFICATION APPROACH
3. APPLICATION: SPATIAL VARIABILITY
4. CONCLUSIONS

MECHANICAL MODEL FOR CLEAR WOOD



- Continuous and homogeneous material
- Orthotropic linear elastic behaviour
  (Guitard, 1987)

MECHANICAL MODEL FOR CLEAR WOOD



- Continuous and homogeneous material
- Orthotropic linear elastic behaviour (Guitard, 1987)

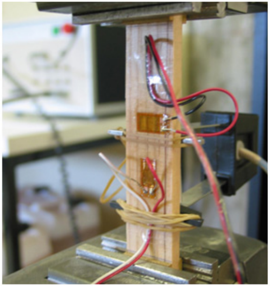
$(L, R) \equiv (1, 2)$ MATERIAL PLANE

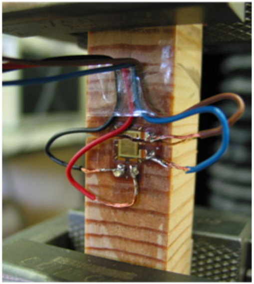
$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} \frac{E_{11}}{(1-\nu_{12}\nu_{21})} & \frac{-\nu_{21}E_{11}}{(1-\nu_{12}\nu_{21})} & 0 \\ \frac{-\nu_{12}E_{22}}{(1-\nu_{12}\nu_{21})} & \frac{E_{22}}{1-\nu_{12}\nu_{21})} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}$$

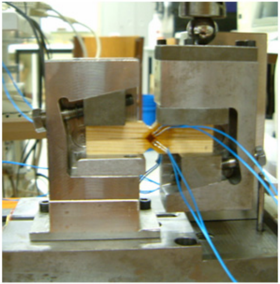
$$= \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}$$

CONVENTIONAL MECHANICAL TESTS

E_{11}, ν_{12} : | E_{22} : | G_{12} :







$\Longrightarrow$

3 Homogeneous tests

4 Elastic properties

SPATIAL VARIABILITY OF WOOD PROPERTIES

- Intra- and inter-variability of wood properties

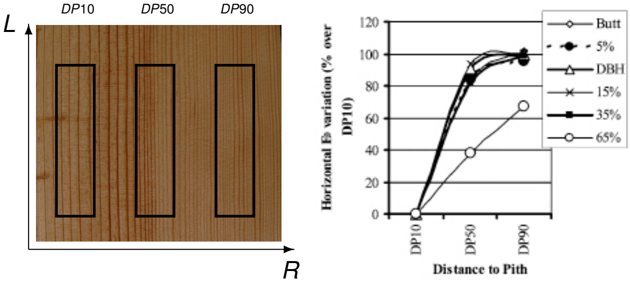
SPATIAL VARIABILITY OF WOOD PROPERTIES

- Intra- and inter-variability of wood properties
- Scarce experimental work: $E_{11} = f(R)$

Machado and Cruz: **Holz Roh Werkst**, 63(2):154-159, 2005

—*P. pinaster* wood

—3-point bending tests



MOTIVATION

GREAT AMOUNT OF EXPERIMENTAL WORK

- Several independent mechanical tests (anisotropy)
- Several specimens need to be tested (variability)

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OBJECTIVES

1. Identification of several *LR* stiffness parameters from a single test using a small specimen
2. Understanding the structural factor interfering in the stiffness spatial variability (Q_{22}, Q_{66})

OUTLINE

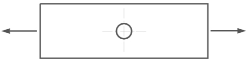
Identification of several elastic parameters by a single test

- Heterogeneous mechanical test

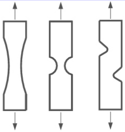
Identification of several elastic parameters by a single test

- Heterogeneous mechanical test

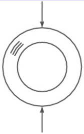
Tensile test:
Open-hole specimen
(Molimard *et al.*, 2005)



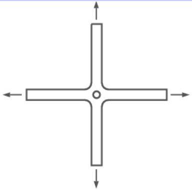
Tensile test:
dog-bone like specimens
(Pannier *et al.*, 2006)



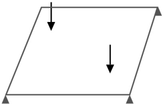
Compression test:
ring
(Moulart *et al.*, 2006)



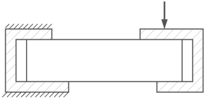
Biaxial tensile tests:
cruciform specimen
(Lecompte, 2007)



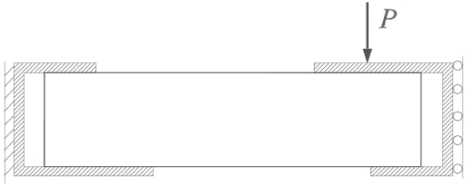
Bending plate test
(Le Magorou *et al.*, 2002)



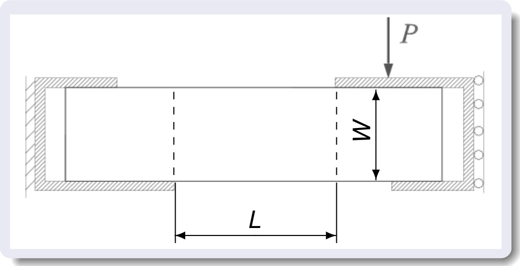
Unnotched Iosipescu test:
(Chalal *et al.*, 2006)



UNNOTCHED IOSIPESCU TEST

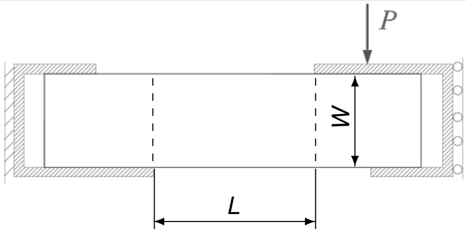


UNNOTCHED IOSIPESCU TEST



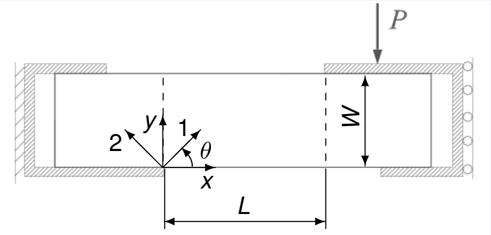
- Region of interest: $L \times W$ (mm^2)

UNNOTCHED IOSIPESCU TEST



- Region of interest: $L \times W$ (mm^2)
- Heterogeneous strain fields: $\varepsilon_i(x, y) \neq 0 \quad (i = 1, 2, 6)$

UNNOTCHED IOSIPESCU TEST



- Region of interest: $L \times W$ (mm^2)
- Heterogeneous strain fields: $\varepsilon_i(x, y) \neq 0 \quad (i = 1, 2, 6)$
- Balanced $\varepsilon_i(x, y)$ distribution: optimisation of L (mm) and θ (°)

IDENTIFICATION OF SEVERAL ELASTIC PARAMETERS BY A SINGLE TEST

- Heterogeneous mechanical test $\mapsto$ Unnotched Iosipescu test
- Full-field optical method

Identification of several elastic parameters by a single test

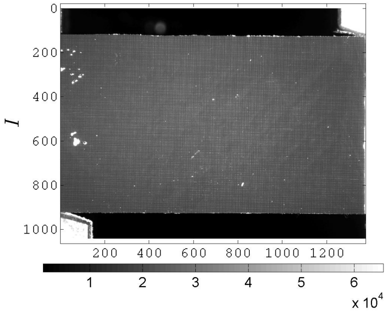
- Heterogeneous mechanical test $\mapsto$ Unnotched Iosipescu test
- Full-field optical method

White-light techniques		
	Periodic pattern	Speckle pattern
u_x, u_y	Grid method	Digital image correlation
		Stereo-correlation ($+\ u_z$)

Interferometric techniques		
	Diffuse light	Diffracted light
u_x, u_y, u_z	Speckle interferometry	Moiré interferometry
$\varepsilon_x, \varepsilon_y, \varepsilon_s$	Speckle shearography	Grating shearography

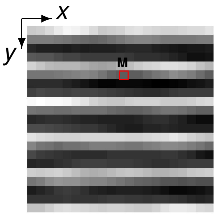
GRID METHOD

Undeformed state (*i*)



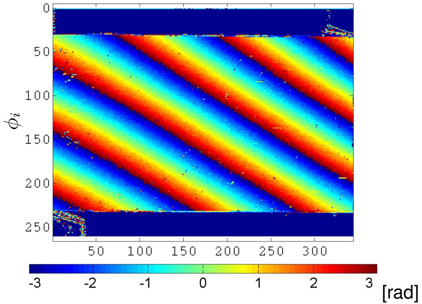
$\Longrightarrow$

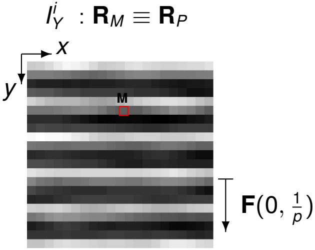
$$I_Y^i \; : \mathbf{R}_M \equiv \mathbf{R}_P$$



GRID METHOD

Spatial phase-shifting method

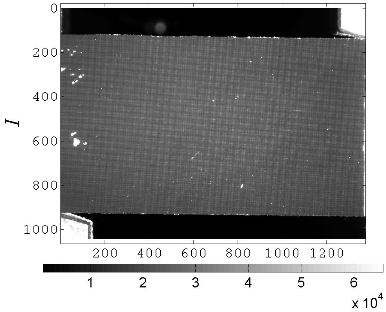




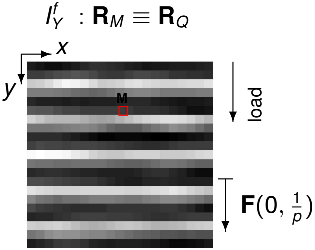
- $\phi_\beta^i(x,y) = 2\pi \mathbf{F}.\mathbf{R}^i$

GRID METHOD

Deformed state (f)



$\Longrightarrow$



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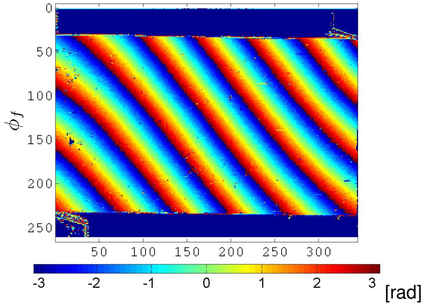
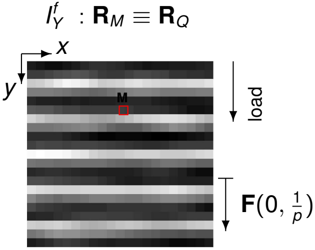
GRID METHOD

Spatial phase-shifting method

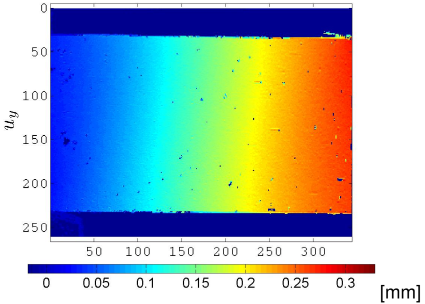


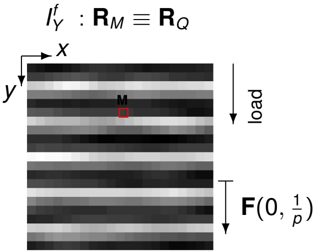


- $\phi_\beta^i(x, y) = 2\pi \mathbf{F}.\mathbf{R}^i$
- $\phi_\beta^f(x, y) = 2\pi \mathbf{F}.\mathbf{R}^f$

GRID METHOD

Phase-displacement relationship

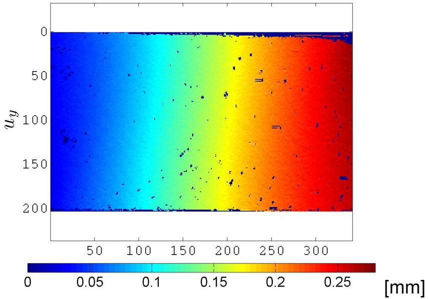




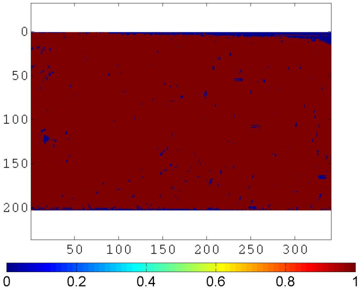
- $\phi^i_\beta(x, y) = 2\pi\mathbf{F}.\mathbf{R}^i$
- $\phi^f_\beta(x, y) = 2\pi\mathbf{F}.\mathbf{R}^f$
- $u_\beta(x, y) = -\frac{p}{2\pi}\Delta\phi^{f-i}_\beta(x, y) \qquad (\beta = x, y)$

STRAIN FIELD RECONSTRUCTION

Raw displacement

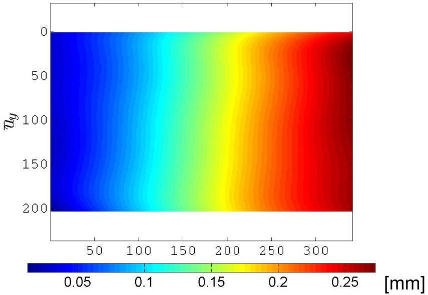


Binary mask



STRAIN FIELD RECONSTRUCTION

Polynomial displacement



LEAST-SQUARES APPROXIMATION SCHEME

$$\min_{\{\mathbf{a}_\beta\}} \ \mathbf{w}\left(\mathbf{u}_\beta - \overline{\mathbf{u}}_\beta\right)^2$$

$$\overline{\mathbf{u}}_\beta = \sum_{i,j}^{d} a_{\beta_{ij}} \, \mathbf{x}^i \, \mathbf{y}^j \qquad (i + j < d)$$

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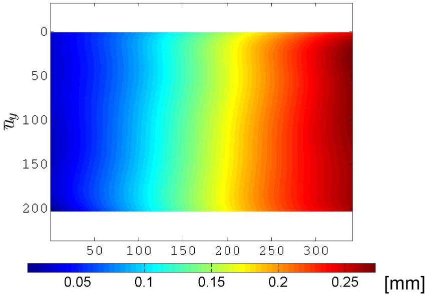
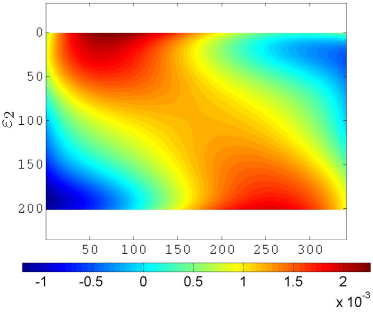


Strain field



LEAST-SQUARES APPROXIMATION SCHEME

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$$\overline{\mathbf{u}}_\beta = \sum_{i,j}^{d} a_{\beta_{ij}} \, \mathbf{x}^i \, \mathbf{y}^j \qquad (i+j<d)$$

$\varepsilon - u$ RELATIONSHIP

$$\varepsilon_{ij} = \frac{1}{2}\left[\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right]$$
$$(i,j = 1,2,6)$$

IDENTIFICATION OF SEVERAL ELASTIC PARAMETERS BY A SINGLE TEST

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- Full-field optical method $\mapsto$ Grid method
- Identification method

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Finite element model updating method	Virtual fields method
↓	↓

Equilibrium equation + Constitutive law:

$$-\int_V Q_{ij}\varepsilon_j\varepsilon_i^\star \, \mathrm{d}V + \int_{S_f} T_\beta(M)u_\beta^\star(M)\, \mathrm{d}S = 0$$

↓	↓
$\min\limits_{Q_{ij}} \lVert U^{\text{num}} - U^{\text{exp}} \rVert$	$Q_{ij} = f(\int_{S_f} T_\beta(M), \varepsilon_j, u_\beta^\star(M), \varepsilon_i^\star)$

VIRTUAL FIELDS METHOD

EQ. I: PRINCIPLE OF VIRTUAL WORK

(PLANE STRESS, STATICS, ABSENCE OF BODY FORCES)

$$\int_S \sigma_i \varepsilon_i^\star \, \mathrm{d}S = \frac{1}{t} \int_{S_f} T_\beta(M, n_\beta) u_\beta^\star(M) \, \mathrm{d}S$$

EQ. II: ORTHOTROPIC LINEAR ELASTIC LAW

$$\sigma_i = Q_{ij} \varepsilon_j$$

VIRTUAL FIELDS METHOD

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Homogenous material

EQS. II → I:

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VIRTUAL FIELDS CHOICE

$$\left(u_\beta^{\star(\alpha)}, \varepsilon_i^{\star(\alpha)} \right) \qquad \alpha = 1, 2, 3, 4$$

VIRTUAL FIELDS METHOD

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$$[P]\{Q\} = \{R\}$$

VIRTUAL FIELDS METHOD

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VIRTUAL FIELDS CHOICE

$$\left(u_\beta^{\star(\alpha)}, \varepsilon_i^{\star(\alpha)}\right) \qquad \alpha = 1, 2, 3, 4$$

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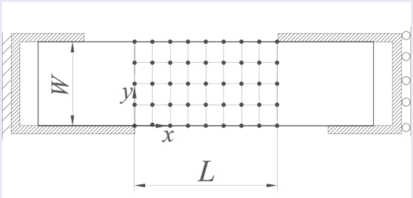
OPTIMISED PIECEWISE SPECIAL VIRTUAL FIELDS



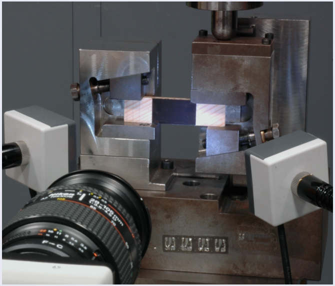
- Automatic construction of virtual fields (Grédiac *et al.*, 2002)
- Piecewise construction of virtual fields (Toussaint *et al.*, 2006)
- Minimisation of the sensitivity of the VFM to noisy data (Avril *et al.*, 2004)

Identification approach

Identification of several elastic parameters by a single test

- Heterogeneous mechanical test $\mapsto$ Unnotched Iosipescu test
- Full-field optical method $\mapsto$ Grid method
- Identification method $\mapsto$ Virtual fields method

$Q_{11}, Q_{22}, Q_{12}, Q_{66}$:



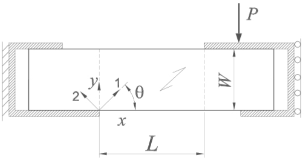
$\Longrightarrow$

1 heterogenous test

several elastic parameters

OUTLINE

- **Validation:** Xavier et al.: **Holzforschung**, 61(5): 573-581, 2007

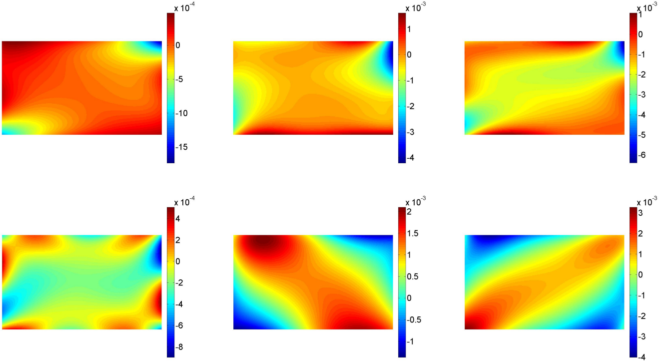


$\mapsto L = 34$ mm, $\theta = \{0°, 45°\}$

ε_1 ε_2 ε_6

0°

45°

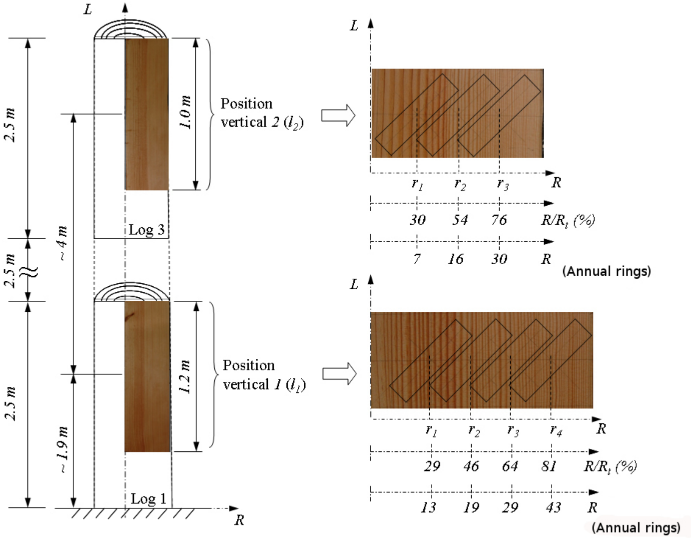


$\Rightarrow \{Q_{11}, Q_{66}\}$

$\Rightarrow \{Q_{22}, Q_{66}\}$

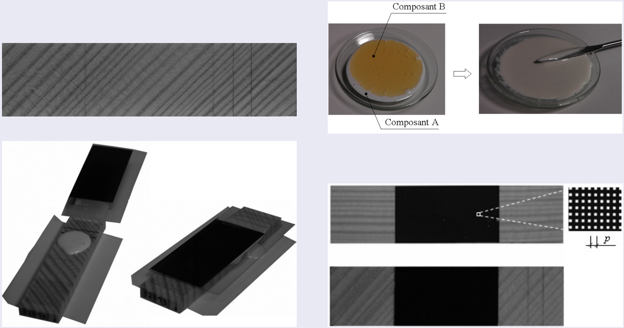
- Application: radial variability within the stem

- **Application:** radial variability within the stem
  $\mapsto$ specimens sampling 45° configuration (Q_{22}, Q_{66})



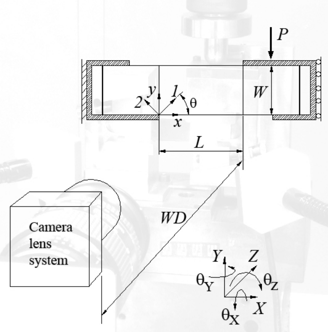
GRID TRANSFER

- Region of interest: $34(L) \times 20(W)$ mm^2
- CCD camera: 1376(H)$\times$1040(V) $\Longrightarrow$ p=0.1 mm
- Pixels/period (N) : 4



MEASUREMENT DETAILS

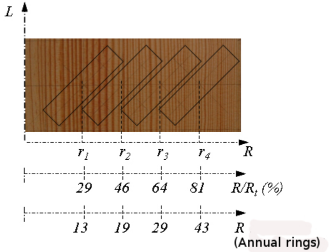
PHOTO-MECHANICAL SET-UP

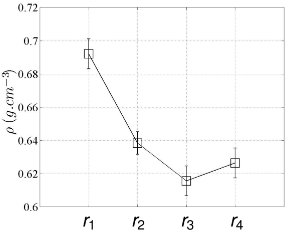


- PCO 12 bit camera
- Nikon AF 28-105 mm IF lens
- 15 mm extension tube
- $m = \frac{6.45\mu m}{25\mu m}$ =0.258 (1:3.9)
- $WD = 450$ mm
- focal length : $f = 100$ mm
- exposure time: $1/8$ =0.125 s
- f-number: $f/8$
- acquisition: 1 load/image per s

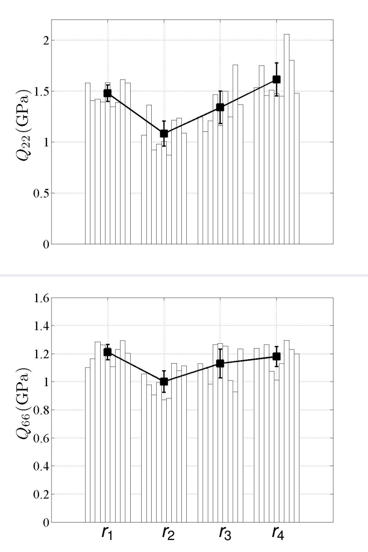
- Spacial resolution: $\sim$ 0.1 mm
- Displacement resolution: $\sigma_u \in$ [0.9, 1.2] μm

IDENTIFICATION RESULTS: RADIAL VARIABILITY





$Q_{ij}(l_1, r_i)$



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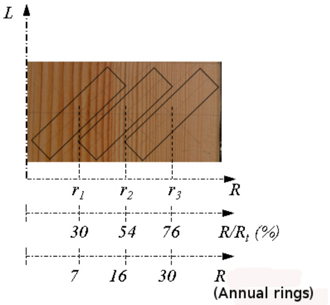
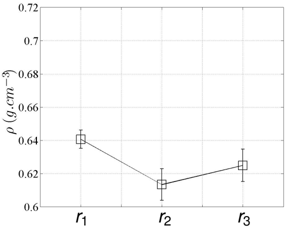
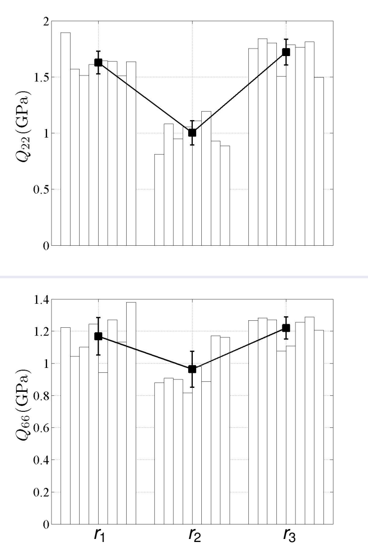


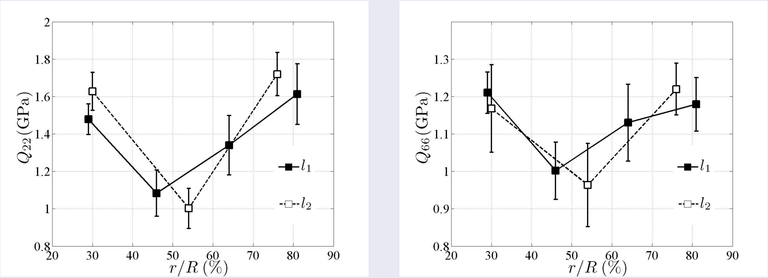


$Q_{ij}(l_2, r_i)$



IDENTIFICATION RESULTS: LONGITUDINAL VARIABILITY

$Q_{ij} = f(l)$



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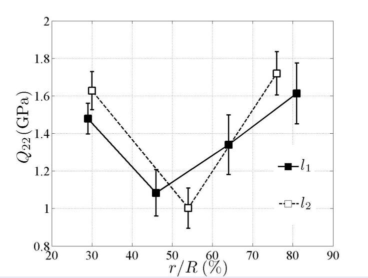

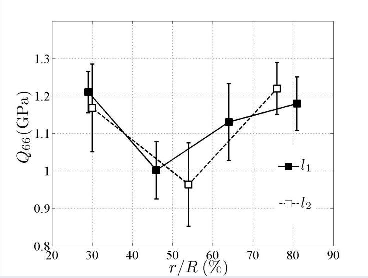

- Q_{22} and Q_{66}: decrease from the pith to about the middle radius of the stem and increase afterwards to the outmost positions:

$$\begin{array}{|ll} Q_{22}: & 49\text{–}72\% \\ Q_{66}: & 18\text{–}27\% \end{array}$$

OUTLINE

CONCLUSIONS

- Extension of the VFM to a complex material: wood
- Specimen configurations allowing the simultaneously identification of Q_{11} and Q_{66} (0° configuration) and Q_{22} and Q_{66} (45° configuration)
- Reference values for the radial variability of Q_{22} (49-72%) and Q_{66} (18-27%)
- Under further investigation:
  - coupling the spatial variability of the elastic parameters with the material morphology
  - parametrisation of the spatial variability from a single plate bending test

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Thank you
for your attention!

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