Modeling R-Curve Toughening Mechanisms with Complex Softening Laws

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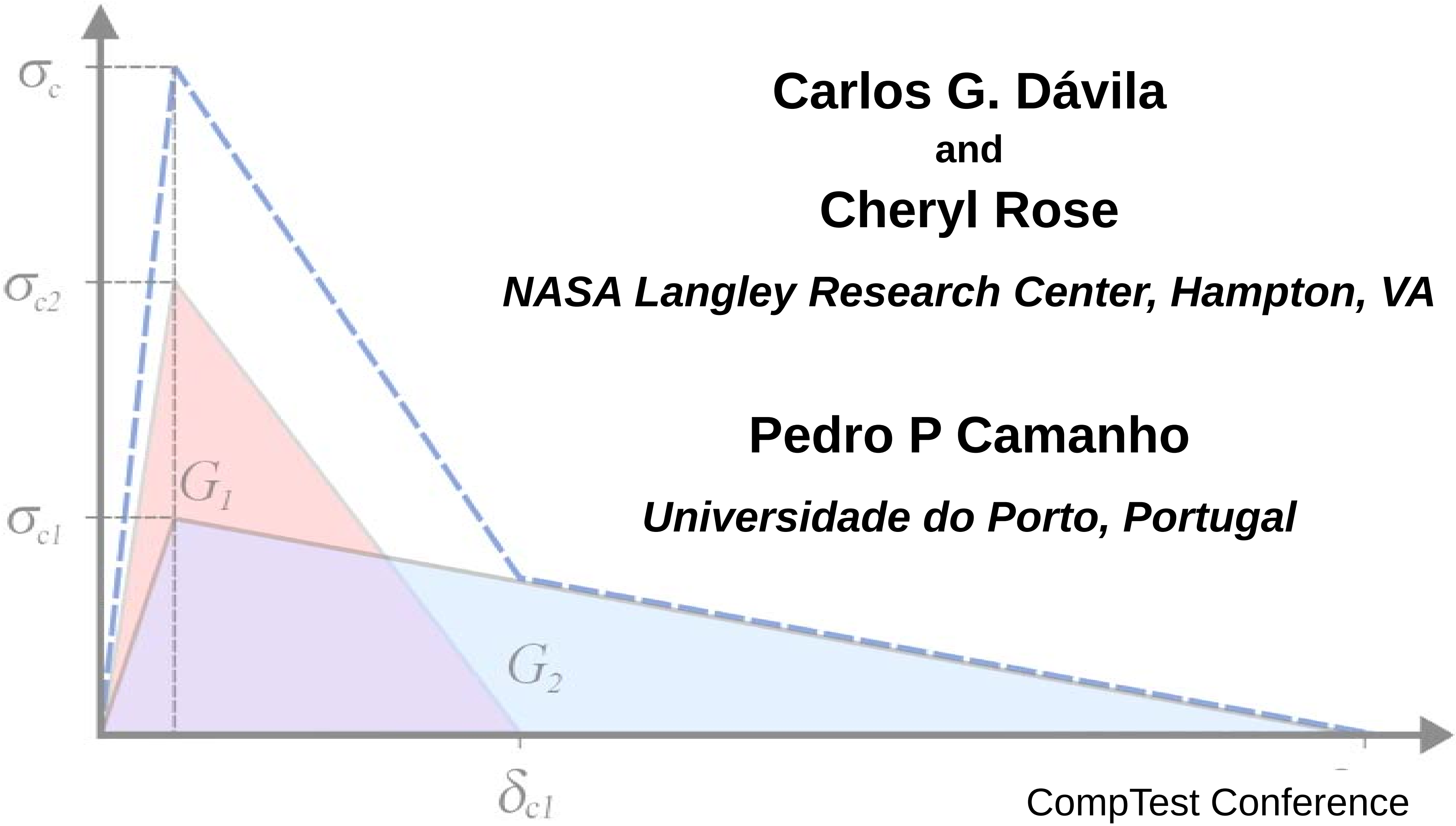


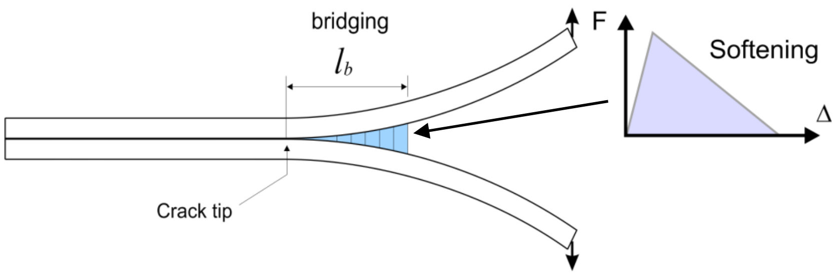
CompTest Conference
Dayton, OH, October 2008

Resistance Curve

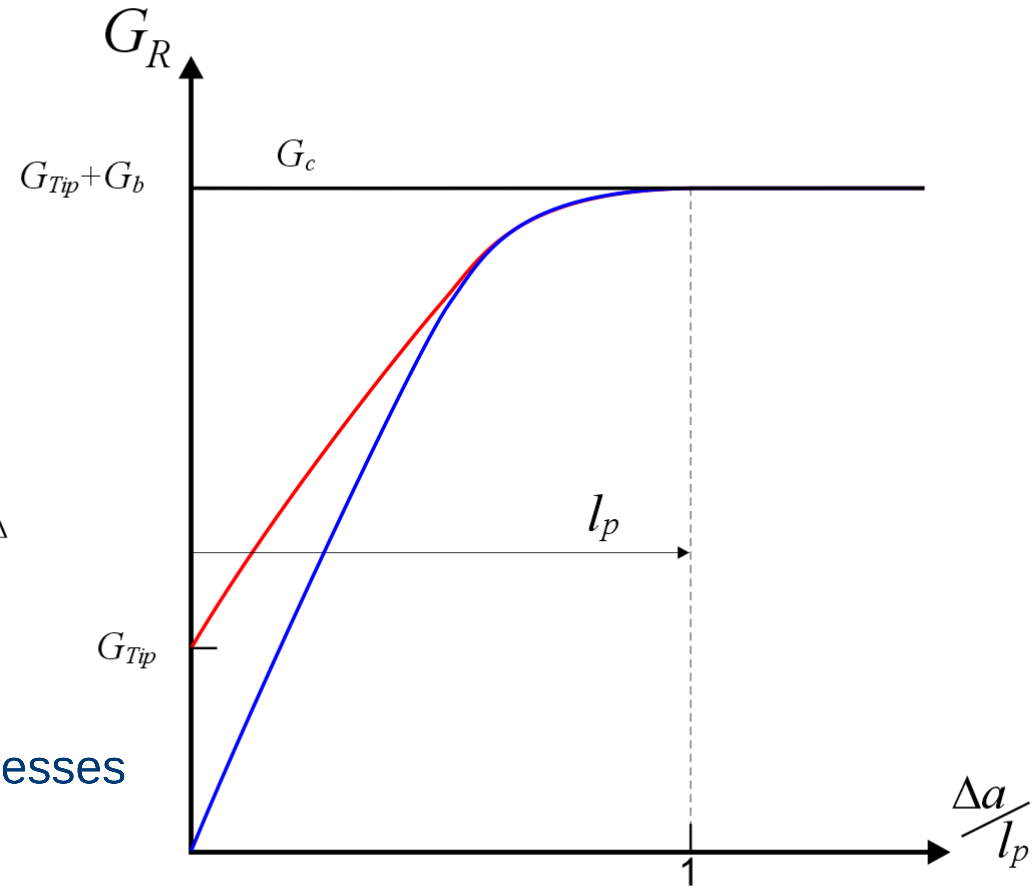
"However complex a fracture process is, a single material property (the fracture toughness) quantifies the resistance to fracture"

- LEFM: Fracture toughness concentrated at crack tip.
  No growth: $G < G_c$

- Toughness from Bridging Stresses
  No growth: $G < G_b(\Delta a)$



- Combination of Tip and Bridging Stresses
  No growth: $G < G_{Tip}+G_b(\Delta a)$



Complex Cohesive Laws for the Fracture of Composites

Objective

Add higher level of fidelity to FE prediction of fracture: R-Curve

Rationale

Relationships between:

- Resistance curve (R-curve)
- Fracture process zone length (l_p)
- Shape of the traction/displacement softening law

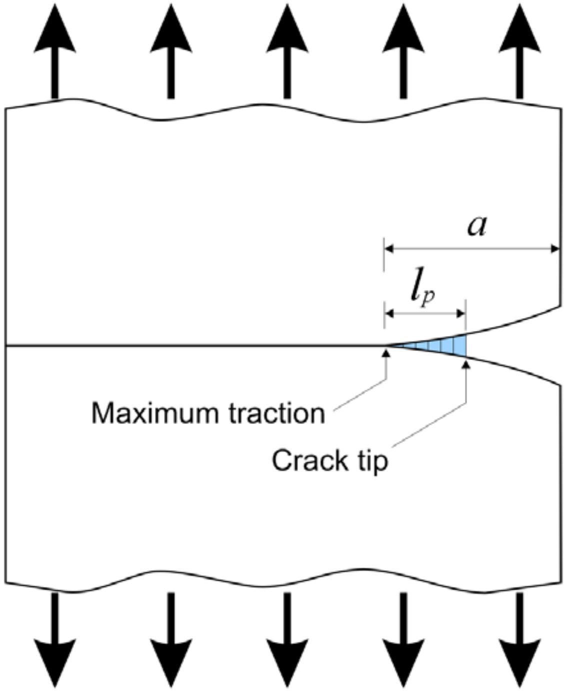
Approach

- Develop equations to predict R-curve for superposed linear cohesive laws
- Demonstrate use of superposed cohesive elements to account for experimental R-curve

Different Failure Theories for Different Structural Scales

Typical values of l_p (Bažant, 2002)

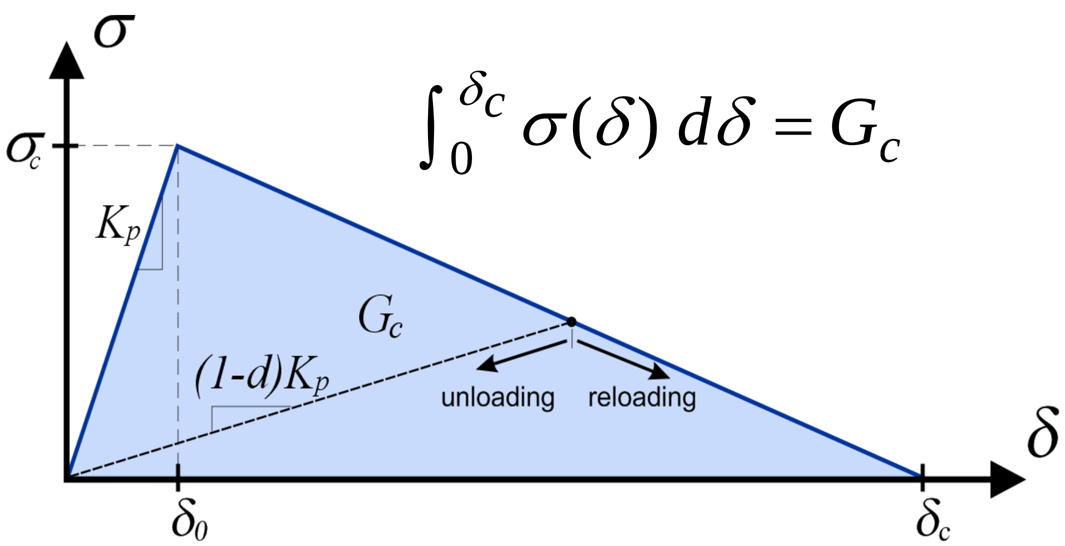
cracks in geological formations:	$l_p \approx$	50 m
dam concrete; sea ice:	$l_p \approx$	3 m
granite; epoxy:	$l_p \approx$	1 mm
silicon oxide ceramic:	$l_p \approx$	0.1 mm



$$\eta = \frac{a}{l_p} \quad \begin{cases} \eta < 5, & \text{ductile damage, plasticity} \\ 5 \le \eta \le 100, & \text{quasi-brittle fracture mechanics} \\ \eta > 100, & \text{brittle (LEFM)} \end{cases}$$

Material Softening Laws

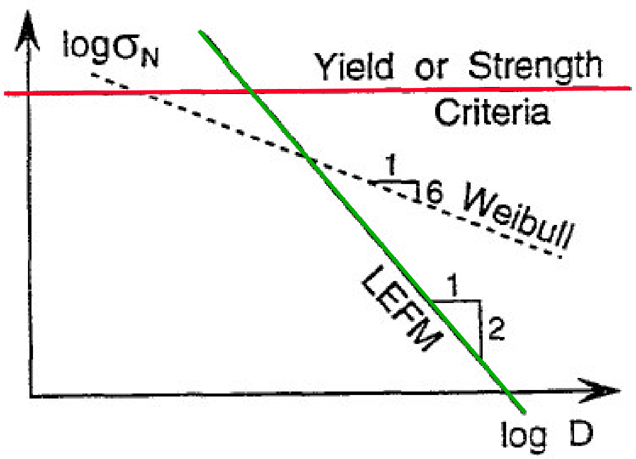
Linear Traction-Displacement Law



$$\int_0^{\delta_c} \sigma(\delta)\, d\delta = G_c$$

Two material properties:

- G_c Fracture toughness
- σ_c Strength

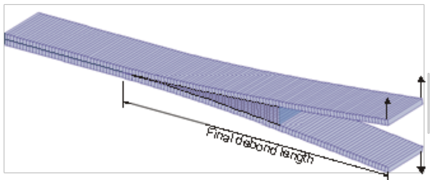


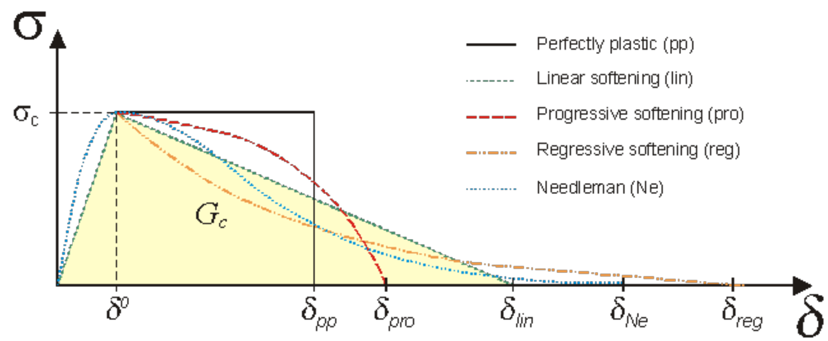
Characteristic Length:

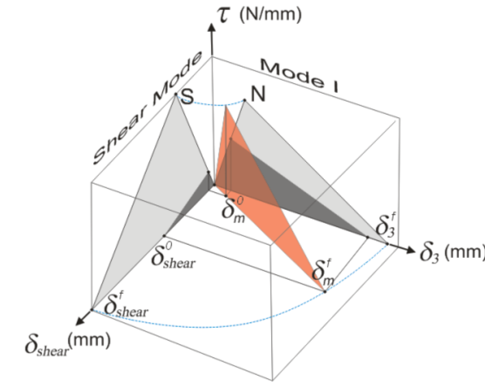
$$l_p = \gamma \frac{E\, G_c}{\sigma_c^2}$$

Material Softening Laws

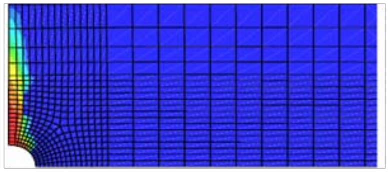
Cohesive elements

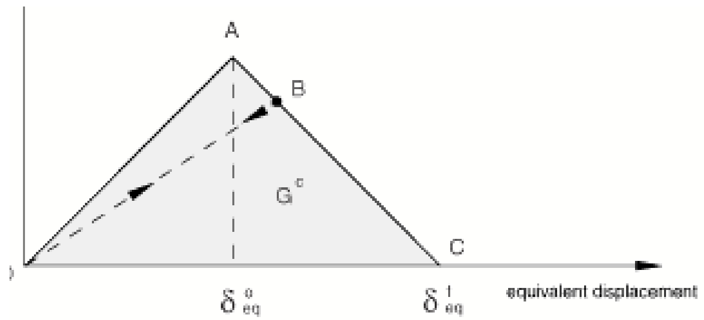






Abaqus Damage Model

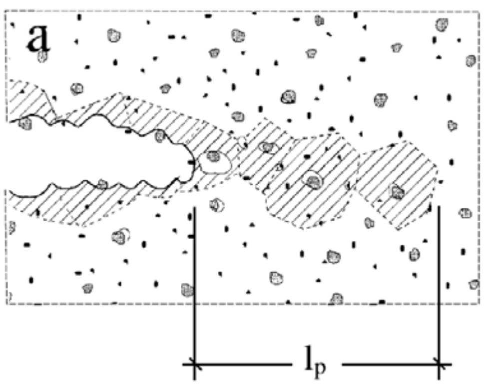


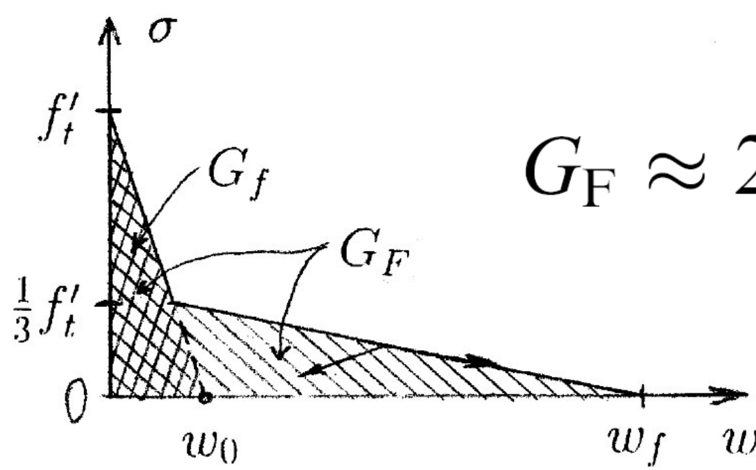


$$\delta_{eq}^{ft} = L^c \sqrt{\langle \varepsilon_{11} \rangle^2 + \alpha \varepsilon_{12}^2},$$

$$\sigma_{eq}^{ft} = \frac{\langle \sigma_{11} \rangle \langle \varepsilon_{11} \rangle + \alpha \tau_{12} \varepsilon_{12}}{\delta_{eq}^{ft} / L^c}.$$

Standards of American Concrete Institute

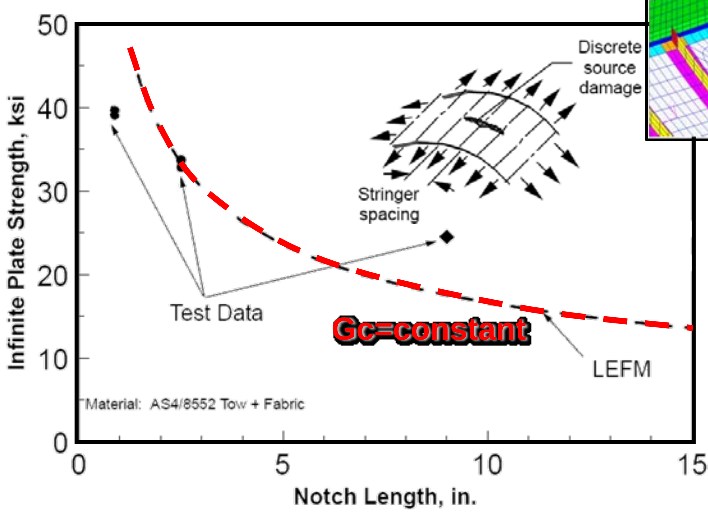


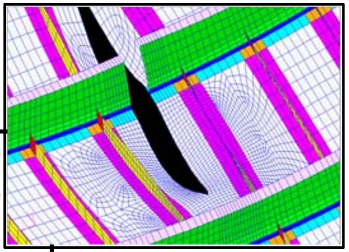


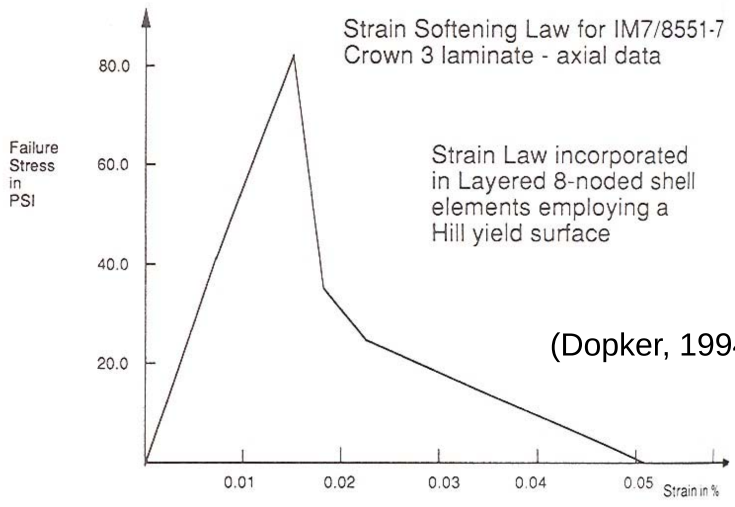
$$G_F \approx 2.5 G_f$$

Material Softening Laws for Composites

NASA-Boeing ATCAS Program







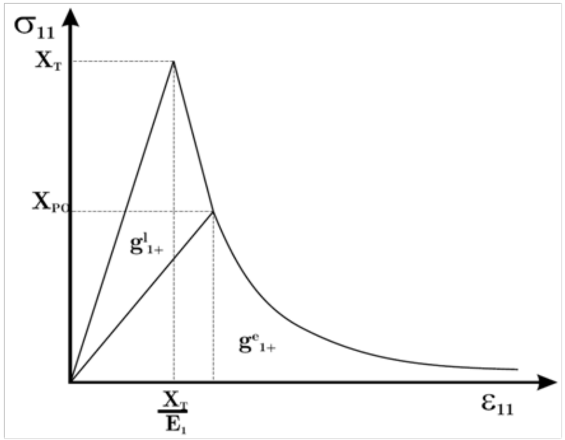
(Dopker, 1994)

Figure 11. A Tytical Advanced Damage Zone Model

LaRC-based damage model (Maimí, 2007)

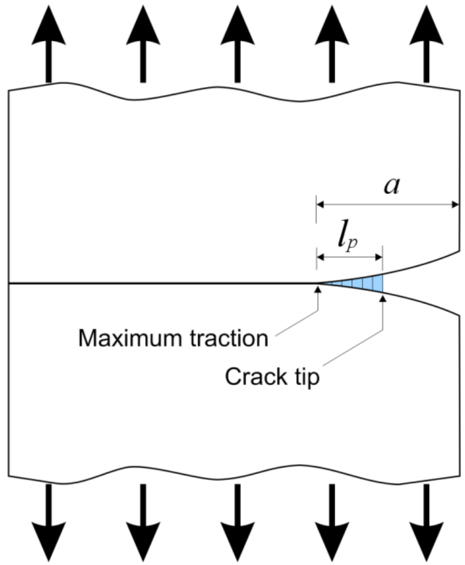
$$d_{1+} = 1 - (1 - d_{1+}^{F}) \frac{r_{L+}^{F}}{r_{L+}} \exp\left[A_{1+}\left(1 - \frac{r_{L+}}{r_{L+}^{F}}\right)\right]$$

r_{L+}: LaRC04 failure criterion for fiber tension



How to obtain the shape of the softening law from experiments?

Length of the Process Zone – Various Models



$$l_p = \gamma \frac{E\,G_c}{\sigma_c^2}$$

what is γ?

Good representation of bilinear cohesive elements

Irwin, 1970	Dugdale, 1960, Barenblatt, 1962	Bao, et al., 1992	Cox, et al., 1994	Rice, 1979	Hillerborg, et al., 1976
$\gamma = \frac{1}{\pi} = 0.32$	$\gamma = \frac{\pi}{8} = 0.39$	$\gamma = 0.73$	$\gamma = \frac{\pi}{4} = 0.79$	$\gamma = 0.884$	$\gamma = 1.0$

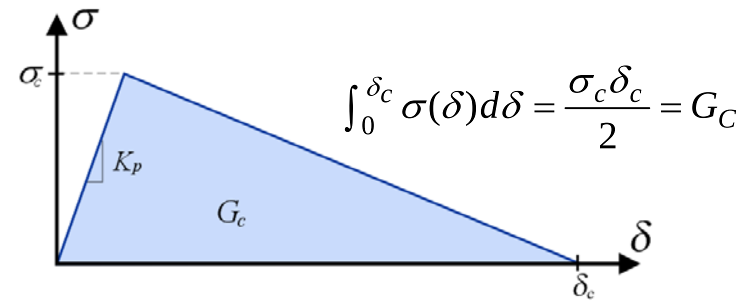
- Irwin, 1970: elliptical notch valid for l_p<<a
- Dugdale, 1960, Barenblatt, 1962: elastic-plastic material
- Bao, et al., 1992: elastic solid w/ bridging stresses
- Cox, et al., 1994: bridging stresses
- Rice, 1979: simplified stress distribution
- Hillerborg, et al., 1976: engineering approximation

Crack Growth Resistance (R-Curve)

Foote et al. ['86]

$$G_R = G_{Tip} + \int_0^{\Delta a} \frac{\sigma_c \delta_c}{l_p}\left(1 - \frac{x}{l_p}\right)dx$$

Linear Softening Law

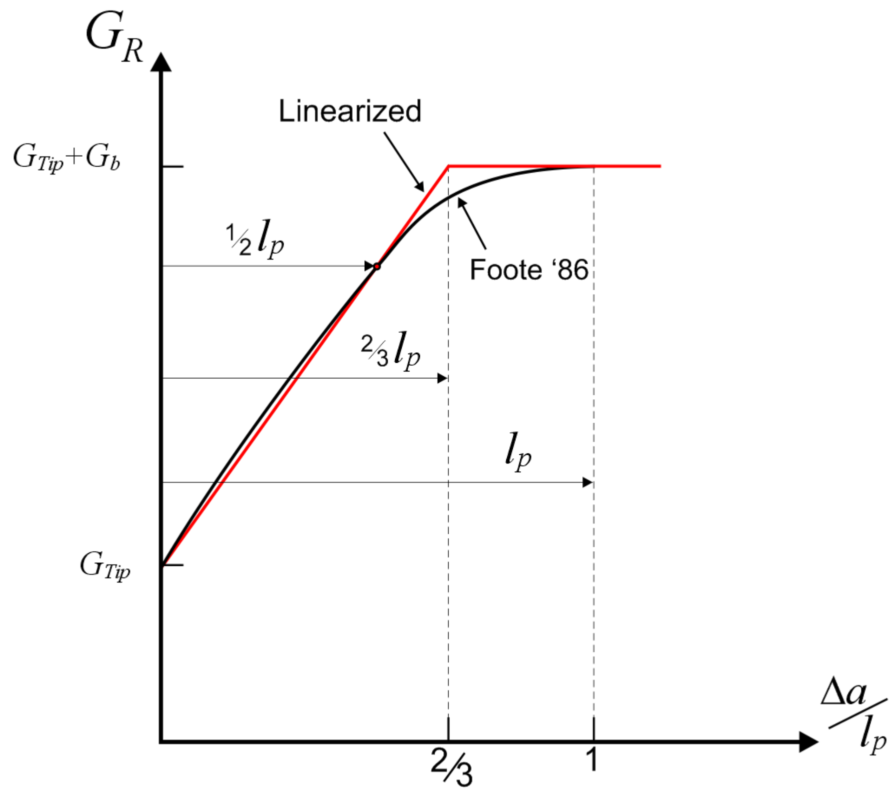


For a bilinear softening law

$$G_R^{NL}(\Delta a) = \begin{cases} G_{Tip} + G_b \frac{\Delta a}{l_p}\left(2 - \frac{\Delta a}{l_p}\right) & for \quad \Delta a < l_p \\ G_{Tip} + G_b & for \quad \Delta a \geq l_p \end{cases}$$

Alternate linearized expression

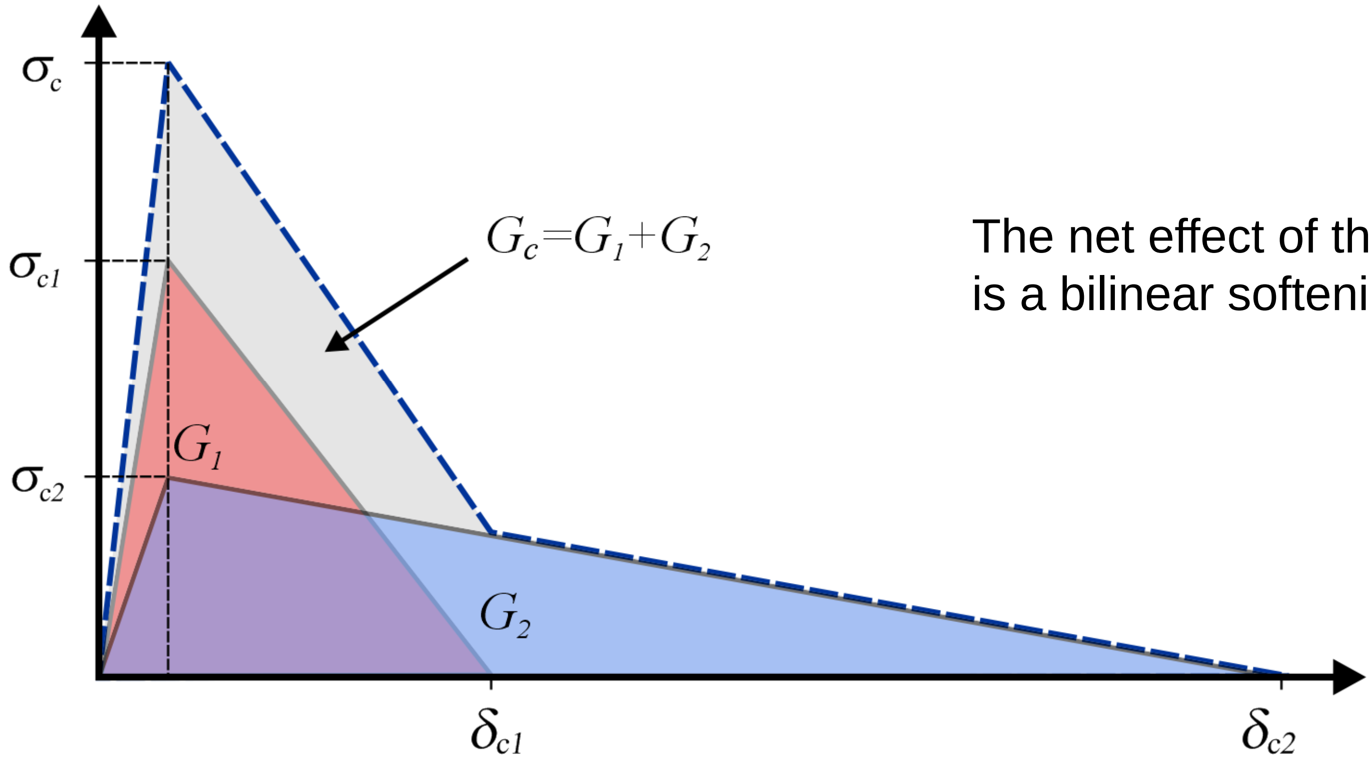
$$G_R^{L}(\Delta a) = \begin{cases} G_{Tip} + \frac{3}{2} G_b \frac{\Delta a}{l_p} & for \quad \Delta a < \frac{2}{3} l_p \\ G_{Tip} + G_b & for \quad \Delta a \geq \frac{2}{3} l_p \end{cases}$$



Bilinear Softening Laws

Consider two simultaneous damage mechanisms:

- Let G_1 represents a mechanism acting close to the tip
- Let G_2 represent a mechanism acting further into the crack wake



The net effect of the superposition is a bilinear softening law

R-Curve for Superposed Linear Softening Laws

R-Curve (linearized)

$$G_R^L(\Delta a) = G_{Tip} + \frac{3}{2} G_b \frac{\Delta a}{l_p}$$

$$\Rightarrow \quad G_R^L(\Delta a) = G_1 \frac{3}{2} \frac{\Delta a}{\beta_1 l_c} + G_2 \frac{3}{2} \frac{\Delta a}{\beta_2 l_c}$$

Choose

$$\beta_1 = \frac{m}{n}; \quad \beta_2 = \frac{1-m}{1-n}$$

where $n = \frac{\sigma_{c1}}{\sigma_c}$ $\quad m = \frac{G_1}{G_c}$

$$\Rightarrow \quad l_p = \beta \frac{\gamma E G_c}{\sigma_c^2} \quad \text{where} \quad \beta = MAX[\beta_1; \beta_2]$$

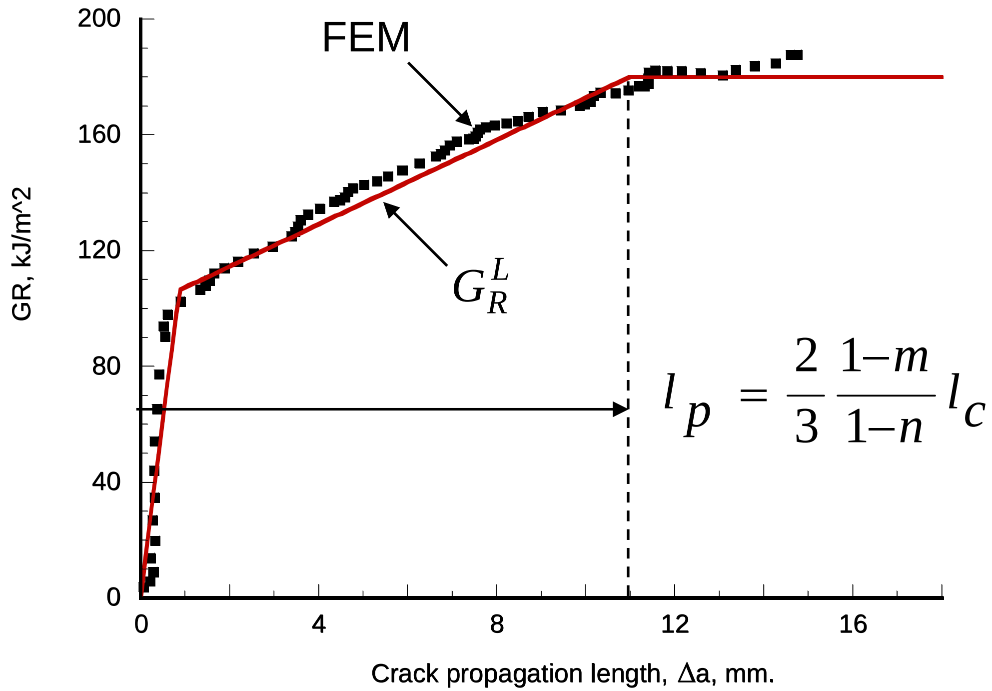
R-Curve for Superposed Linear Softening Laws

$$G_R^L(\Delta a) = MIN\left(G_1;\ nG_c \frac{3}{2}\frac{\Delta a}{l_c}\right) + MIN\left(G_2;\ (1-n)G_c \frac{3}{2}\frac{\Delta a}{l_c}\right)$$

(Dávila, 2008)

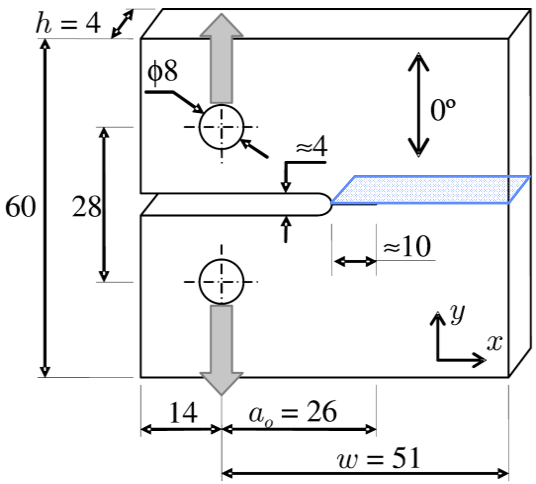
where $l_c = \gamma E G_c / \sigma_c^2$ $\quad n = \frac{\sigma_{c1}}{\sigma_c}$ $\quad m = \frac{G_1}{G_c}$

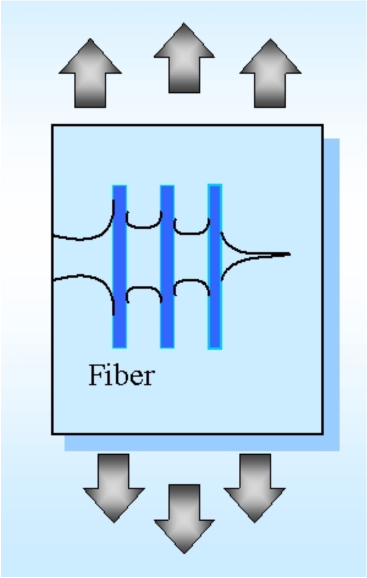
R-Curves

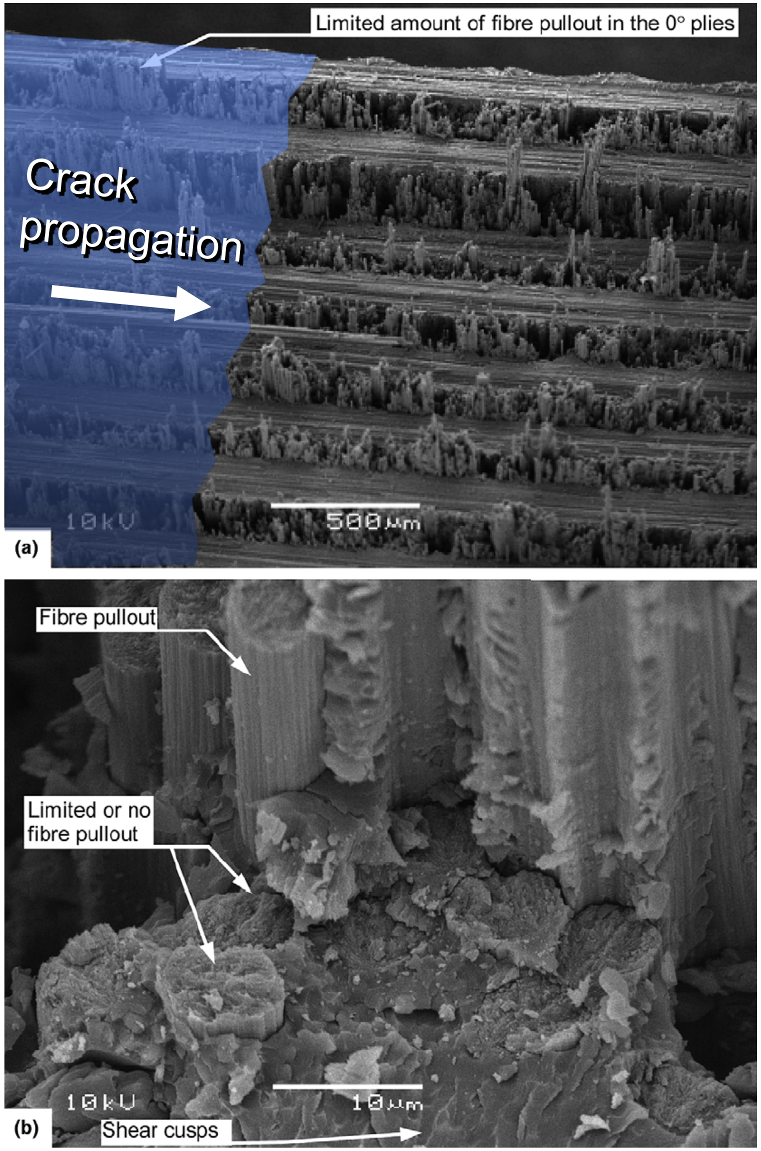


Measuring Toughness of Fiber Fracture

Compact Tension Specimen $[90/0]_{ns}$



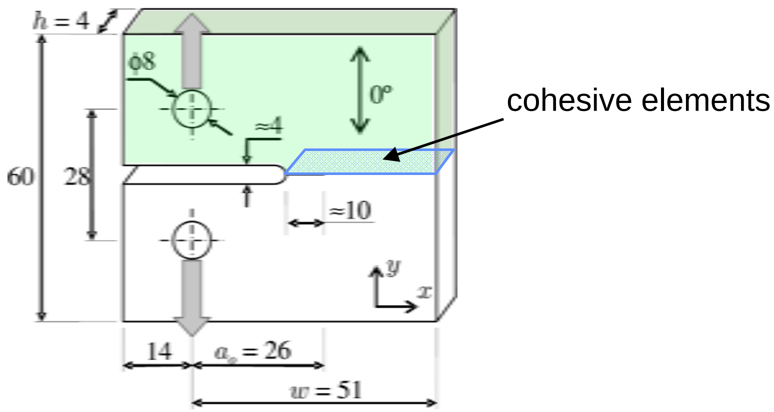




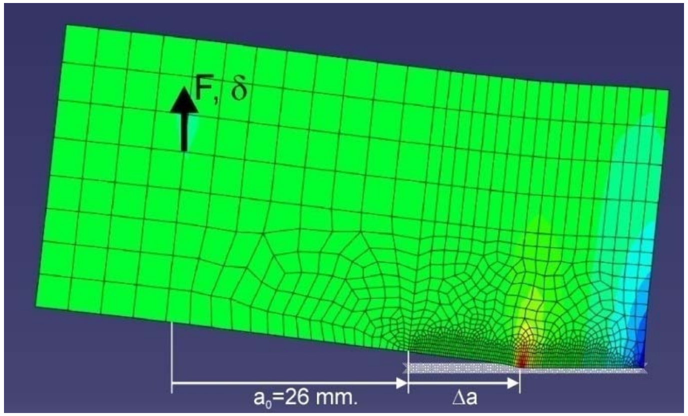
Pinho, 2006

Analysis of Compact Tension Specimen

Compact Tension Specimen

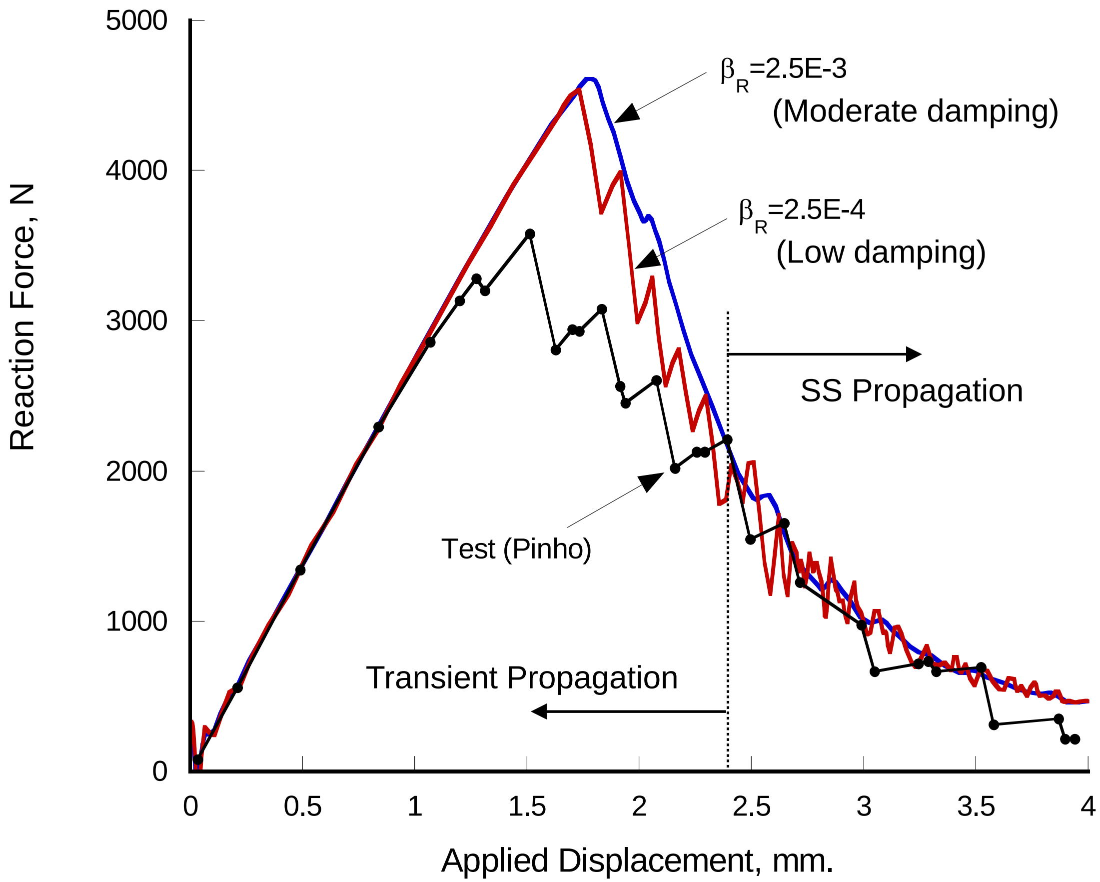


Shell model of CT Specimen



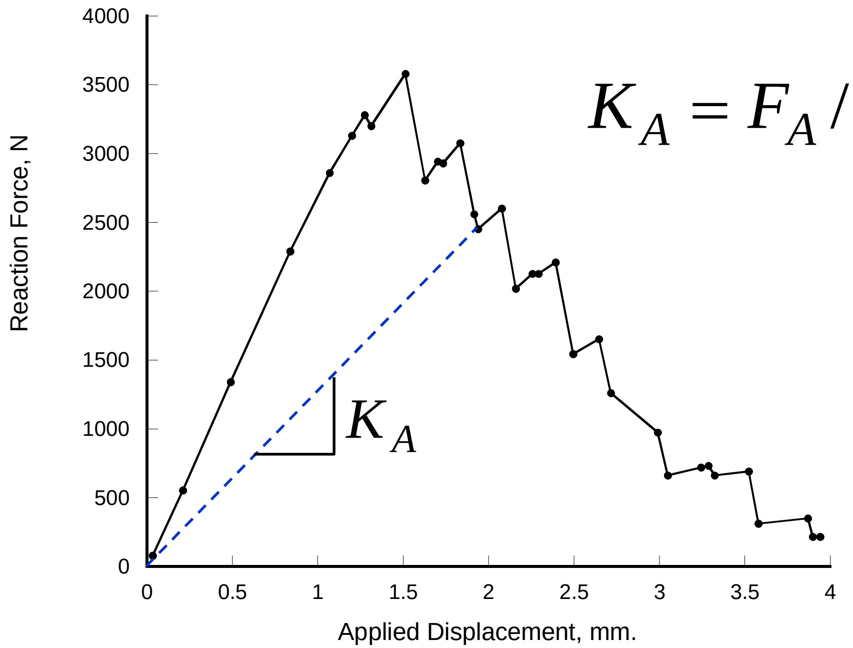
Finite Element Analysis

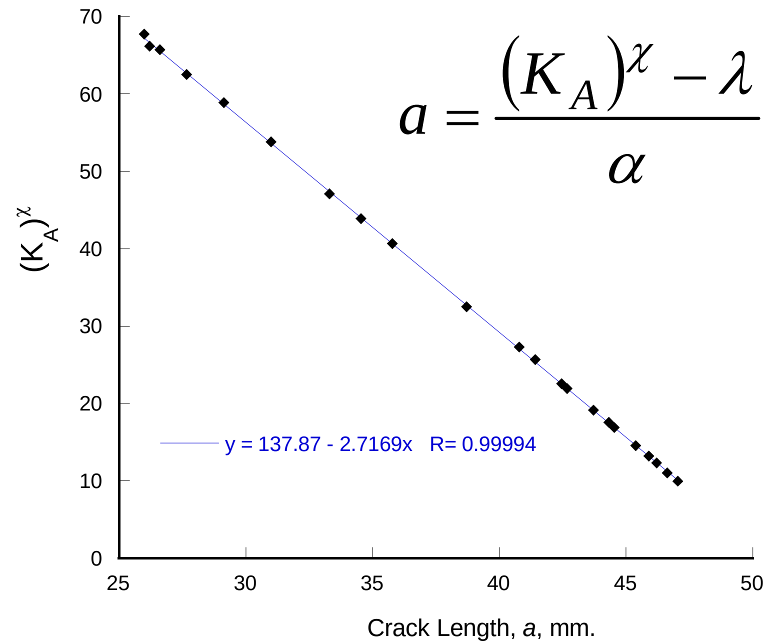
- Cohesive elements (linear softening).
- G_c=180 kJ/m^2 (from Pinho).
- Implicit dynamic analysis for improved convergence rate.
- Low and moderate Raleigh damping.



Extracting R-Curves by MCC Method

Determining crack length as a function of K_A



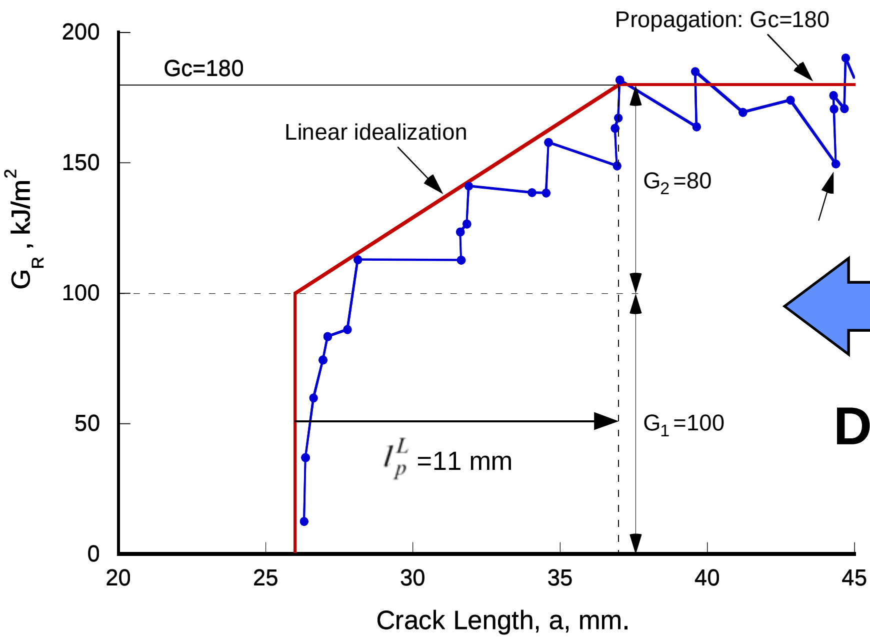


Modified Compliance Calibration

$$G_R(a) = -\frac{F_A^2}{2 \cdot t^0} \frac{\alpha(\lambda + \alpha a)^{-\left(1 + \frac{1}{\chi}\right)}}{\chi}$$

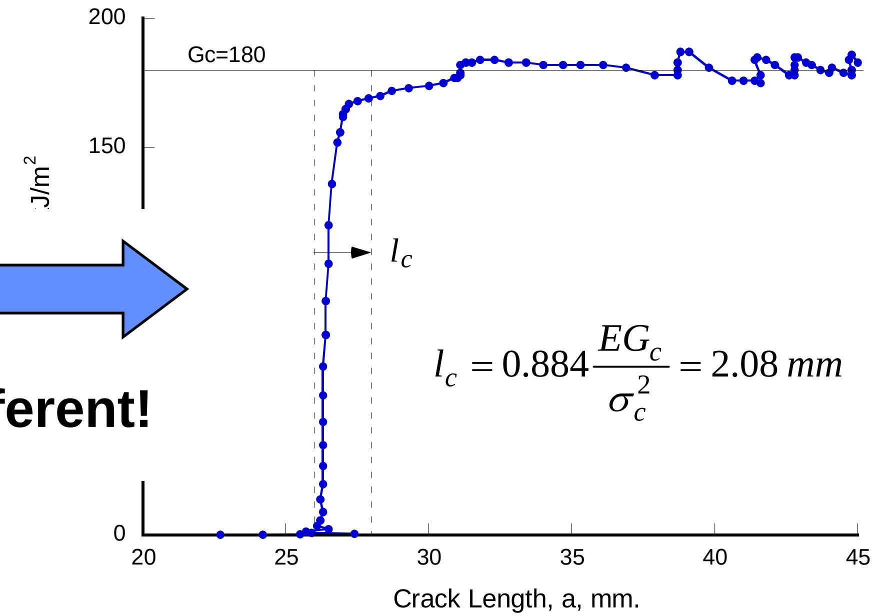
Comparing R-Curves from Test and Analysis

Experiment



Finite Element Model

Linear cohesive law w/ Gc=180 kJ/m^2



Different!

More than one linear softening law is necessary to represent experimental R-curve

Use:

G_1=100 kJ/m^2

G_2=80 kJ/m^2

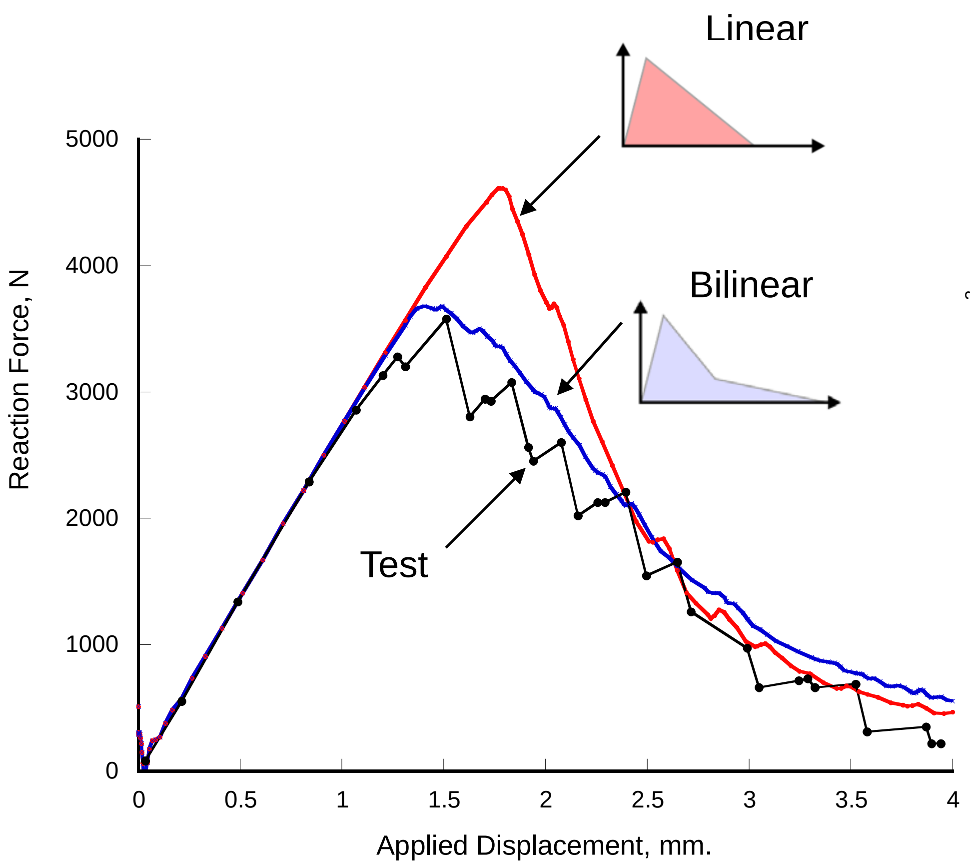
$l_p^L = 11\,\text{mm}$

Solve: n, m using G_R^L

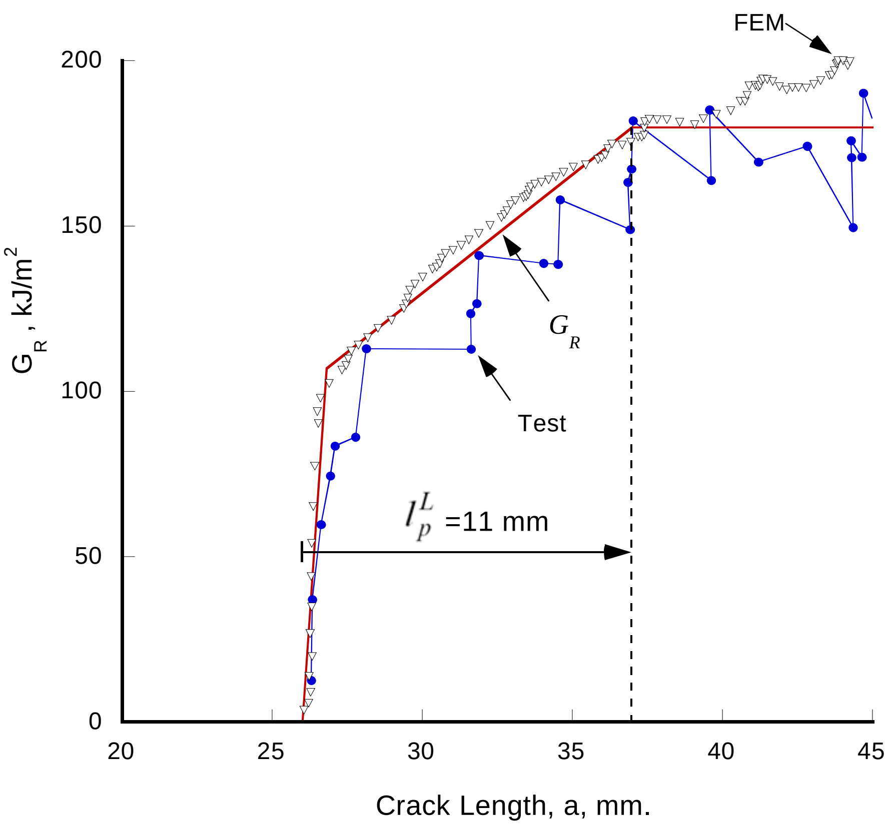
Analysis with Superposed Cohesive Elements

Solve for n from:

$$l_p^L = \frac{2}{3}\frac{1-m}{1-n}l_c \quad \Rightarrow n = 0.943$$



Comparison of R-curves



Concluding Remarks

- The importance of l_p & the R-curve, on the prediction of fracture of a composite laminate was examined.
- Two new alternate equations for the R-curve of superposed linear softening laws were proposed: $G_R^{NL}(\Delta a)$ and $G_R^{L}(\Delta a)$
- Fracture of a CT specimen was analyzed with cohesive elements.
  - A linear softening law is insufficient: fiber bridging and fiber pullout result in R-curve.
  - $G_R^{L}(\Delta a)$ was used for determining the parameters of the bilinear softening law.
- Compared to linear softening, bilinear softening reduces the error in the strength of the CT specimen from 29% to 2.8% .