Modeling Off-Axis Notch Sensitivity of Fiber Metal Laminates

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Outline

1. Background

- Notched strengths of composites
- Objectives

2. Experiments

- Glare-3
- Off-axis tension tests on notched specimens

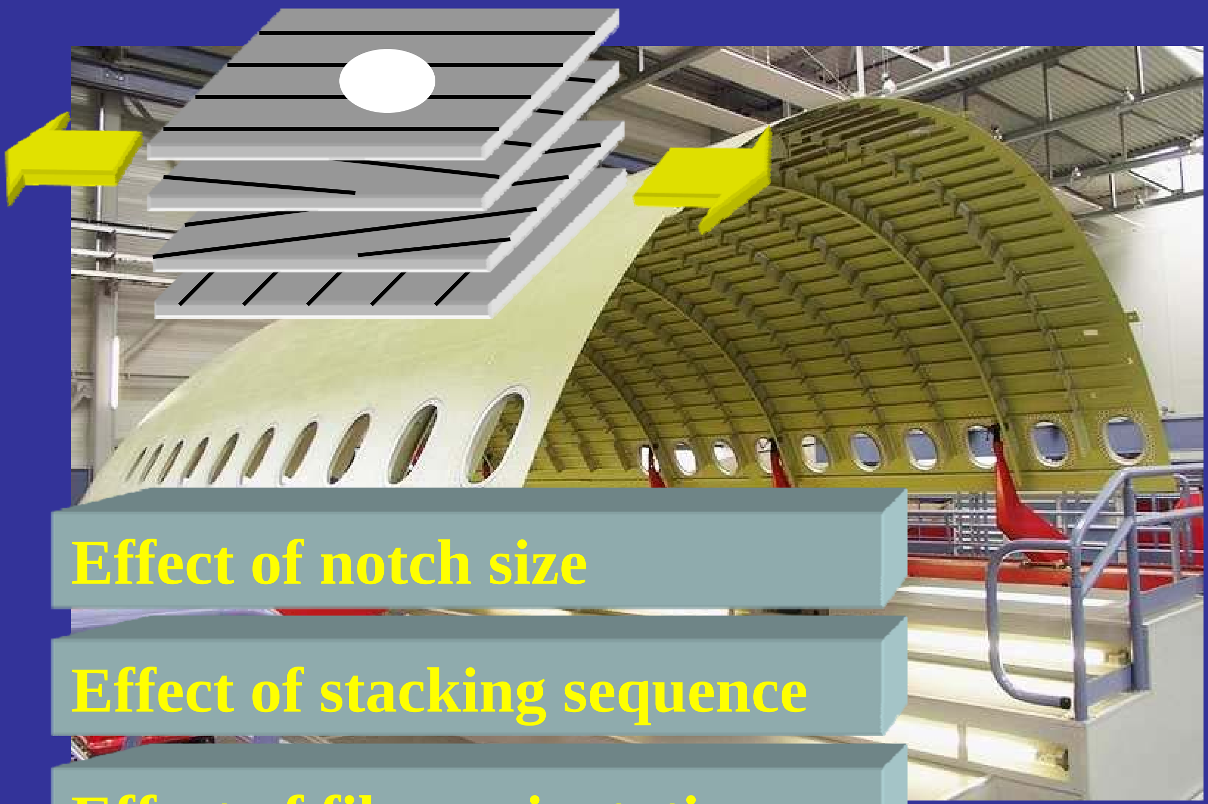
3. Analytical Modeling & Verification

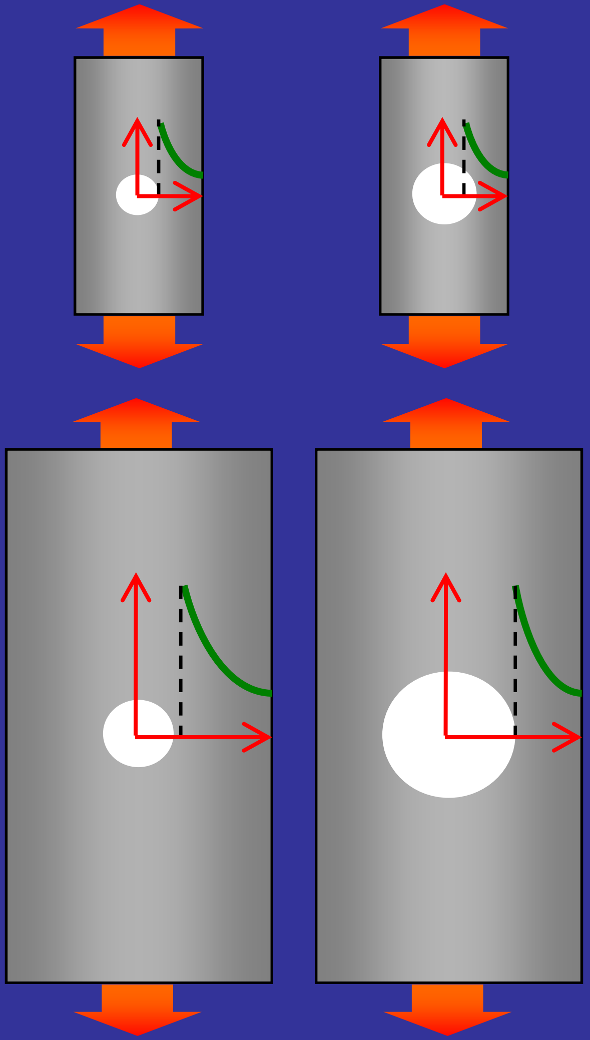
- Multiaxial notch sensitive insensitive failure criteri
- A formula for off-axis notched strength
- Comparison with experimental results

4. Conclusions

Notch Sensitivity of Composites

It is one of the most important engineering issues to establish a reliable method for predicting the notched strengths of composites.



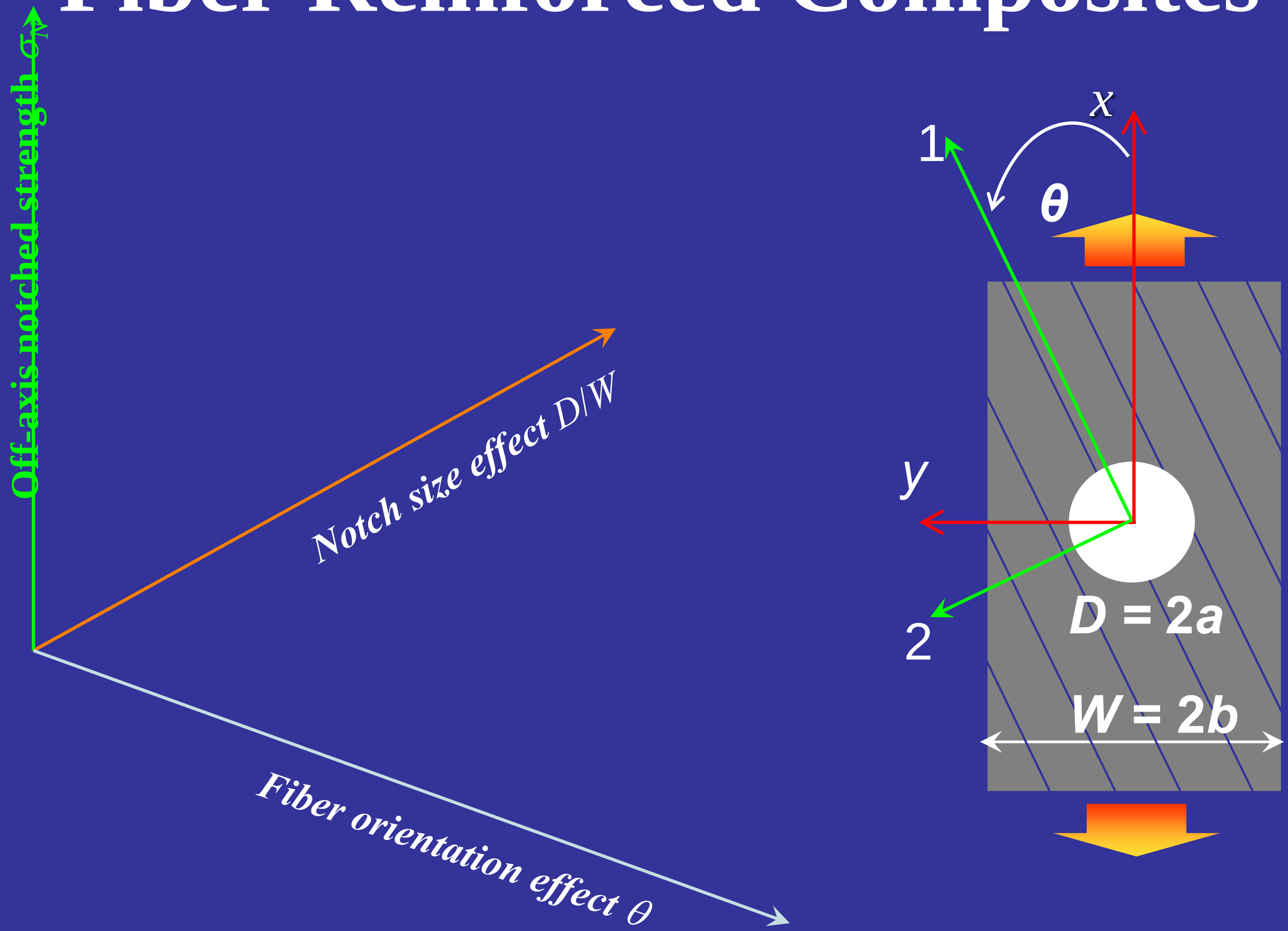


Effect of notch size

Effect of stacking sequence

Effect of fiber orientation

Off-Axis Notched Strengths of Fiber Reinforced Composites



Objectives

Modeling of the off-axis notched strengths of fiber metal laminate GLARE-3

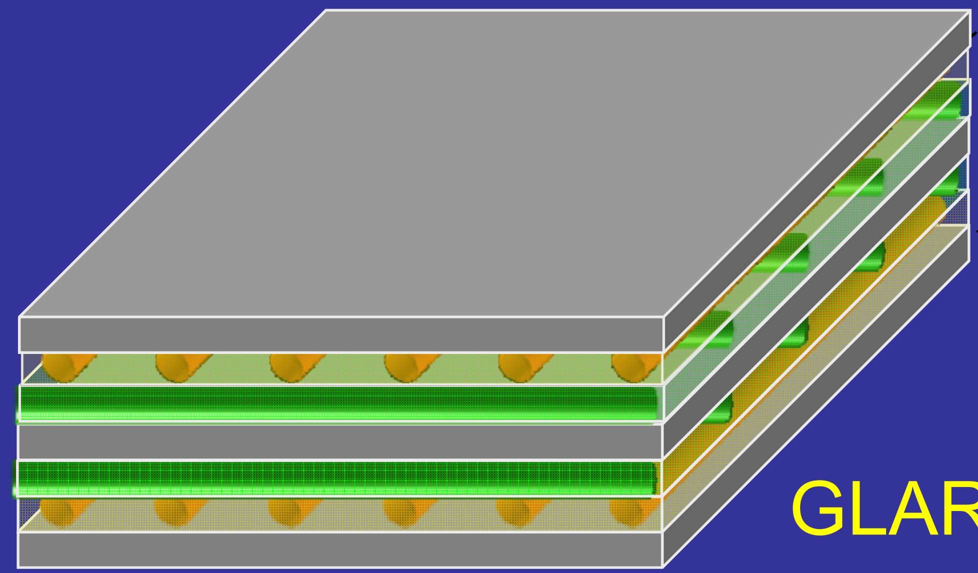
EXPERIMENTAL:

1. Notch size effect
2. Fiber orientation dependence
3. Off-axis notch sensitivity

THEORETICAL:

1. Formulation of a multiaxial failure criterion
    - A criterion for ductile failure
    - A criterion for brittle failure
    - A criterion for transitional ductile-brittle failure
2. A formula for off-axis notched strength prediction

FML (Fiber Metal Laminates)

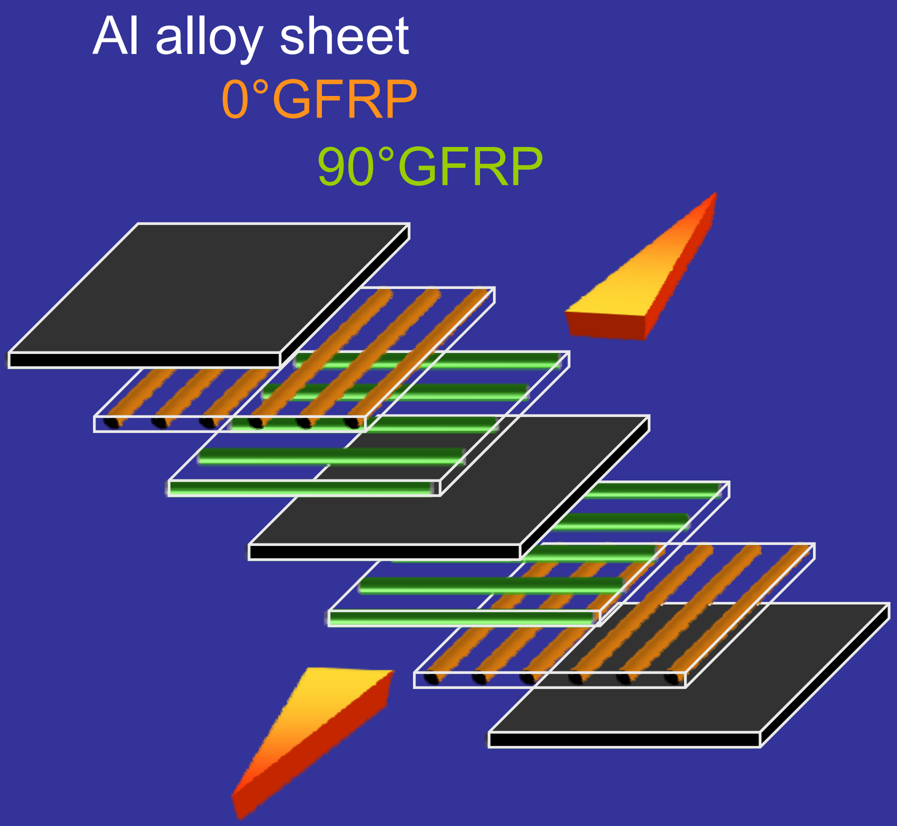


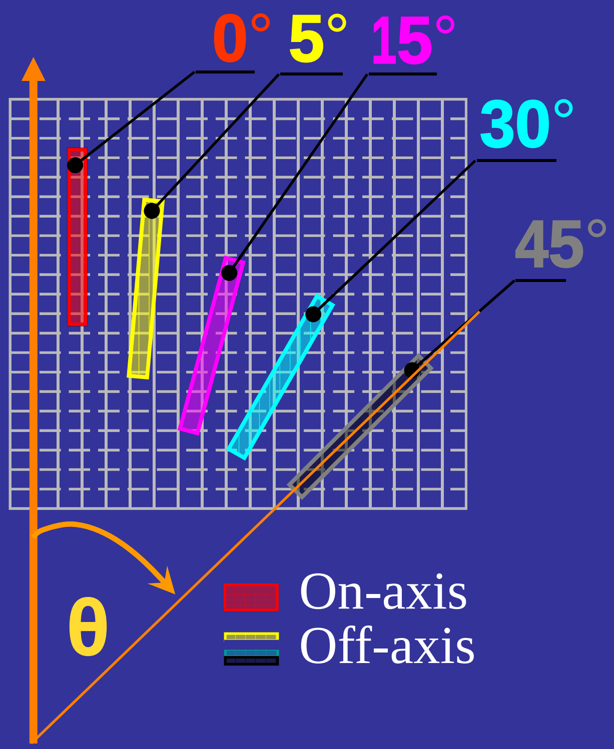
- **GLARE**

Al alloy + GFRP (Glass Fiber Reinforced Plastic)

Material

► GLARE-3

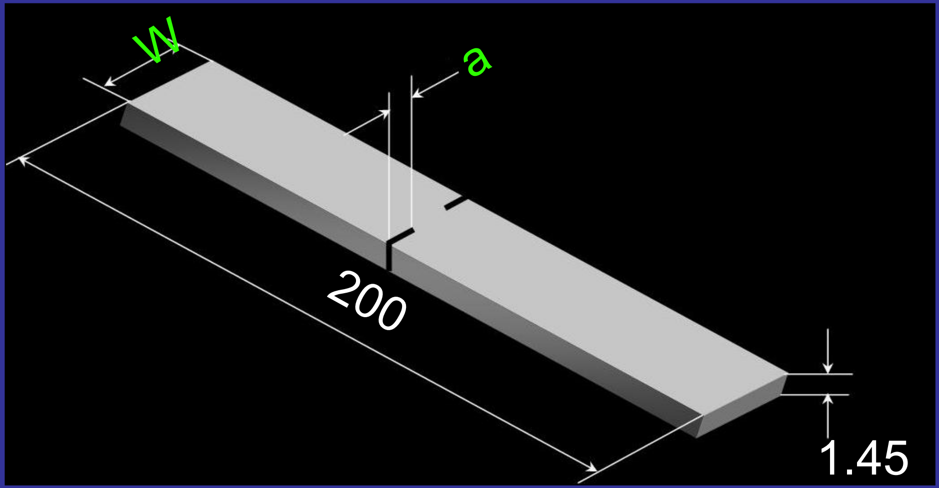




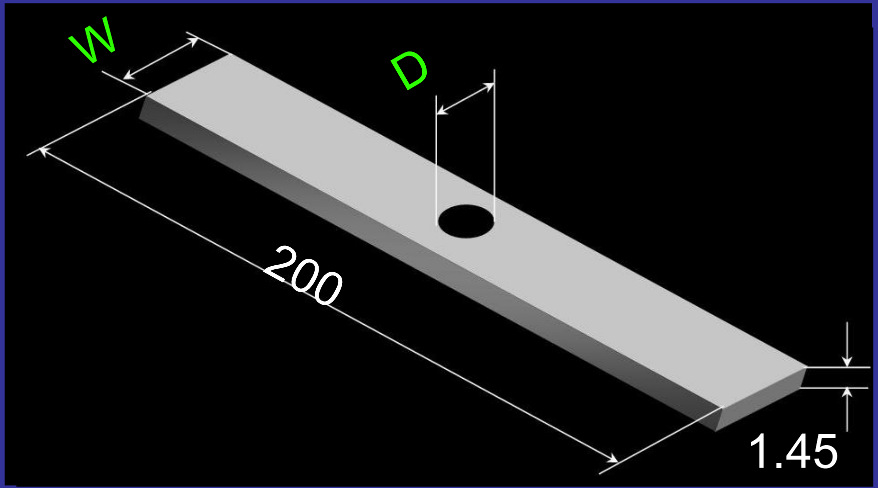
Notched Specimens

Off-axis angle	θ	:	0, 5,15, 30, 45, 90°
Width	W	:	10, 20, 30 mm
Normalized width	D/W	:	0.0, 0.1, 0.2, 0.4
	$2a/W$	:	

Double edge notches



Center circular hole



Test Procedure

JIS K7073

- Temperature
  RT
- Rate
  1.0 mm/min
- Strain Measurement
  Strain Gauge
  DVE
  Disp
  Extensometer

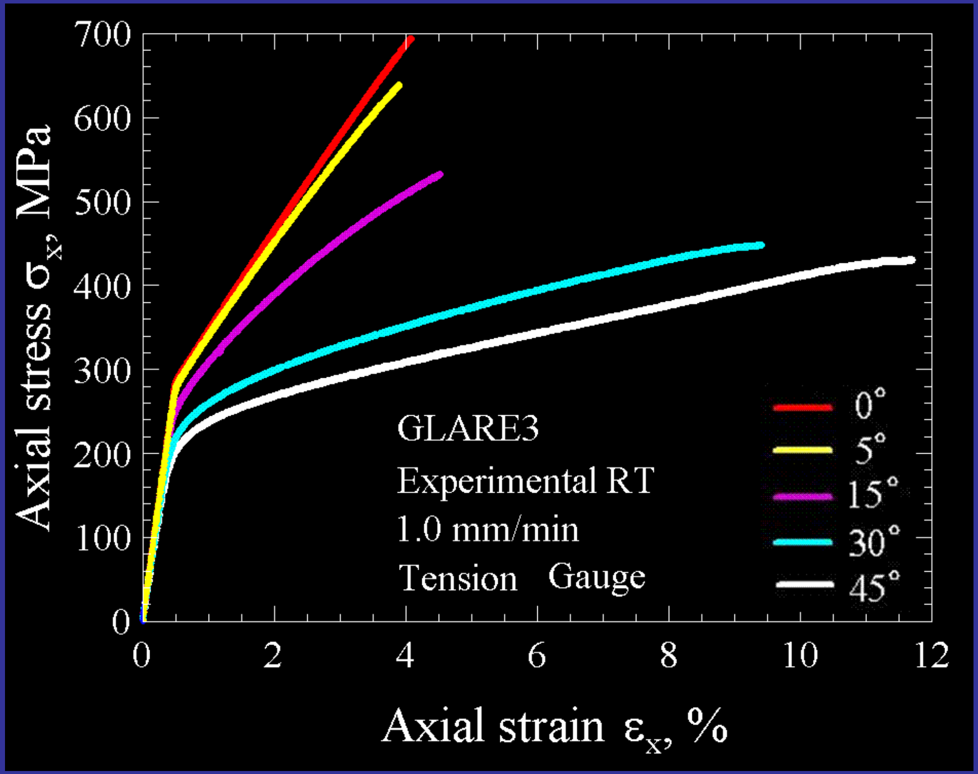
MTS Test Star 810

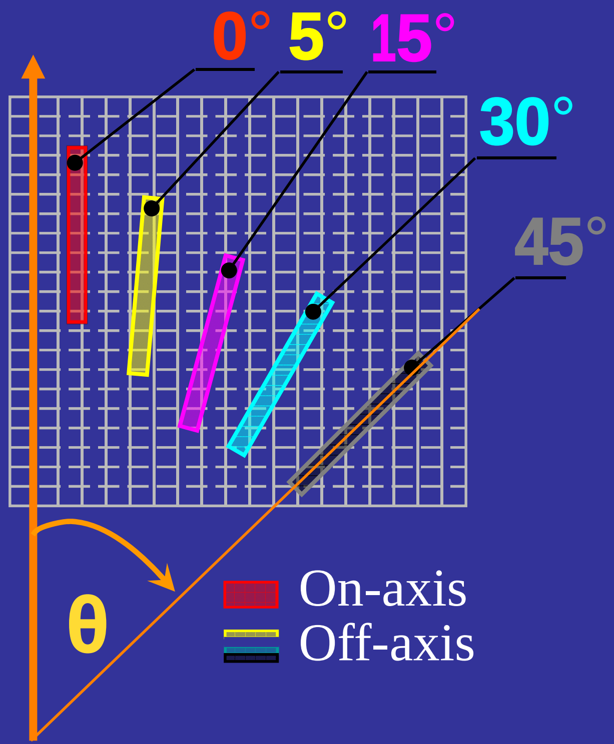
Extensometer

Experimental Results

Off-axis Stress-Strain Curves

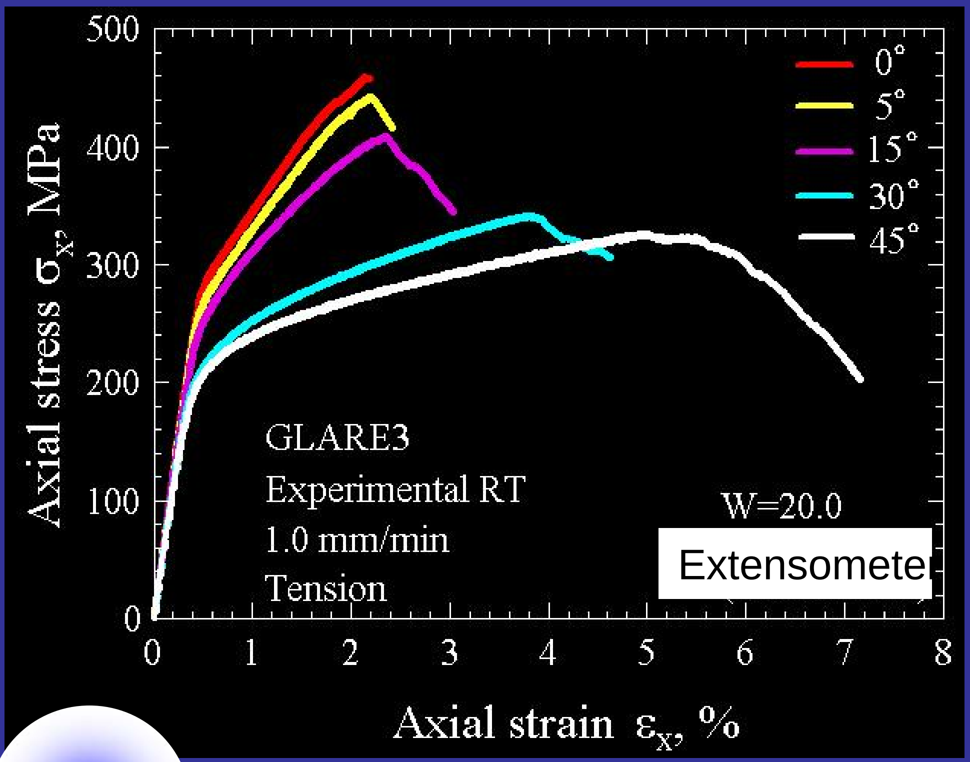
Unnotched behavior



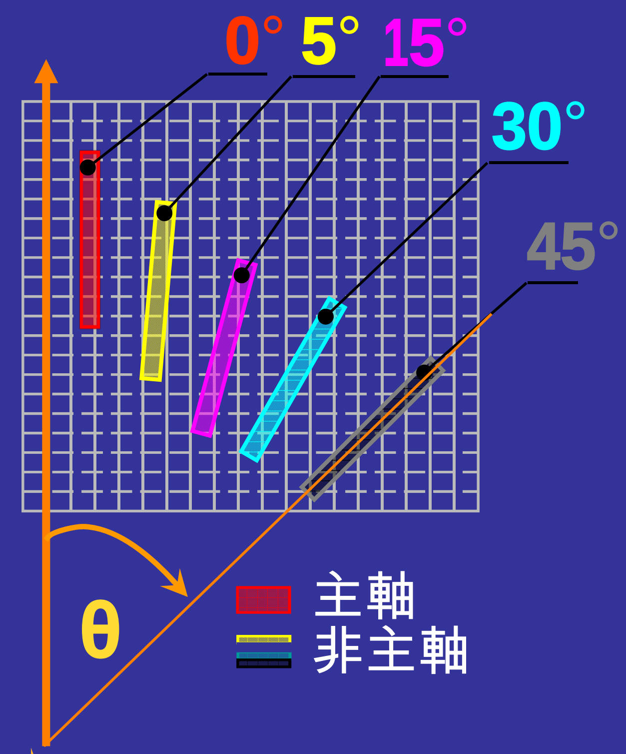


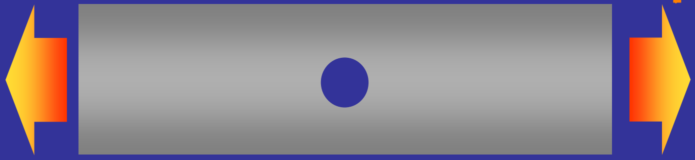
Off-Axis Stress-Strain Curves

Notched behavior



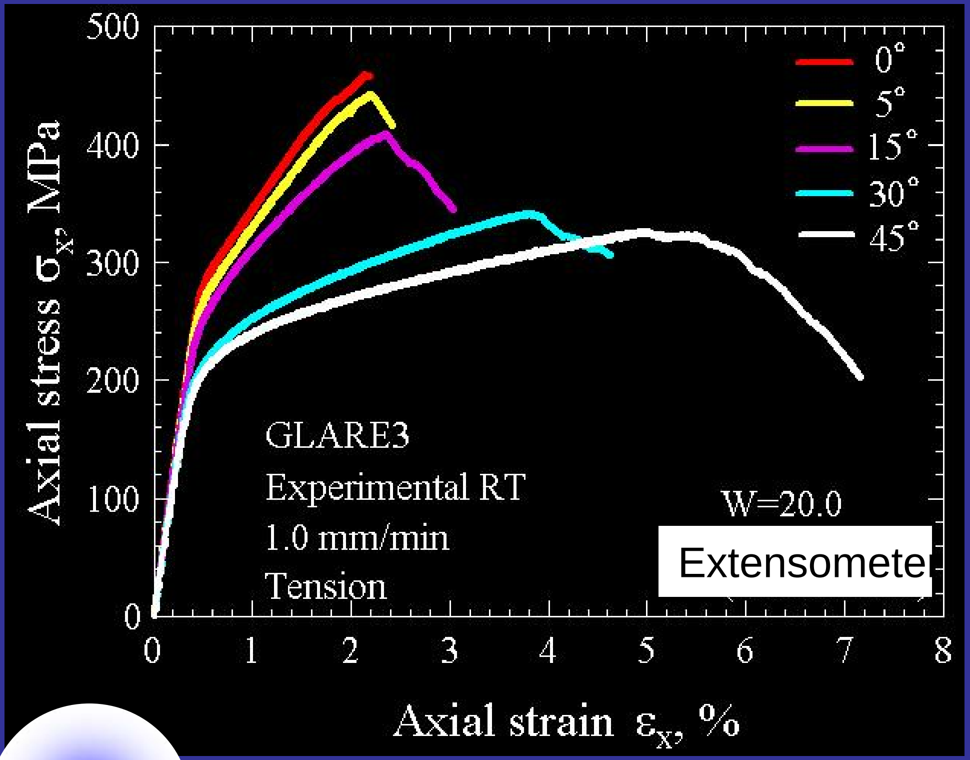
D = 2.0



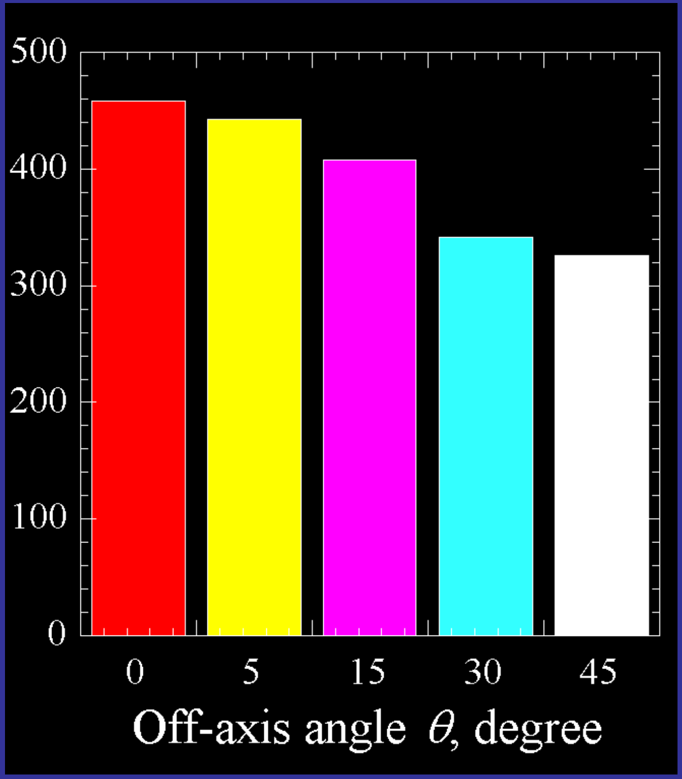


Off-Axis Stress-Strain Curves

Notched behavior



Notched strength



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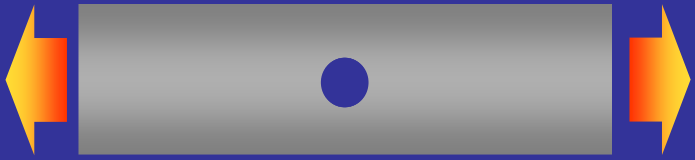

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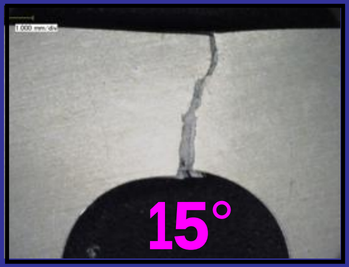
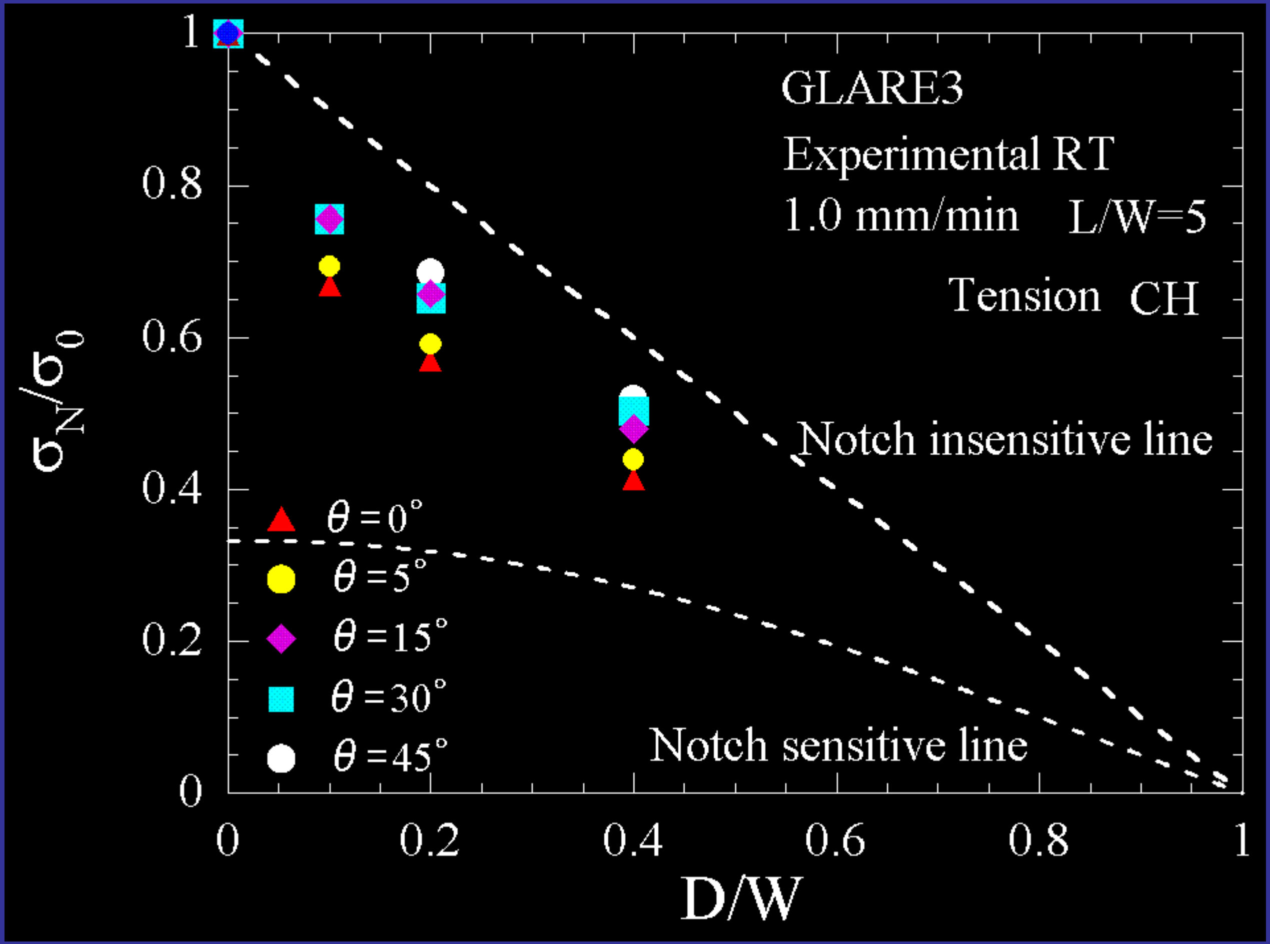
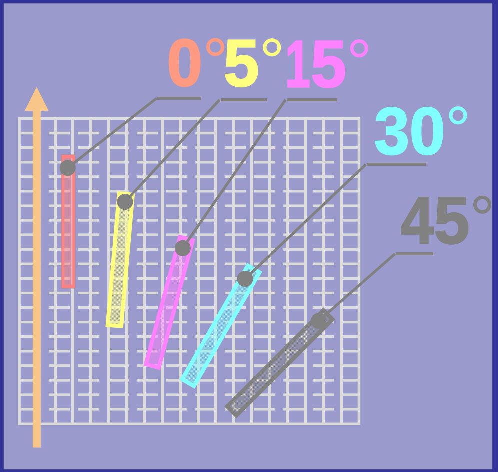
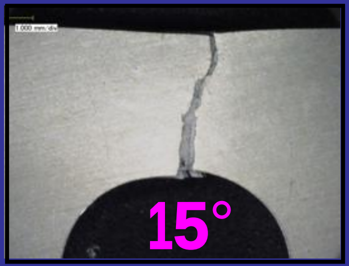
Off-Axis Notch Sensitivity Curve

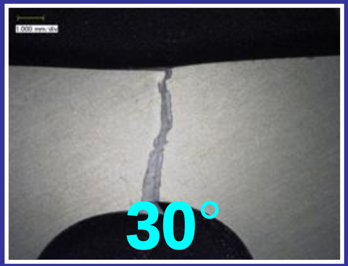


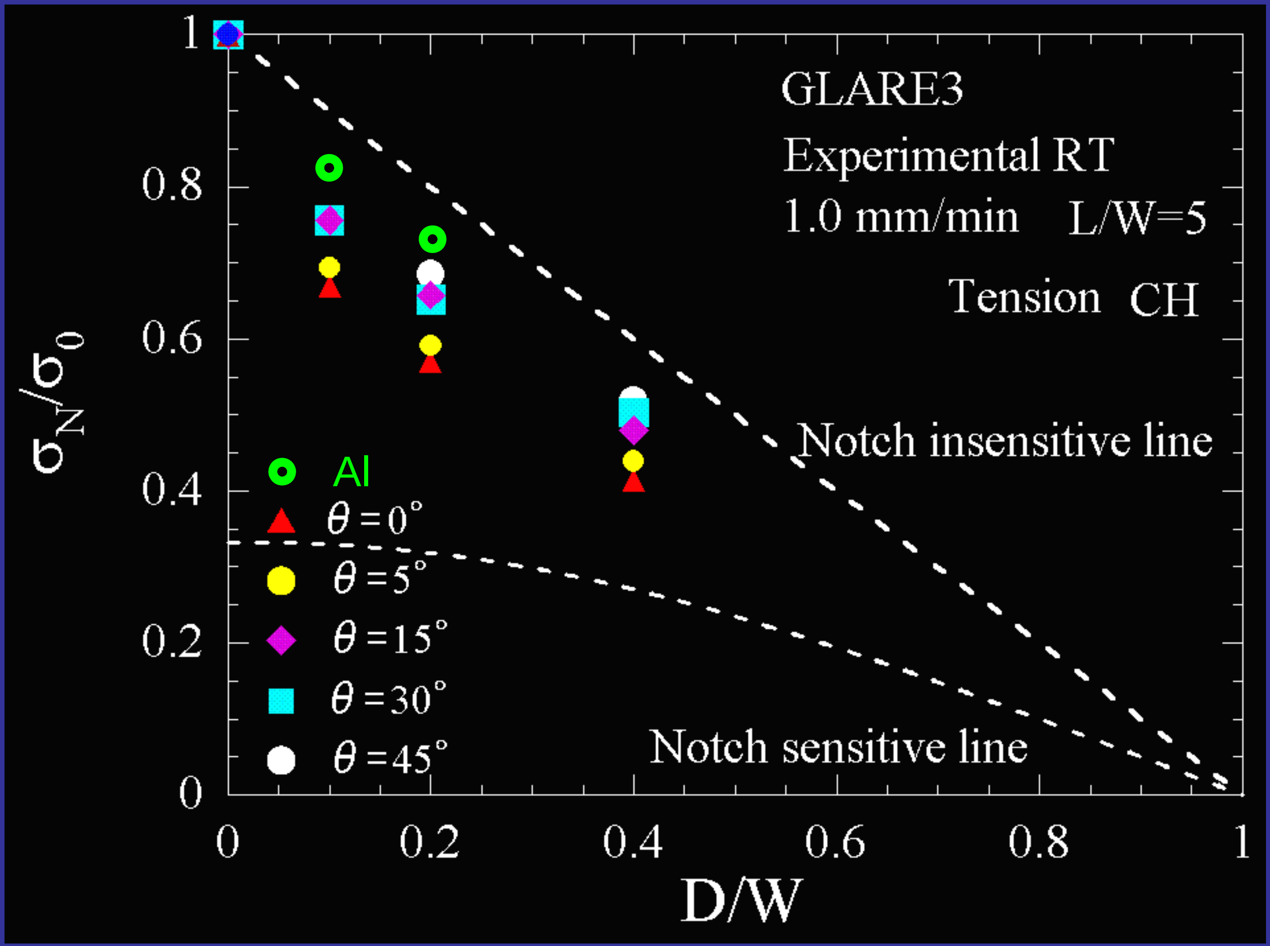


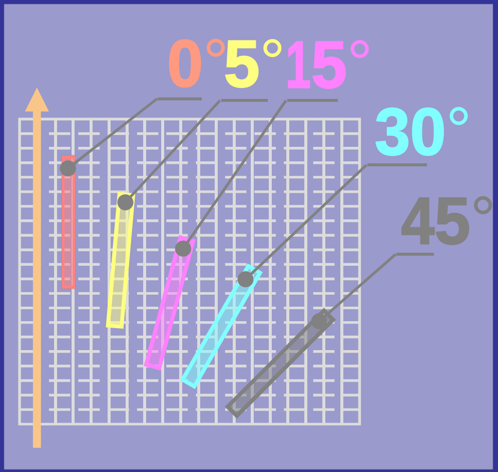


Off-Axis Notch Sensitivity Curve









Application of existing fracture criteria

- **Semi-Empirical Criteria:**
  - Point-stress (PS) criterion
  - Average stress (AS) criterion
  - Modified PS criterion
- **Fracture Mechanics Criteria:**
  - Inherent flaw criterion
  - Cohesive zone criterion
  - R-curve based criterion

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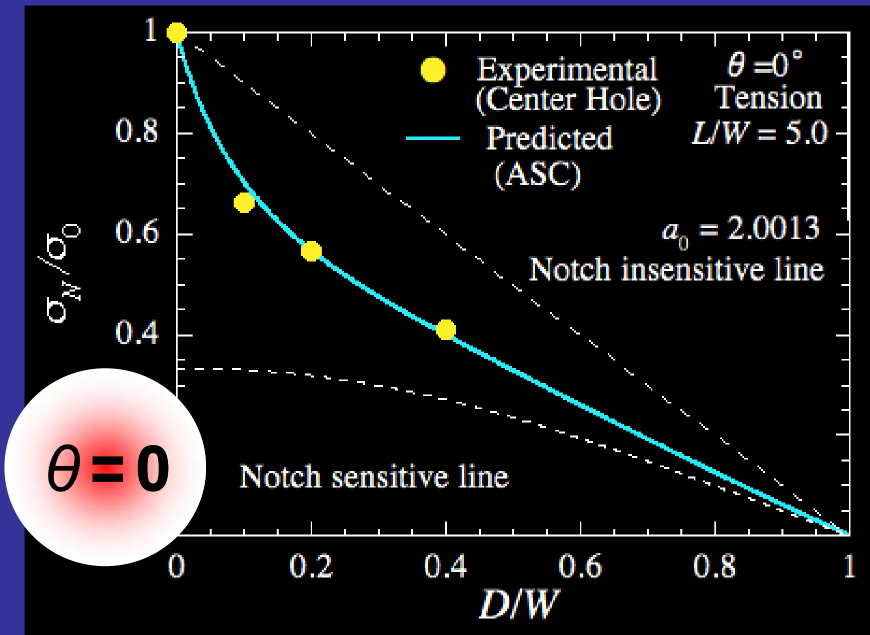
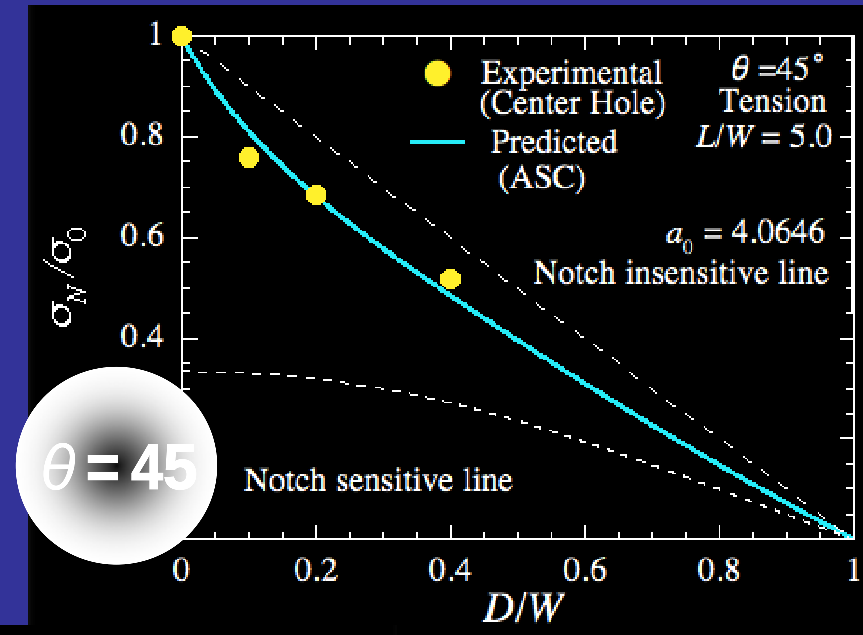
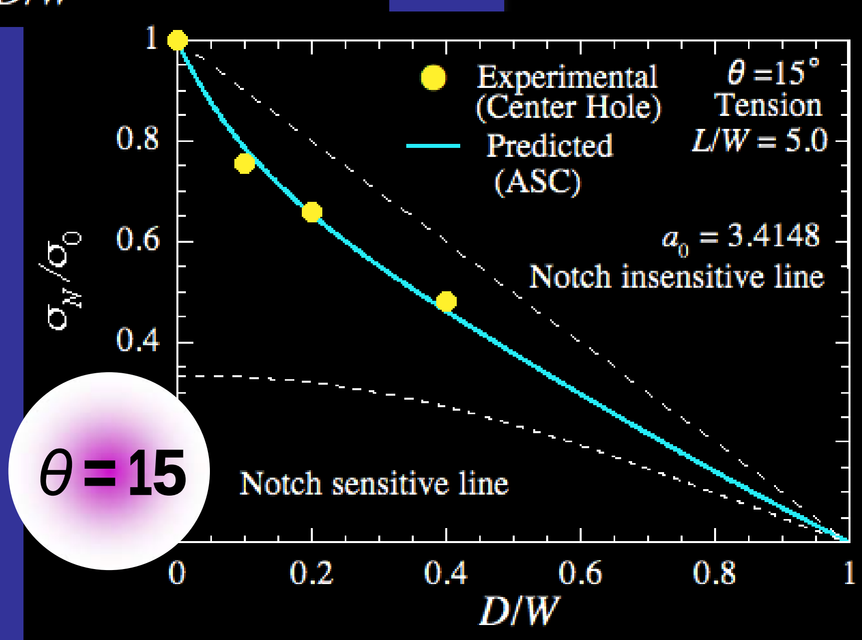
Average Stress (AS) Criterion







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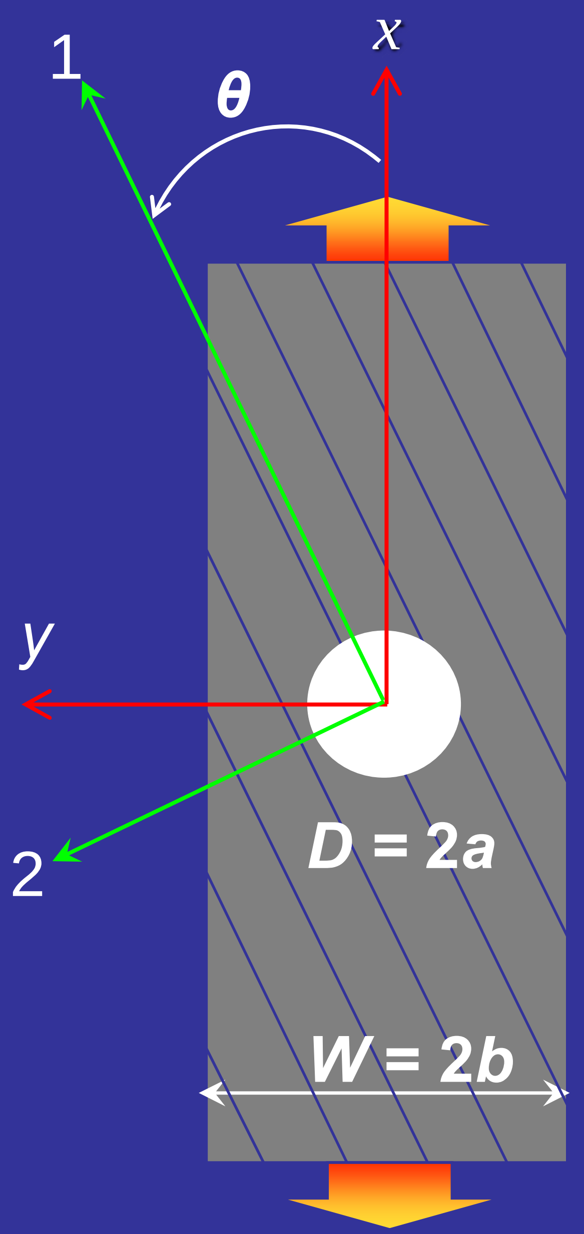
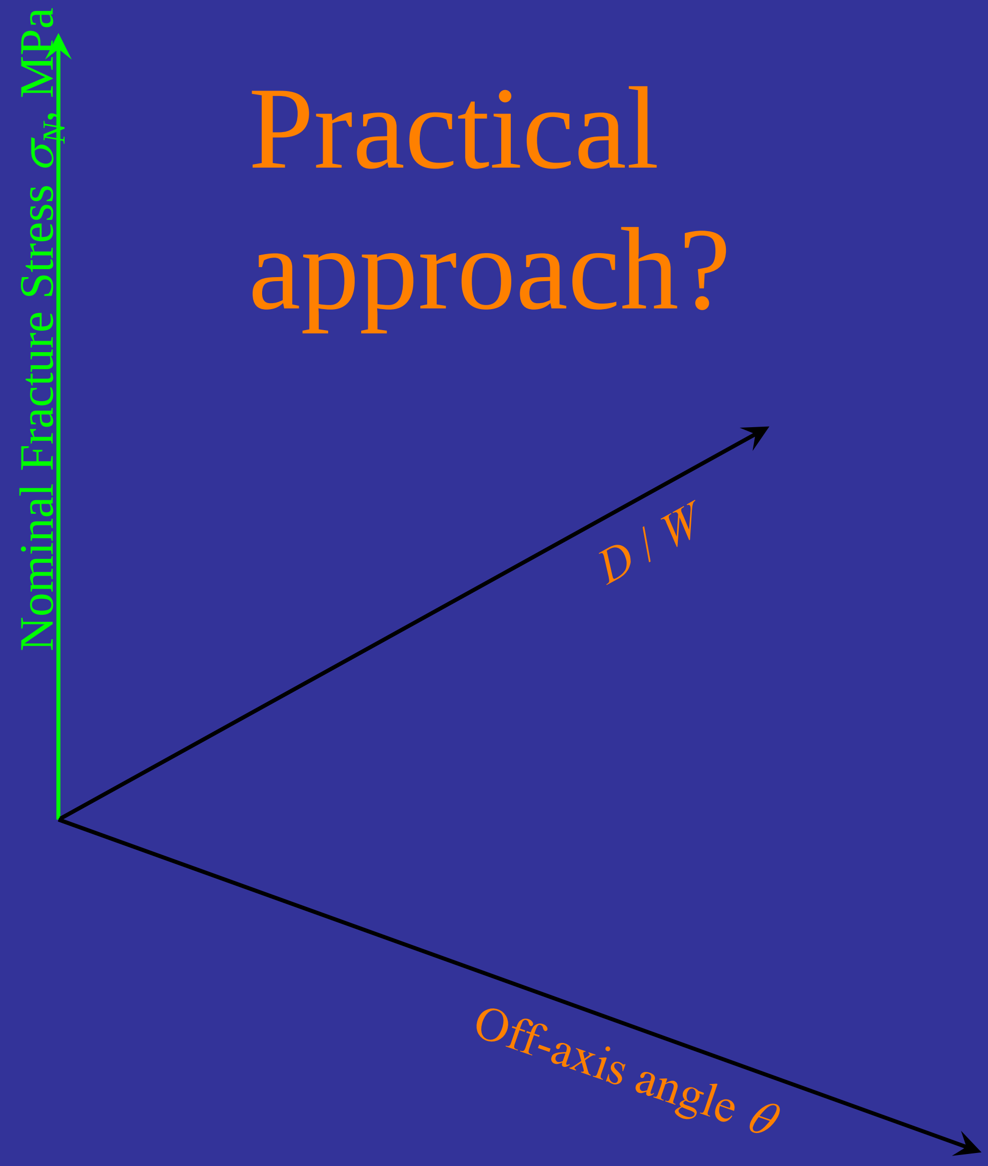
Off-Axis Notch Size Effect





Multiaxial Notch Sensitivity Modeling

- A ductile failure criterion
- A brittle failure criterion
- A brittle-ductile failure criterion
- A formula for off-axis notched strength

A Ductile Failure Criterion

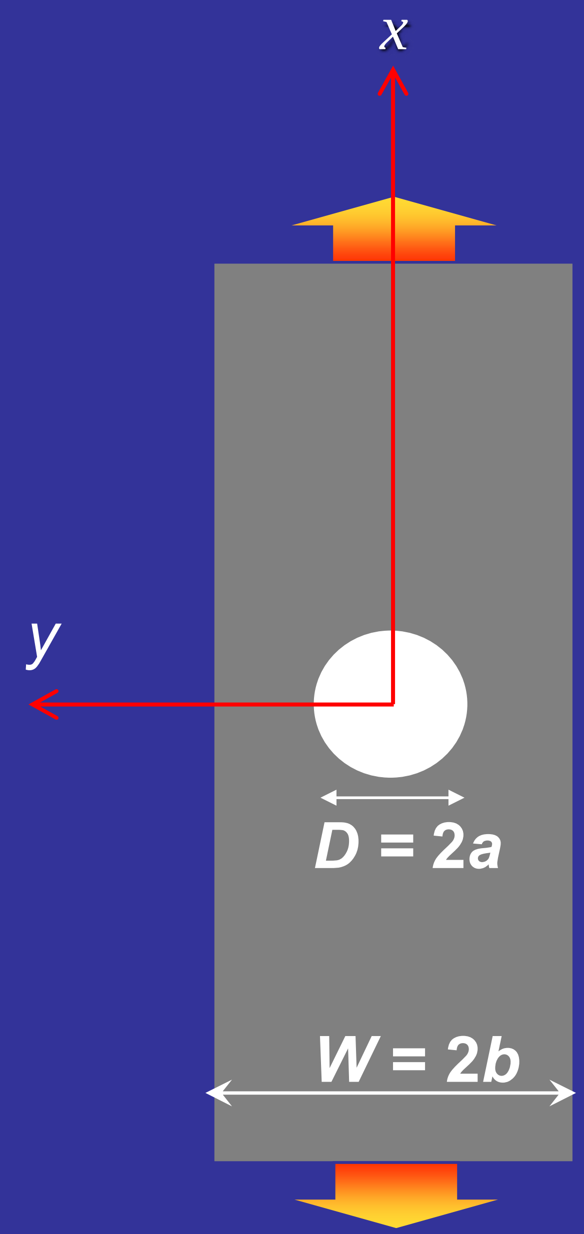
The Net Section Stress Criterion:

$$\frac{\sigma_N}{\sigma_0} = 1 - \frac{a}{b} = 1 - \alpha$$

σ_N : Remote stress at failure (notched strength)

σ_0 : Material strength (unnotched strength)

α : Diameter to width ratio (= $D/W = a/b$)



A Multiaxial Ductile Failure Criterion

$$\frac{\sigma_N}{\sigma_0} = 1 - \frac{a}{b} = 1 - \alpha$$

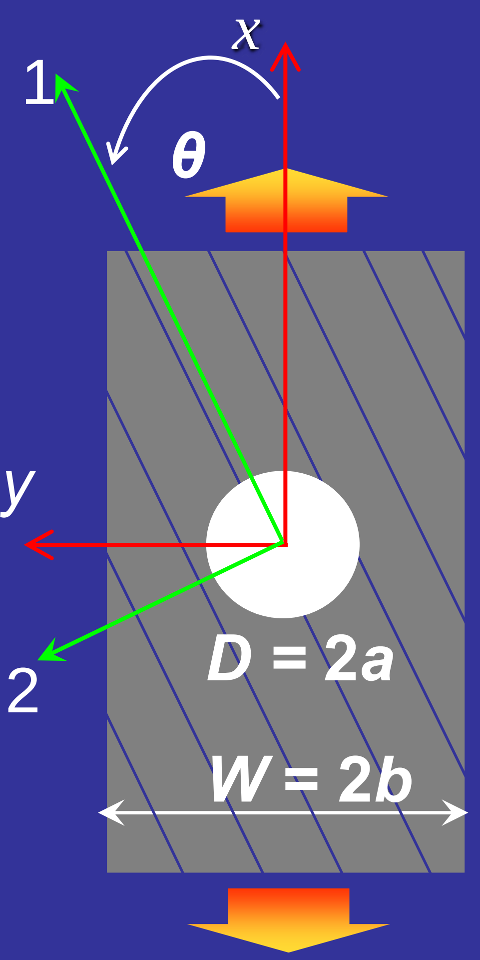
The Net Section Stress Criterion:

$$\frac{\sigma_{net}}{\sigma_0} = \frac{1}{1-\alpha}\frac{\sigma_x}{\sigma_0} = 1$$

$$f_{NI} = \left(\frac{1}{1-\alpha}\sigma^*\right)^2 = 1$$

where

$$\sigma^* = \sqrt{\left(\frac{\sigma_{11}}{X}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{X^2} + \left(\frac{\sigma_{22}}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2}$$



Non-Dimensional Effective Stress

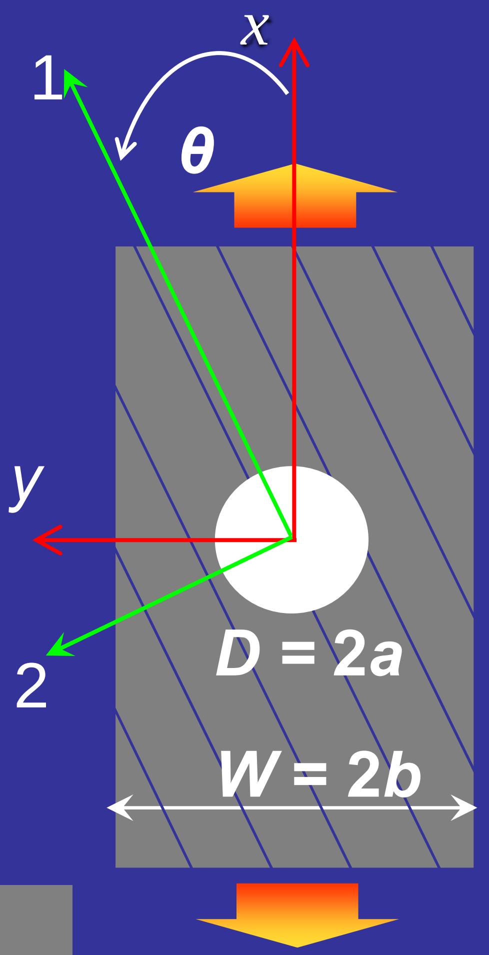
Tsai-Hill Static Failure Criterion:

$$\left(\frac{\sigma_{11}}{X}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{X^2} + \left(\frac{\sigma_{22}}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 = 1$$

X: Longitudinal strength

Y: Transverse strength

S: Shear strength



Non-Dimensional Effective Stress:

$$\sigma^* \equiv \sqrt{\left(\frac{\sigma_{11}}{X}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{X^2} + \left(\frac{\sigma_{22}}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2}$$

Theoretical Stress Ratio

Off-Axis Loading on UD Composites

Non-Dimensional Effective Stress

$$\sigma^* = \Omega(\theta)\sigma_x \quad \Rightarrow \quad \text{Static Failure Condition: } \sigma^* = \Omega(\theta)\sigma_x = 1$$

$$\therefore \quad \sigma_0 = \frac{1}{\Omega(\theta)}$$

where $$\Omega(\theta) = \frac{1}{\sqrt{\left(\frac{\cos^2\theta}{X}\right)^2 - \frac{\cos^2\theta\sin^2\theta}{X^2} + \left(\frac{\sin^2\theta}{Y}\right)^2 + \left(\frac{-\cos\theta\sin\theta}{S}\right)^2}}$$

Theoretical stress ratio for off-axis loading:

$$\sigma^* = \Omega(\theta)\sigma_x = \frac{\sigma_x}{1/\Omega(\theta)} = \frac{\sigma_x}{\sigma_0}$$

A Brittle Failure Criterion

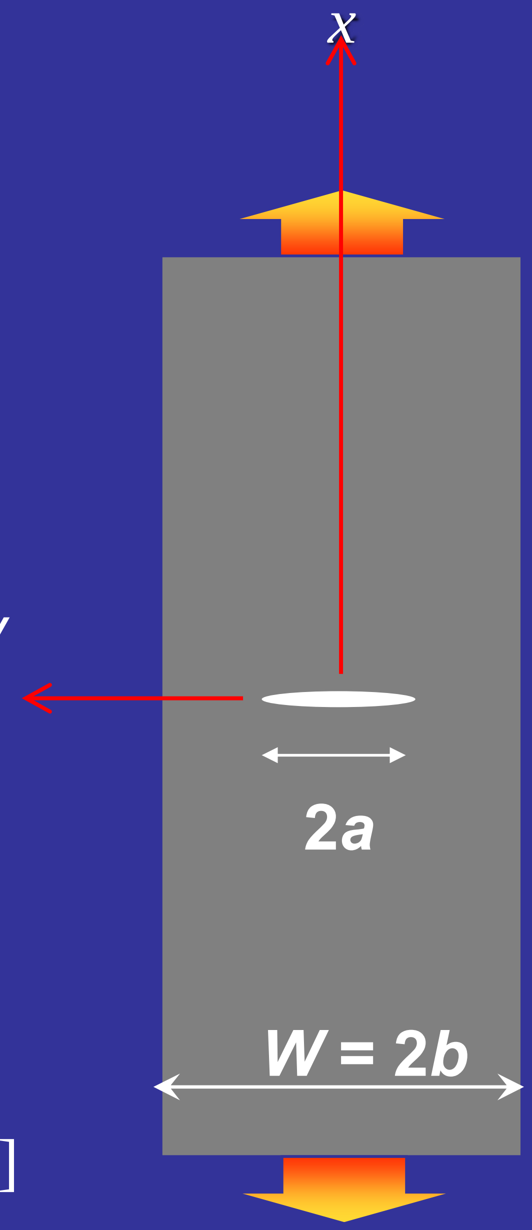
Fracture Mechanics Criterion:

$$\frac{\sigma_N}{\sigma_0} = \frac{K_{IC}}{F\sigma_0\sqrt{\pi a}}$$

σ_N : Remote stress at failure (notched strength)

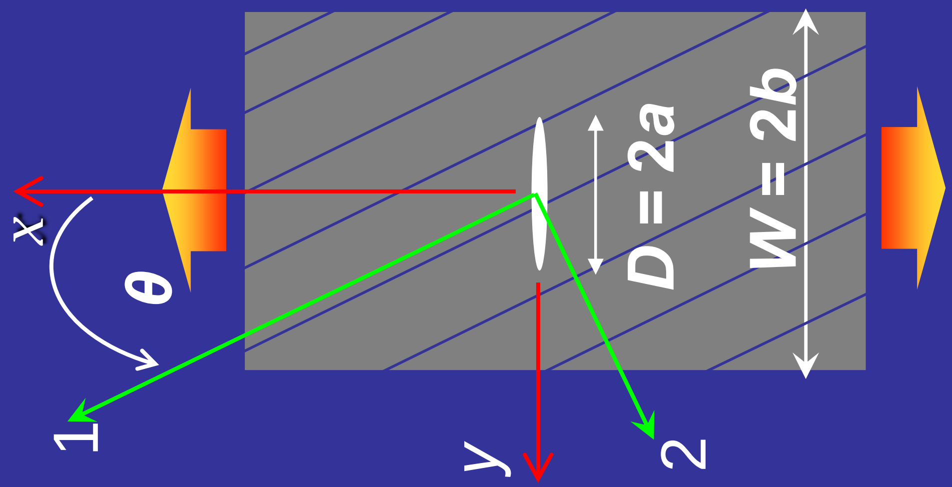
σ_0 : Material strength (unnotched strength)

F : Finite width correction factor [$F = F(2a/W) = F(a/b) = F(\alpha)$]



A Multiaxial Brittle Failure Criterion

$$\frac{\sigma_N}{\sigma_0} = \frac{K_{IC}}{\sigma_0 F\sqrt{\pi a}}$$



Fracture Mechanics Criterion:

$$\frac{K_I}{K_{IC}} = \frac{\sigma_x F\sqrt{\pi a}}{K_{IC}} = 1$$

$$f_{NS} = f_{NS}(k_{11}, k_{22}, k_{12}) = \left(\frac{k_{11}}{K_{IC}^{11}}\right)^2 - \frac{k_{11}k_{22}}{\left(K_{IC}^{11}\right)^2} + \left(\frac{k_{22}}{K_{IC}^{22}}\right)^2 + \left(\frac{k_{12}}{K_{IIC}^{12}}\right)^2 = 1$$

where

$$\begin{pmatrix} k_{11} \\ k_{22} \\ k_{12} \end{pmatrix} = F_I\sqrt{\pi a}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \chi \end{bmatrix}\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{pmatrix}$$

A Multiaxial Brittle Failure Criterion

Normalized principal fracture toughness

(Principal stress brittleness numbers)

$$\frac{K_{IC}^{11}}{X\sqrt{b}} \equiv \kappa_{11}$$

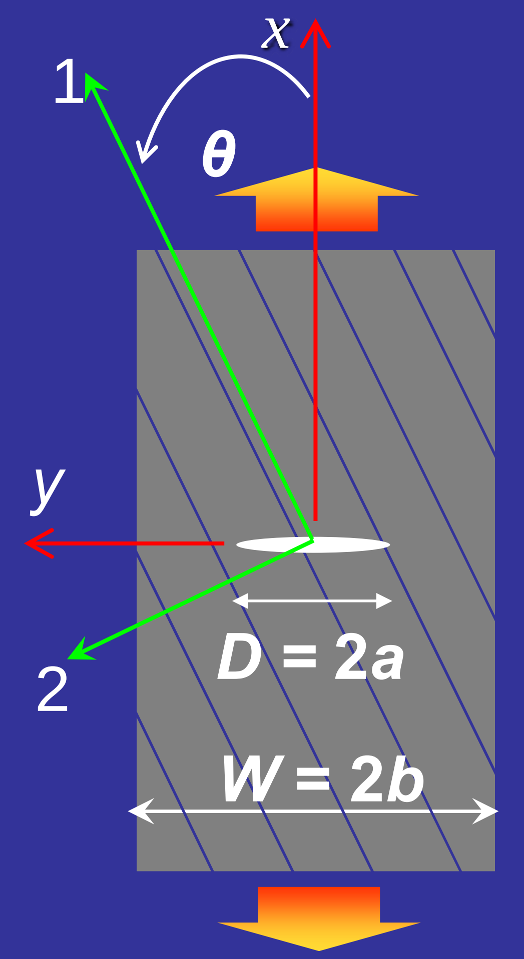
$$\frac{K_{IC}^{22}}{Y\sqrt{b}} \equiv \kappa_{22}$$

$$\frac{K_{IC}^{12}}{S\sqrt{b}} \equiv \kappa_{12}$$

extension

$$s = \frac{K_{IC}}{\sigma_u \sqrt{b}}$$

(Carpinteri)



$$f_{NS} = F_I^2 \pi \alpha \left[\left(\frac{\sigma_{11}}{X\kappa_{11}} \right)^2 - \frac{\sigma_{11}\sigma_{22}}{\left(X\kappa_{11}\right)^2} + \left(\frac{\sigma_{22}}{Y\kappa_{22}} \right)^2 + \chi^2 \left(\frac{\sigma_{12}}{S\kappa_{12}} \right)^2 \right]$$

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A Multiaxial Brittle-Ductile Failure Criterion

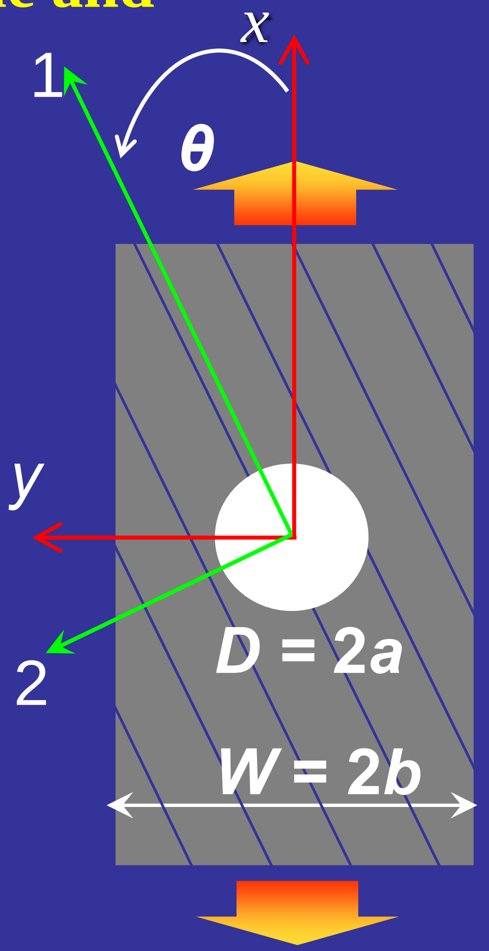
A multiaxial failure criterion for orthotropic solids with any notch sensitivity bounded by the ductile and brittle limits:

$$f = f_{NI} + f_{NS} = 1$$

where

$$f_{NI} = \left(\frac{\sigma^{*}}{1-\alpha} \right)^{2}$$

$$f_{NS} = F_{I}^{2} \pi \alpha \left[\left(\frac{\sigma_{11}}{X\kappa_{11}} \right)^{2} - \frac{\sigma_{11}\sigma_{22}}{(X\kappa_{11})^{2}} + \left(\frac{\sigma_{22}}{Y\kappa_{22}} \right)^{2} + \chi^{2} \left(\frac{\sigma_{12}}{S\kappa_{12}} \right)^{2} \right]$$



A Formula for Off-Axis Notched Strength

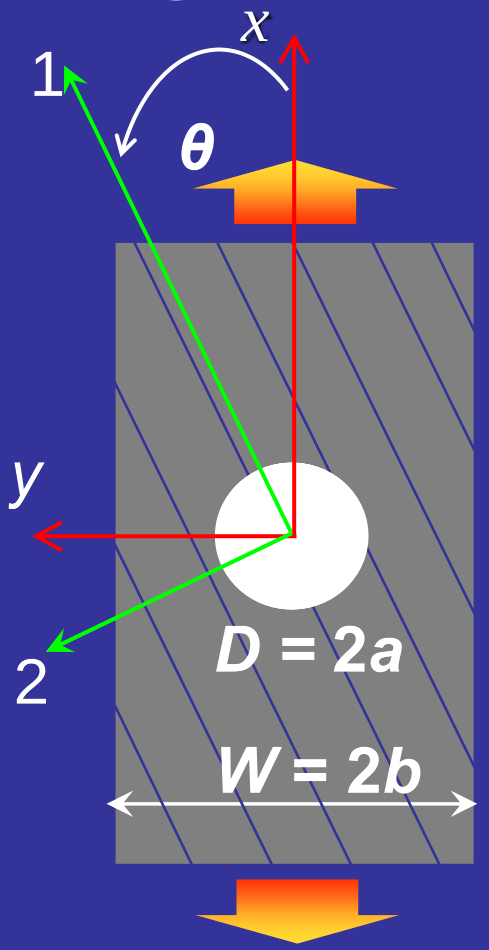
$$f = f_{NI} + f_{NS} = 1$$

$$\frac{\sigma_N}{\sigma_0} = \frac{1}{\sqrt{\left(\frac{1}{1-\alpha}\right)^2 + \left(\frac{\Omega_{NS}}{\Omega_0}\right)^2}}$$

where

$$\Omega_0 = \sqrt{\frac{m^4}{X^2} - \frac{m^2 n^2}{X^2} + \frac{n^4}{Y^2} + \frac{m^2 n^2}{S^2}} = \frac{1}{\sigma_0}$$

$$\Omega_{NS} = \sqrt{F_I^2 \pi \alpha \left\{ \frac{m^4 - m^2 n^2}{(X\kappa_{11})^2} + \frac{n^4}{(Y\kappa_{22})^2} + \chi^2 \frac{m^2 n^2}{(S\kappa_{12})^2} \right\}}$$



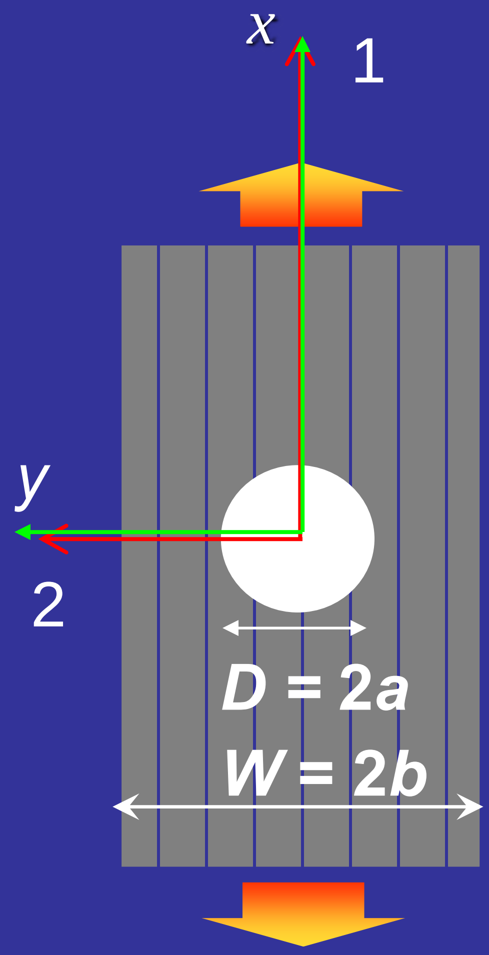
$$\begin{cases} m = \cos\theta \\ n = \sin\theta \end{cases}$$

Principal Notched Strengths

$$\frac{X_N}{X} = \frac{1}{\sqrt{\frac{1}{(1-\alpha)^2} + \frac{F_I{}^2 \pi \alpha}{\kappa_{11}{}^2}}}$$

$$\frac{Y_N}{Y} = \frac{1}{\sqrt{\frac{1}{(1-\alpha)^2} + \frac{F_I{}^2 \pi \alpha}{\kappa_{22}{}^2}}}$$

$$\frac{S_N}{S} = \frac{1}{\sqrt{\frac{1}{(1-\alpha)^2} + \chi^2 \frac{F_I{}^2 \pi \alpha}{\kappa_{12}{}^2}}}$$



(a generalization of the Suo-Ho-Gong model, 1993)

Verification

Application to GLARE-3

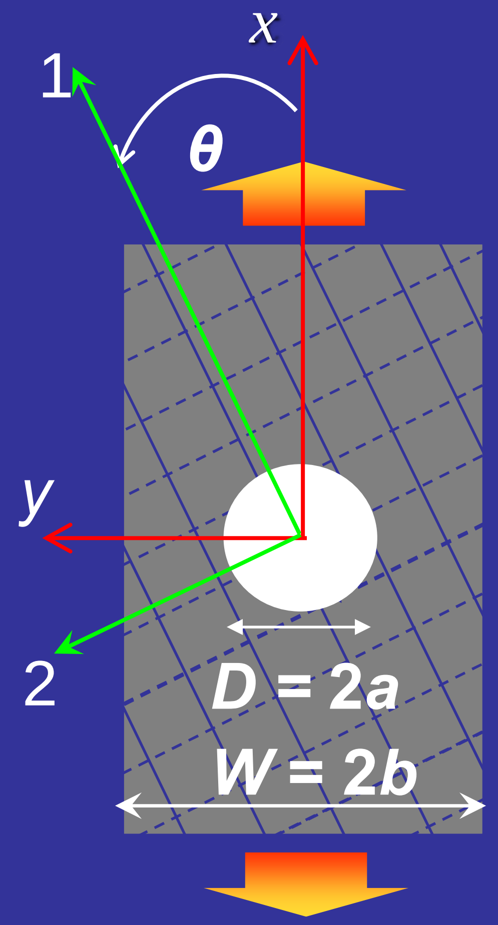
$$\frac{X_N}{X} = \frac{1}{\sqrt{\dfrac{1}{(1-\alpha)^2} + \dfrac{F_I^{\,2}\pi\alpha}{\kappa_I^{\,2}}}} = \frac{Y_N}{Y}$$

$$\frac{S_N}{S} = \frac{1}{\sqrt{\dfrac{1}{(1-\alpha)^2} + \chi^2\dfrac{F_I^{\,2}\pi\alpha}{\kappa_{II}^{\,2}}}}$$

where

$$F_I = \left(1 - 0.025\alpha^2 + 0.06\alpha^4\right)\sqrt{\sec\frac{\pi\alpha}{2}}$$

$\chi \approx 0.6$ (Sih and Chen, 1973)



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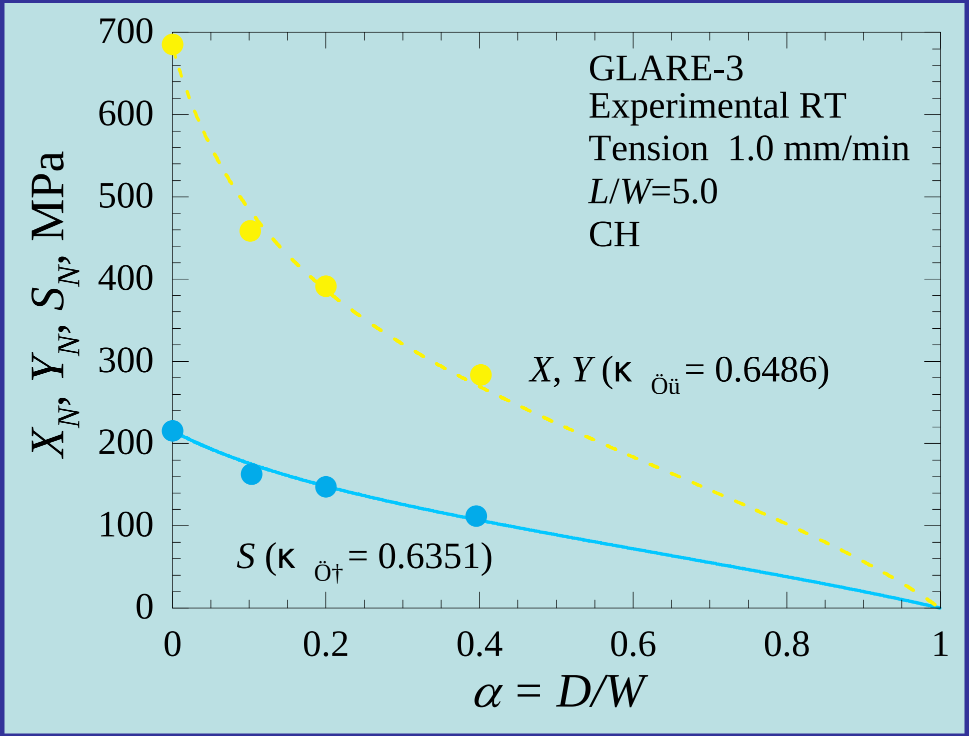
Principal Notch Sensitivity in GLARE-3

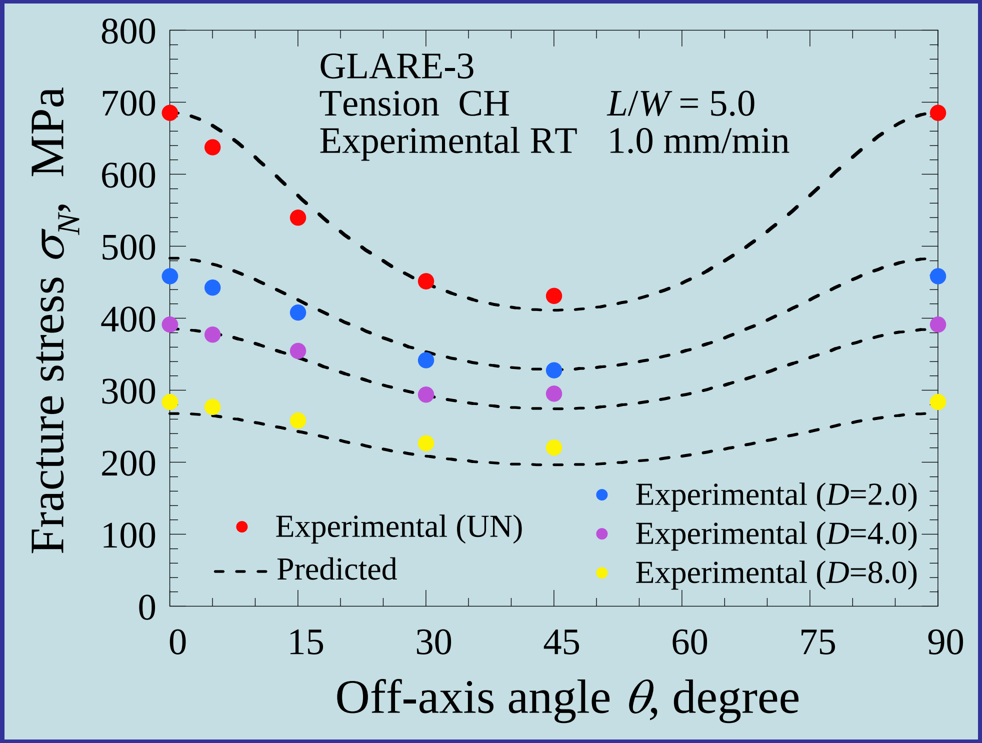


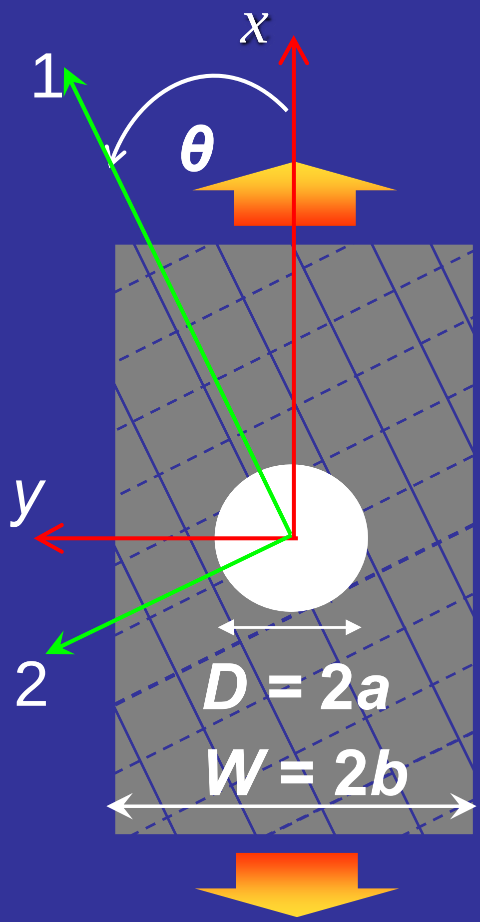
$$\frac{X_N}{X} = \frac{1}{\sqrt{\frac{1}{(1-\alpha)^2} + \frac{F_I^{\,2}\pi\alpha}{\kappa_I^{\,2}}}} = \frac{Y_N}{Y}$$

$$\frac{S_N}{S} = \frac{1}{\sqrt{\frac{1}{(1-\alpha)^2} + \chi^2\frac{F_I^{\,2}\pi\alpha}{\kappa_{II}^{\,2}}}}$$

Off-Axis Notched Strength of GLARE-3

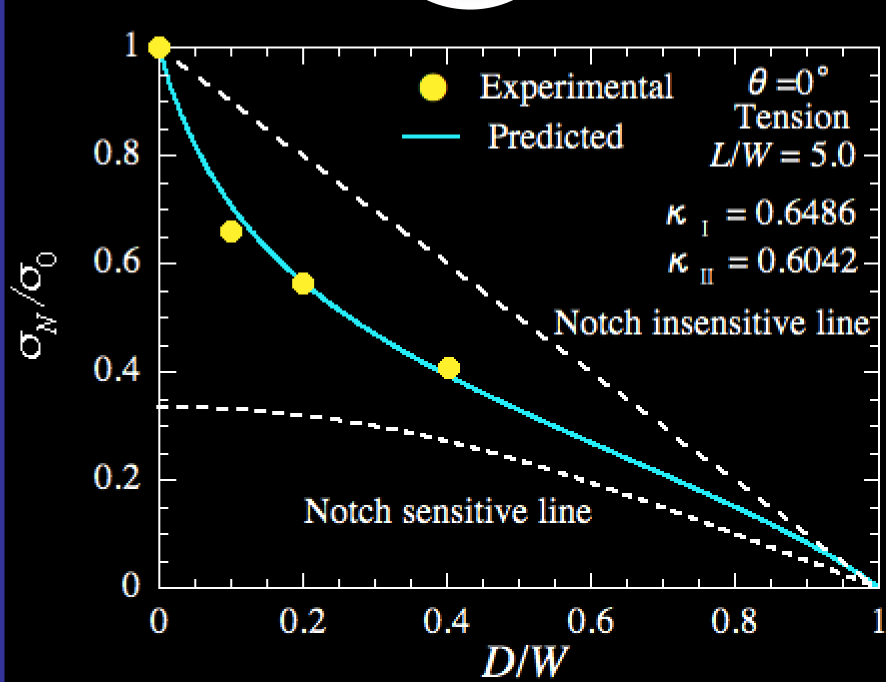
Predicted



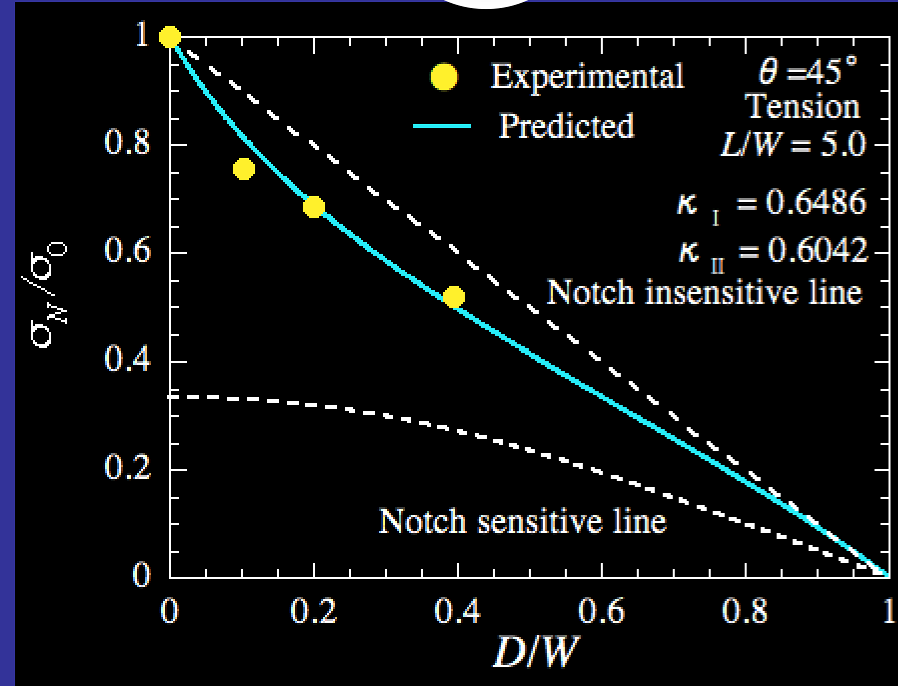


Predicted Off-Axis Notch Sensitivity in GLARE-3

$\theta = 0$



$\theta = 45$



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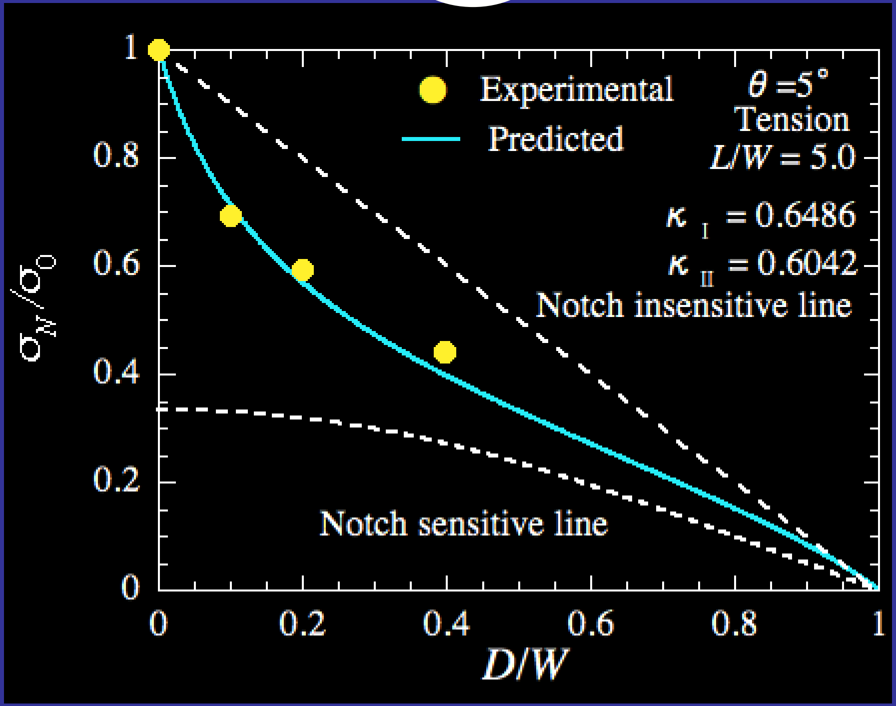
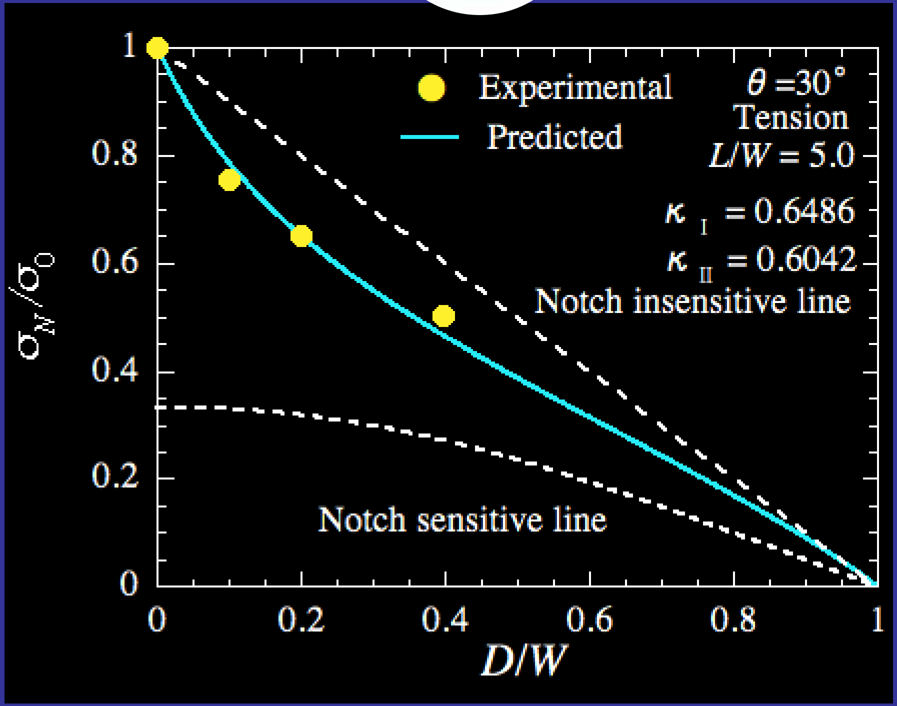
Predicted Off-Axis Notch Sensitivity in GLARE-3

$\theta = 5$

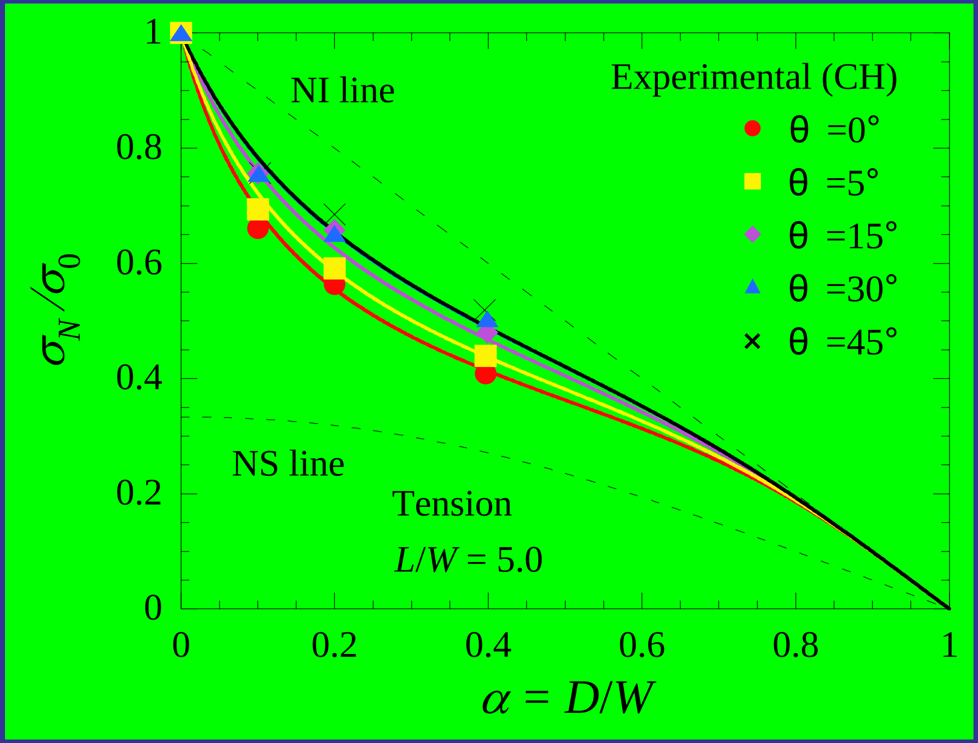


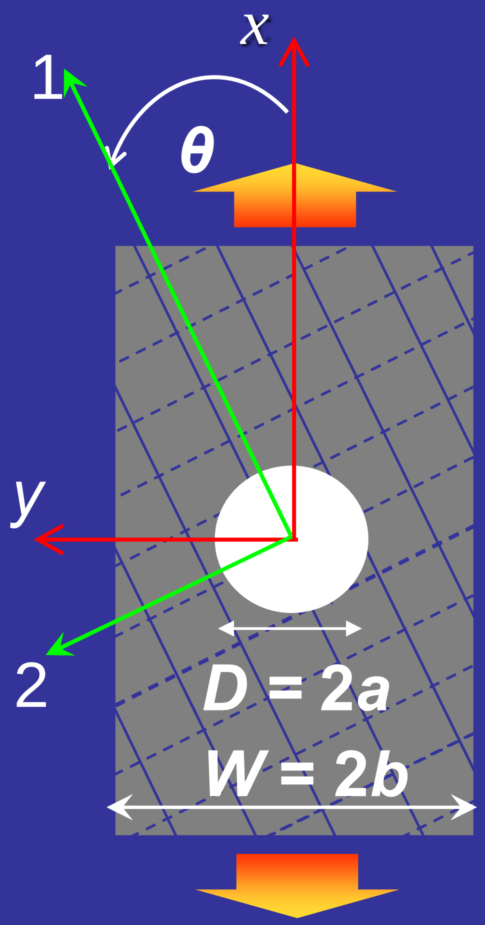
$\theta = 30$



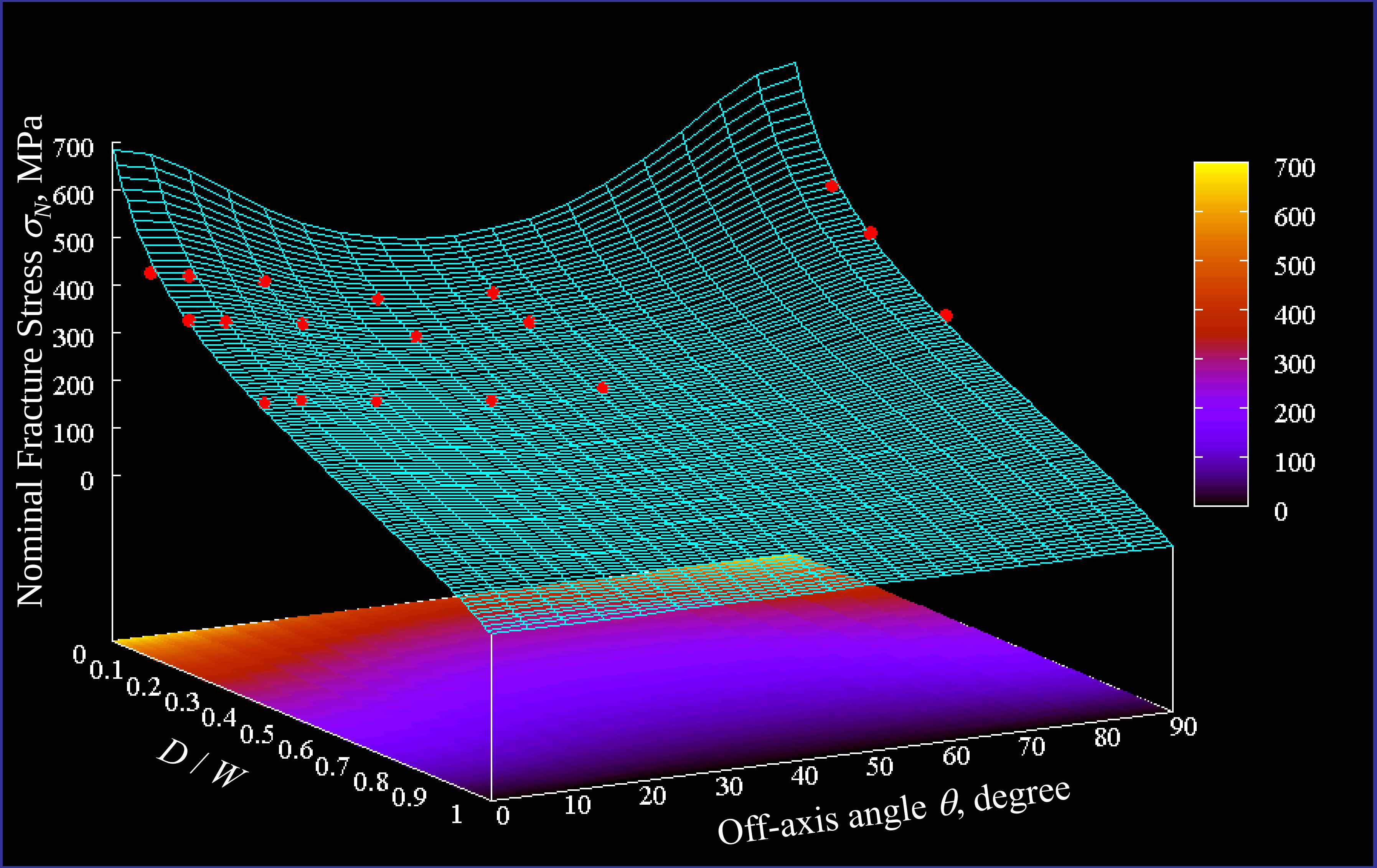
Fiber Orientation Dependence of Off-Axis Notch Sensitivity in GLARE-3

Predicted

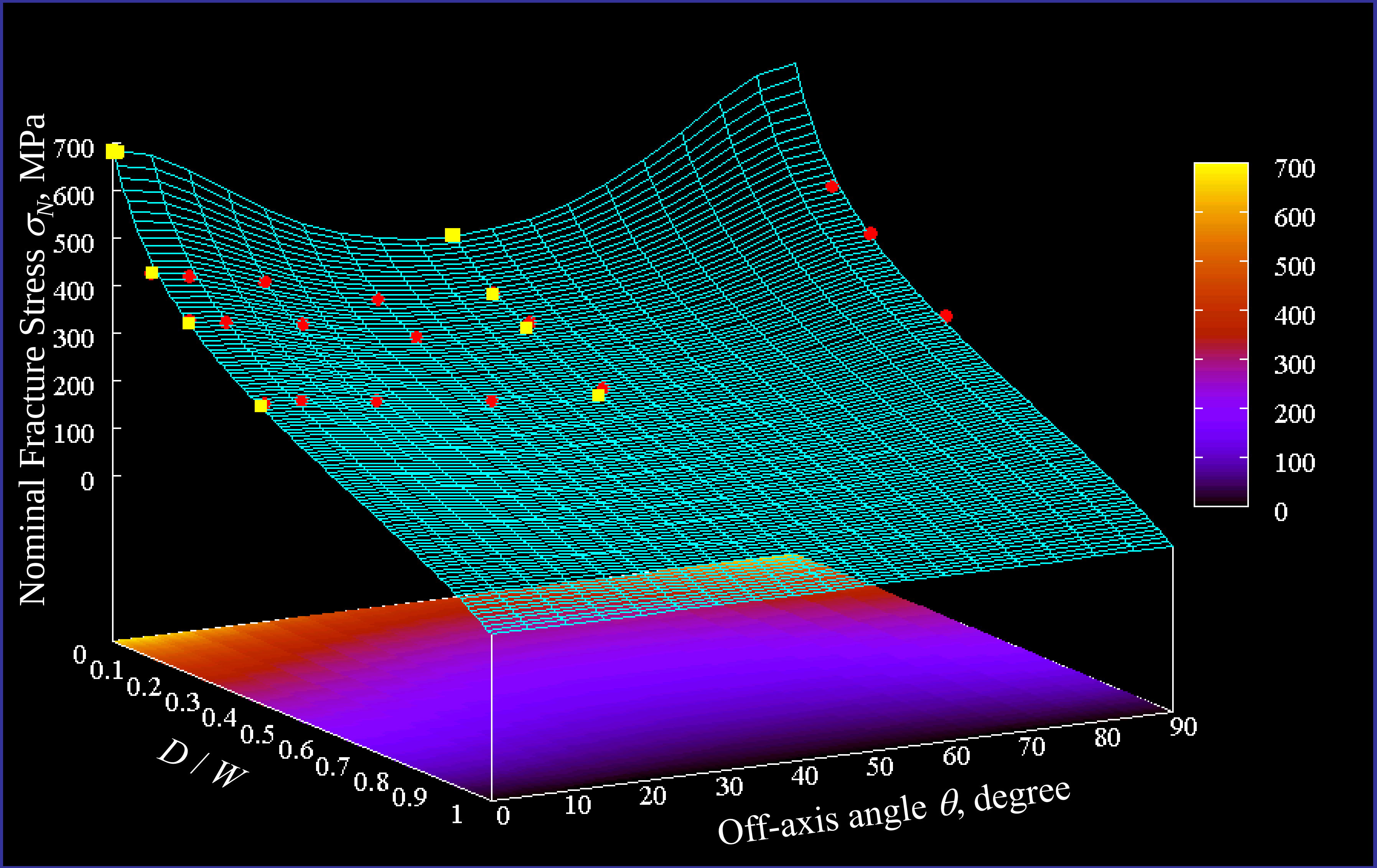




Usefulness of the Formula



Usefulness of the Formula



Conclusions (1/2)

Experimental study was conducted on the off-axis specimens with central open circular holes for different notch sizes and fiber orientations in order to identify the notch sensitivity in GLARE-3. **Theoretical** attempt at general formulation of a failure criterion applicable to notched orthotropic composites was also made.

EXPERIMENTAL

- The notched strength of Glare-3 decreases with increasing notch dimension, regardless of the fiber orientation.
- The off-axis notched strength of Glare-3 is bounded by the notch insensitive and sensitive limits, regardless of the fiber orientation, indicating moderate sensitivity to a notch.
- The notch sensitivity of Glare-3 depends on the fiber orientation of the GFRP layers, and the notch sensitivity of Glare-3 is highest in the fiber direction and lowest in the 45° direction.

Conclusions (2/2)

THEORETICAL

- A new multiaxial failure criterion for notched orthotorpic materials with any notch sensitivity was developed.
- It was formulated by combining a notch insensitve (ductile) and sensitive (brittle) failure criteria based on the net section stress criterion and the fracture mechanics criterion, respectively.
- A formula was derived to efficiently predict the off-axis notched strength of orthotropic fiber reinforced composites for any length of a notch as well as for any fiber orientation.
- The proposed failure criterion succeeded in accurately and efficiently describing the notch size effect and the fiber orientation dependence of the off-axis notched strength of the fiber metal laminate GLARE-3.

Thank you for your kind attention !