Length of the cohesive zone in delaminated composite materials

<u>A.Turon</u>, J.Costa, P.Maimí
AMADE, Polytechnic School, University of Girona

P.P.Camanho
DEMEGI, Faculdade de Engenharia, Universidade do Porto

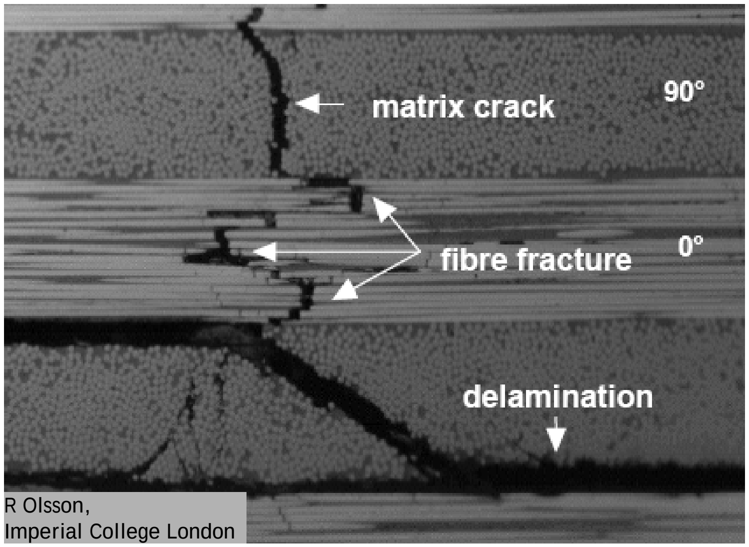
AMADE
ANALYSIS AND ADVANCED MATERIALS FOR STRUCTURAL DESIGN

Universitat de Girona

Outline

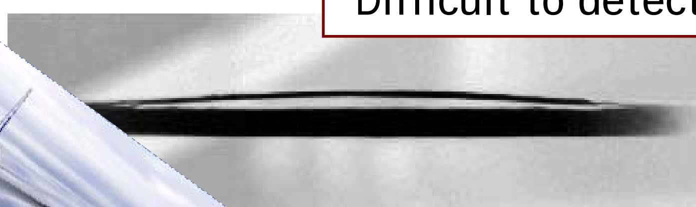
- Introduction
- Length of Cohesive Zone
    - Isotropic materials
    - Orthotropic materials
    - Finite sized geometries
- Simulation using cohesive elements
    - Mode I, Mode II and Mixed Mode
- Delamination simulation using coarse meshes
- Conclusions

Introduction



R Olsson,
Imperial College London

Difficult to detect



Causes of delamination:

- Low velocity impacts
- Manufacturing defects
- In service loads

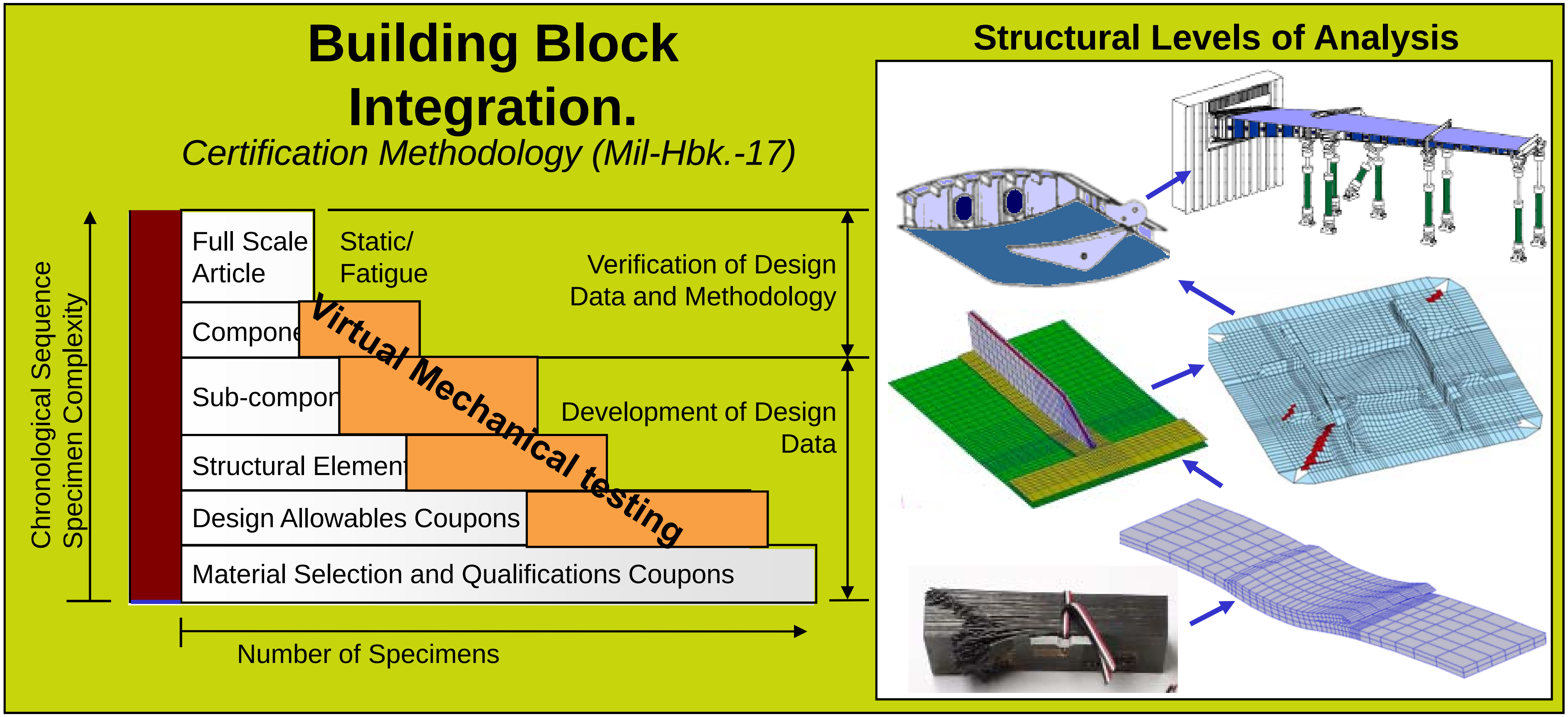
Consequences:

- Loss of stiffness
- Loss of Compression strength
- Structural Collapse
- Water intake

...

Delamination has been often referred to as *"the most feared failure mode to attack a structural composite"* *(Pagano and Schoeppner, 2000)"*

Virtual testing

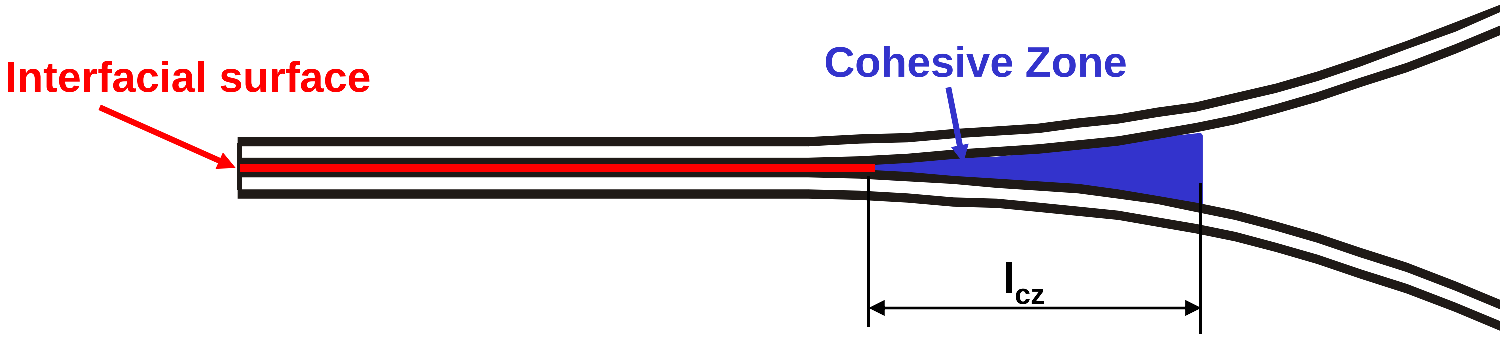


ANALYSIS AND ADVANCED MATERIALS FOR STRUCTURAL DESIGN
Universitat de Girona

Simulation of delamination

FE using CZM approach

- Fracture properties → Cohesive surface
- Properties of the bulk material → Continuum regions

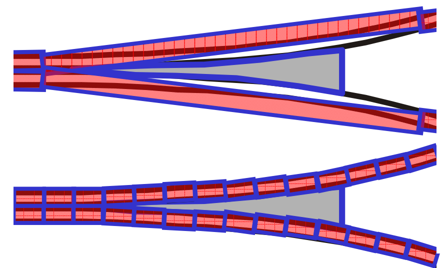


Necessary conditions for accurate simulations:

- Global compliance is unaffected by the interface.
- Mesh size smaller than the cohesive zone length (3-4 elements).

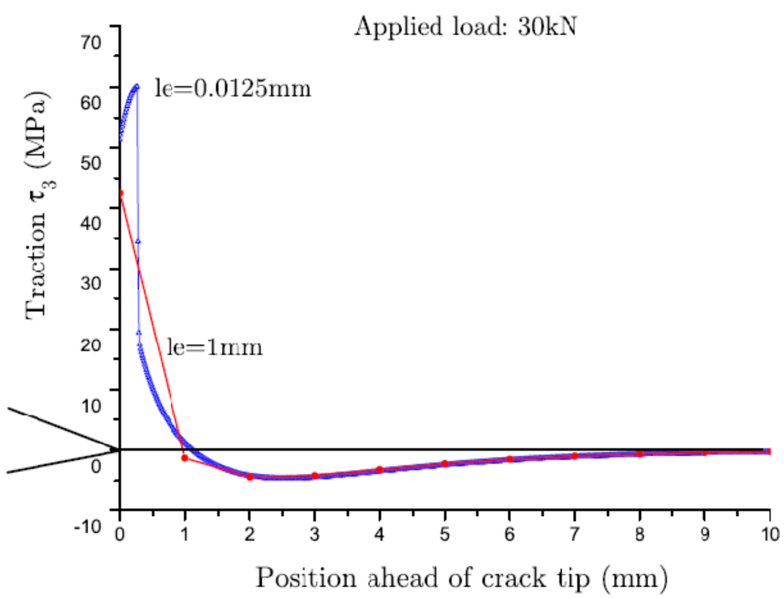
Mesh size and Cohesive Zone length

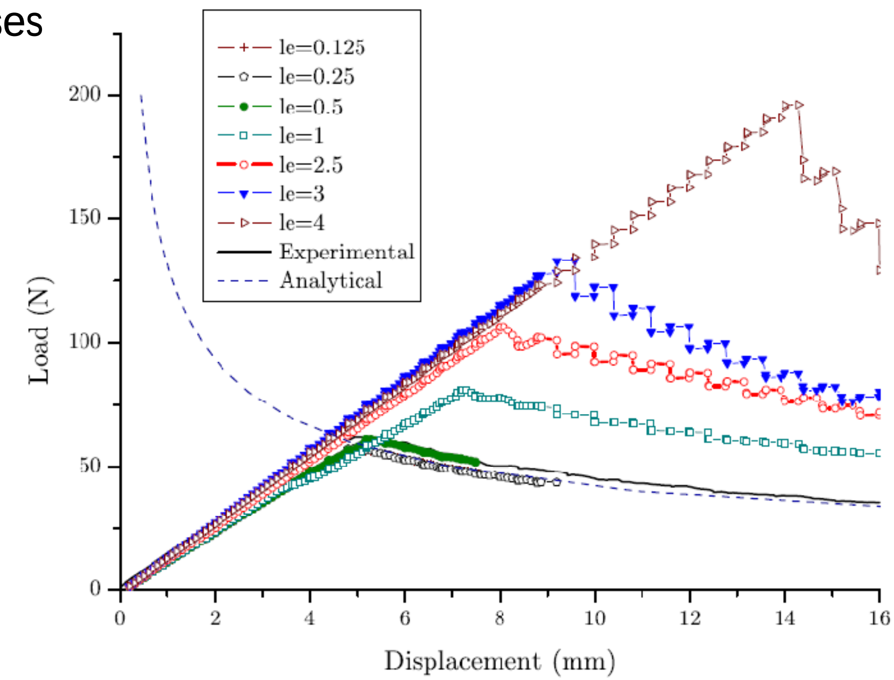
If the mesh size is larger than the cohesive zone:



- The softening region ahead of the crack tip is not captured.
- Fracture Energy is not represented accurately.
- Not converged results.

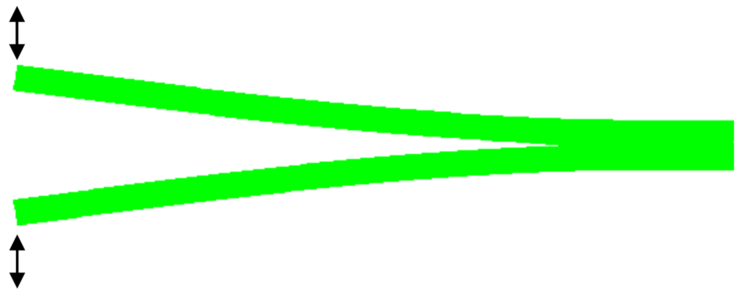
Coarse meshes also induce non-smooth responses

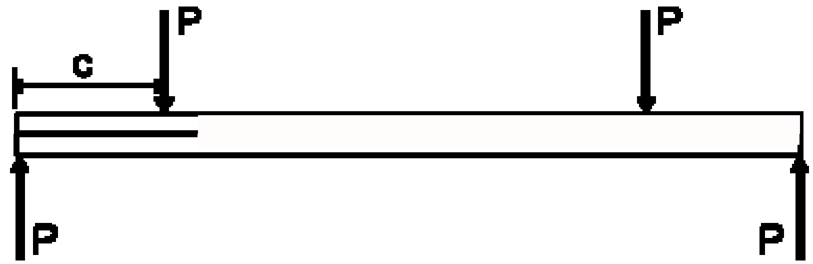


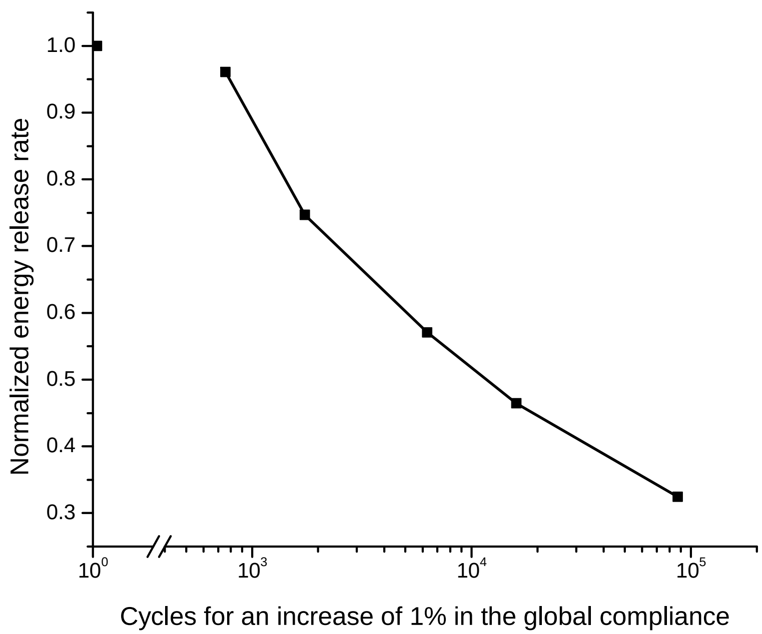


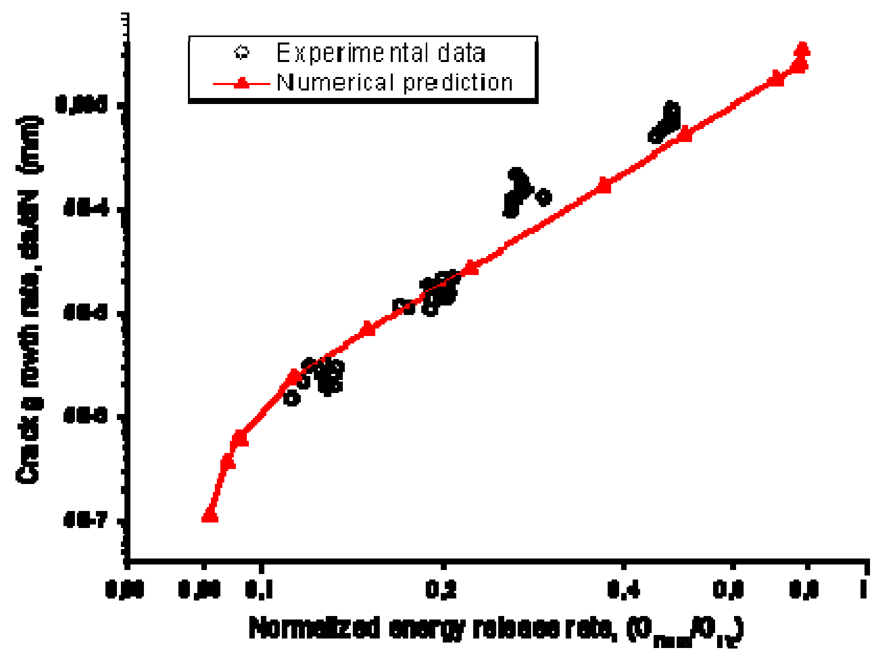
A minimum number of elements is needed in the cohesive zone $N_e = \frac{l_{cz}}{l_e}$

Simulation of delamination under fatigue loading & cohesive zone length







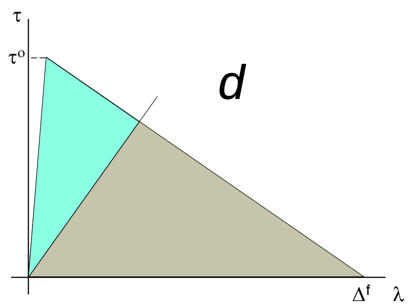


ONSET

PROPAGATION

Simulation of delamination under fatigue loading & cohesive zone length

Degradation of the interface due to fatigue loading



$$\frac{\partial d}{\partial N} ?$$

⇒ related to the crack growth rate (experimental data)

$$\frac{da}{dN}(G_{\max}, R, B)$$

$$\frac{\partial d}{\partial N} = \frac{\partial d}{\partial A_d}\frac{\partial A_d}{\partial N}$$

⇒

$$\frac{\partial \mathrm{d}}{\partial N} = \frac{1}{A_{CZ}} \frac{(\Delta^f(1-\mathrm{d}) + \mathrm{d}\Delta^o)^2}{\Delta^f \Delta^o} \frac{\partial A}{\partial N}$$

area of the cohesive zone

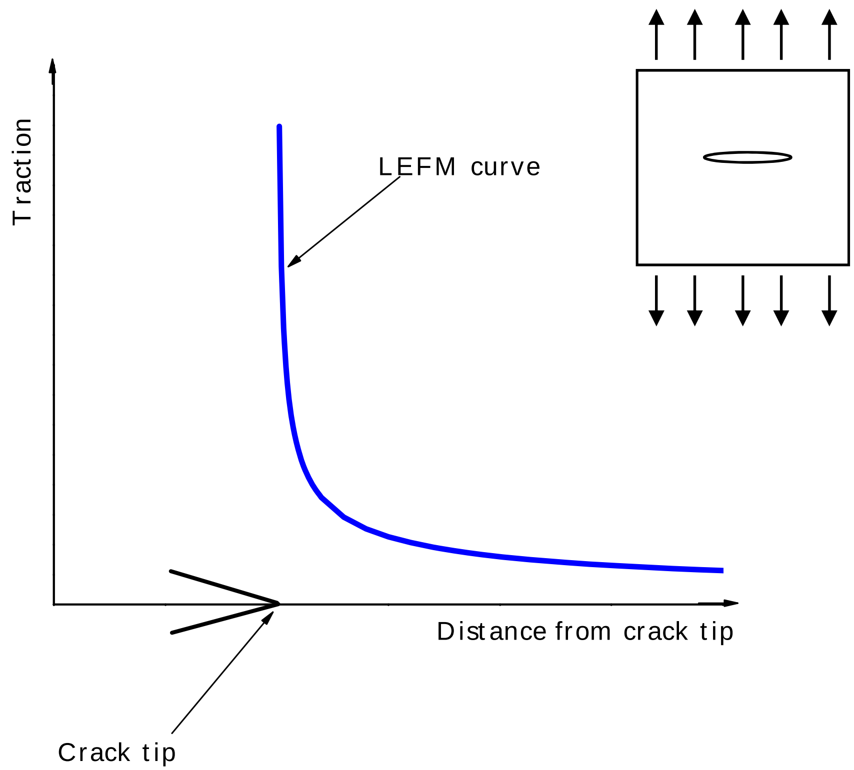
Motivation of the work

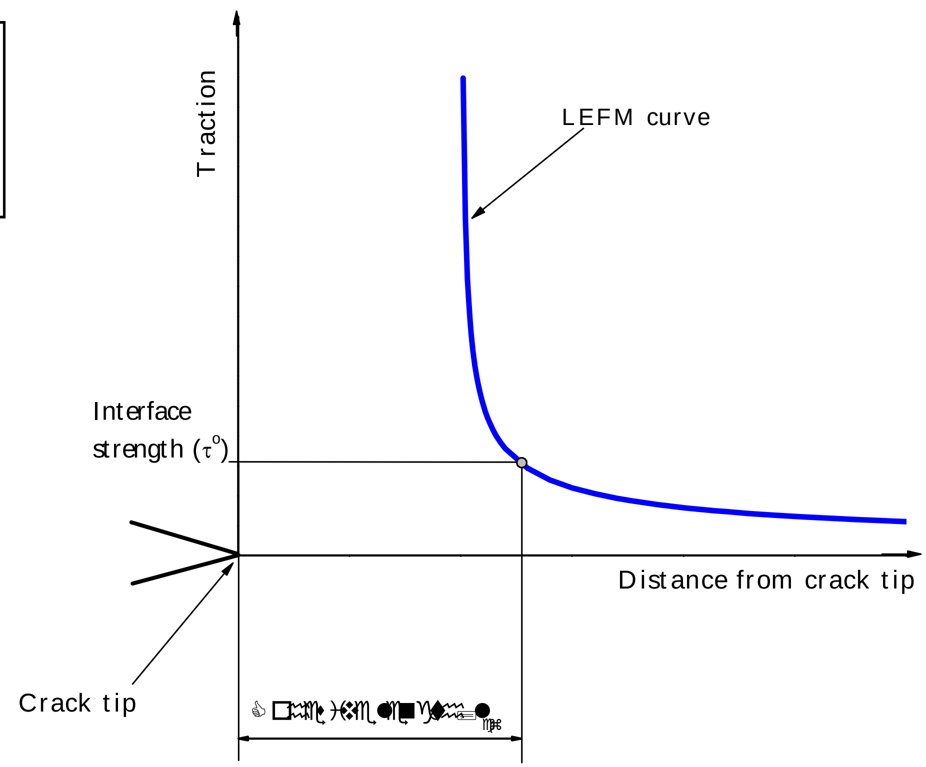
It is important to predict the **length of the cohesive zone** to simulate **delamination** under **static** and **fatigue** loading and get **accurate results.**

Length of cohesive zone: Isotropic materials

Different approaches to determine the length of the cohesive zone:

- Irwin's approach:





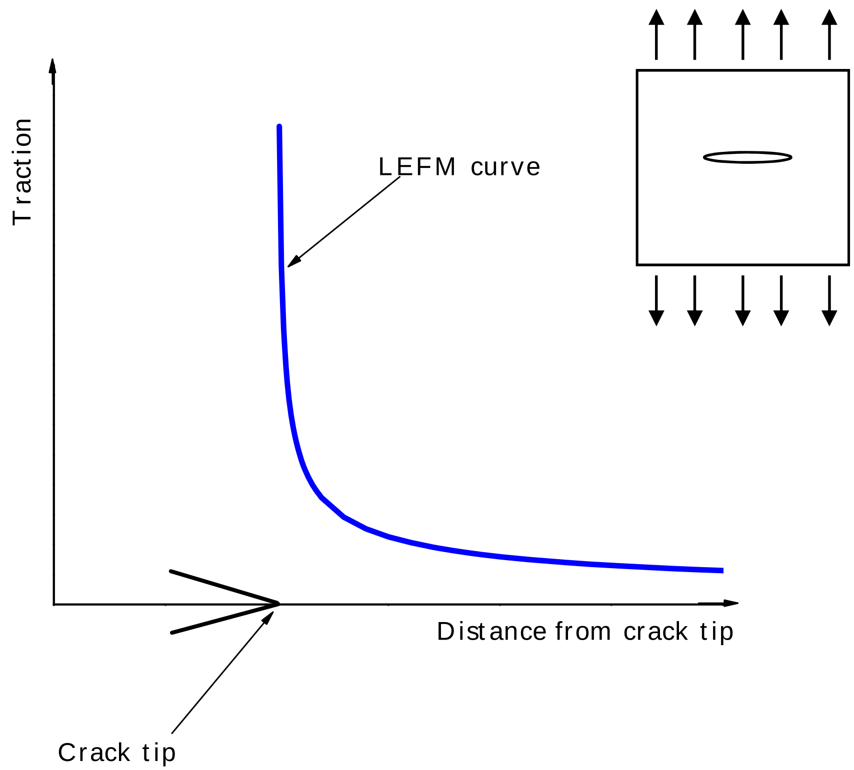
LEFM - no cohesive zone

Irwin model. Ductile materials

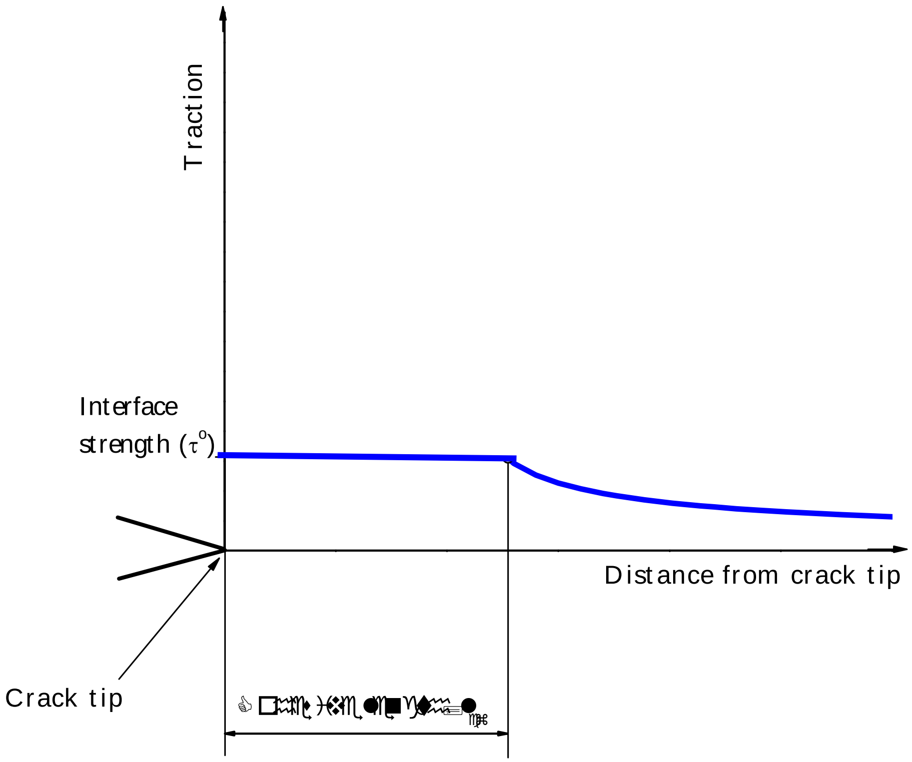
Length of cohesive zone: Isotropic materials

Different approaches to determine the length of the cohesive zone:

- Irwin's approach:

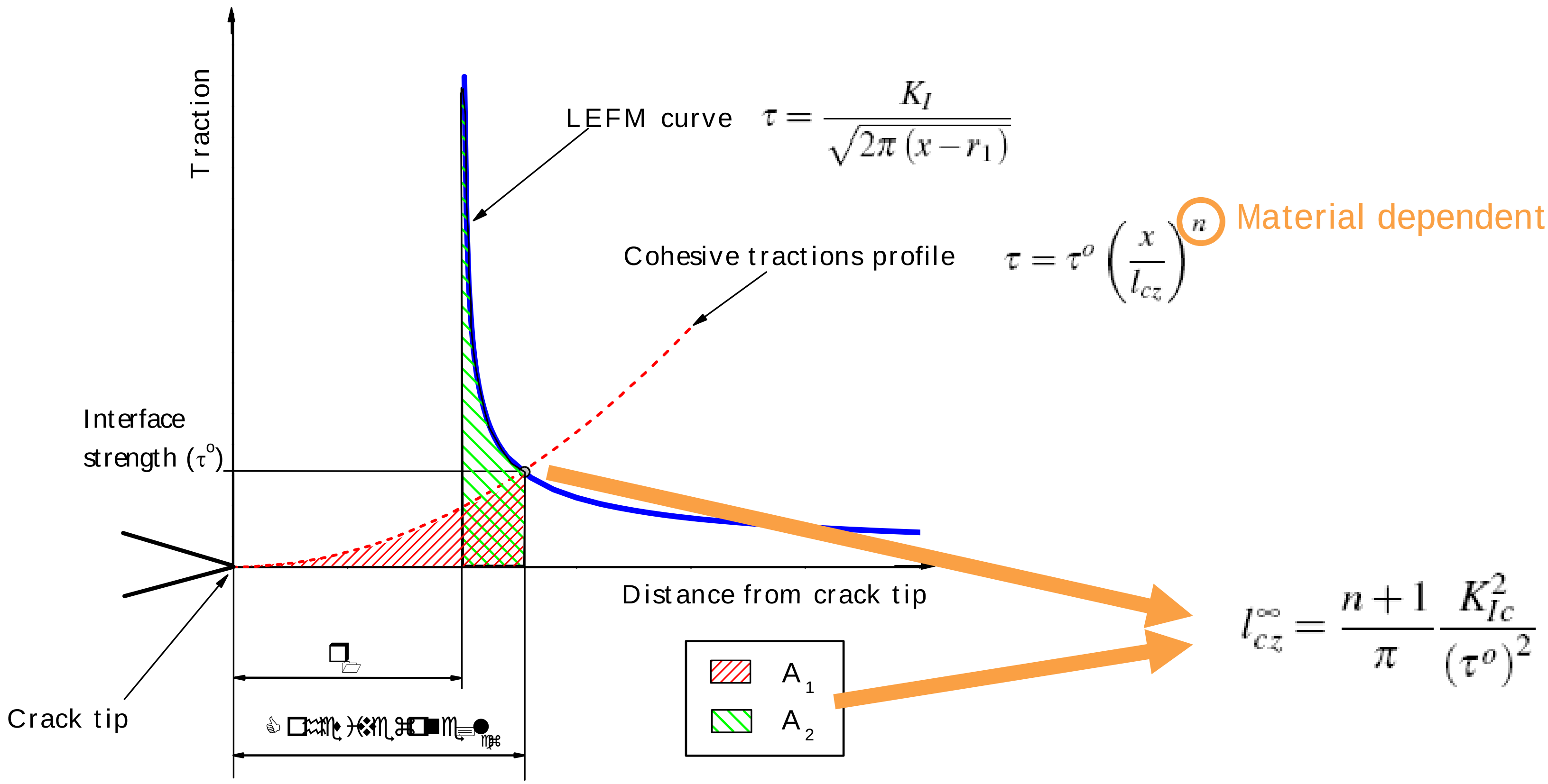


LEFM - no cohesive zone



Irwin model. Ductile materials

Length of cohesive zone: Isotropic materials



Bazant model (quasi-brittle materials)

Length of cohesive zone: Isotropic materials

Other approaches to determine the length of the cohesive zone:

- Dugdale's approach: By linear superposition of the crack tip stress intensity factors produced by the external loads and by the internal cohesive tractions

$$l_{cz}^{\infty} = \frac{\pi}{8}\frac{K_{Ic}^2}{(\tau^o)^2}$$

- Cox and Marshall: stress intensity factor at the crack tip equal to the critical value and COD at the beginning of the cohesive zone equal to the critical value.

$$l_{cz}^{\infty} = \frac{\pi}{4}\frac{K_{Ic}^2}{(\tau^o)^2}$$

$$l_{cz}^{\infty} = M \frac{K_{Ic}^2}{(\tau^o)^2}$$

Hillerborg characteristic length of the material

$$G_I = \frac{K_I^2}{E'}$$

$$l_{cz}^{\infty} = M \frac{E' G_{Ic}}{(\tau^o)^2}$$

	M
Hui et al. [12]	$\frac{2}{3\pi} = 0.21$
Irwin [13]	$\frac{1}{\pi} = 0.31$
Bažant et al. [2]	$\frac{n+1}{\pi}$
Dugdale [7], Barenblatt [1]	$\frac{\pi}{8} = 0.39$
Cox and Marhall [5]	$\frac{\pi}{4} = 0.785$
Rice [16], Falk et al. [8]	$\frac{9\pi}{32} = 0.88$

Length of cohesive zone: Orthotropic materials

Isotropic materials

$$G_I = \frac{K_I^2}{E'}$$

Orthotropic materials

$$G_I = K_I^2 \left(\frac{a_{11}a_{22}}{2}\right)^{\frac{1}{2}} \left[\left(\frac{a_{22}}{a_{11}}\right)^{\frac{1}{2}} + \frac{2a_{12}+a_{66}}{2a_{11}}\right]^{\frac{1}{2}}$$

$$G_{II} = K_{II}^2 \frac{a_{11}}{\sqrt{2}} \left[\left(\frac{a_{22}}{a_{11}}\right)^{\frac{1}{2}} + \frac{2a_{12}+a_{66}}{2a_{11}}\right]^{\frac{1}{2}}$$

Length of cohesive zone: Orthotropic materials

Isotropic materials

$$l_{cz}^{\infty} = M \frac{E' G_{Ic}}{(\tau^o)^2}$$

Orthotropic materials

$$l_{Icz}^{\infty} = M_I \frac{E'_I G_{Ic}}{(\tau_3^o)}$$

$$l_{IIcz}^{\infty} = M_{II} \frac{E'_{II} G_{IIc}}{(\tau_{shear}^o)}$$

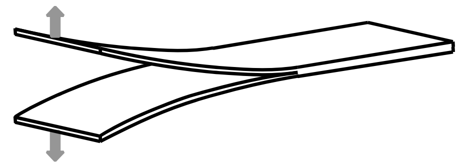
$$l_{cz}^{\infty} = M E_m \frac{G_c}{(\tau^o)^2}$$

$$E'_I = \left(\frac{a_1}{2} \cdots \left[\left(a_{11} \right) \cdots \frac{a_{12} + a_{66}}{2a_{11}} \right]^{-\frac{1}{2}}$$

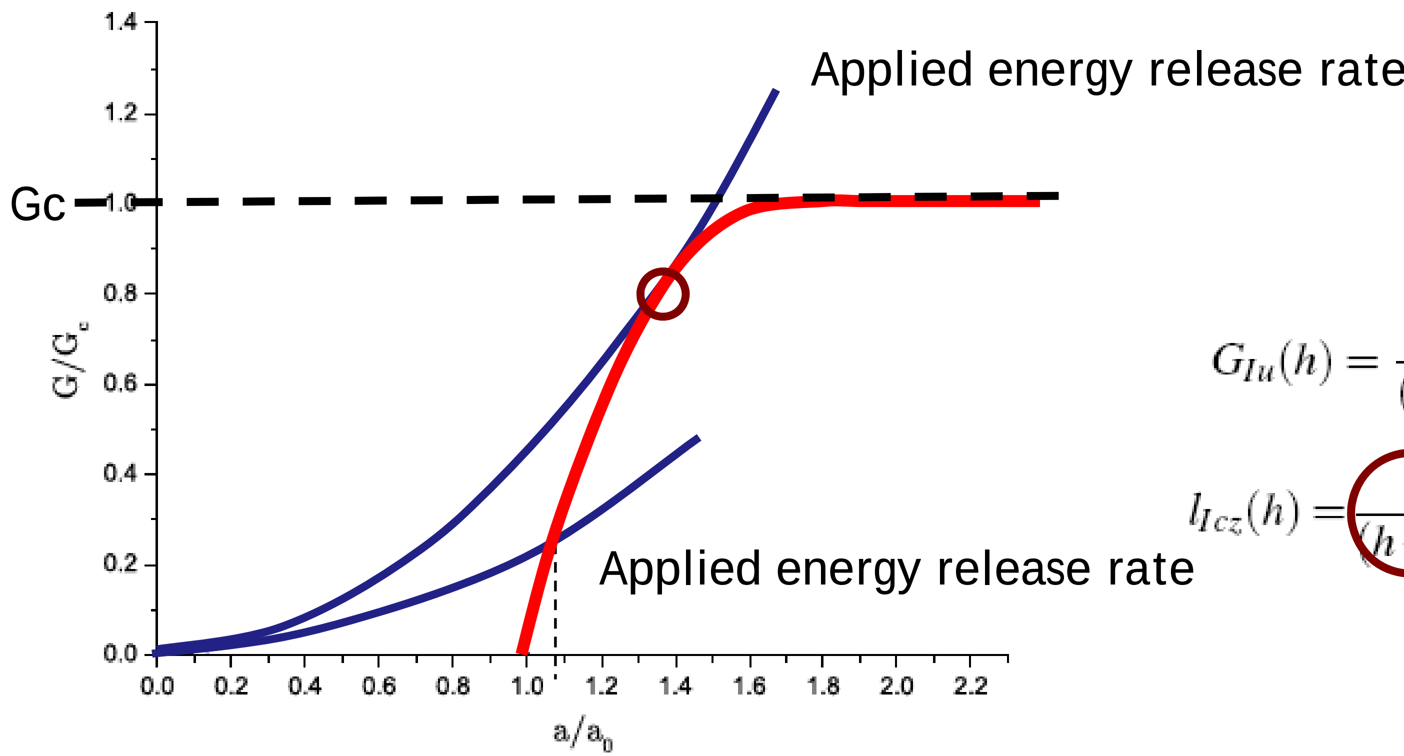
$$E_m = E_I (1 - B) + E_{II} B$$

$$E'_{II} = \left(\cdots \frac{\cdots_{66}}{} \right]^{-\frac{1}{2}}$$

Finite-sized geometries



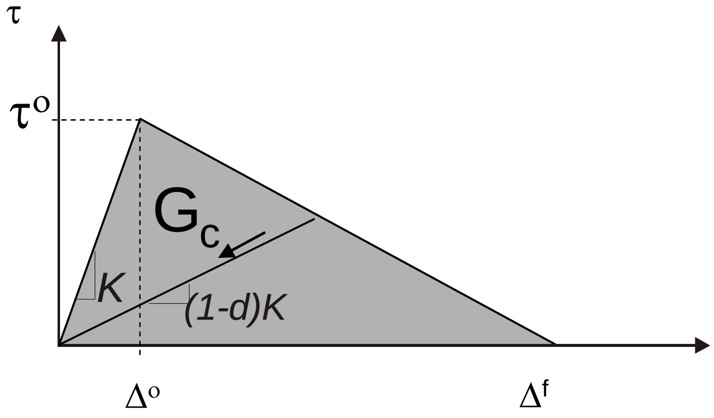
Crack propagates for lower values than Fracture Toughness

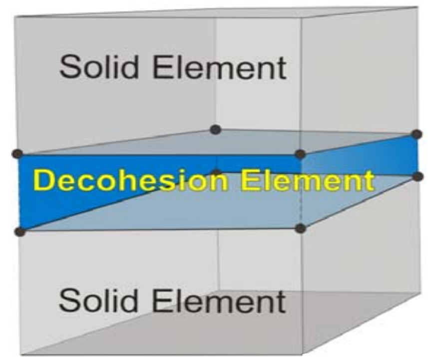


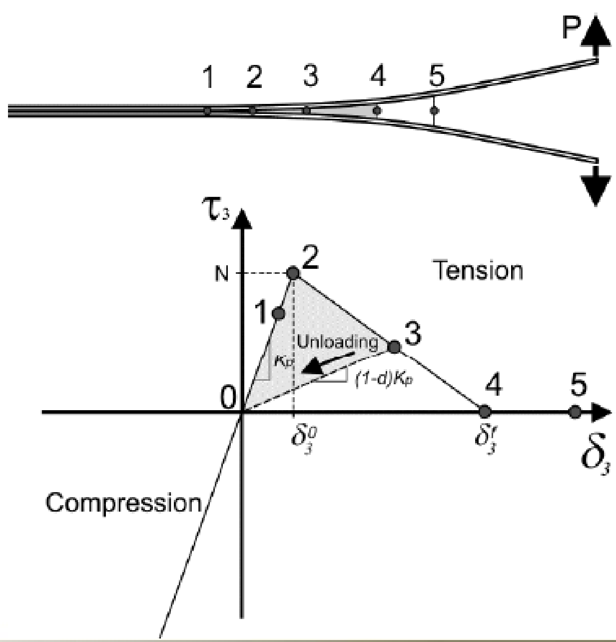
$$G_{Iu}(h) = \frac{G_{Ic} h}{(h + h_I)}$$

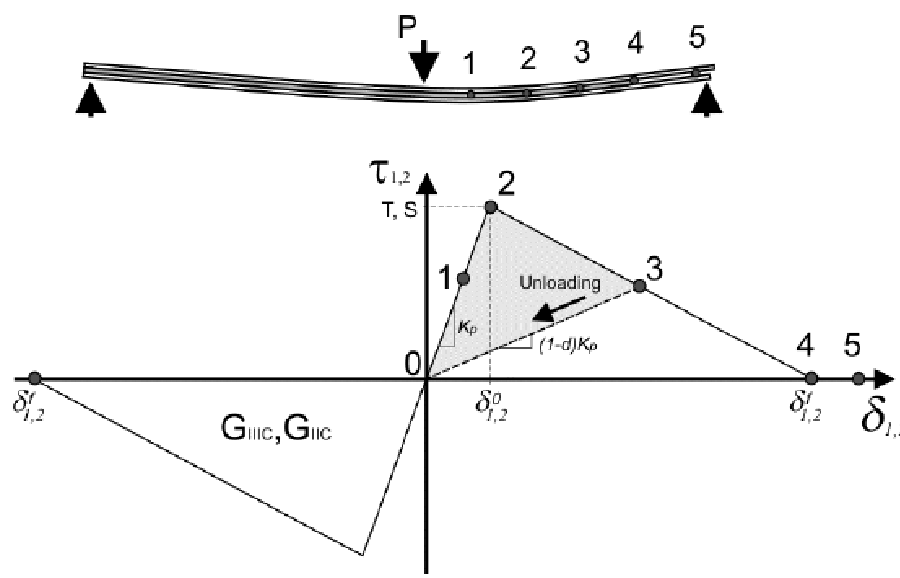
$$l_{Icz}(h) = \frac{h}{(h + h_I)} l_{Icz}^{\infty}$$

Simulation by means of cohesive elements









Cohesive elements. Static formulation.

Free Energy $\psi(\Delta, \mathsf{d}) = (1-\mathsf{d})\,\psi^0(\Delta_i) - \mathsf{d}\psi^0(\delta_{3i}\langle -\Delta_3\rangle)$

Constitutive equation $\tau_i = \frac{\partial \psi}{\partial \Delta_i} = (1-\mathsf{d})\,\delta_{ij} K \Delta_j - \mathsf{d}\delta_{ij} K \delta_{3j}\langle -\Delta_3\rangle$

Displacement jump norm $\lambda = \sqrt{\langle \Delta_3\rangle^2 + (\Delta_{shear})^2}$

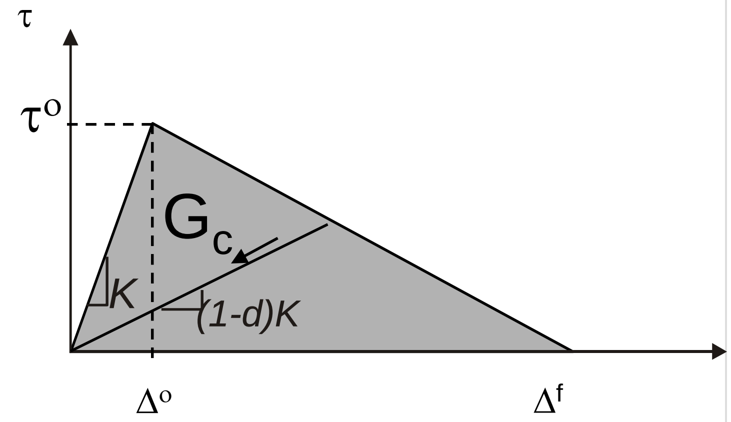
Damage criterion $\bar{F}(\lambda^{\mathsf{t}}, r^{\mathsf{t}}) := \mathsf{G}(\lambda^{\mathsf{t}}) - \mathsf{G}(r^{\mathsf{t}}) \leq 0 \qquad \forall \mathsf{t} \geq 0$

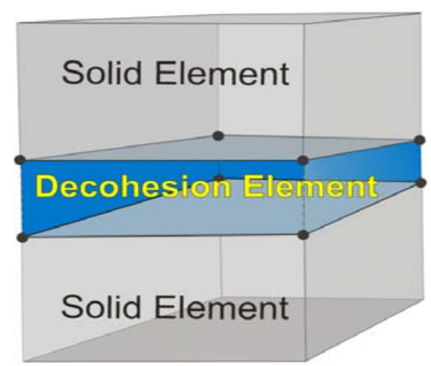
$$\mathsf{G}(\lambda) = \frac{\Delta^f(\lambda - \Delta^o)}{\lambda(\Delta^f - \Delta^o)}$$

Evolution law $\dot{\mathsf{d}} = \dot{\mu}\frac{\partial \bar{F}(\lambda, r)}{\partial \lambda} = \dot{\mu}\frac{\partial \mathsf{G}(\lambda)}{\partial \lambda}$

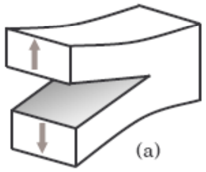
Load/unload conditions $\dot{\mu} \geq 0 \;;\; \bar{F}(\lambda^{\mathsf{t}}, r^{\mathsf{t}}) \leq 0 \;;\; \dot{\mu}\bar{F}(\lambda^{\mathsf{t}}, r^{\mathsf{t}}) = 0$

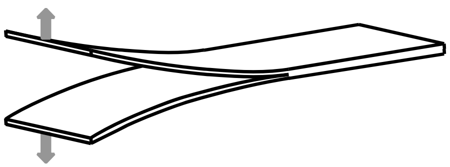
$$r^{\mathsf{t}} = \max\{\Delta^o, \max_s \lambda^s\} \quad 0 \leq s \leq \mathsf{t}$$

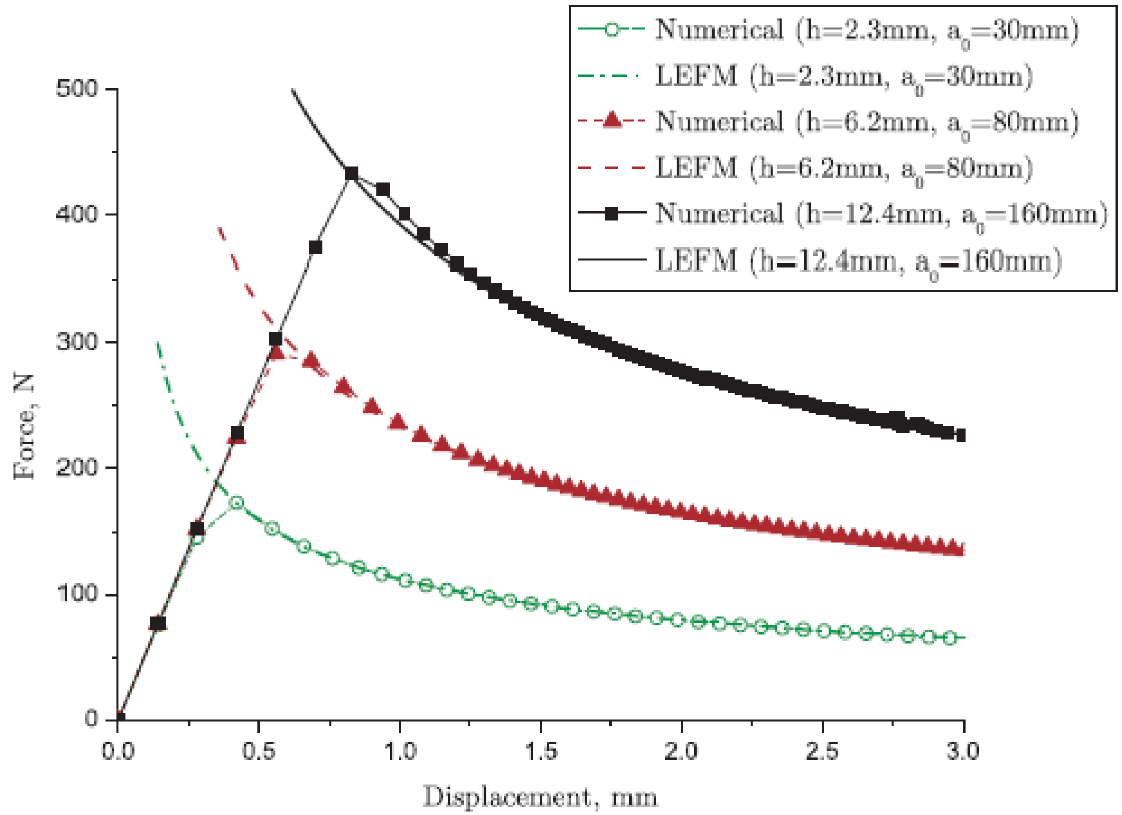


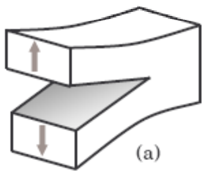


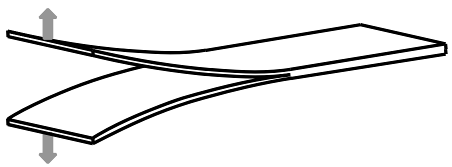
Implemented as a User element in ABAQUS

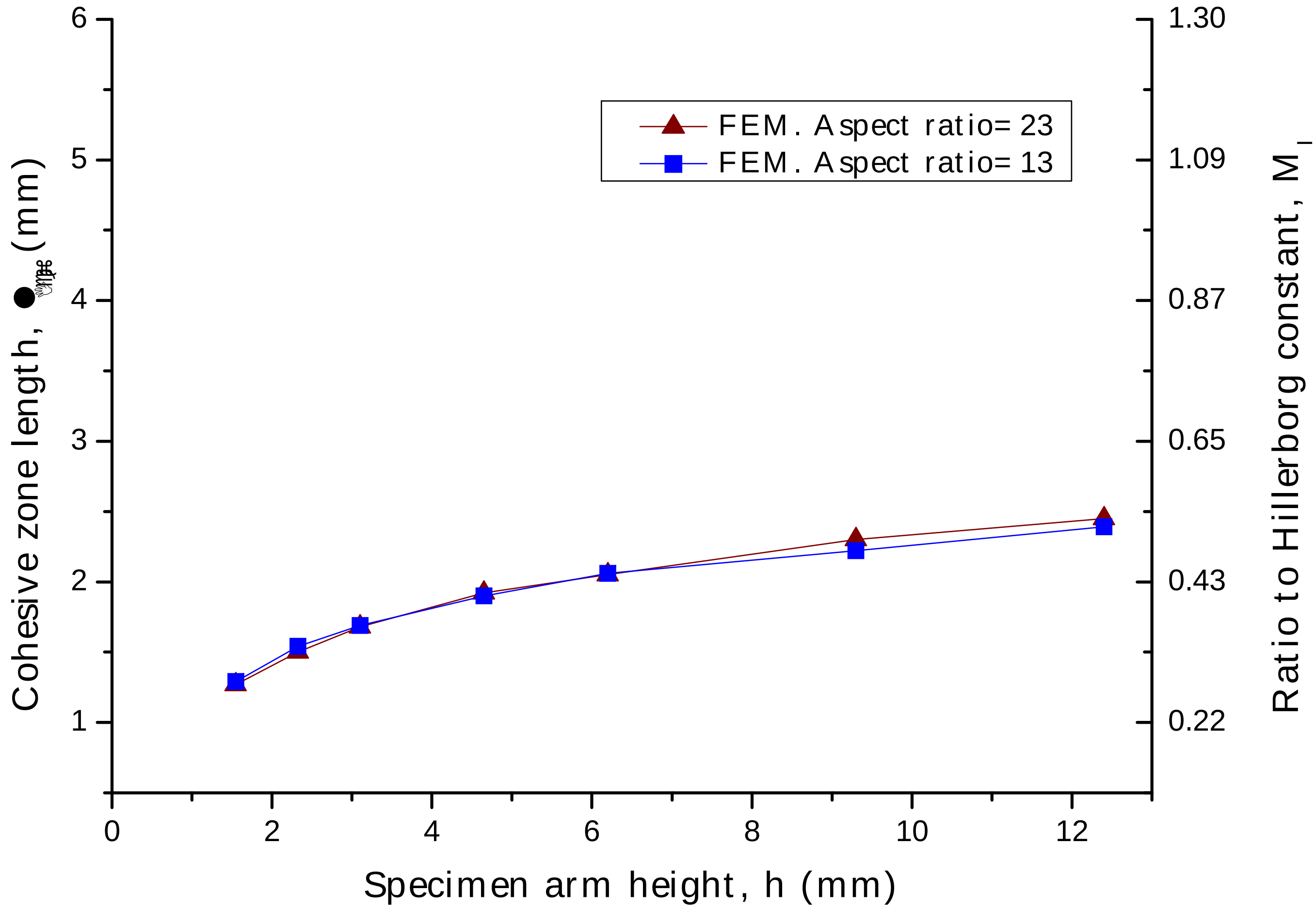

(a)

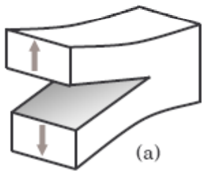


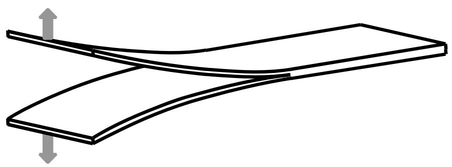

Numerical (h=2.3mm, a0=30mm)
LEFM (h=2.3mm, a0=30mm)
Numerical (h=6.2mm, a0=80mm)
LEFM (h=6.2mm, a0=80mm)
Numerical (h=12.4mm, a0=160mm)
LEFM (h=12.4mm, a0=160mm)
Force, N
Displacement, mm
0
100
200
300
400
500
0.0
0.5
1.0
1.5
2.0
2.5
3.0

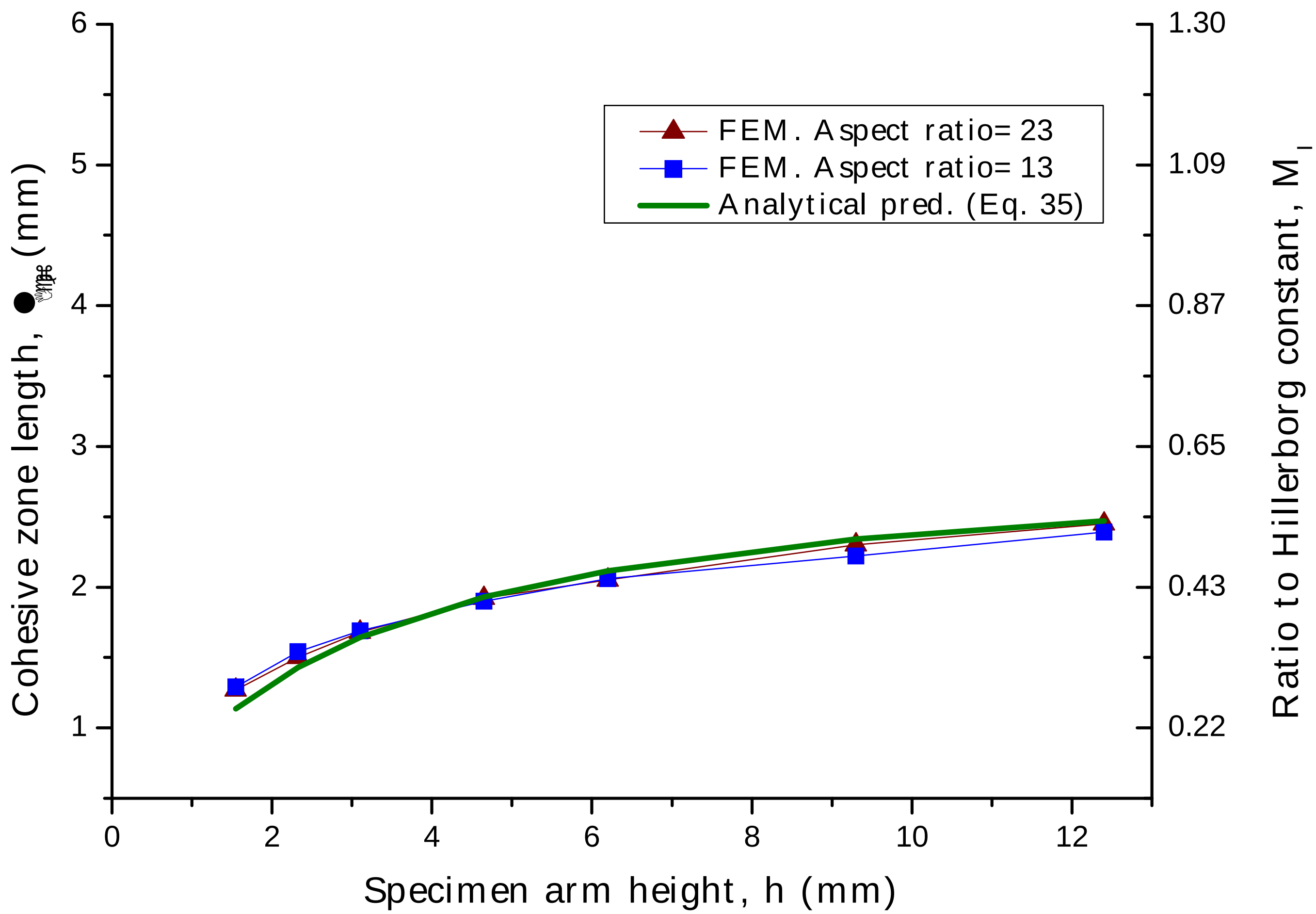

(a)

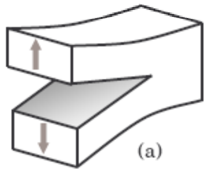


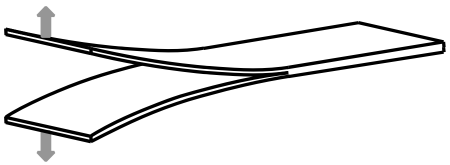

FEM. Aspect ratio= 23
FEM. Aspect ratio= 13
Cohesive zone length, (mm)
Ratio to Hillerborg constant, M_I
Specimen arm height, h (mm)
0
2
4
6
8
10
12
1
2
3
4
5
6
0.22
0.43
0.65
0.87
1.09
1.30

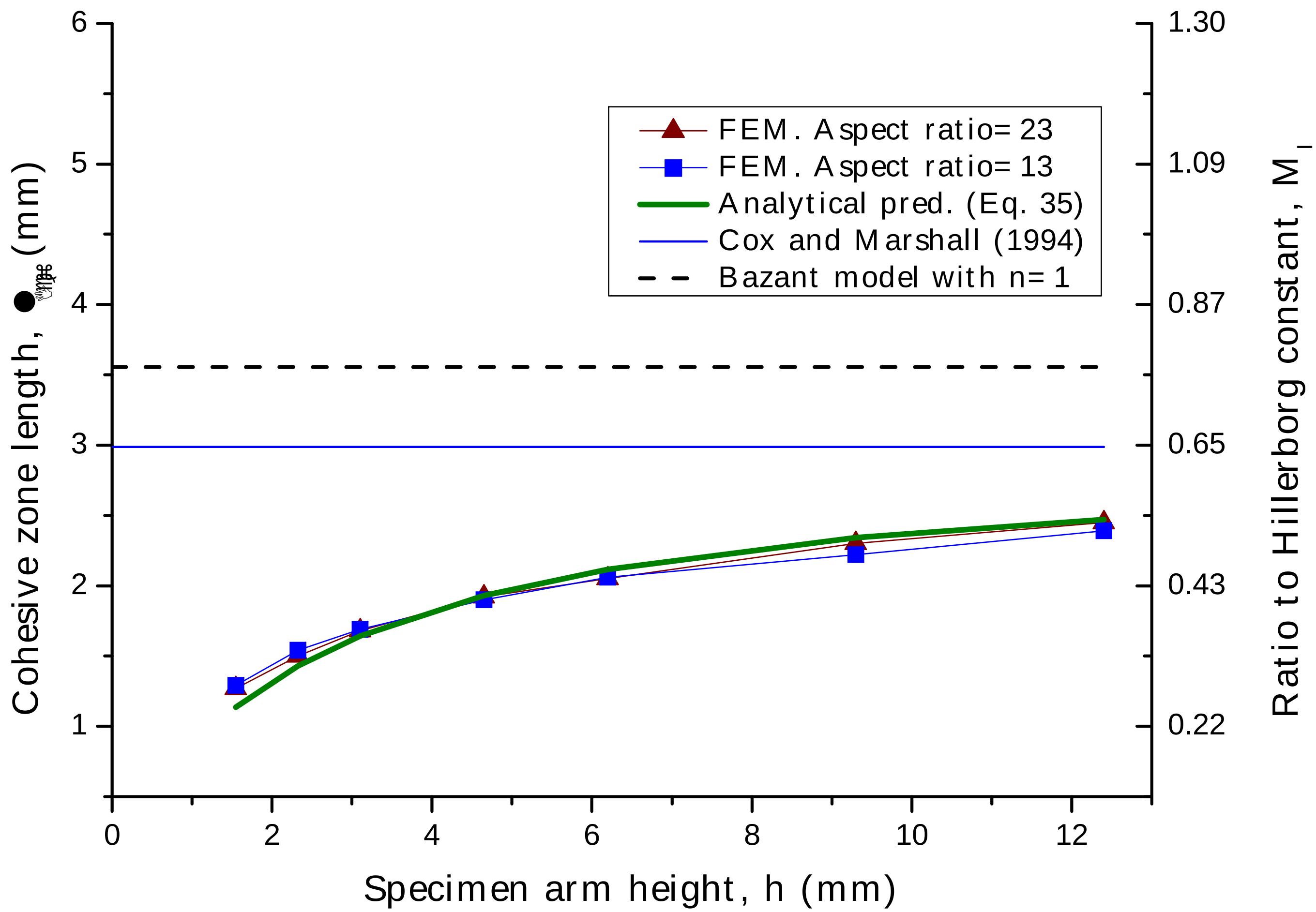

(a)



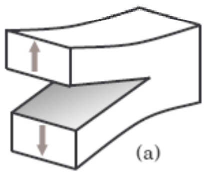

FEM. Aspect ratio= 23
FEM. Aspect ratio= 13
Analytical pred. (Eq. 35)
Cohesive zone length, (mm)
Ratio to Hillerborg constant, M_I
Specimen arm height, h (mm)
0
2
4
6
8
10
12
1
2
3
4
5
6
0.22
0.43
0.65
0.87
1.09
1.30

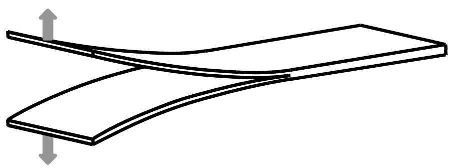

(a)

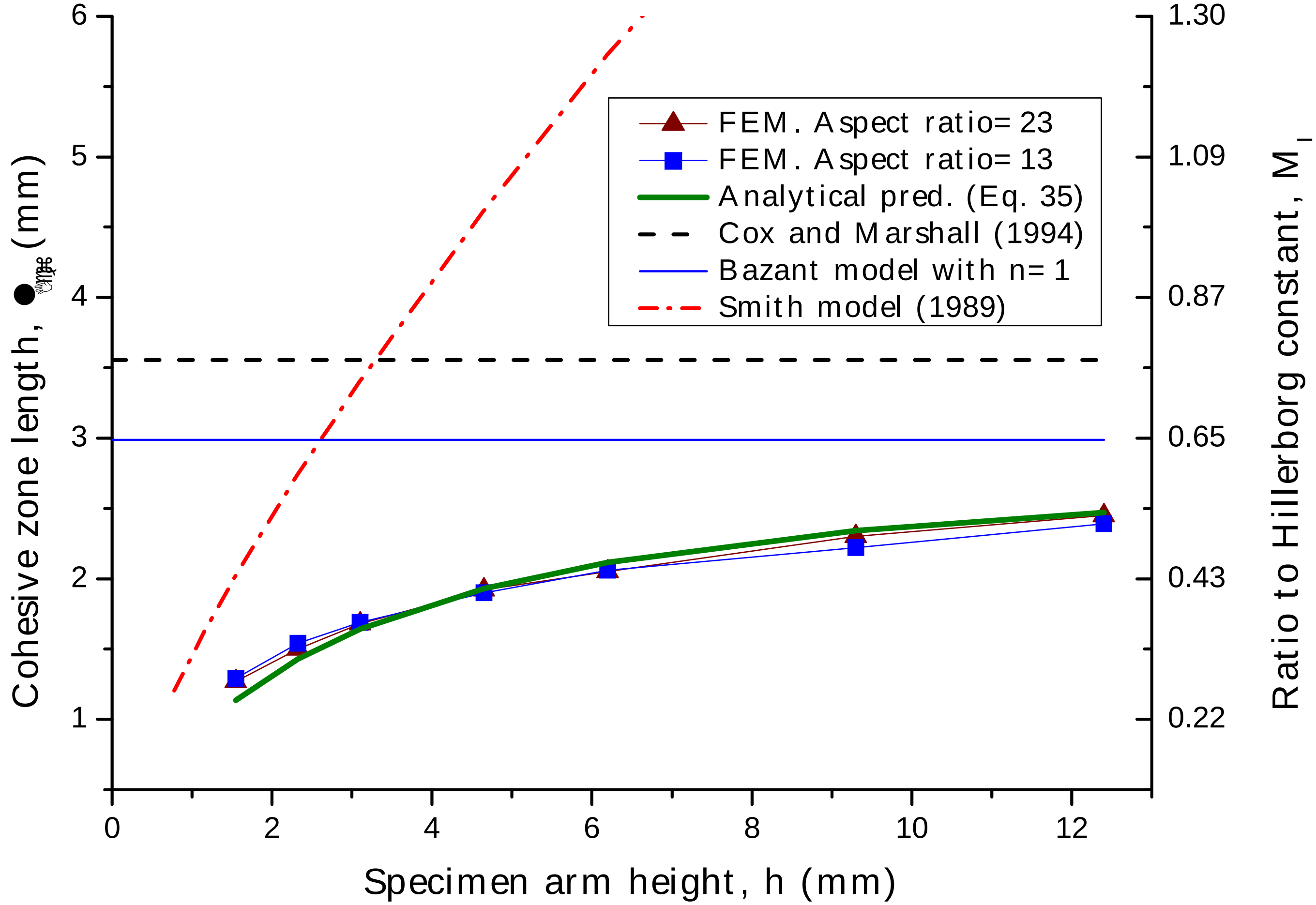



FEM. Aspect ratio= 23
FEM. Aspect ratio= 13
Analytical pred. (Eq. 35)
Cox and Marshall (1994)
Bazant model with n= 1
Cohesive zone length, (mm)
Ratio to Hillerborg constant, M
Specimen arm height, h (mm)
0
2
4
6
8
10
12
1
2
3
4
5
6
0.22
0.43
0.65
0.87
1.09
1.30

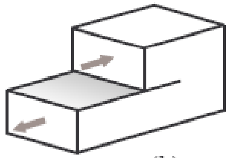
IT NETWORK OF INNOVATION SUPPORT CENTRES
CIDEM



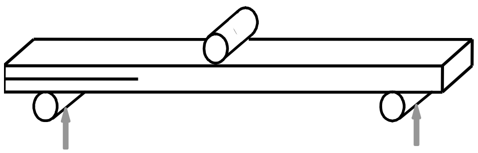


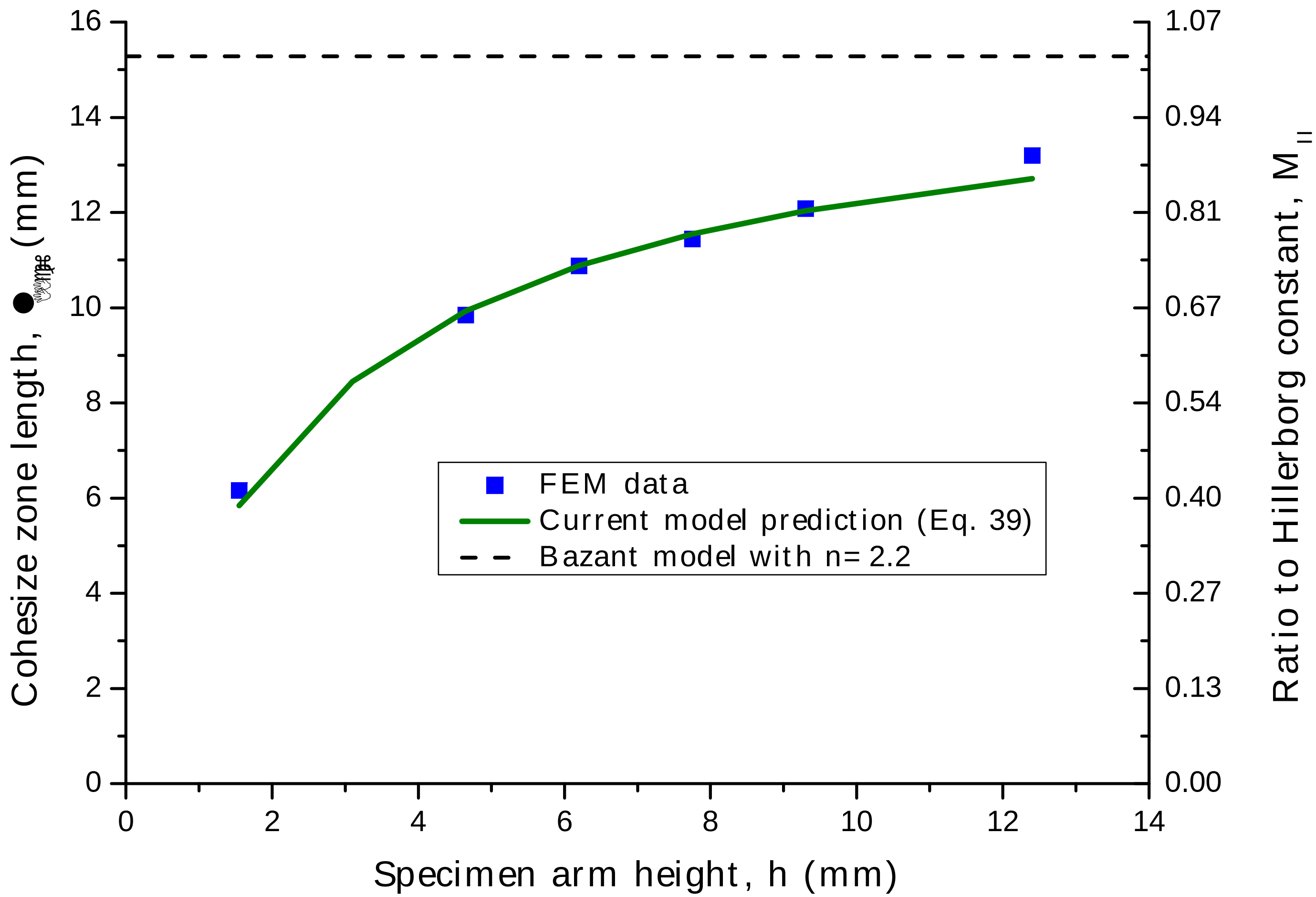


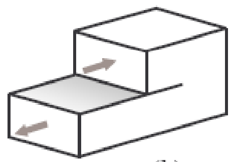
$$l_{Icz}(h) = \left(\frac{G_{Ic}E'}{(\tau_3^o)^2}\right)^{\frac{1}{3}} h^{\frac{3}{4}}$$



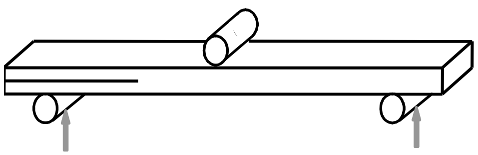
(b)



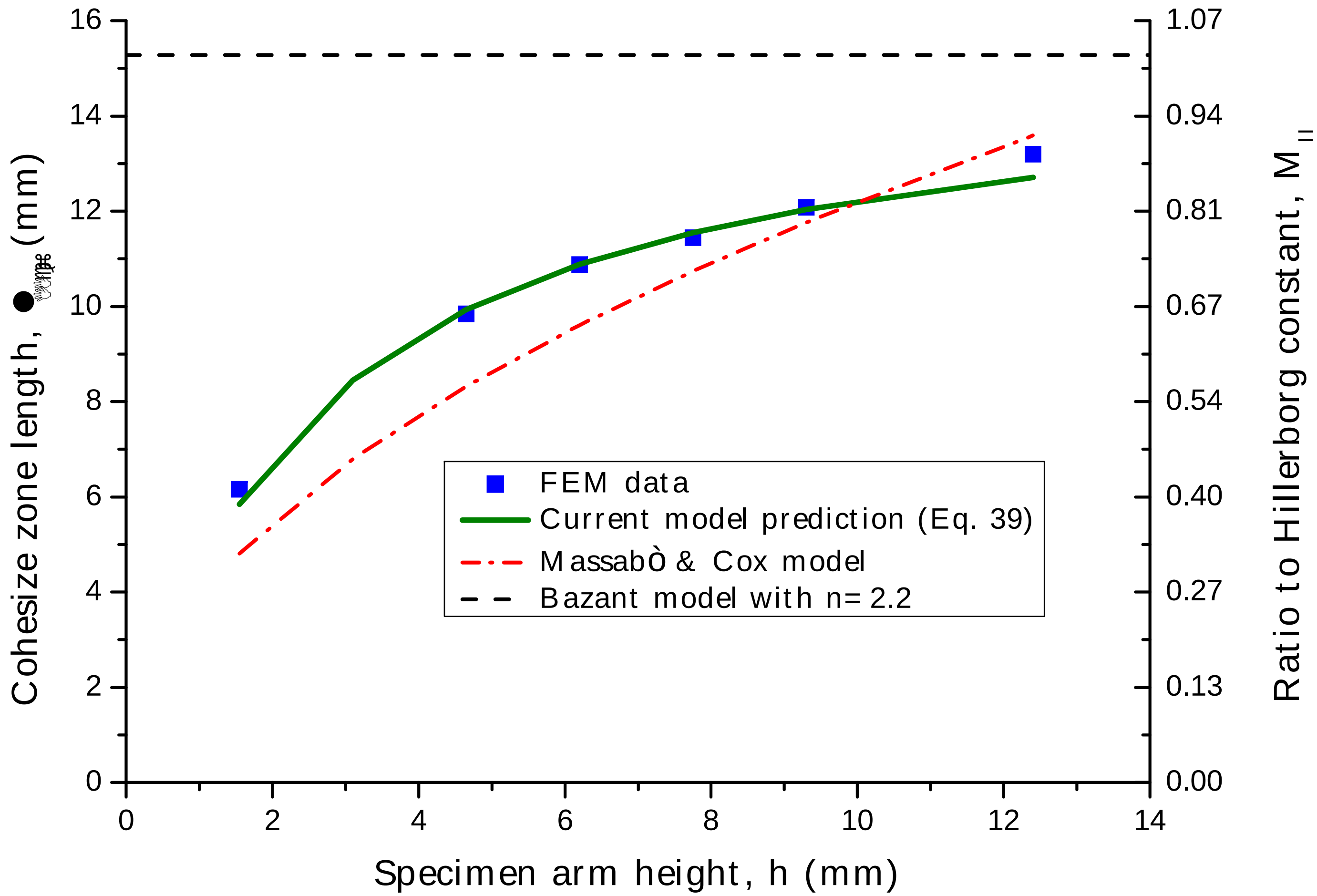

Cohesize zone length, (mm)
Specimen arm height, h (mm)
Ratio to Hillerborg constant, M_{II}
FEM data
Current model prediction (Eq. 39)
Bazant model with n= 2.2
0
2
4
6
8
10
12
14
16
0.00
0.13
0.27
0.40
0.54
0.67
0.81
0.94
1.07

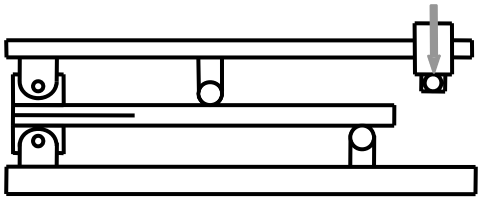


(b)

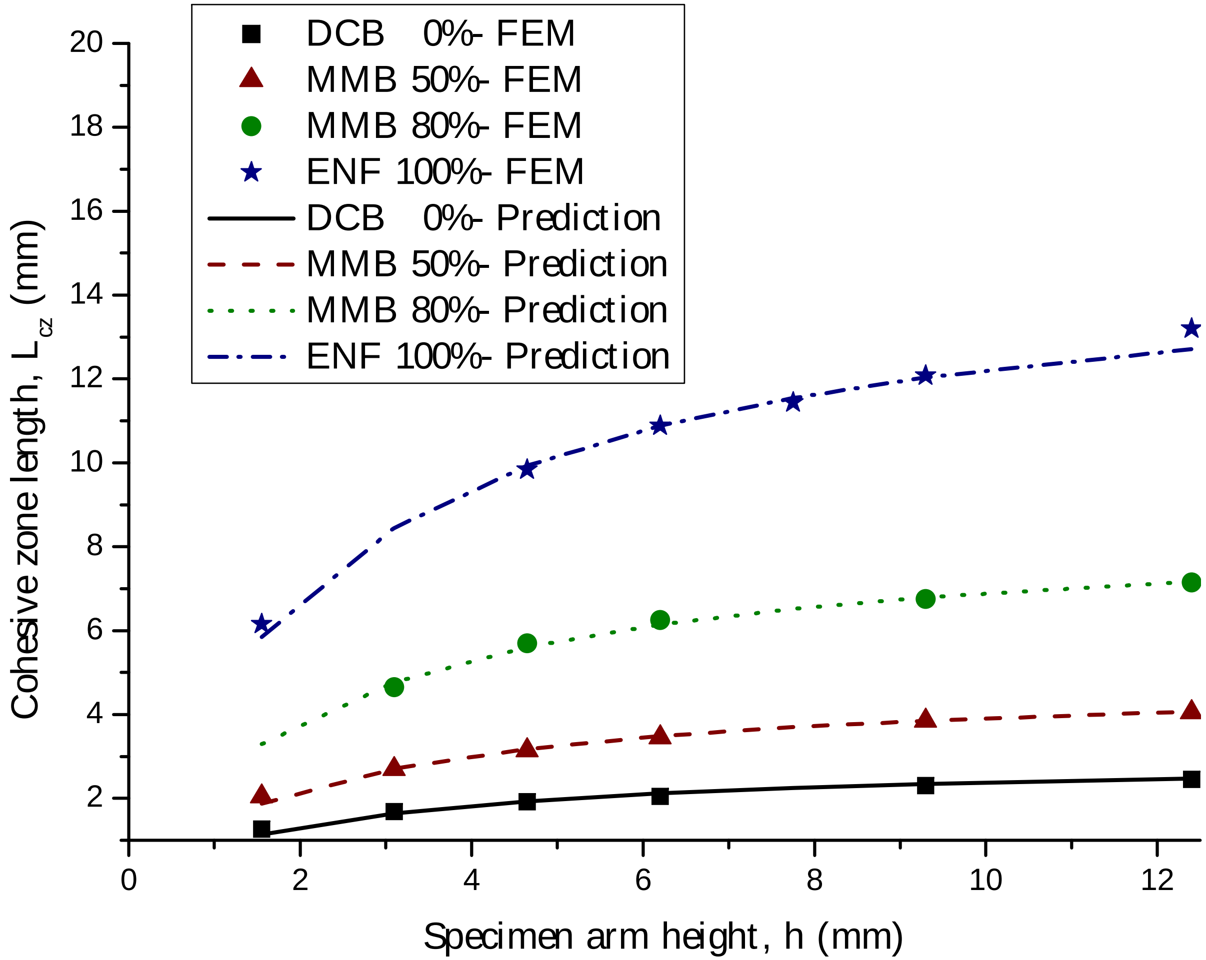


$$l_{IIcz}(h) = \sqrt{\left(\frac{G_{IIc}E'}{(\tau_3^o)^2}\right) h}$$



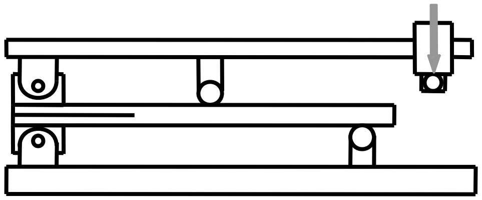


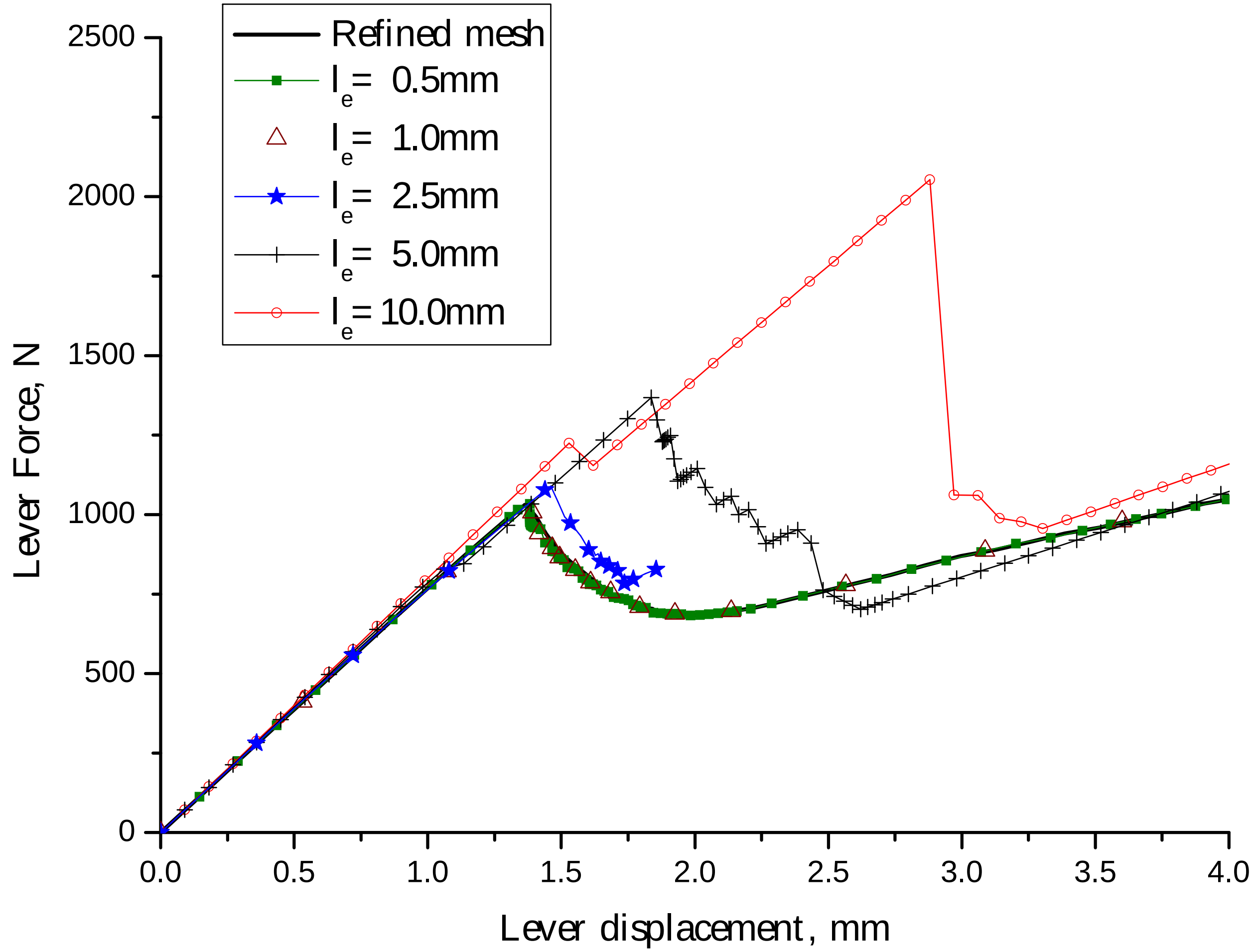
M_I	$M_{50\%}$	$M_{80\%}$	M_{II}
0.66	0.51	0.66	1.03



Delamination simulation using coarse meshes

Mixed-mode loading (50%)





AMADE
ANALYSIS AND ADVANCED MATERIALS FOR STRUCTURAL DESIGN

Universitat de Girona

Delamination simulation using coarse meshes

What can we do to improve the solution?

$$l_{cz}(h) = \frac{h}{(h+h_0)} M E_m \frac{G_c}{(\tau^o)^2}$$

$$G_C \uparrow \Rightarrow l_{cz} \uparrow$$
$$\tau^o \uparrow \Rightarrow l_{cz} \downarrow\downarrow$$

Engineering solution:

Reduce the interfacial strength in order to assure a minimum number of elements, N_e, in the cohesive zone:

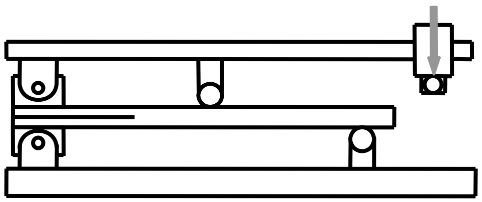
$$N_e = \frac{l_{cz}}{l_e}$$

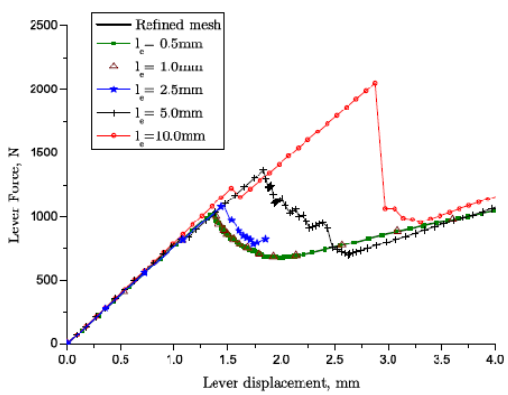
$$f_r = \frac{\sqrt{\frac{Mh}{h+h_0} \frac{E_m G_C}{N_e l_e}}}{\tau^o}$$

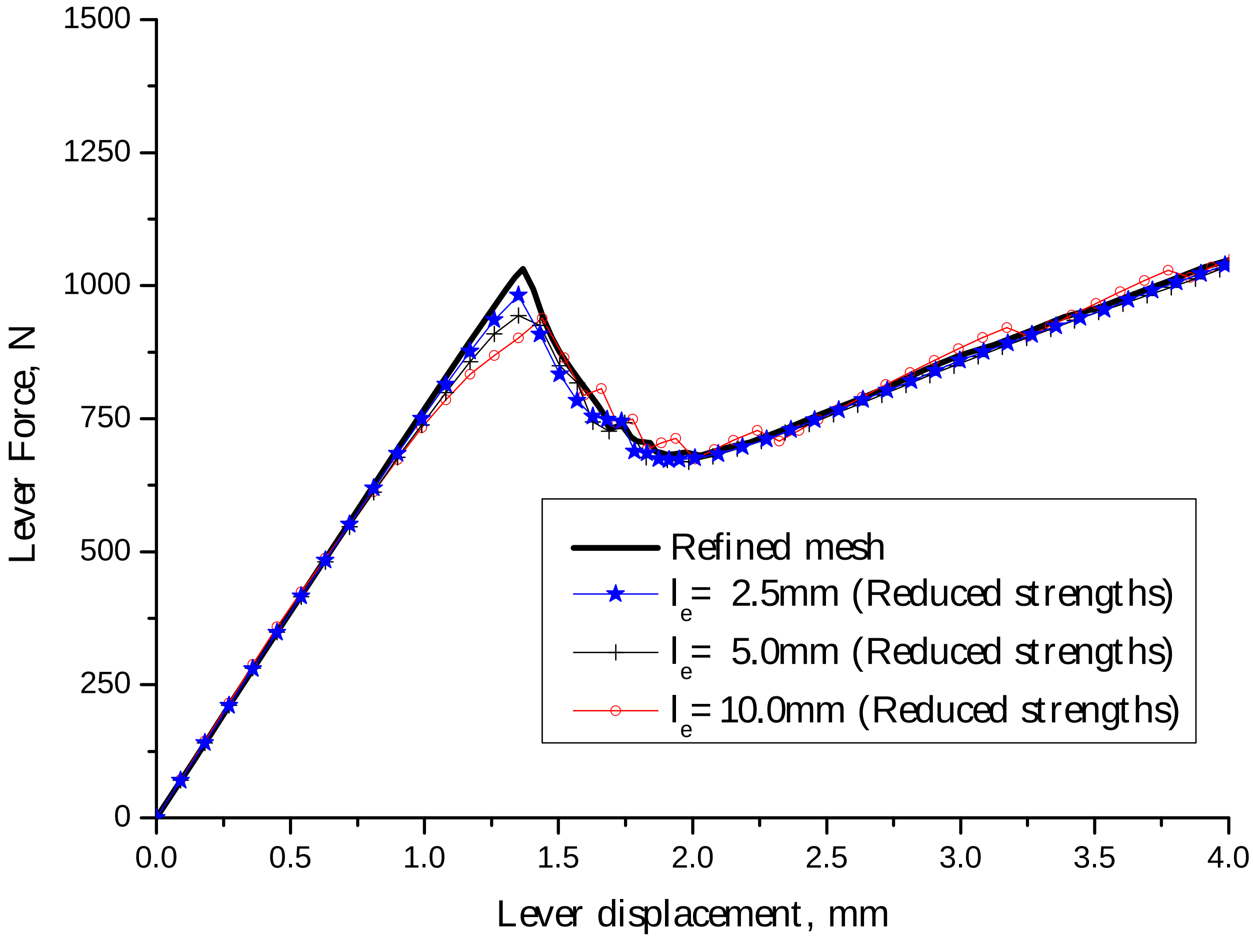
$$\tau_3^o = \min\left(\tau_3^o, f_r \tau_3^o\right)$$
$$\tau_{shear}^o = \min\left(\tau_{shear}^o, f_r \tau_{shear}^o\right)$$

Delamination simulation using coarse meshes

Mixed-mode loading (50%)



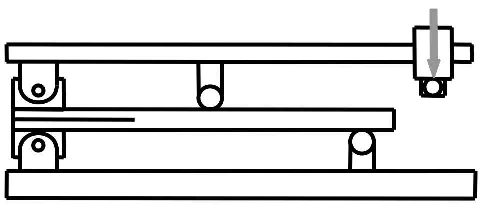




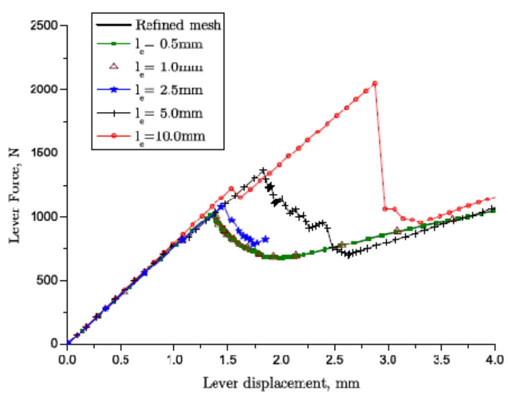
Delamination simulation using coarse meshes

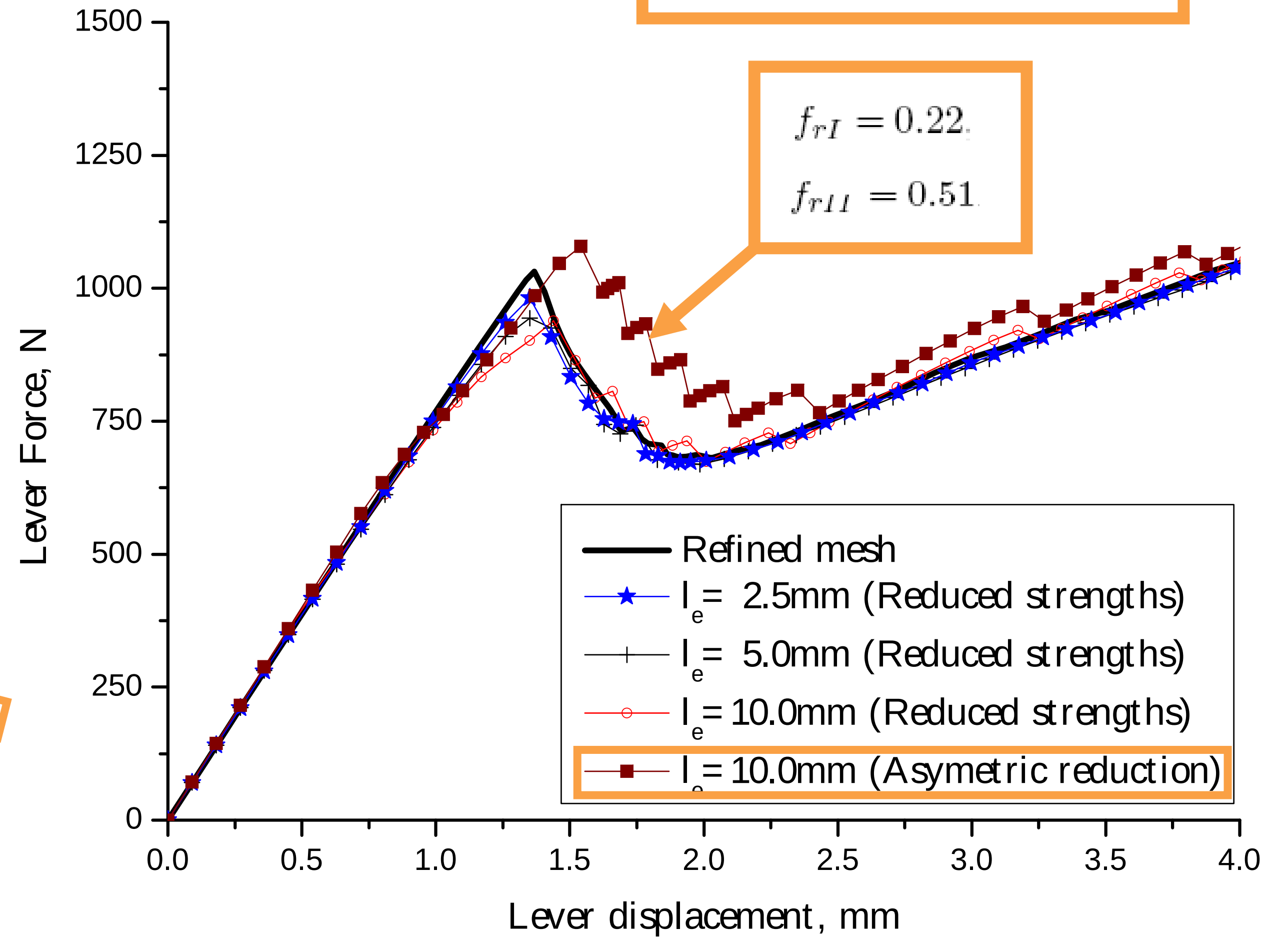
$$\tau_3^o = \min\left(\tau_3^o, f_r \tau_3^o\right)$$
$$\tau_{shear}^o = \min\left(\tau_{shear}^o, f_r \tau_{shear}^o\right)$$

Mixed-mode loading (50%)



$$f_{rI} = 0.22$$
$$f_{rII} = 0.51$$





Conclusions

- Length of Cohesive Zone depends on the material properties, the geometry/size of the structure, and on the loading mode.
- New expressions for finites-sized specimens have been presented.
- Expressions proposed have been used to simulate delamination under mixed-mode loading conditions using coarse meshes.
- Ongoing work: The implementation of the presented expressions within the fatigue model.

AMADE

ANALYSIS AND ADVANCED MATERIALS
FOR STRUCTURAL DESIGN

Universitat de Girona

Els colors "corporatius" d'AMADE en RGB són:

verd: 90/88/60

roig: 128/0/0

taronja: 250/160/068

Length of cohesive zone: Orthotropic materials

Mixed-Mode loading

$$\tau^2 = \tau_3^2 + \tau_{shear}^2$$

$$K^2 = K_I^2 + K_{II}^2$$

$$K^2 = GE_m$$

$$E_m G = E_I G_I + E_{II} G_{II}$$

$$G = G_I + G_{II}$$

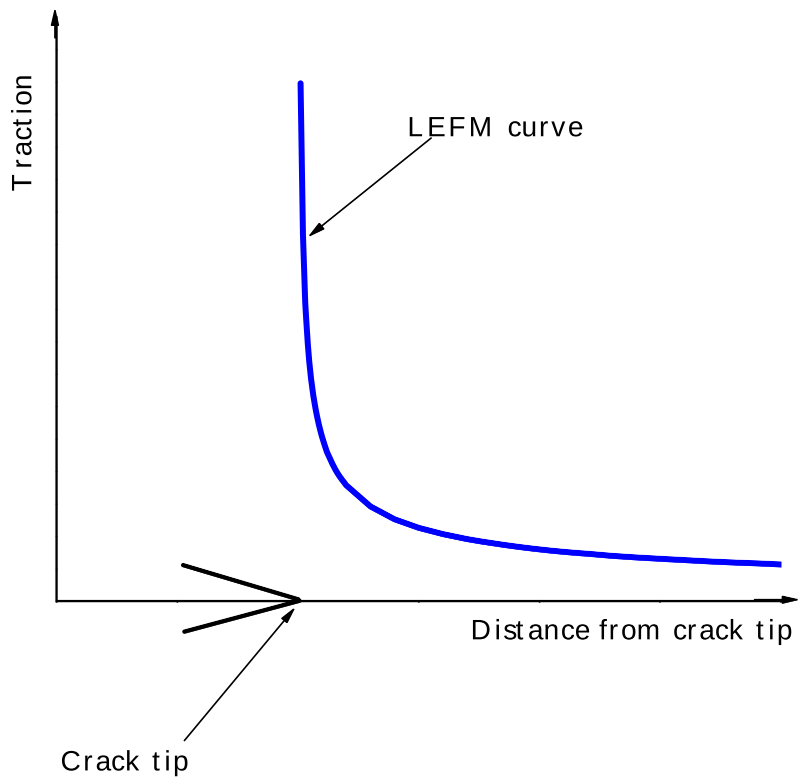
$$E_m = E_I(1-B) + E_{II}B$$

$$B = \frac{G_{II}}{G_I + G_{II}}$$

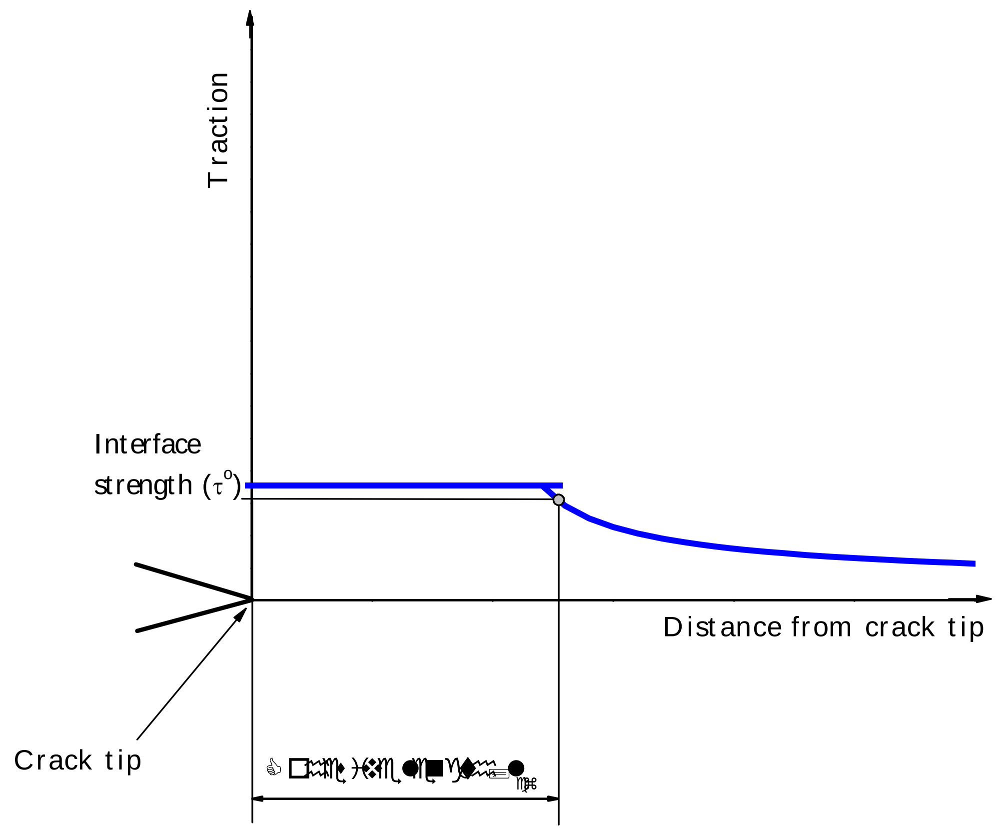
$$l_{cz}^{\infty} = M E_m \frac{G_c}{(\tau^o)^2}$$

$$G_c = G_{Ic} + (G_{IIc} - G_{Ic})B^{\eta}$$

$$(\tau^o)^2 = (\tau_3^o)^2 + \left[(\tau_{shear}^o)^2 - (\tau_3^o)^2\right]B^{\eta}$$



LEFM - no cohesive zone



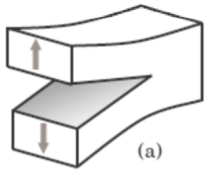
Irwin model (Ductile materials)

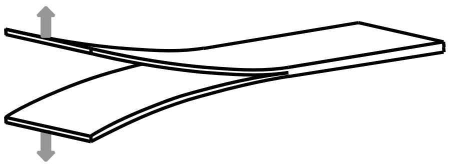
The S.Pellegrino World's 50 Best Restaurants.

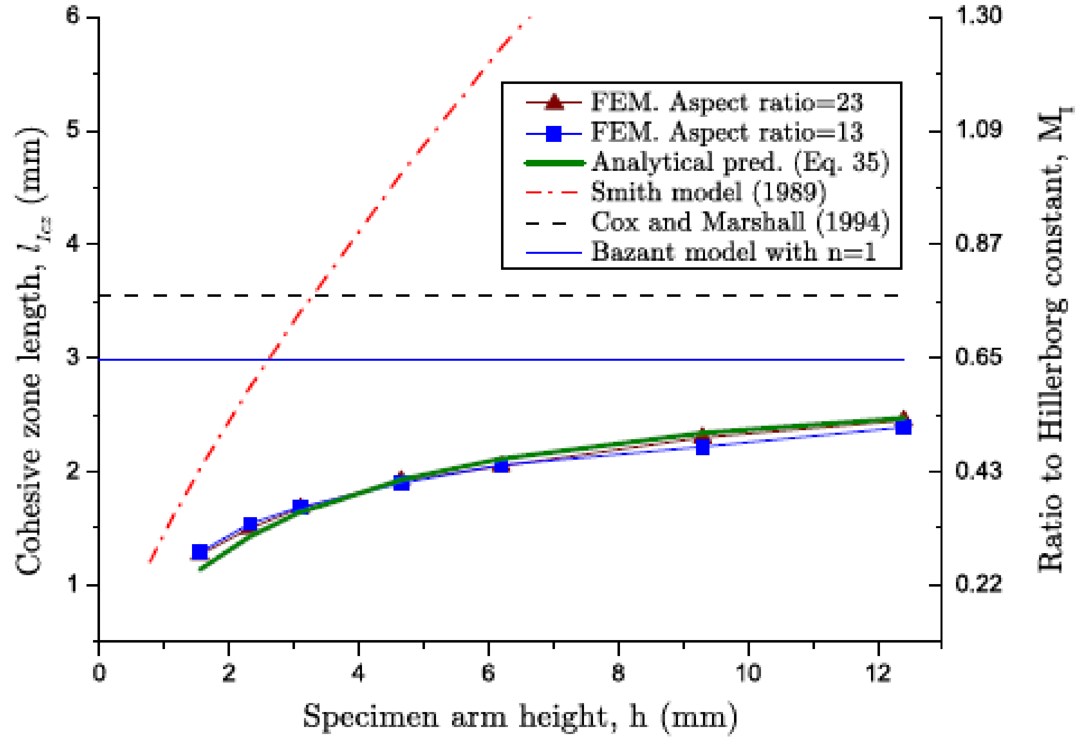
HOME
ABOUT US & OUR PARTNERS
PAST LISTS
LIFETIME ACHIEVEMENT AWARD WINNERS

The 2007 List

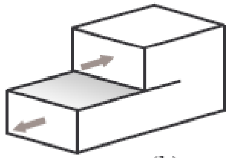
1	El Bulli	Spain	World's Best Restaurant Best in Europe
11 ▲10	El Celler de Can Roca	Spain	
22 ▼11	Can Fabes	Spain	



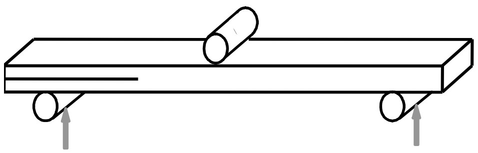


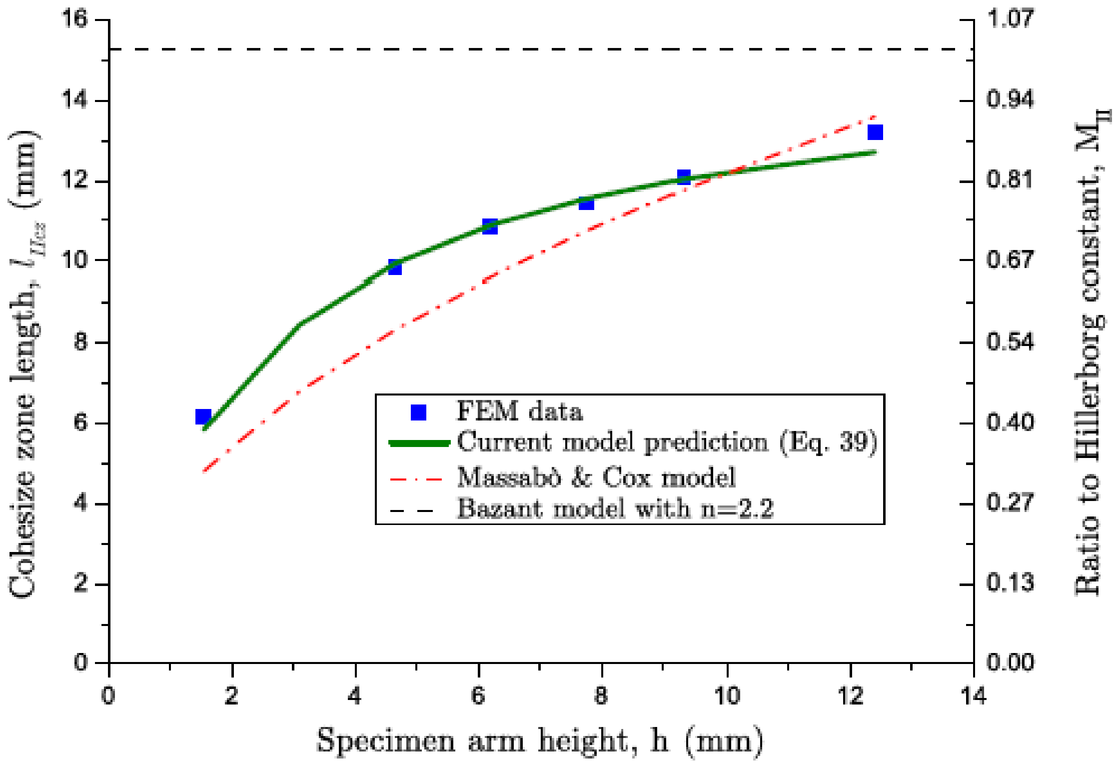


$$l_{Icz}(h) = \left(\frac{G_{Ic}E'}{(\tau_3^o)^2}\right)^{\frac{1}{3}} h^{\frac{3}{4}}$$



(b)





$$l_{IIcz}(h) = \sqrt{\left(\frac{G_{IIc}E'}{(\tau_3^o)^2}\right) h}$$