Identification of the through-thickness shear modulus of composites from full-field surface deformation measurements

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INTRODUCTION

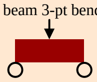
Identification of the through-thickness shear moduli of composites

- Iosipescu test

- Torsion on rectangular bars

These techniques require relatively thick specimens (need to bond a strain gauge)

- Short beam 3-pt bend test



- Double notch specimen

Not suitable for shear modulus

Statement from recent literature

"A noticeable void in current literature is the lack of a test method for determining the interlaminar shear modulus. This is largely attributable to the fact that conventional methods of direct stress and strain measurements cannot be easily adapted for the measurement of interlaminar properties.

Many techniques have been developed over the years but:

For example, utilising these conventional methods for determining the interlaminar shear modulus requires extremely thick composite coupons to be manufactured, which has proven to be very difficult and costly."

A. Chan, W.K. Chiu, and X.L. Liu. Determining the elastic interlaminar shear modulus of composite laminates. Composite Structures, 80:396–408, 2007.

THEORY

Higher order shear theory (pure bending)

- Deformation field

$$\begin{cases} u(x,y,z) = -z\dfrac{\partial w(x,y)}{\partial x} + f(z)\gamma_x^0(x,y) \\ v(x,y,z) = -z\dfrac{\partial w(x,y)}{\partial y} + f(z)\gamma_y^0(x,y) \\ w(x,y,z) = w(x,y) \end{cases}$$

- Strain field

$$\begin{cases} \varepsilon_{xx}(x,y,z) = -z\dfrac{\partial^2 w(x,y)}{\partial x^2} + f(z)\dfrac{\partial \gamma_x^0(x,y)}{\partial x} \\ \varepsilon_{yy}(x,y,z) = -z\dfrac{\partial^2 w(x,y)}{\partial y^2} + f(z)\dfrac{\partial \gamma_y^0(x,y)}{\partial y} \\ \varepsilon_{zz}(x,y,z) = 0 \\ 2\varepsilon_{xy}(x,y,z) = -2z\dfrac{\partial^2 w(x,y)}{\partial x \partial y} + 2f(z)\left(\dfrac{\partial \gamma_x^0(x,y)}{\partial y} + \dfrac{\partial \gamma_y^0(x,y)}{\partial y}\right) \\ 2\varepsilon_{xz}(x,y,z) = f'(z)\gamma_x^0(x,y) \\ 2\varepsilon_{yz}(x,y,z) = f'(z)\gamma_y^0(x,y) \end{cases}$$

- Conditions on f(z)

Free edge condition $f'(h/2) = f'(-h/2) = 0$

Zero in-plane displacement at mid-plane $f(0) = 0$

f must be an odd function $\begin{matrix} u(x,y,h/2) = -u(x,y,-h/2) \\ v(x,y,h/2) = -v(x,y,-h/2) \end{matrix}$

f must be C^1

- Several popular f functions

$f(z) = 0$ Love-Kirchhoff thin plate theory

$f(z) = z$ Mindlin-Reissner plate theory

$f(z) = z\left(1 - \dfrac{4z^2}{3h^2}\right)$ Schmidt-Reddy theory (parabolic TT shear strain distribution)

$f(z) = \dfrac{h}{\pi}\sin\left(\dfrac{h}{\pi}z\right)$ Touratier theory (cosine TT shear strain distribution)

Average TT shear strain through the thickness

$$\begin{cases} \dfrac{1}{h}\displaystyle\int_{-h/2}^{h/2} 2\varepsilon_{xz}(x,y,z)dz = \dfrac{1}{h}\gamma_x^0(x,y)\int_{-h/2}^{h/2} f'(z)dz \\ \dfrac{1}{h}\displaystyle\int_{-h/2}^{h/2} 2\varepsilon_{yz}(x,y,z)dz = \dfrac{1}{h}\gamma_y^0(x,y)\int_{-h/2}^{h/2} f'(z)dz \end{cases}$$

f is an odd function

$$\begin{cases} \dfrac{1}{h}\displaystyle\int_{-h/2}^{h/2} 2\varepsilon_{xz}(x,y,z)dz = \dfrac{2}{h}\gamma_x^0(x,y)f(h/2) \\ \dfrac{1}{h}\displaystyle\int_{-h/2}^{h/2} 2\varepsilon_{yz}(x,y,z)dz = \dfrac{2}{h}\gamma_y^0(x,y)f(h/2) \end{cases}$$

- Surface measurements

$$\begin{cases} u_s(x,y) = -\dfrac{h}{2}\dfrac{\partial w(x,y)}{\partial x} + f(h/2)\gamma_x^0(x,y) \\ v_s(x,y) = -\dfrac{h}{2}\dfrac{\partial w(x,y)}{\partial y} + f(h/2)\gamma_y^0(x,y) \end{cases}$$

$$\begin{cases} f(h/2)\gamma_x^0(x,y) = u_s(x,y) + \dfrac{h}{2}\dfrac{\partial w(x,y)}{\partial x} \\ f(h/2)\gamma_y^0(x,y) = u_s(x,y) + \dfrac{h}{2}\dfrac{\partial w(x,y)}{\partial y} \end{cases}$$

- Substitute in TT shear strain average

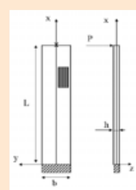
$$\begin{cases} \dfrac{1}{h}\displaystyle\int_{-h/2}^{h/2} 2\varepsilon_{xz}(x,y,z)dz = \dfrac{2}{h}u_s(x,y) + \dfrac{\partial w(x,y)}{\partial x} \\ \dfrac{1}{h}\displaystyle\int_{-h/2}^{h/2} 2\varepsilon_{xz}(x,y,z)dz = \dfrac{2}{h}v_s(x,y) + \dfrac{\partial w(x,y)}{\partial y} \end{cases} \quad \text{(Eq. 1)}$$

$u_s(x,y)$ and $v_s(x,y)$ surface in-plane deformations

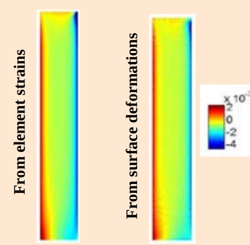
$\dfrac{\partial w(x,y)}{\partial x}$ and $\dfrac{\partial w(x,y)}{\partial y}$ surface slopes

These quantities can be measured experimentally (3D DIC, speckle interferometry…)

FE validation (beam)



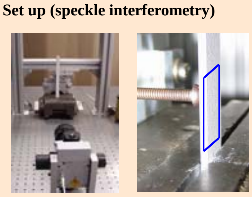
3D FE model (8-noded bricks)
L = 40 mm, b = 14 mm, h=2 mm
Carbon/epoxy UD
Half model (symmetry)

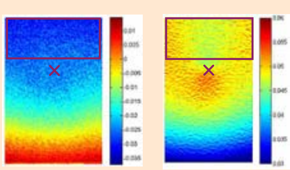


$\dfrac{1}{h}\displaystyle\int_{-h/2}^{h/2} 2\varepsilon_{xz}(x,y,z)dz$

EXPERIMENTAL VALIDATION

Set up (speckle interferometry)





Problem : u_s (and w) relative not absolute !

Determination of the constant u^0

No shear force above loading point (rectangular areas on figures above)

$$\frac{2}{h}(u_s + u^0) + \frac{\partial w}{\partial x} = 0$$

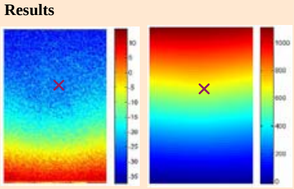
$$G_{xz}\int_S \left(\frac{2}{h}u_s + \frac{\partial w}{\partial x}\right)dxdy = \frac{PL}{h}$$

Specimen

- T300/914 0° UD, P=237 N
- L_0 = 20 mm, b = 14 mm, h=2 mm

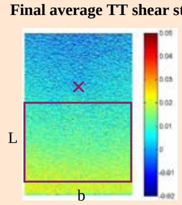
E_{xx} = 130 GPa
G_{xy} = 5 GPa
Expected deflection: 520 µm

Results



u_s in microns — w in microns

Final average TT shear strain map



Expected TT average shear strain: **1.7 10^{-3}**

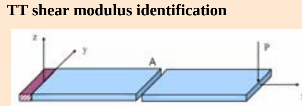
$$\overline{\frac{2}{h}u_s + \frac{\partial w}{\partial x}} = \frac{P}{bhG_{xz}}$$

Experimental TT average shear strain: **1.5 10^{-2}** !!!

Does not work…why ??

IDENTIFICATION

TT shear modulus identification



Over section A

$$\int_{A_2} \sigma_{xz}dydz = P$$

Substituting the stress

$$\int_{A_2} G_{xz}\varepsilon_{xz}dydz = P$$

Homogeneous material

$$G_{xz}\int_{A_2} \varepsilon_{xz}dydz = P$$

Integrating over x (between L_1 and L_2, $L = L_2 - L_1$)

$$G_{xz}\int_{L_1}^{L_2}\int_A \varepsilon_{xz}dxdydz = PL$$

Integrating over z explicitly using Eq. 1

$$G_{xz}\int_S \left(\frac{2}{h}u_s + \frac{\partial w}{\partial x}\right)dxdy = \frac{PL}{h}$$

CONCLUSION

Summary

- Identification of the TT shear modulus of thin composites from surface measurements
- Requires 3 deformation components at the surface
- Validated on 3D FE model
- Problem of absolute displacement measurements: solved
- Experimental results disappointing

Future work

- More detailed FE validation
- Look at possible parasitic effects (large unbalance between in-plane displacements and deflection)

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