Imperial College London

The Composites Centre

for research, modelling, testing and training in advanced composites

Micromechanics of kink-band formation

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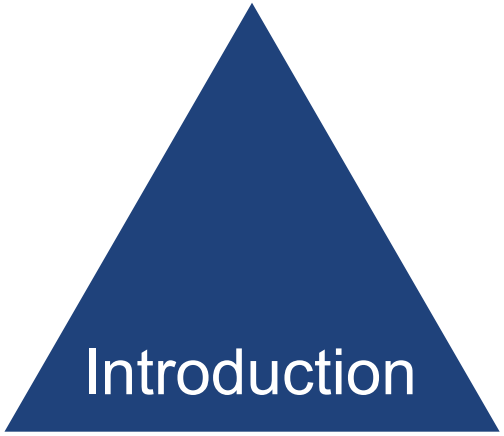
AIRBUS

RENAULT F1 Team

Wednesday, 22 October 2008

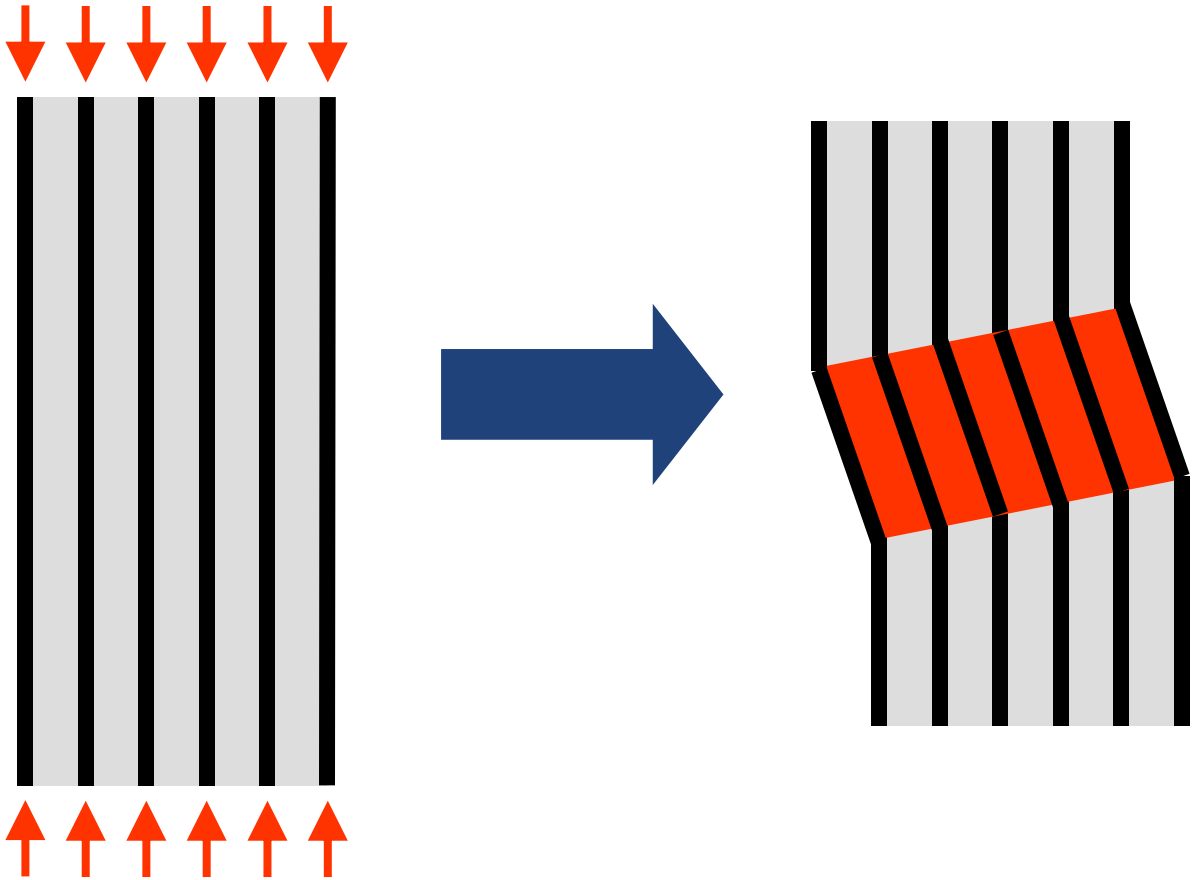
Outline

- Introduction
- Experimental work
- Numerical simulations
- Analytical model
- Conclusions


Introduction

What is a kink band?

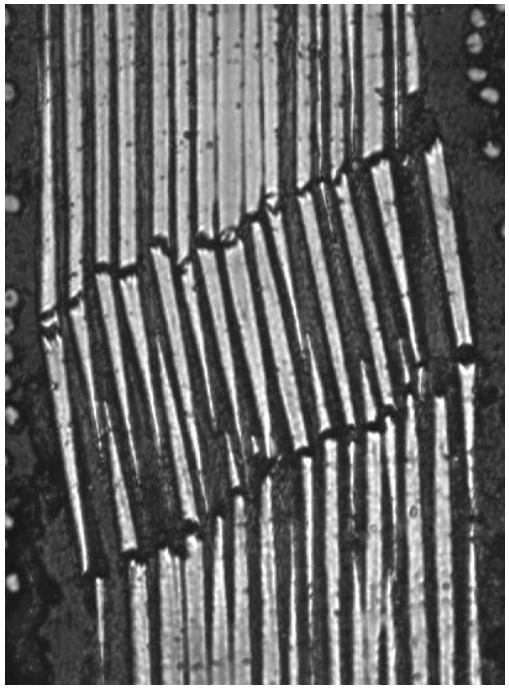
- Composite under longitudinal compression



What is a kink band?

- Layered materials

- rocks

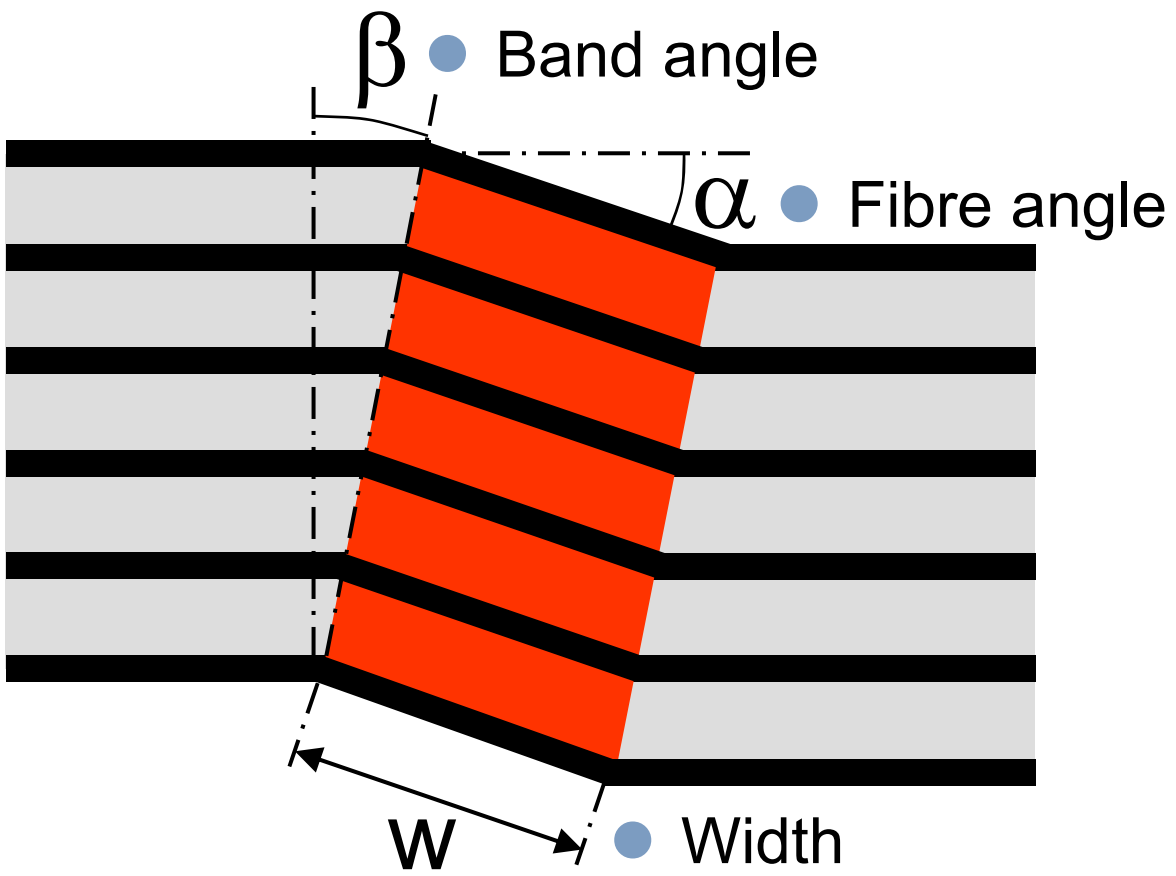


- composites

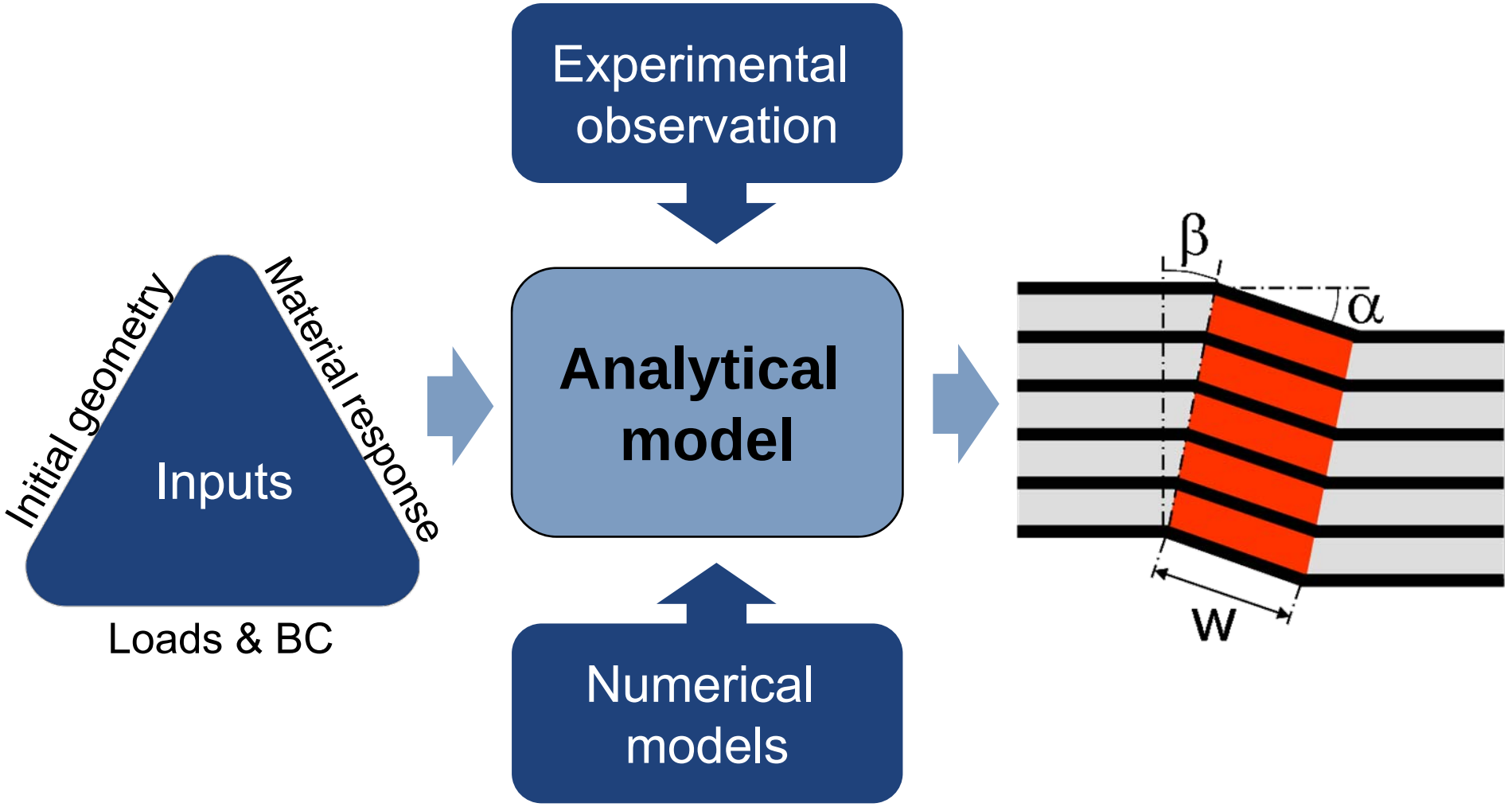
- paper

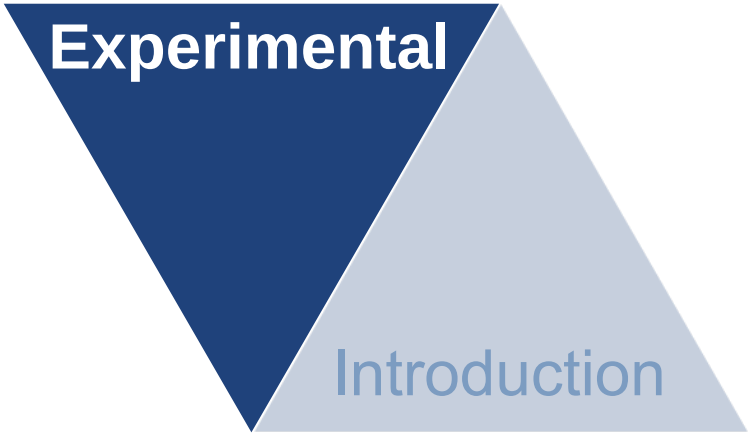
What is a kink band?

- Kink band geometry



Objective


Experimental observation
Analytical model
Numerical models
Inputs
Initial geometry
Material response
Loads & BC
β
α
w


Experimental
Introduction

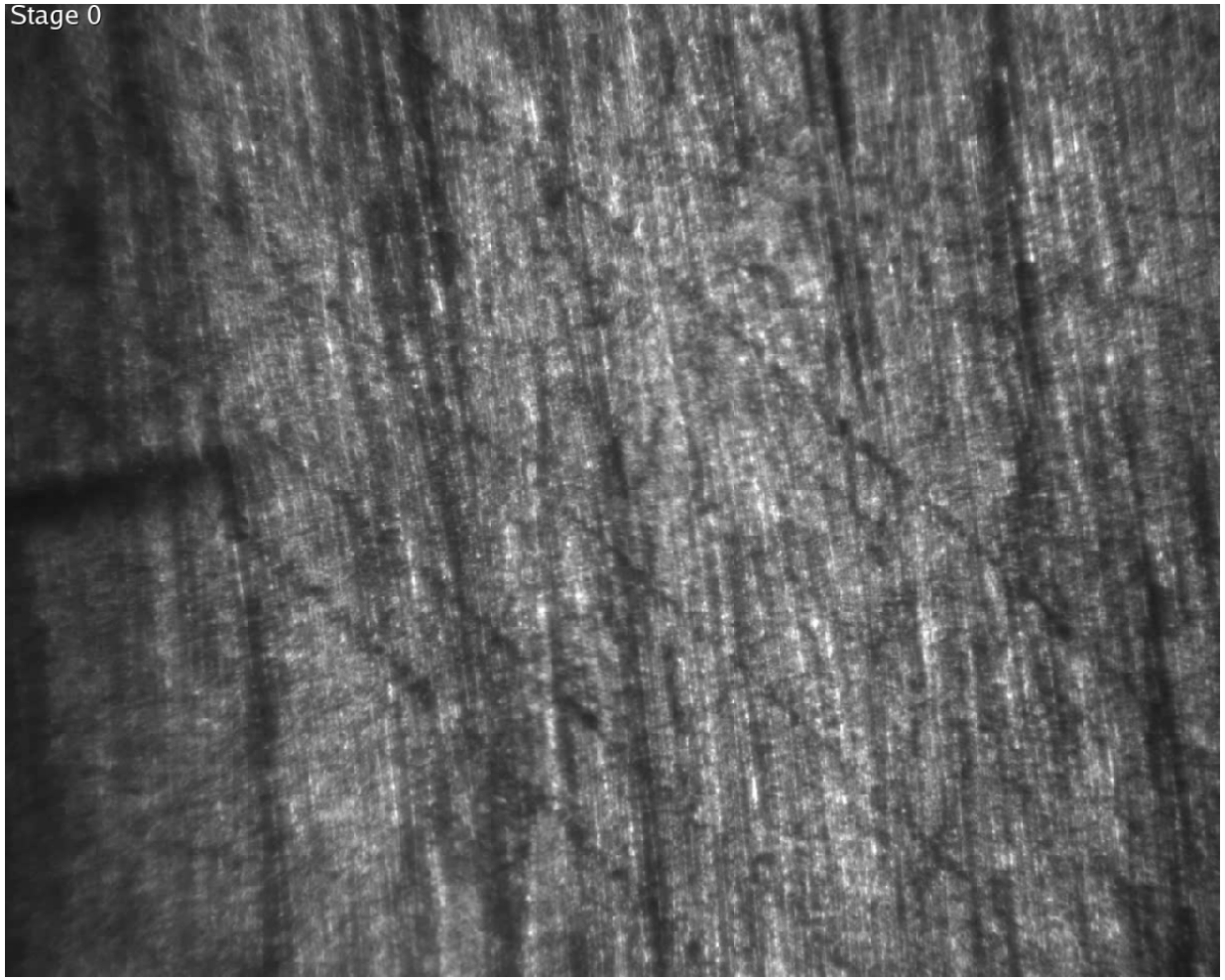
Objective

- Understand mechanics of kinking
  - Sequence of events

- Observe real kink bands
  - High magnification & resolution
  - Loaded

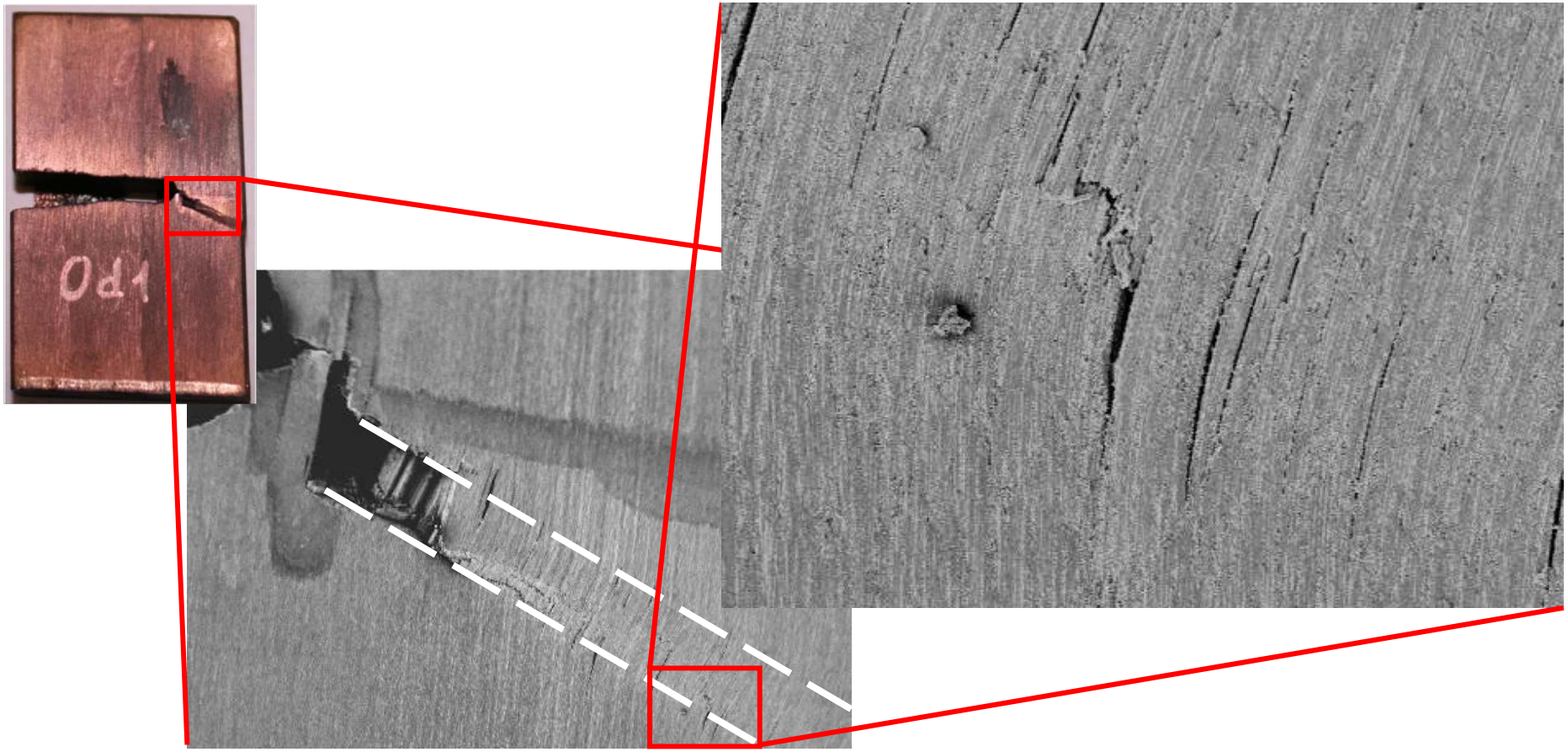
Results

- Overview on propagation



Results

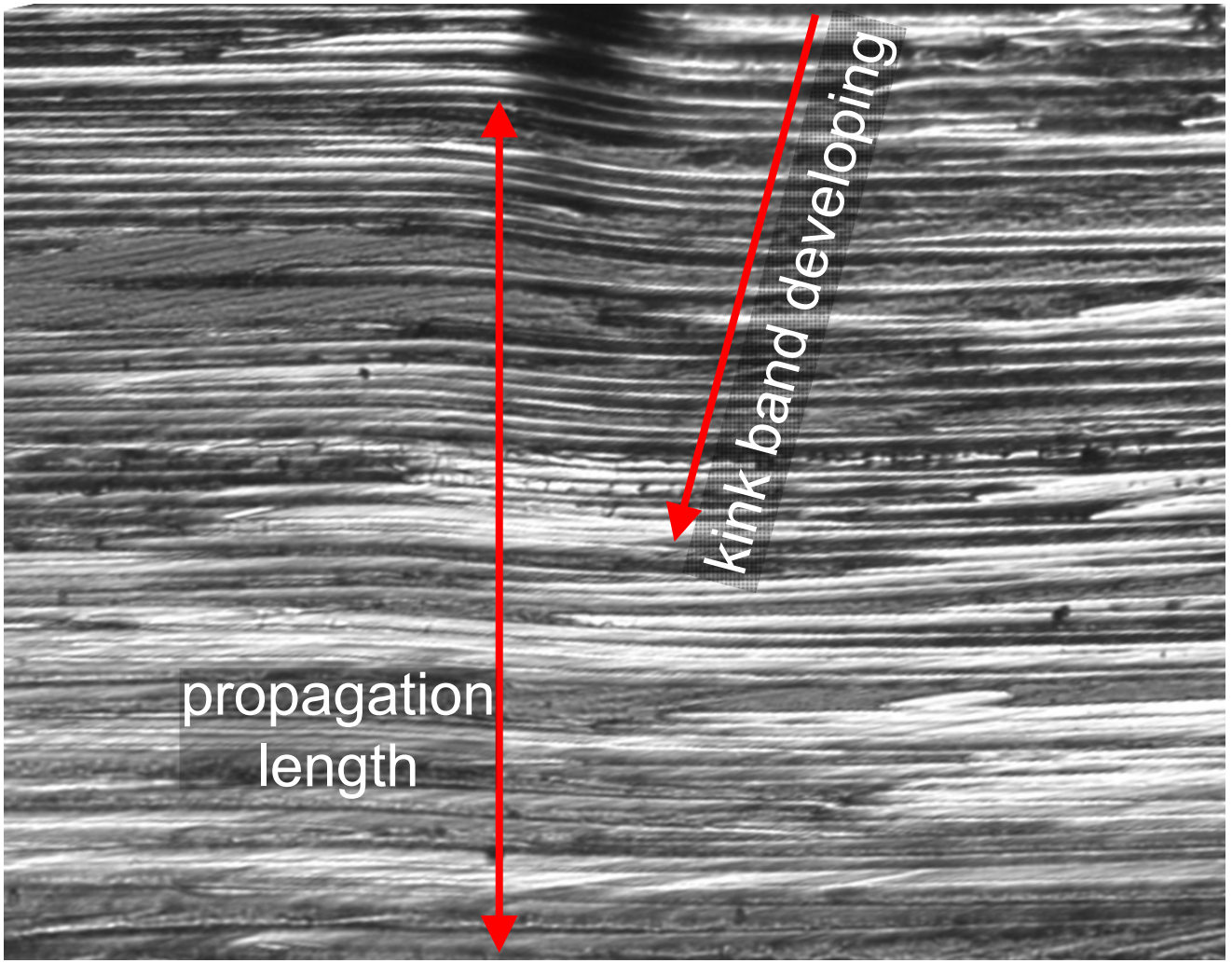
- Macroscopic kink band without fibre failure



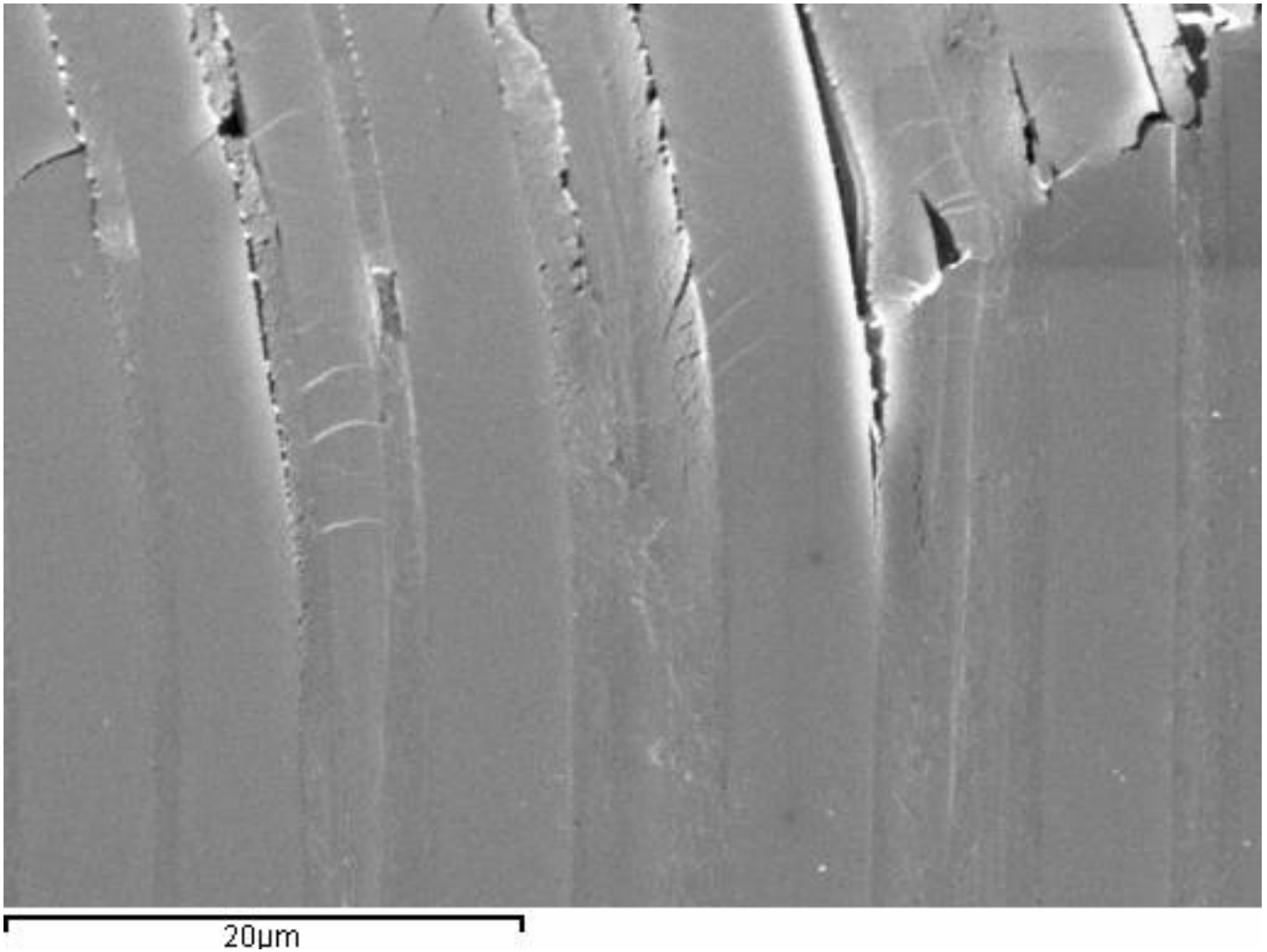
Experimental Results

- Propagation

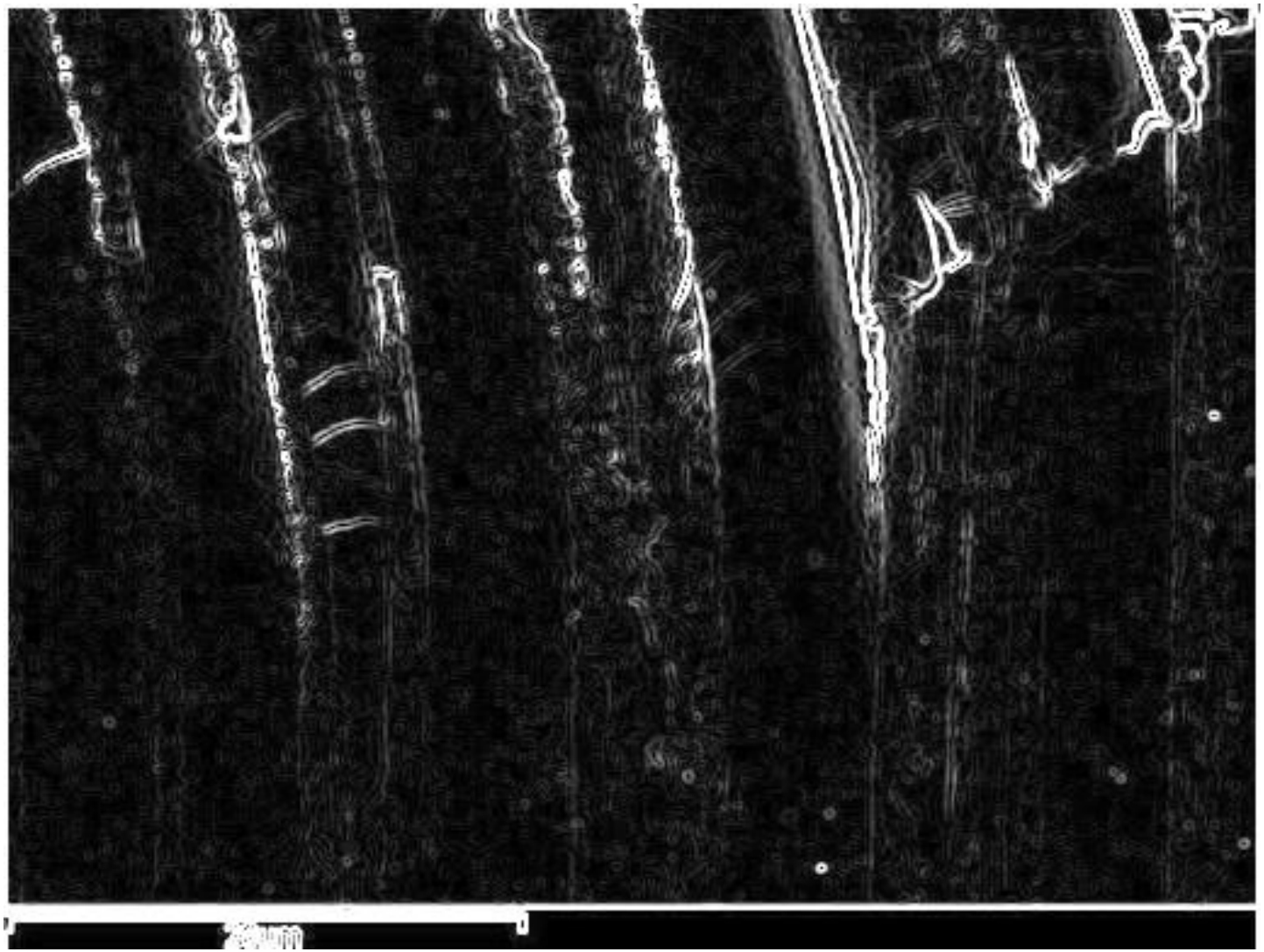
  (loaded)



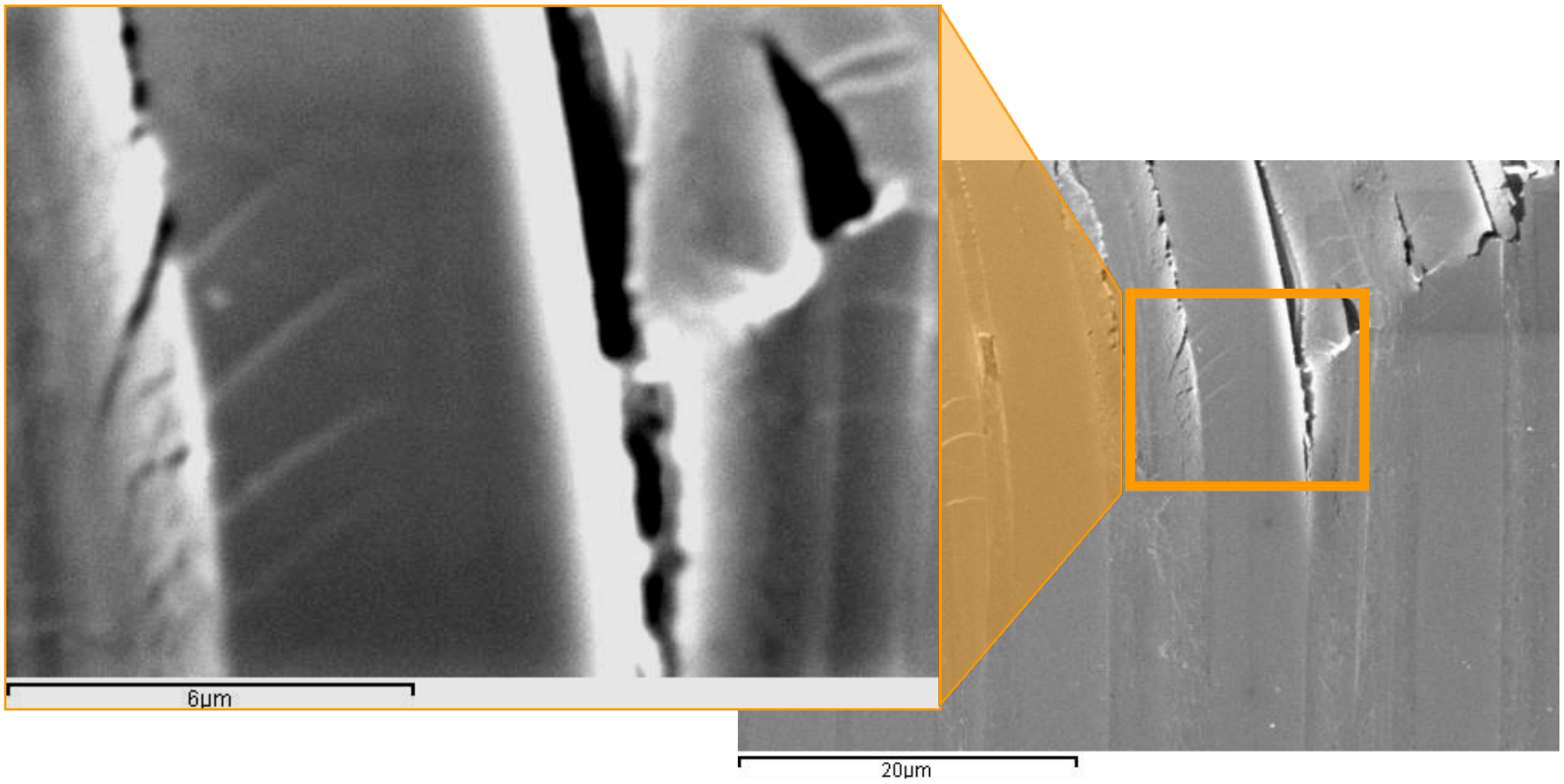
Experimental Results



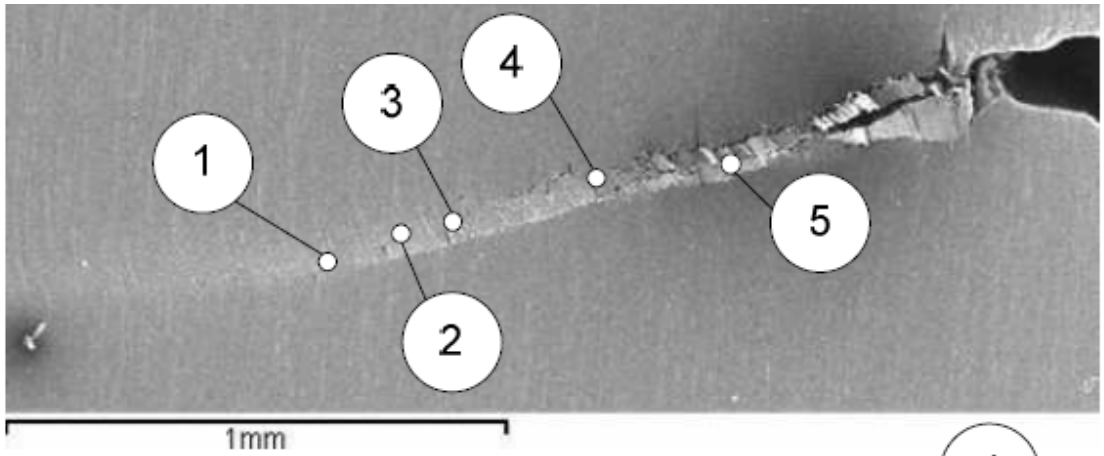
Experimental Results

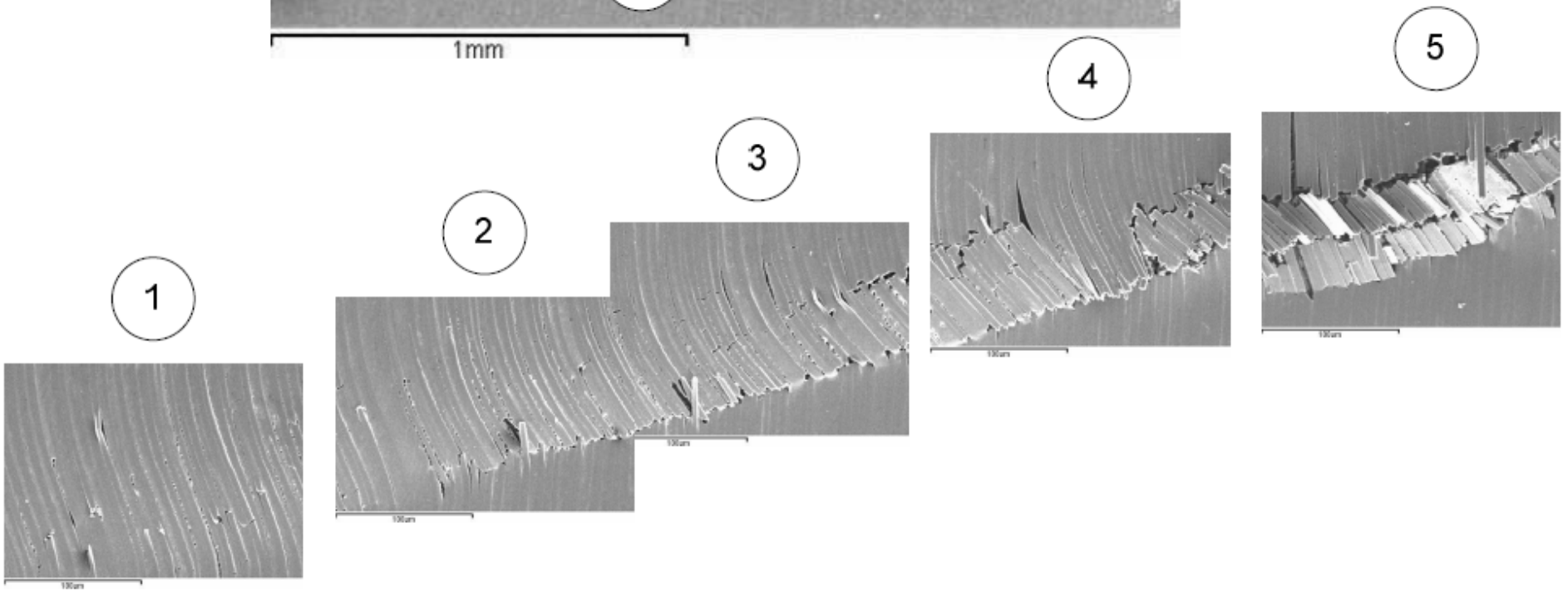


Experimental Results

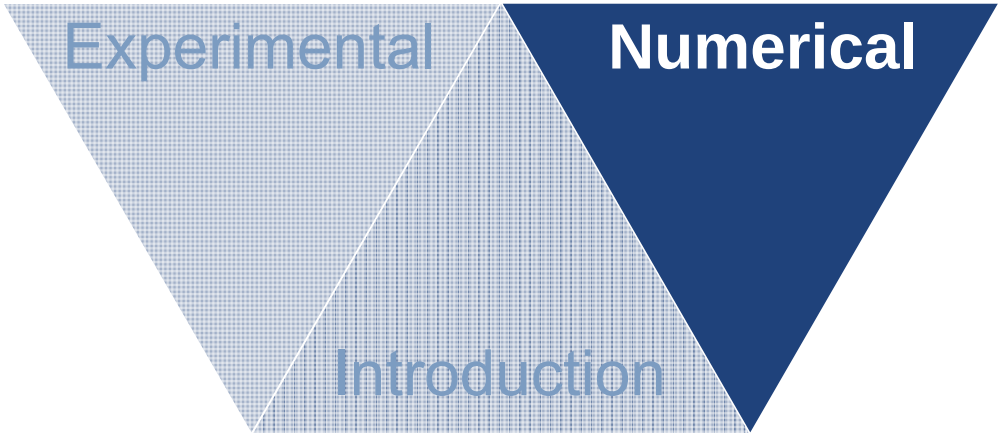


Conclusions





Numerical

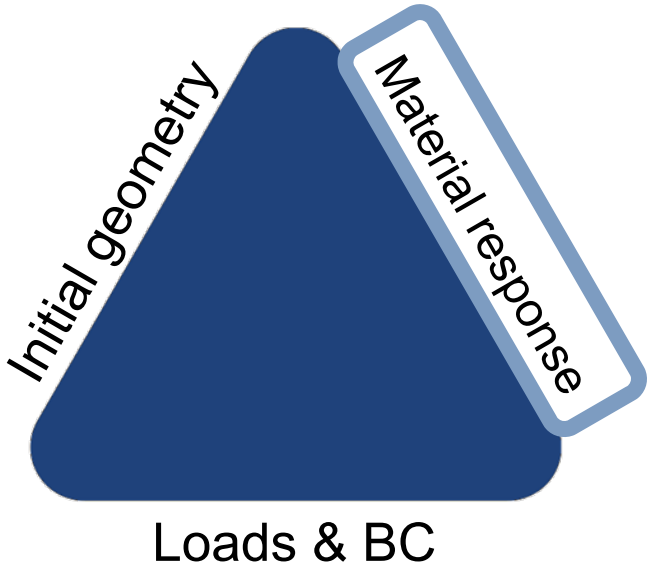


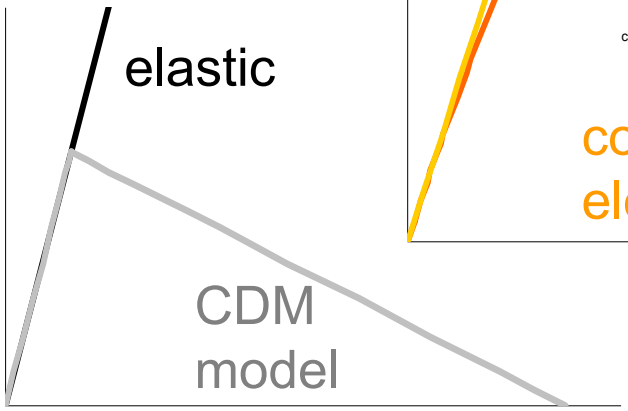
Objective

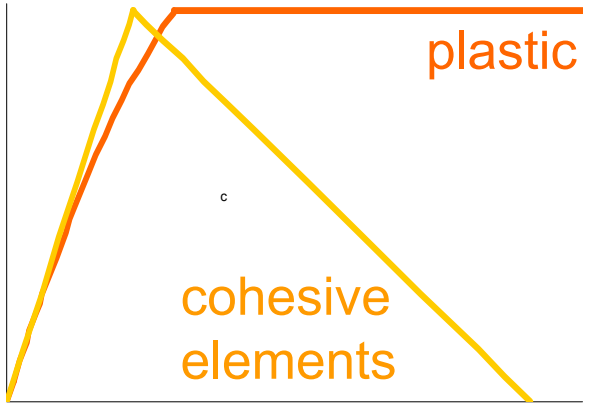
- Understand mechanics of kinking
  - Stress & displacement fields
- Verify hypotheses for analytical model

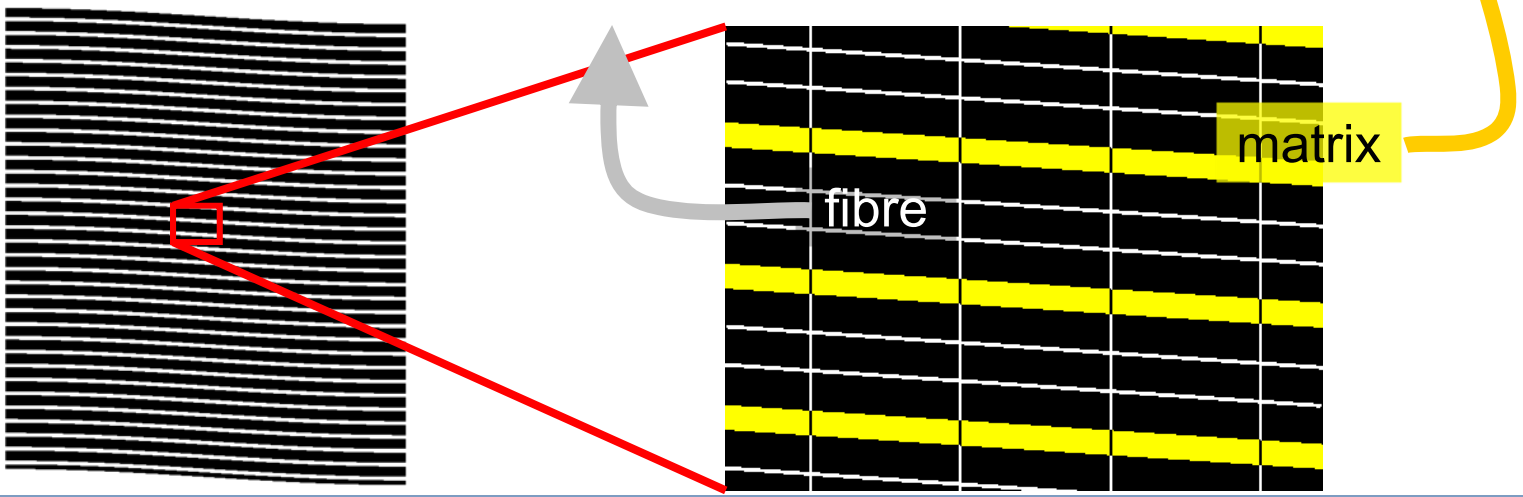
- Simulate kink bands at the micro-scale

Modelling strategy



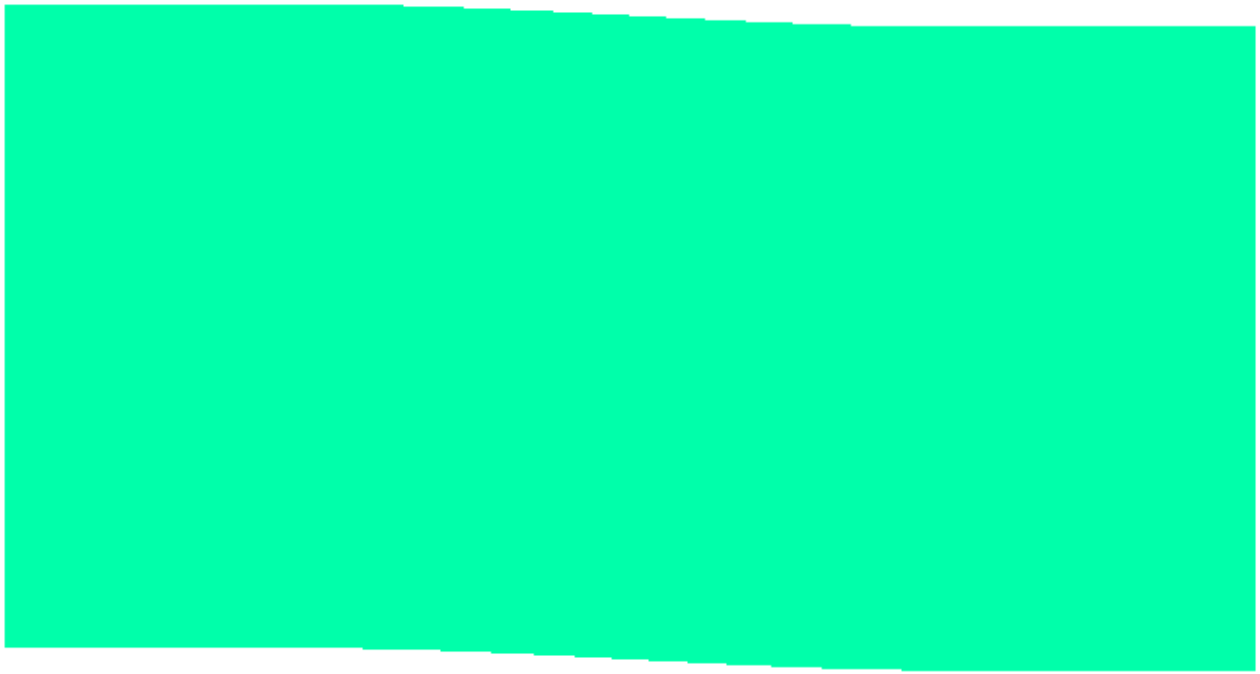






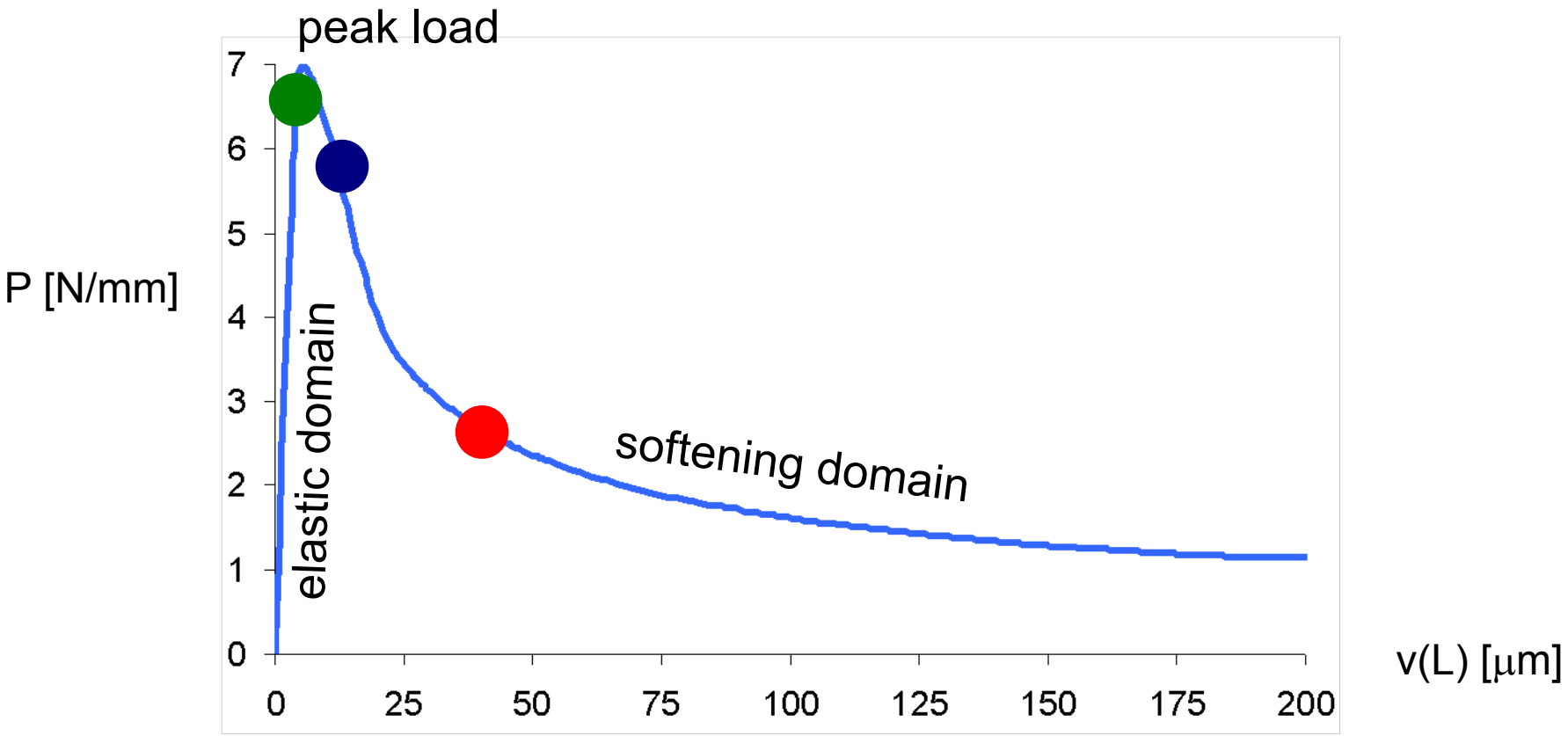
Overview of formation

- σ_1 in fibres
  - CDM in fibres
  - global waviness
  - kink-band formation & broadening



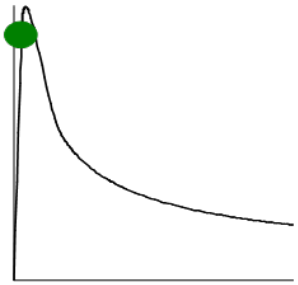
Overview of formation

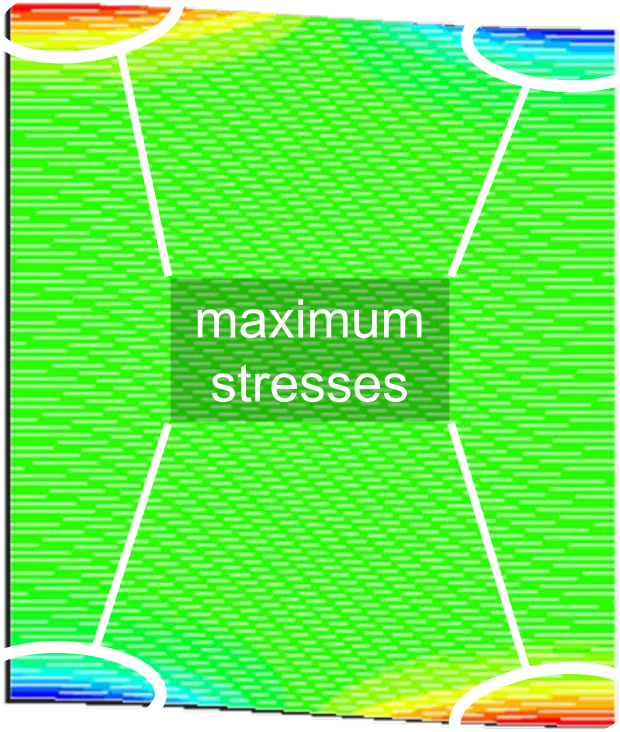
- Load vs. displacement curve

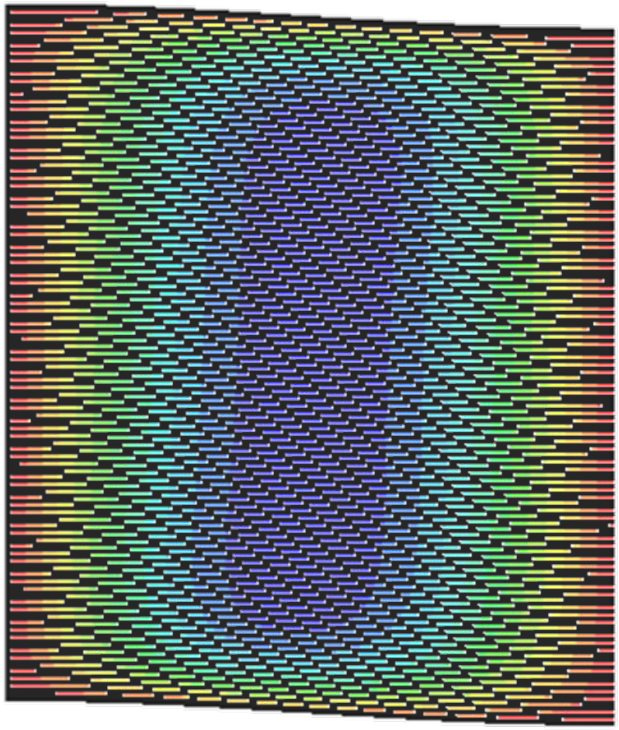


Stress fields during formation

- Before peak load
  - σ_1 in fibres
  - τ_{12} in matrix

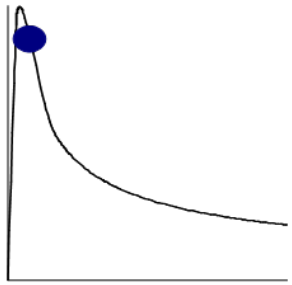


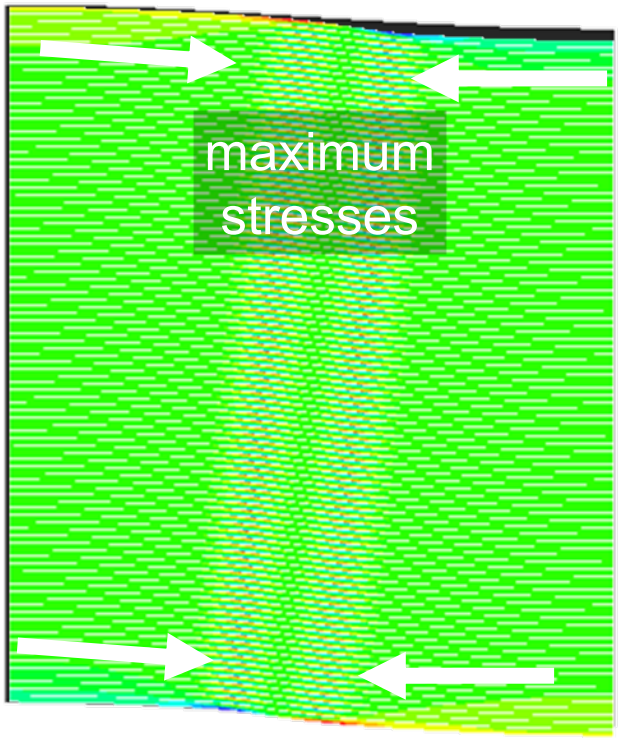


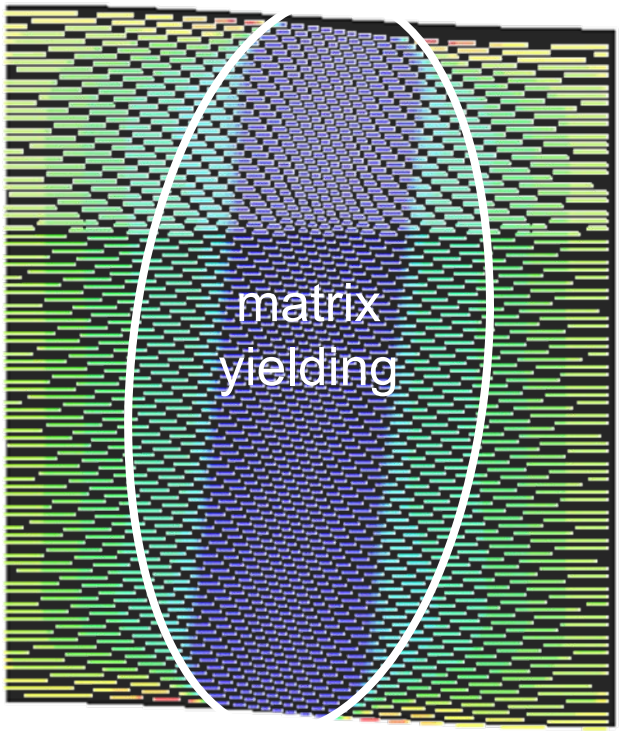


Stress fields during formation

- After peak load
  - σ_1 in fibres
  - τ_{12} in matrix

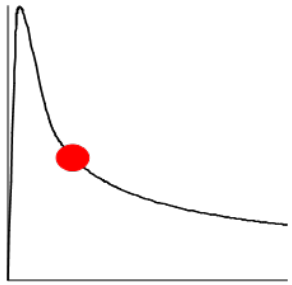


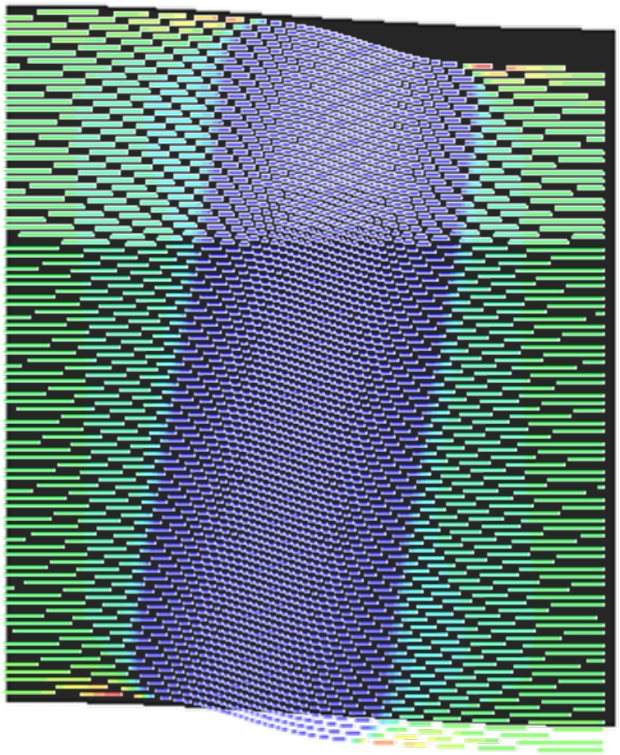




Stress fields during formation

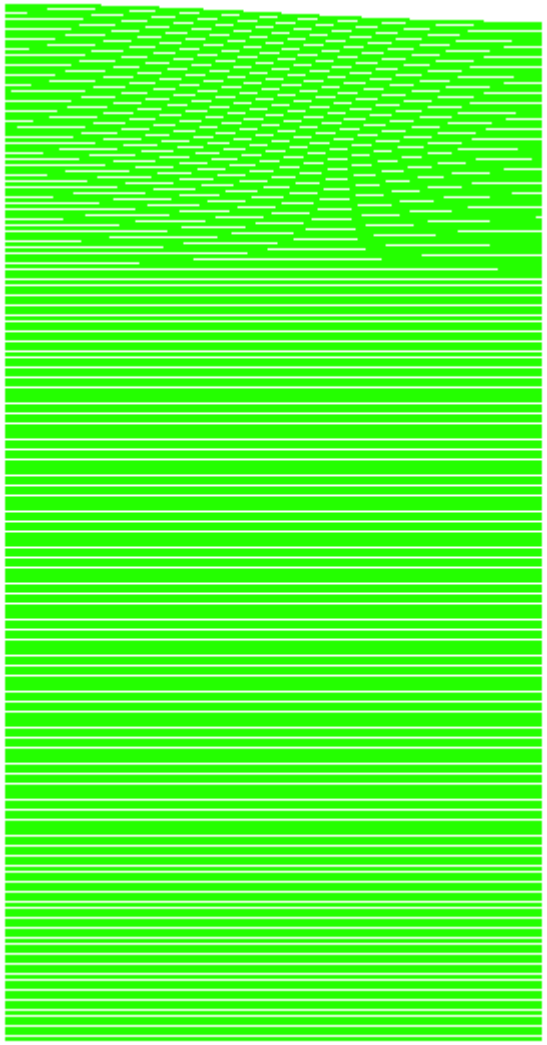
- Onset fibre failure
  - σ_1 in fibres
  - τ_{12} in matrix





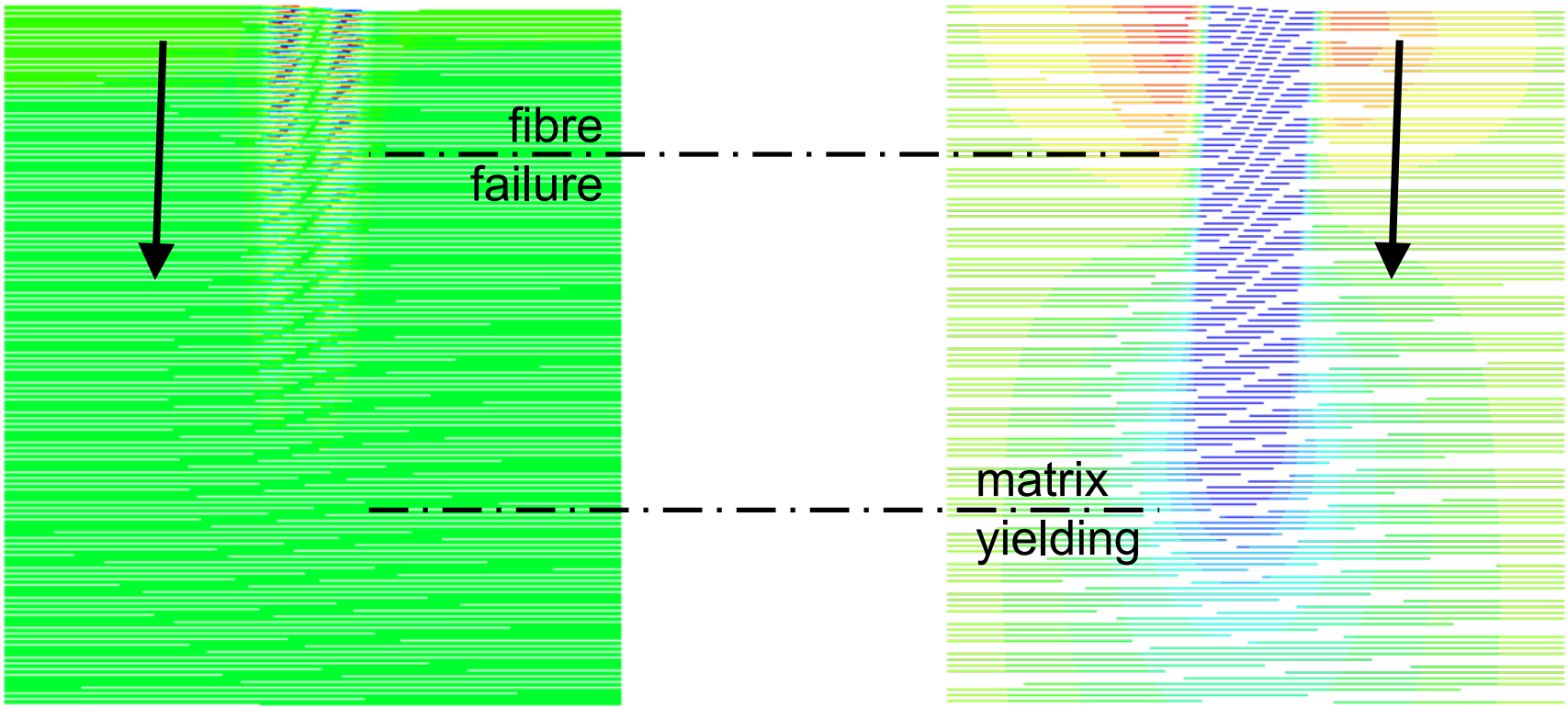
Propagation

- σ_1 in fibres
  - kinking of misaligned fibres
  - propagation along perfect fibres
  - band broadening



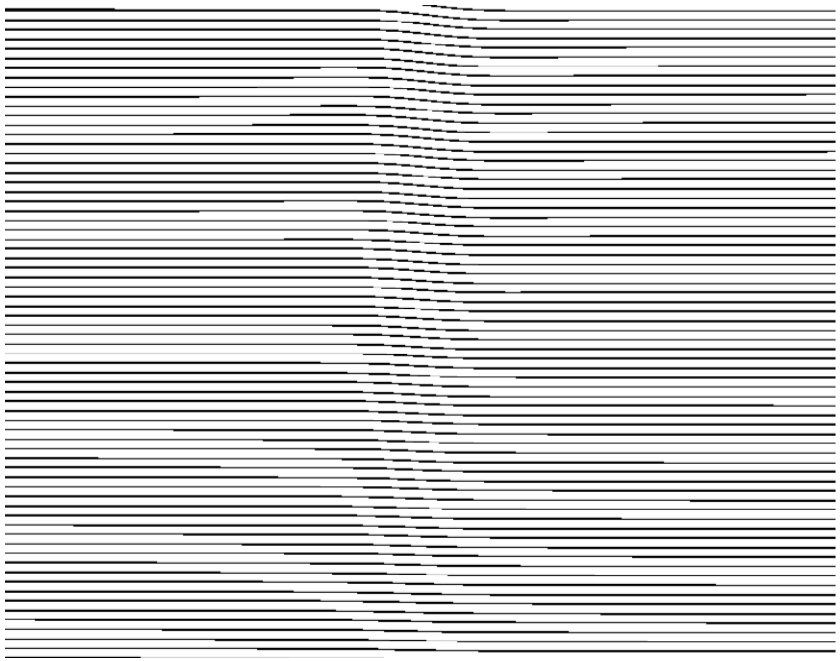
Propagation

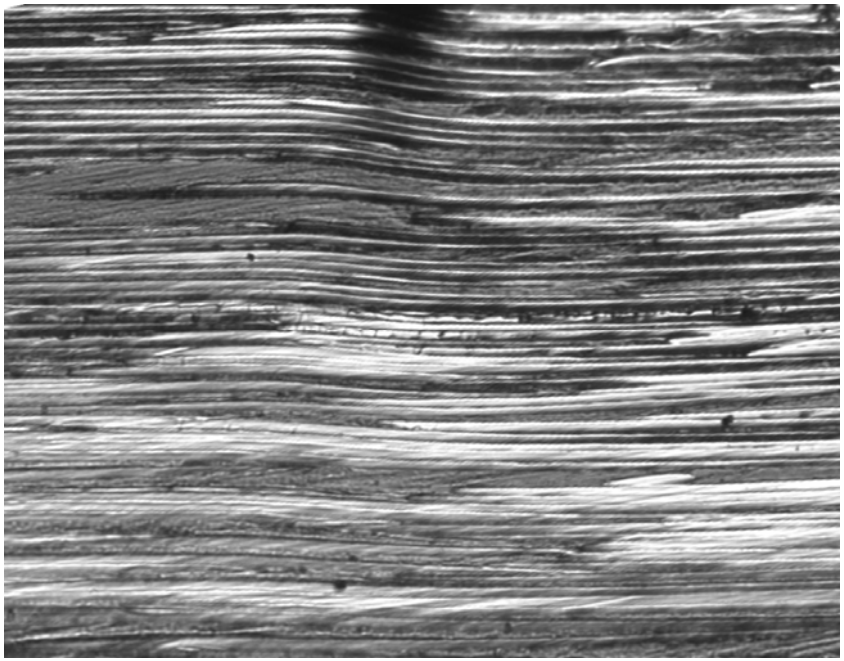
- Kinking of initially straight fibres
  - σ_1 in fibres
  - τ_{12} in matrix



Conclusions

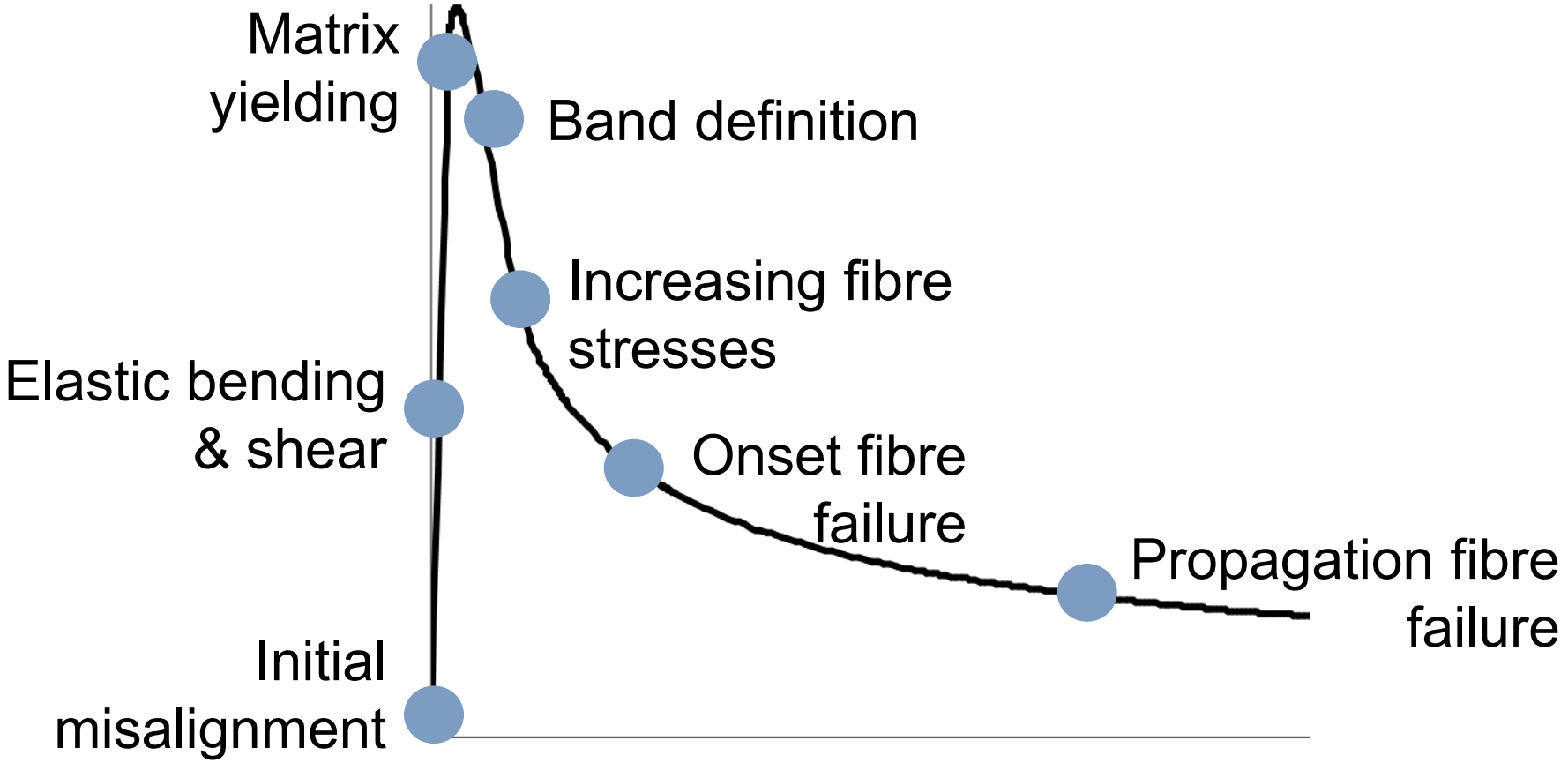
- Representativeness
  - Agreement with experimental results





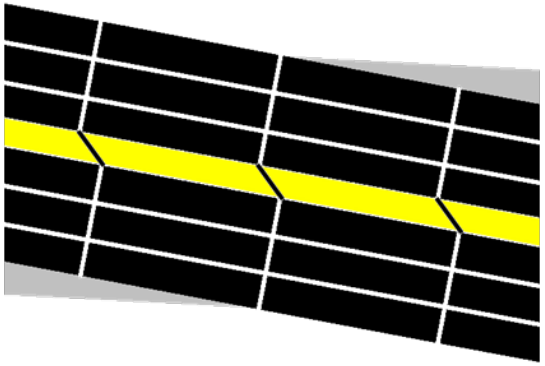
Conclusions

- Sequence of events

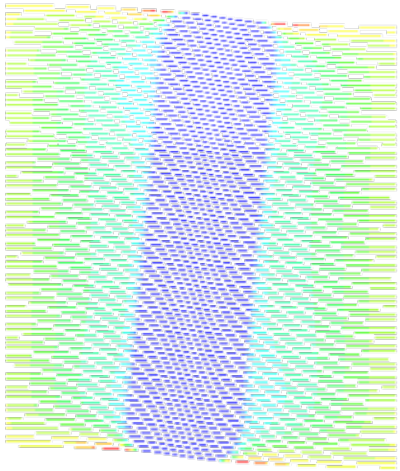


Conclusions

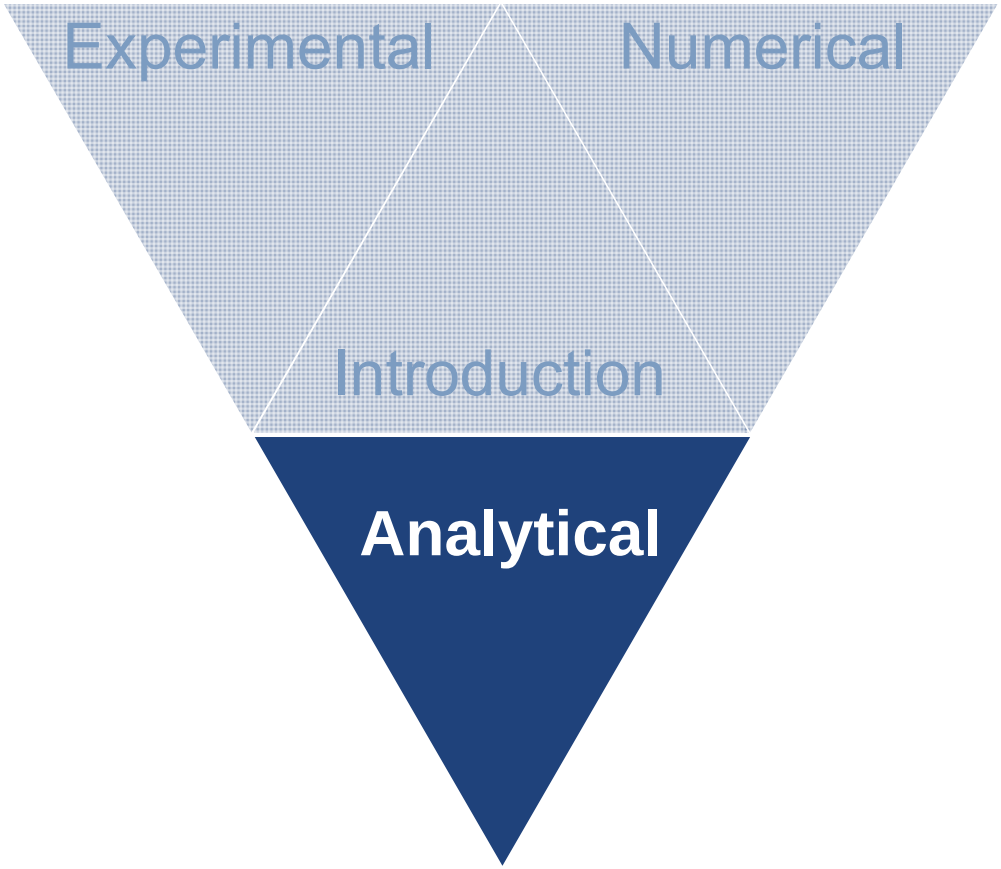
- Micromechanics
  - Response
    - Fibres ➔ bending
    - Matrix ➔ interface in shear



  - Features
    - Initial misalignment
    - Matrix yielding

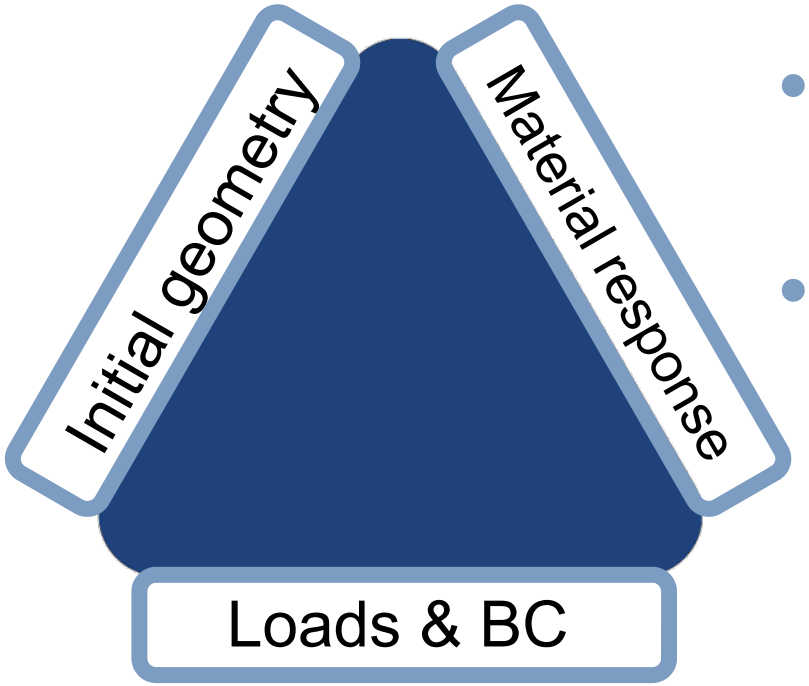


Analytical



Assumptions & Applicability

- 2D model
- Sinusoidal imperfection
- Small rotations

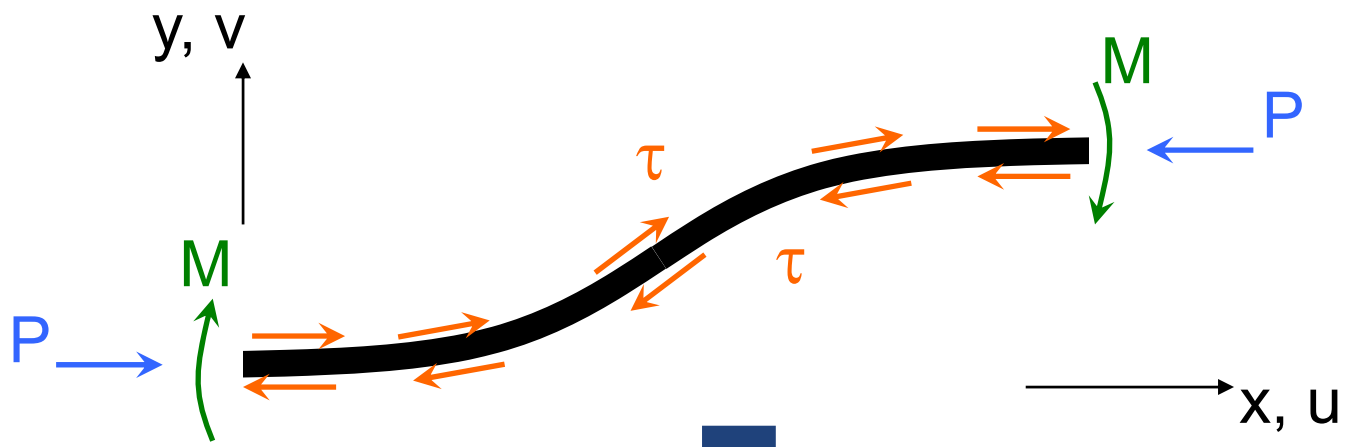


- Fibre ← compression + bending
- **Matrix** ← shear

- **Pure compression**
- Non-rotating ends

Development of the model

- Equilibrium of the fibre



$$\partial M + P \cdot \partial y - \tau \cdot \phi_f \cdot \partial s = 0$$

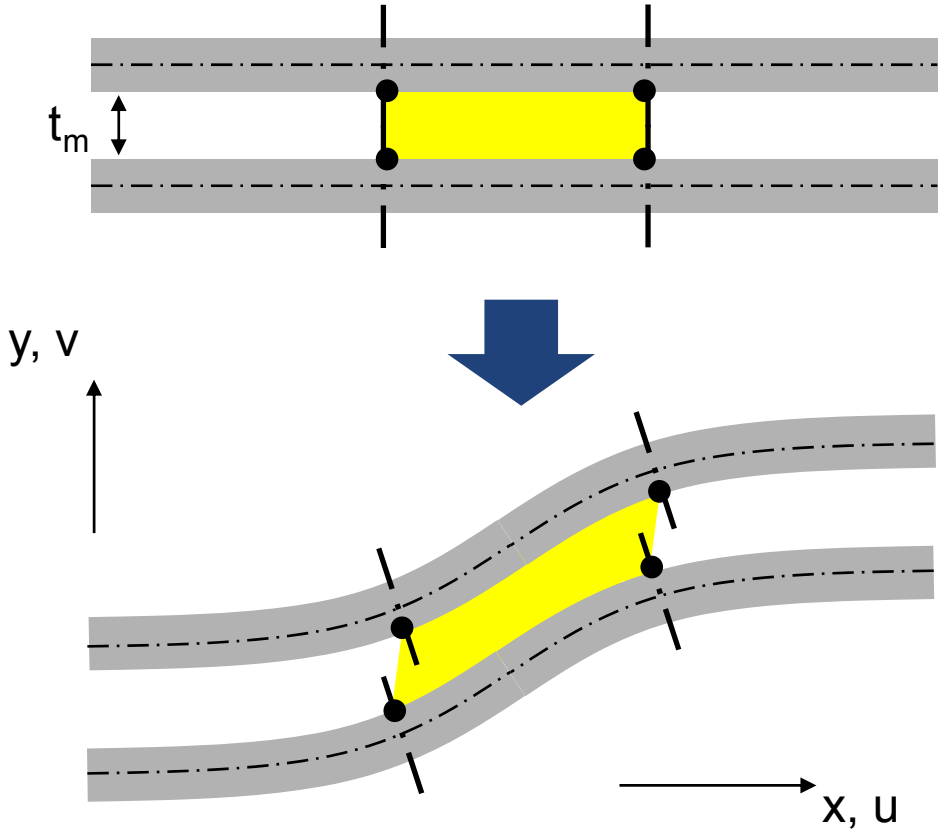
$$M = E_f \cdot I_f \frac{\partial^2 v}{\partial x^2}$$

?

∂x

Development of the model

- Shear in the matrix

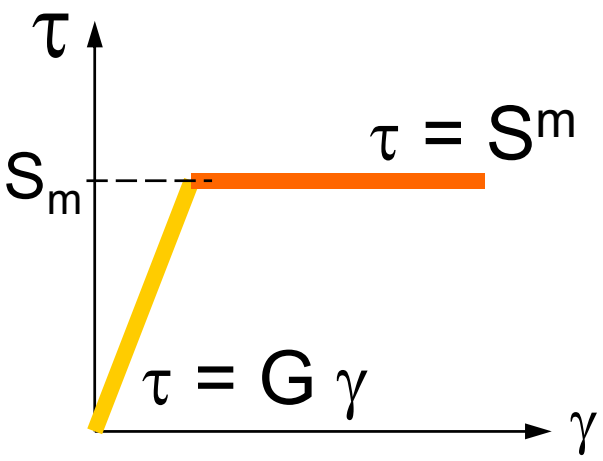


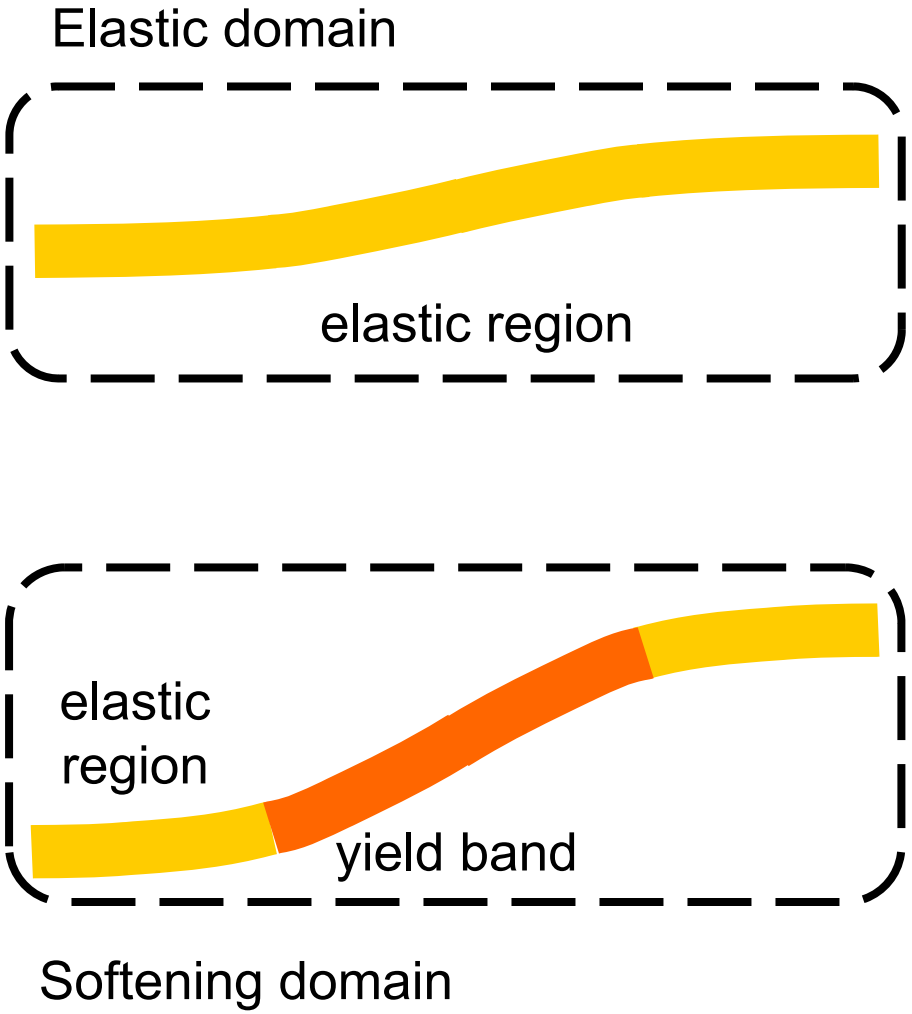
- Shear deformation

$$\gamma = \left(1 + \frac{\phi_f}{t_m}\right)\frac{\partial v}{\partial x}$$

Development of the model

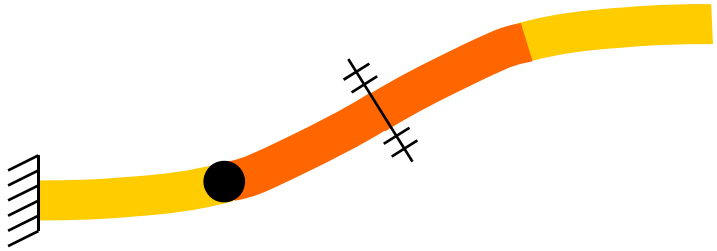
- Shear in the matrix
  - Constitutive law





Development of the model

- Differential equations
  - Elastic domain
  - Softening domain



elastic region

$$E_f \cdot I_f \frac{\partial^3 v}{\partial x^3} - \left(G^{2D} \cdot \phi_f - P\right)\frac{\partial v}{\partial x} = -P\frac{\partial y_0}{\partial x}$$

yield band

$$E_f \cdot I_f \frac{\partial^3 v}{\partial x^3} + P\frac{\partial v}{\partial x} = -P\frac{\partial y_0}{\partial x} + \phi_f \cdot S_m$$

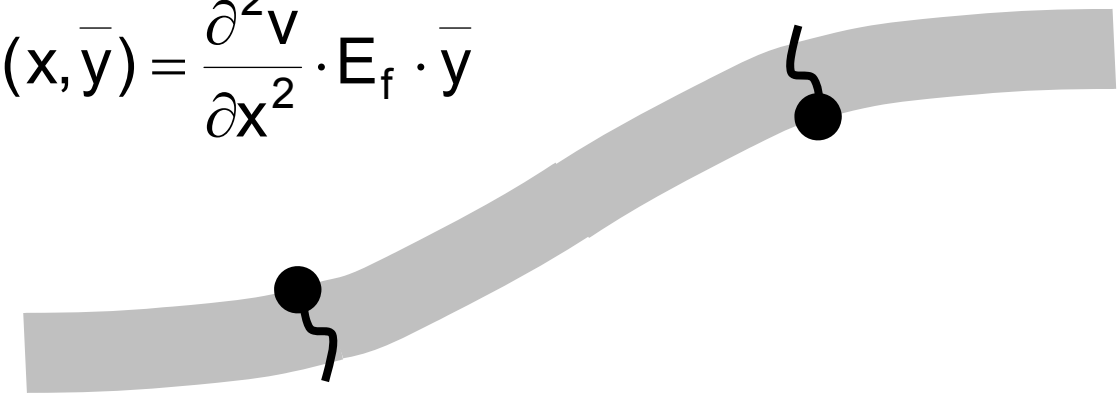
Development of the model

- Axial stresses and fibre failure

$$\sigma_1(x,\bar{y}) = \sigma_1^{P} + \sigma_1^{M}$$

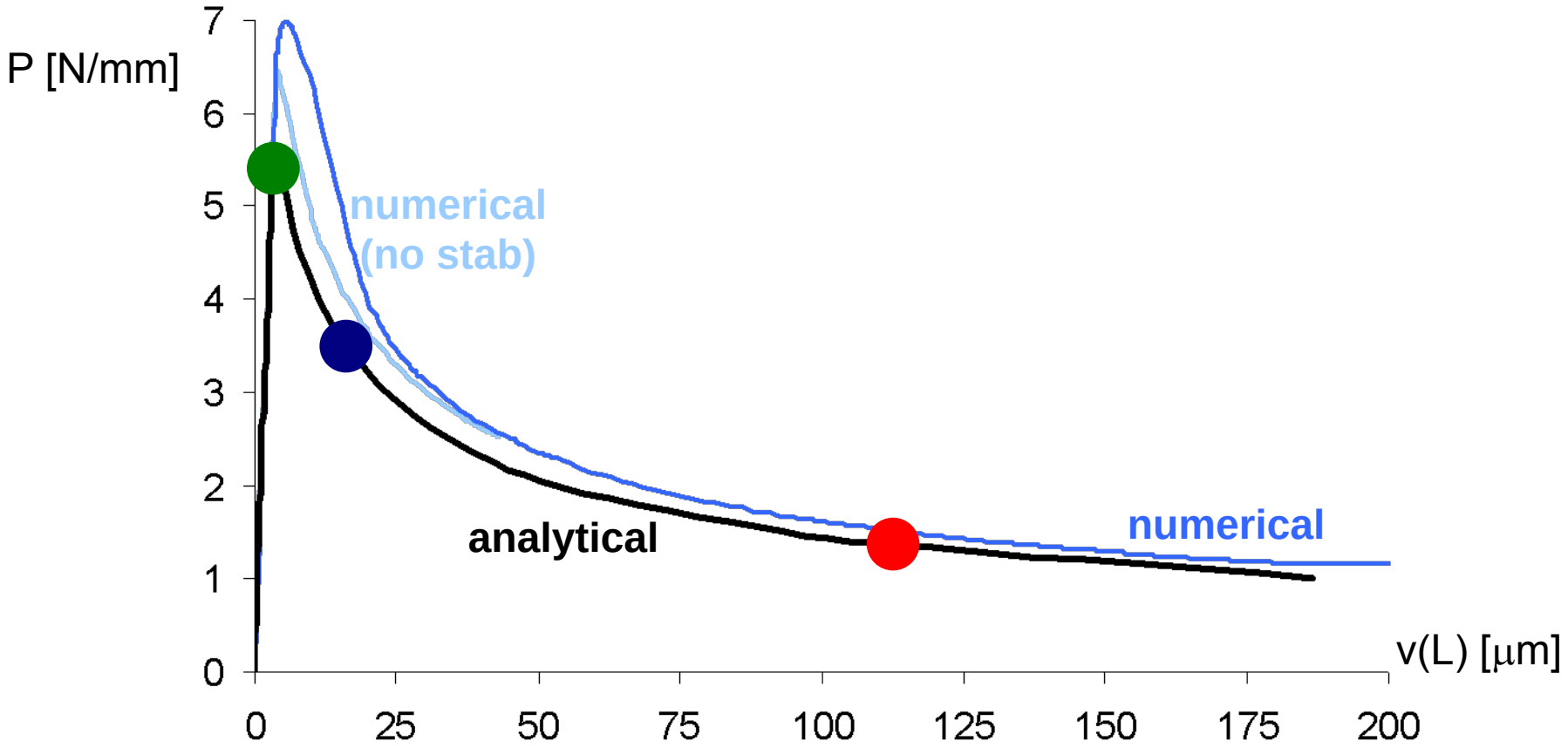
$$\sigma_1^{P} = \frac{P}{A_f} < 0$$

$$\sigma_1^{M}(x,\bar{y}) = \frac{\partial^2 v}{\partial x^2} \cdot E_f \cdot \bar{y}$$



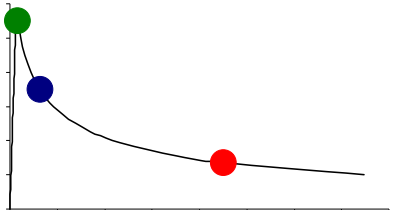
Results

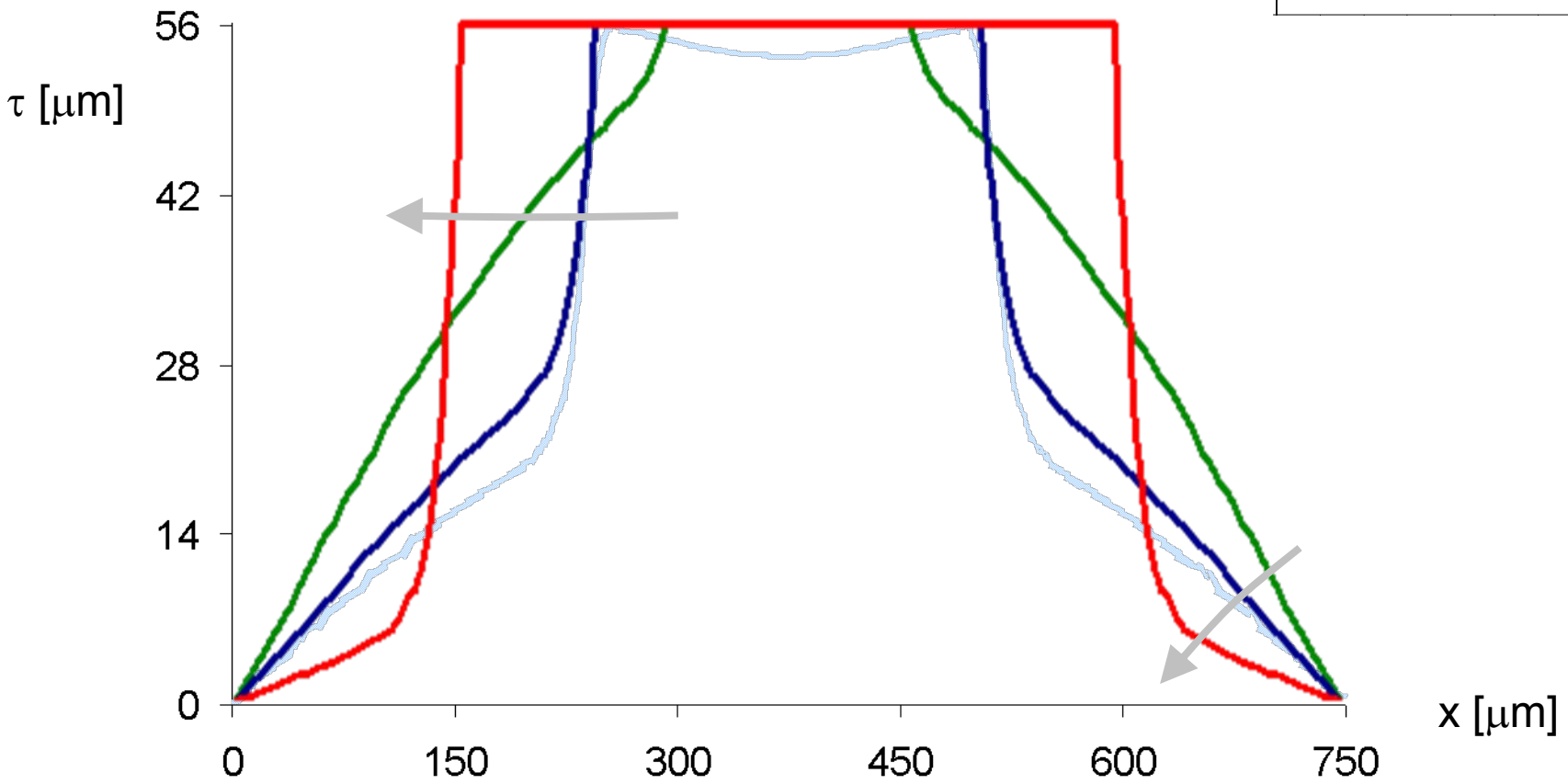
- Load versus displacement



Results

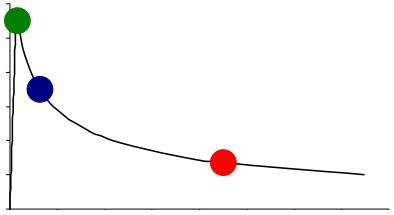
- Shear stresses

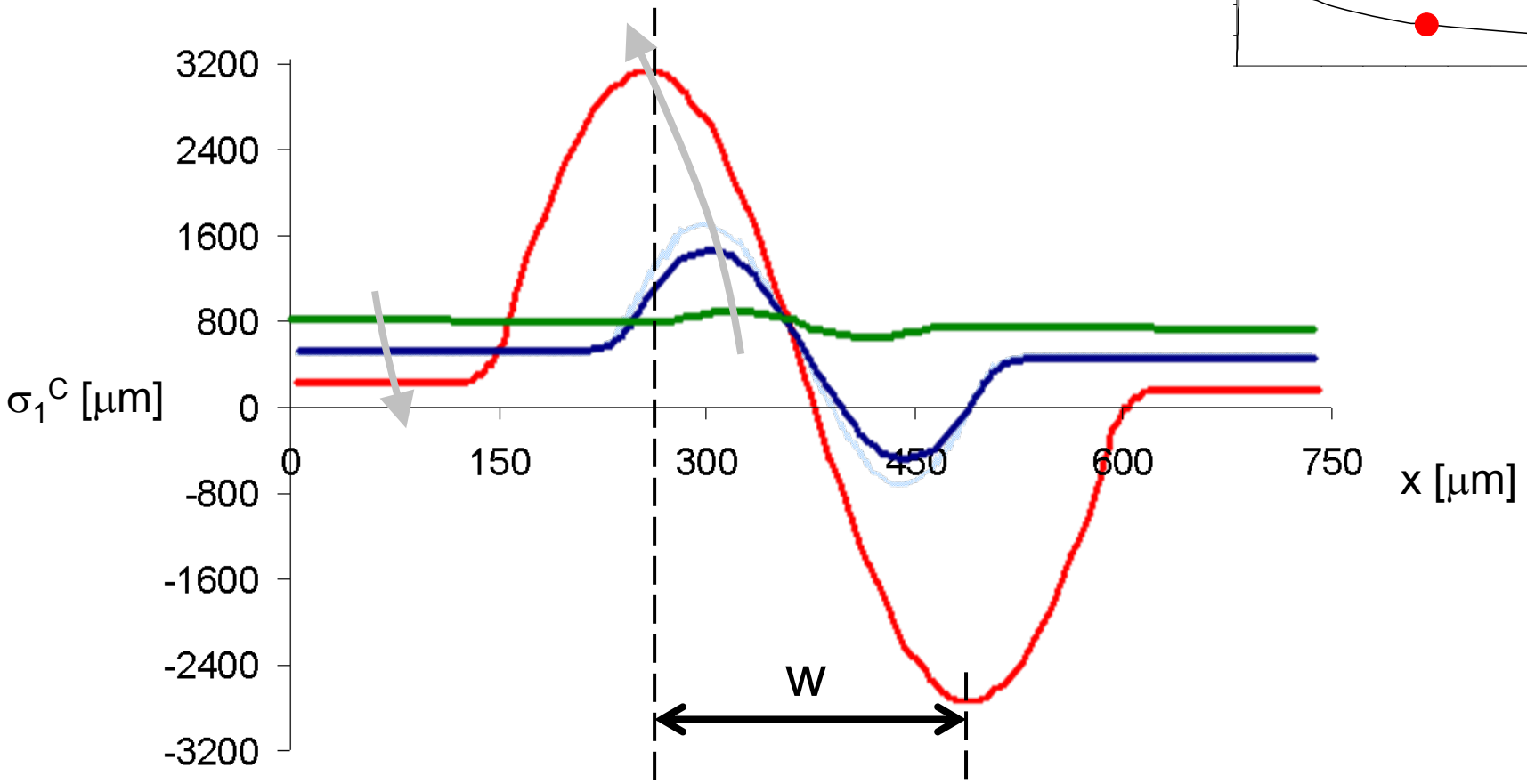


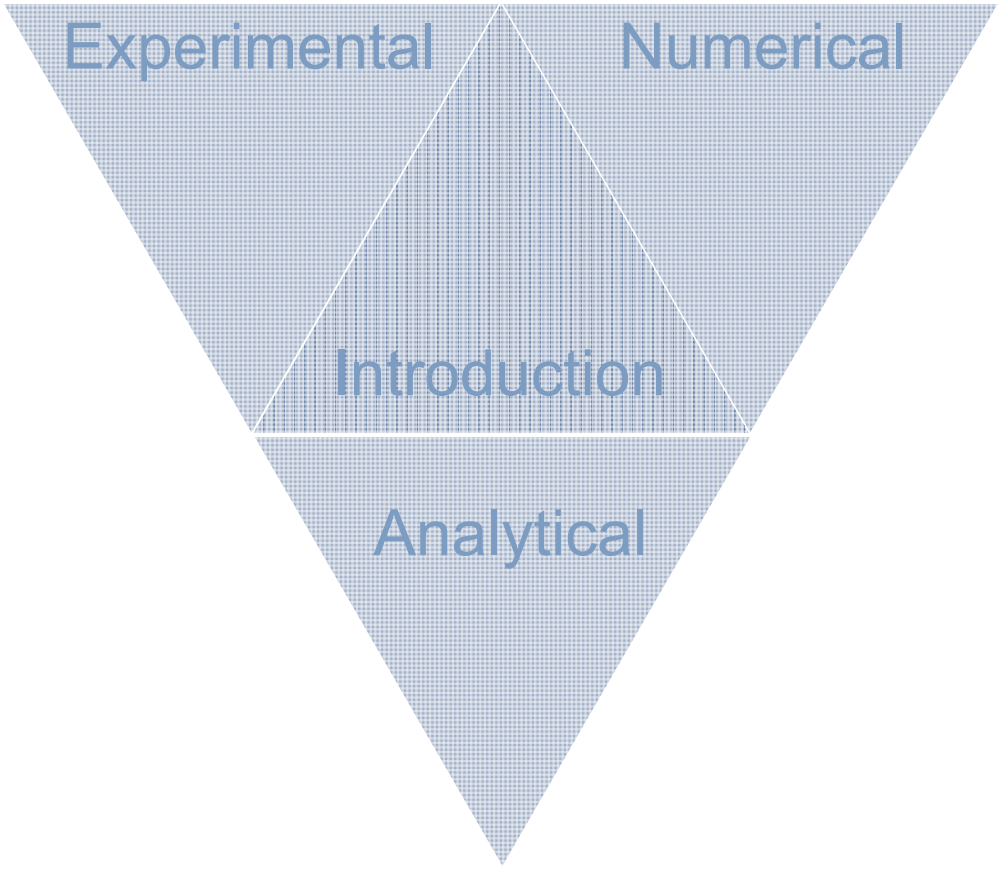


Results

- Axial stresses







Conclusions

Analytical model

- Limitations
  - Pure compression ➔ 3D
  - Not entirely closed form ➔ simplified expressions ?
  - Continuity along the matrix ➔ ✖ splitting

Analytical model

- Achievements
  - ✓ Outputs:
    - ✓ P^{peak} in closed form
    - ✓ **w** & *fibre rotation* @ fibre failure
    - ✓ **σ & τ & v fields**
  - ✓ Suitable for layered media & hydrostatic pressure
  - ✓ No in-phase constrains
  - ✓ No final shape imposed
  - ✓ **Agreement with FE & experiments**

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for research, modelling, testing and training in advanced composites

Composites2009

2nd **ECCOMAS** Thematic Conference on the

Mechanical Response of Composites

1-3 April 2009
Imperial College London, UK

Imperial College London

ECCOMAS Thematic Conference

- Looking forward to seeing you in London in April 2009

2nd Call for Papers

ICCM17 Edinburgh

17th International Conference on Composite Materials

27-31 July 2009
Edinburgh, UK

- Session: Failure Criteria for Design