Modeling Off-Axis Notch Sensitivity of Fiber Metal Laminates

M. Kawai & Y. Arai

Department of Engineering Mechanics and Energy, University of Tsukuba, Japan
Outline

1. Background
   - Notched strengths of composites
   - Objectives

2. Experiments
   - Glare-3
   - Off-axis tension tests on notched specimens

3. Analytical Modeling & Verification
   - Multiaxial notch sensitive insensitive failure criteria
   - A formula for off-axis notched strength
   - Comparison with experimental results

4. Conclusions
It is one of the most important engineering issues to establish a reliable method for predicting the notched strengths of composites.
Off-Axis Notched Strengths of Fiber Reinforced Composites

Notch size effect $D/W$

Fiber orientation effect $\theta$

$D = 2a$

$W = 2b$
Objectives

Modeling of the off-axis notched strengths of fiber metal laminate GLARE-3

EXPERIMENTAL:

1. Notch size effect
2. Fiber orientation dependence
3. Off-axis notch sensitivity

THEORETICAL:

1. Formulation of a multiaxial failure criterion
   - A criterion for ductile failure
   - A criterion for brittle failure
   - A criterion for transitional ductile-brittle failure
2. A formula for off-axis notched strength prediction
FML (Fiber Metal Laminates)

GLARE

Al alloy + GFRP (Glass Fiber Reinforced Plastic)
Material

GLARE-3

Al alloy sheet
0°GFRP
90°GFRP

On-axis
Off-axis

University of Tsukuba
Notched Specimens

- Off-axis angle: $\theta = 0, 5, 15, 30, 45, 90^\circ$
- Width: $W = 10, 20, 30$ mm
- Normalized width: $D/W = 0.0, 0.1, 0.2, 0.4$
- $2a/W$:
Test Procedure

JIS K7073

- Temperature
  - RT
- Rate
  - 1.0 mm/min
- Strain Measurement
  - Strain Gauge
  - DVE
  - Disp
  - Extensometer

MTS Test Star 810

Extensometer
Experimental Results
Off-axis Stress-Strain Curves

Unnotched behavior

Axial stress $\sigma_{x^y}$, MPa
Axial strain $\varepsilon_{x^y}$, %

GLARE3
Experimental RT
1.0 mm/min
Tension Gauge

0° 5° 15° 30° 45°

On-axis
Off-axis
Off-Axis Stress-Strain Curves

Notched behavior

Axial stress $\sigma_{xy}$, MPa

Axial strain $\varepsilon_{xz}$, %

GLARE3
Experimental RT
1.0 mm/min Tension

Extensometer $D = 2.0$

W = 20.0
Off-Axis Stress-Strain Curves

Notched behavior

- GLARE3 Experimental RT
- 1.0 mm/min Tension

Notched strength

$D = 2.0$
Off-Axis Notch Sensitivity Curve

GLARE3
Experimental RT
1.0 mm/min  L/W=5
Tension  CH

\[ \frac{\sigma_N}{\sigma_0} \]

Notch insensitive line

Notch sensitive line

\[ D/W \]
Off-Axis Notch Sensitivity Curve

GLARE3
Experimental RT
1.0 mm/min L/W=5
Tension CH

$\sigma_N / \sigma_0$

0 0.2 0.4 0.6 0.8 1

D/W

Notch insensitive line

Al
$\theta = 0^\circ$
$\theta = 5^\circ$
$\theta = 15^\circ$
$\theta = 30^\circ$
$\theta = 45^\circ$

Notch sensitive line
Application of existing fracture criteria

- **Semi-Empirical Criteria:**
  - Point-stress (PS) criterion
  - Average stress (AS) criterion
  - Modified PS criterion

- **Fracture Mechanics Criteria:**
  - Inherent flaw criterion
  - Cohesive zone criterion
  - R-curve based criterion
Average Stress (AS) Criterion

\[ \theta = 45^\circ \]

\[ \theta = 0^\circ \]

\[ \theta = 15^\circ \]

\[ a_0 = 2.0013 \]

\[ a_0 = 4.0646 \]

\[ a_0 = 3.4148 \]

Notch insensitive line

Notch sensitive line

\( \frac{\sigma_N}{\sigma_0} \)

\( \frac{\sigma_Y}{\sigma_0} \)
Off-Axis Notch Size Effect

Nominal Fracture Stress $\sigma_N$, MPa

Practical approach?

$D = 2a$

$W = 2b$

$\theta$
Multiaxial Notch Sensitivity Modeling

- A ductile failure criterion
- A brittle failure criterion
- A brittle-ductile failure criterion
- A formula for off-axis notched strength
A Ductile Failure Criterion

The Net Section Stress Criterion:

\[ \frac{\sigma_N}{\sigma_0} = 1 - \frac{a}{b} = 1 - \alpha \]

- \( \sigma_N \): Remote stress at failure (notched strength)
- \( \sigma_0 \): Material strength (unnotched strength)
- \( \alpha \): Diameter to width ratio \( (= D/W = a/b) \)
A Multiaxial Ductile Failure Criterion

\[
\frac{\sigma_N}{\sigma_0} = 1 - \frac{a}{b} = 1 - \alpha
\]

The Net Section Stress Criterion:

\[
\frac{\sigma_{\text{net}}}{\sigma_0} = \frac{1}{1 - \alpha} \frac{\sigma_x}{\sigma_0} = 1
\]

\[
f_{NI} = \left( \frac{1}{1 - \alpha} \sigma^* \right)^2 = 1
\]

where

\[
\sigma^* = \sqrt{ \left( \frac{\sigma_{11}}{X} \right)^2 - \frac{\sigma_{11} \sigma_{22}}{X^2} + \left( \frac{\sigma_{22}}{Y} \right)^2 + \left( \frac{\tau_{12}}{S} \right)^2 }
\]
Non-Dimensional Effective Stress

Tsai-Hill Static Failure Criterion:
\[
\left( \frac{\sigma_{11}}{X} \right)^2 - \frac{\sigma_{11} \sigma_{22}}{X^2} + \left( \frac{\sigma_{22}}{Y} \right)^2 + \left( \frac{\tau_{12}}{S} \right)^2 = 1
\]

- \( X \): Longitudinal strength
- \( Y \): Transverse strength
- \( S \): Shear strength

Non-Dimensional Effective Stress:
\[
\sigma^* \equiv \sqrt{\left( \frac{\sigma_{11}}{X} \right)^2 - \frac{\sigma_{11} \sigma_{22}}{X^2} + \left( \frac{\sigma_{22}}{Y} \right)^2 + \left( \frac{\tau_{12}}{S} \right)^2}
\]
Theoretical Stress Ratio

Off-Axis Loading on UD Composites

Non-Dimensional Effective Stress

\[ \sigma^* = \Omega(\theta)\sigma_x \]

Static Failure Condition: \[ \sigma^* = \Omega(\theta)\sigma_x = 1 \]

\[ \therefore \sigma_0 = \frac{1}{\Omega(\theta)} \]

where \[
\Omega(\theta) = \sqrt{\left(\frac{\cos^2 \theta}{X}\right)^2 - \frac{\cos^2 \theta \sin^2 \theta}{X^2} + \left(\frac{\sin^2 \theta}{Y}\right)^2 + \left(\frac{-\cos \theta \sin \theta}{S}\right)^2}
\]

Theoretical stress ratio for off-axis loading:

\[ \sigma^* = \Omega(\theta)\sigma_x = \frac{\sigma_x}{1/\Omega(\theta)} = \frac{\sigma_x}{\sigma_0} \]
**A Brittle Failure Criterion**

**Fracture Mechanics Criterion:**

\[
\frac{\sigma_N}{\sigma_0} = \frac{K_{IC}}{F \sigma_0 \sqrt{\pi a}}
\]

- \(\sigma_N\): Remote stress at failure (notched strength)
- \(\sigma_0\): Material strength (unnotched strength)
- \(F\): Finite width correction factor

\[F = F(2a/W) = F(a/b) = F(\alpha)\]
\[ \frac{\sigma_N}{\sigma_0} = \frac{K_{IC}}{\sigma_0 F \sqrt{\pi a}} \]

Fracture Mechanics Criterion:
\[ \frac{K_I}{K_{IC}} = \frac{\sigma F \sqrt{\pi a}}{K_{IC}} = 1 \]

\[ f_{NS} = f_{NS}(k_{11}, k_{22}, k_{12}) = \left( \frac{k_{11}}{K_{11}^{IC}} \right)^2 - \frac{k_{11} k_{22}}{\left( K_{IC}^{11} \right)^2} + \left( \frac{k_{22}}{K_{IC}^{22}} \right)^2 + \left( \frac{k_{12}}{K_{IC}^{12}} \right)^2 = 1 \]

where
\[ \begin{pmatrix} k_{11} \\ k_{22} \\ k_{12} \end{pmatrix} = F \sqrt{\pi a} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \chi \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{pmatrix} \]
Normalized principal fracture toughness
(Principal stress brittleness numbers)

\[
\frac{K_{IC}^{11}}{X \sqrt{b}} \equiv \kappa_{11}
\]

\[
\frac{K_{IC}^{22}}{Y \sqrt{b}} \equiv \kappa_{22}
\]

\[
\frac{K_{IC}^{12}}{S \sqrt{b}} \equiv \kappa_{12}
\]

\[
s = \frac{K_{IC}}{\sigma_u \sqrt{b}}
\]

(Carpinteri)

\[
f_{NS} = F_1^2 \pi \alpha \left[ \left( \frac{\sigma_{11}}{X \kappa_{11}} \right)^2 - \frac{\sigma_{11} \sigma_{22}}{(X \kappa_{11})^2} + \left( \frac{\sigma_{22}}{Y \kappa_{22}} \right)^2 + \chi^2 \left( \frac{\sigma_{12}}{S \kappa_{12}} \right)^2 \right]
\]
A multiaxial failure criterion for orthotropic solids with any notch sensitivity bounded by the ductile and brittle limits:

\[ f = f_{NI} + f_{NS} = 1 \]

where

\[ f_{NI} = \left( \frac{\sigma^*}{1-\alpha} \right)^2 \]

\[ f_{NS} = F_1^2 \pi \alpha \left[ \left( \frac{\sigma_{11}}{X\kappa_{11}} \right)^2 - \frac{\sigma_{11}\sigma_{22}}{(X\kappa_{11})^2} + \left( \frac{\sigma_{22}}{Y\kappa_{22}} \right)^2 + \chi^2 \left( \frac{\sigma_{12}}{S\kappa_{12}} \right)^2 \right] \]
A Formula for Off-Axis Notched Strength

\[ f = f_{NI} + f_{NS} = 1 \]

\[ \frac{\sigma_N}{\sigma_0} = \sqrt{\left( \frac{1}{1 - \alpha} \right)^2 + \left( \frac{\Omega_{NS}}{\Omega_0} \right)^2} \]

where

\[ \Omega_0 = \sqrt{\frac{m^4}{X^2} - \frac{m^2 n^2}{X^2} + \frac{n^4}{Y^2} + \frac{m^2 n^2}{S^2} = \frac{1}{\sigma_0}} \]

\[ \Omega_{NS} = \sqrt{F_1^2 \pi \alpha \left\{ \frac{m^4 - m^2 n^2}{(X\kappa_{11})^2} + \frac{n^4}{(Y\kappa_{22})^2} + \chi^2 \frac{m^2 n^2}{(S\kappa_{12})^2} \right\}} \]

\[ \begin{cases} m = \cos \theta \\ n = \sin \theta \end{cases} \]
Principal Notched Strengths

\[ \frac{X_N}{X} = \frac{1}{\sqrt{\left(1 - \alpha\right)^2 + \frac{F_I^2 \pi \alpha}{\kappa_{11}^2}}} \]

\[ \frac{Y_N}{Y} = \frac{1}{\sqrt{\left(1 - \alpha\right)^2 + \frac{F_I^2 \pi \alpha}{\kappa_{22}^2}}} \]

\[ \frac{S_N}{S} = \frac{1}{\sqrt{\left(1 - \alpha\right)^2 + \chi^2 \frac{F_I^2 \pi \alpha}{\kappa_{12}^2}}} \]

(a generalization of the Suo-Ho-Gong model, 1993)

\[ D = 2a \]

\[ W = 2b \]
Verification
Application to GLARE-3

\[ \frac{X_N}{X} = \frac{1}{\sqrt{\left(1 - \alpha \right)^2 + \left(\frac{F_1^2 \pi \alpha}{\kappa_I^2} \right)^2}} = \frac{Y_N}{Y} \]

\[ \frac{S_N}{S} = \frac{1}{\sqrt{\left(1 - \alpha \right)^2 + \left(\frac{\chi^2 F_1^2 \pi \alpha}{\kappa_{II}^2} \right)^2}} \]

where

\[ F_I = \left(1 - 0.025 \alpha^2 + 0.06 \alpha^4\right)^\frac{1}{2} \sec \frac{\pi \alpha}{2} \]

\[ \chi \approx 0.6 \quad \text{(Sih and Chen, 1973)} \]
Principal Notch Sensitivity in GLARE-3

\[
\frac{X_N}{X} = \frac{1}{\sqrt{\left(1 - \alpha\right)^2 + \frac{F_I^2 \pi \alpha}{\kappa_I^2}}} = \frac{Y_N}{Y}
\]

\[
\frac{S_N}{S} = \frac{1}{\sqrt{\left(1 - \alpha\right)^2 + \chi^2 \frac{F_I^2 \pi \alpha}{\kappa_{II}^2}}}
\]

GLARE-3 Experimental RT
Tension 1.0 mm/min
\(L/W=5.0\)
CH

\[
X, Y (\kappa_{\delta u} = 0.6486)
\]

\[
S (\kappa_{\delta v} = 0.6351)
\]
Off-Axis Notched Strength of GLARE-3

**Experimental**

- GLARE-3 Tension CH
- $L/W = 5.0$
- Experimental RT 1.0 mm/min

**Predicted**

- $D = 2a$
- $W = 2b$

Fracture stress $\sigma_N$, MPa

Off-axis angle $\theta$, degree
Predicted Off-Axis Notch Sensitivity in GLARE-3

\[ \theta = 0 \]

\[ \theta = 45 \]

\( \kappa_1 = 0.6486 \)

\( \kappa_2 = 0.6042 \)

Notch insensitive line

Notch sensitive line

Experimental

Predicted

Tension

\( L/W = 5.0 \)
Predicted Off-Axis Notch Sensitivity in GLARE-3

\[ \theta = 5 \]

\[ \theta = 30 \]

\[ \kappa_1 = 0.6486 \]
\[ \kappa_2 = 0.6042 \]

\[ \frac{\sigma_N}{\sigma_0} \] vs \[ D/W \]

Notch insensitive line

Notch sensitive line

Experimental vs Predicted
Fiber Orientation Dependence of Off-Axis Notch Sensitivity in GLARE-3

Predicted

\[
\frac{\sigma_N}{\sigma_0} = \frac{\alpha}{1 - \alpha}
\]

where

- \( \sigma_N \) is the notch stress,
- \( \sigma_0 \) is the tensile strength,
- \( \alpha = \frac{D}{W} \) is the ratio of the notch depth to the width,
- \( D = 2a \) is the notch depth,
- \( W = 2b \) is the width.

Graph showing variations of notch stress with tension for different fiber orientations (\( \theta \)).

Plot includes:
- NI line (Experimental (CH))
- NS line
- Tension
- Experimental data points for different fiber angles (\( \theta = 0^\circ, 5^\circ, 15^\circ, 30^\circ, 45^\circ \)).
Nominal Fracture Stress $\sigma_N$, MPa

Usefulness of the Formula

$D/W$ vs. Off-axis angle $\theta$, degree
Usefulness of the Formula

Nominal Fracture Stress $\sigma_N$, MPa
Conclusions (1/2)

Experimental study was conducted on the off-axis specimens with central open circular holes for different notch sizes and fiber orientations in order to identify the notch sensitivity in GLARE-3. Theoretical attempt at general formulation of a failure criterion applicable to notched orthotropic composites was also made.

EXPERIMENTAL

- The notched strength of Glare-3 decreases with increasing notch dimension, regardless of the fiber orientation.
- The off-axis notched strength of Glare-3 is bounded by the notch insensitive and sensitive limits, regardless of the fiber orientation, indicating moderate sensitivity to a notch.
- The notch sensitivity of Glare-3 depends on the fiber orientation of the GFRP layers, and the notch sensitivity of Glare-3 is highest in the fiber direction and lowest in the 45° direction.
A new multiaxial failure criterion for notched orthotropic materials with any notch sensitivity was developed.

It was formulated by combining a notch insensitive (ductile) and sensitive (brittle) failure criteria based on the net section stress criterion and the fracture mechanics criterion, respectively.

A formula was derived to efficiently predict the off-axis notched strength of orthotropic fiber reinforced composites for any length of a notch as well as for any fiber orientation.

The proposed failure criterion succeeded in accurately and efficiently describing the notch size effect and the fiber orientation dependence of the off-axis notched strength of the fiber metal laminate GLARE-3.
Thank you for your kind attention!