Competition Policy and Agreements Between Firms

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Abstract

The European Commission recently considered abolishing the notification system and focusing on a regime of ex post investigations.

Our objective is to show that a better understanding of markets can justify this reform. When the precision of the competition authority's analysis is not high enough, an ex ante control, like the notification system, is preferable. On the other hand, if the decision accuracy improves, an ex post investigation regime achieves a higher social welfare.

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Non-Technical Summary

The European Commission (EC) recently considered updating the procedure for the detection of anti-competitive practices and abuses of dominant position (articles 81 and 82 of the European Community Treaty). This reform, initiated by the publication of a white paper (European Commission, 1999), consists of modifying the Council Regulation 17 applying since 1962. The idea is to give-up the actual notification system in favour of an *ex-post* offences repression regime and to decentralize the competition rules enforcement by strengthening the Member States authorities and jurisdiction's power.

When the Council Regulation was discussed in 1962, two options were proposed to the Member States deputies. Either they chose a directly applicable exception system (*ex-post* control) where all signed agreements were legal until they were upheld by a Court. Or they voted for an authorization regime (*ex-ante* control) with mandatory notification, where agreements were null and void unless they were approved by the EC.

The final choice was a centralized administrative authorization regime. It can be explained by the desire of the EC to understand markets' behaviour, but also for helping firms in determining their agreements' impact on the consumers' welfare. The implementation of articles 81 and 82 by the Council Regulation 17 clearly states that all firms desiring to enforce their agreements should first notify the EC. This means that firms that have signed an agreement should file and fill in forms at the EC. Then, this agreement becomes enforceable by a Court only if approved by the EC. However, the EC decision can be more or less informal, ranging from a simple administrative comfort letter which leaves the firm without any legal security (the firm can be investigated and convicted when applying) to an individual exemption published in the Official Journal of the European Communities (OJEC) which offers strong legal protection to firms.

This system allowed the EC to develop coherent law enforcement (decisions were centralized at the EC level) and to diffuse a competition culture within each of the Member States (EC directives, European Court of Justice rulings). Nevertheless, as the Council Regulation 17 imposes a response for each notification, the EC has quickly been overwhelmed with pending files: only 4 years after the Regulation came in force, more than 37450 files were pending (all types of cases included). In order to fight this crowding effect, the EC thought about different solutions to limit the number of notifications and to speed its analysis. Therefore, the EC defined more precisely its judging criteria (concept of appreciable effect on competition which allow minor cases to be removed, general notices to clarify exemption conditions, block exemption regulations) and developed the comfort letter system (informal decision which allows delays due the publication in the

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OJEC to be avoided).

However, after more than 35 years of existence, it seems that the EC has not reached its objectives with such a system of mergers and restraints control. These objectives were twofold: to guarantee an efficient protection of competition and to ensure a sufficient legal security to firms. Even though notification requests are now stable (216 cases in 1998) and decisions ruled are increasing (50 % more than in 1996), these figures mask the quality of the decisions taken by the EC. Indeed, more than 90 % of cases are closed in an informal way, not allowing the expected legal security to firms. Moreover, the efficient protection of competition has not even been achieved: in the last five years, only 0.5 % of notifications have lead to a prohibition of the agreement. An alternative interpretation could be that the agreements harmful to welfare are not notified to the EC.

All these facts pushed the EC to think of reforming the system. According to the EC, a directly applicable exception system could be more efficient in protecting Competition Law because the EC could choose which agreements to investigate (based on third party complaints for instance).

From an economic point of view, the move from a notification system to a legal exemption regime can be justified by learning about the general market structure. A better knowledge of market organisation and competition (demand functions, firms costs functions, price formation, ...) and of the impact on consumers' welfare of agreements could be such explanations. As long as the number of cases judged by the EC increases, the EC rules better and better.

However, it seems clear that this legal exemption system (*ex-post*) can correctly be implemented only when EC ruling quality is sufficient. The Council Regulation 17 that came in force in 1962 could then be seen as a first step: to collect sufficient information in order to rule better to implement the second step: a directly applicable legal exception regime with decentralized authorities.

Our objective in this paper is twofold: first, we try to provide theoretical justifications for this reform and then, we determine the optimal policy combining notifications and ex post audits.

We show that the move from a notification system to a legal exemption regime can be justified by learning about the general market structure. When it was initially created, the EC had poor knowledge of the types of agreements firms had the opportunity to sign and of their impact on consumers' welfare. As a consequence it made misjudgments. In such a context, we show that it is better to adopt a notification system. After 35 years of practice, the EC rules better and better. The decisions and judgments are of "higher quality". Therefore, it is better for its to focus on an ex post control system since using

fines, the EC is able to discourage bad agreements without deterring good ones.

Finally, using the tools available to the European Commission, we show that it is optimal to choose a mixed regime combining both types of control, even though, in most cases the optimal mixed regime looks more like a pure system. This mixed regime allows the Competition Authority to screen the different types of firms more often. To do this, the CA uses the *ex post* control as a threat to give incentives to the firms not to sign a *bad* agreement.

1　Introduction

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However, after more than 35 years of existence, it seems that the EC has not reached its objectives with such a system of mergers and restraints control. These objectives were twofold: to guarantee an efficient protection of competition and to ensure a sufficient legal security to firms. Even though notification requests are now stable (216 cases in 1998) and decisions ruled are increasing (50 % more than in 1996), these figures mask the quality of the decisions taken by the EC. Indeed, more than 90 % of cases are closed in an informal way, not allowing the expected legal security to firms. Moreover, the efficient protection of competition has not even been achieved: in the last five years, only 0.5 % of notifications have lead to a prohibition of the agreement. An alternative interpretation could be that the agreements harmful to welfare are not notified to the EC.[1]

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From an economic point of view, the move from a notification system to a legal exemption regime can be justified by learning about the general market structure. A better knowledge of market organisation and competition (demand functions, firms costs functions, price formation, ...) and of the impact on consumers' welfare of agreements could be such explanations. As long as the number of cases judged by the EC increases, the EC rules better and better. The decisions and judgements are of "higher quality". Therefore, it is interesting for the EC to focus on an *ex-post* control system because of the fines (with such an instrument, the EC will be able to discourage *bad* agreements without deterring *good* ones).

However, it seems clear that this legal exemption system (*ex-post*) can correctly be implemented only when EC ruling quality is sufficient. The Council Regulation 17 that came in force in 1962 could then be seen as a first step: to collect sufficient information in order to rule better to implement the second step: a directly applicable legal exception regime with decentralized authorities.

Contrary to the literature on regulation theory, only a few theoretical works have analysed the design of optimal competition laws and implementation issues. Cartel prosecution is probably the sole area that has been given attention by economic theory. Even though a real consensus among economist on the pro- and anti-competitive effects of vertical agreements does not exist, they usually agree that price-fixing or market-sharing

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[1]See Neven, Papandropoulos and Seabright (1998) for a detailled analysis of the Council Regulation 17, its weaknesses and for numerous applied cases.

cartels, or more generally horizontal agreements, are harmful for society. The existing theoretical literature has therefore focused on the ways to fight against cartels and on the design of optimal cartel prosecution laws.

Besanko and Spulber (89), in a static framework, look at the optimal design of antitrust policy when collusive behavior is unobservable and production costs are private information. They show that it is too costly to prevent all the cartel agreements and that an optimal antitrust policy should tolerate some collusion. To limit collusion, they consider the possibility of running a random audit to prevent the least efficient firms signing an agreement.[2]

More recently, some papers have considered the effects of possible communication between a Competition Authority (CA henceforth) and the firms. They have tried to analyse, from a theoretical point of view, how incentives to collude are modified with the introduction of leniency programs. The idea is to be more lenient towards members of cartel organisation who are willing to disclose information to the CA. In this case, the CA could trade-off between the cost of detecting and proving collusion between firms with the cost of providing incentives to reveal collusive agreements.[3]

However, the existing literature has not considered the question of competition law design or implementation issues regarding vertical agreements between firms. Many papers have analysed the welfare effects of different kinds of vertical restraints such as exclusive territories, exclusive dealing or resale price maintenance in different contexts[4], but none of them provides the means of fighting against harmful agreements.

The objective of this paper is to provide a first theoretical analysis of the different ways to deal with vertical agreements for competition authorities. In particular, this paper is a first attempt to understand the reasons which have dictated the European Commission choices in 1962, as well as today.

The paper is organised as follows: section 2 presents the framework of the analysis. In order to better understand how each regime works, we analyze sequentially the two possible options, authorization (section 3) and exception regimes (section 4), and compare their respective efficiencies in terms of social welfare. Finally in section 5, we extend the analysis and consider a mixed system combining both types of investigations, and

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[2]See also Baron and Besanko (1984), Reinganum and Wilde (1985), Pénard and Souam (1999) or Souam (2001) for studies on cartel prosecution under asymmetric information.

[3]See for example Motta and Polo (2000) or Spagnolo (2000). Rey (2000) provides a detailed presentation of competition policies design and implementation issues, and proposes a broad review of the existing litterature.

[4]See for example Caballero-Sanz and Rey (1996) and Rey (1997) for a review of the litterature on vertical restraints.

determine the optimal budget allocation between notifications treatment (*ex ante* control) and *ex post* investigations. Section 6 concludes.

2   The Framework

We consider an economy consisting of two different types of agents: firms, which have the opportunity to sign an agreement that can be either welfare improving or harmful for consumers, and a Competition Authority trying to maximise the social welfare.

2.1   Firms

There exists a continuum of firms, whose size is, without any loss of generality, normalized to 1. Each firm has the opportunity to sign an agreement. Two different types of agreements exist defined by their impact on social welfare.

A *bad* agreement is typically a contract whose object is to reduce competition and has therefore a negative impact on the consumers' welfare. This can, for example, be any kind of agreement which restricts the consumers' choice when the objective is to limit the number of suppliers available to a retailer, or when the contract includes the provision of an exclusive territory for a distributor. This framework also allows us to consider as a *bad* agreement any contract which restricts trade within the Common Market. On the contrary, a *good* agreement is a contract which increases the firms' profit, but also enhances consumers' surplus.

We assume that a proportion η_G of firms, $\eta_G \in]0, 1[$, have the opportunity to sign a *good* agreement (or type "G" agreement). If the firm signs this deal, it earns an extra profit π_G and the social welfare increases by W_G. If the firm does not eventually sign the contract, the effects on the firm's profit and social welfare are null. We denote by γ the proportion of firms that sign a *good* agreement when they have this opportunity.

On the other side, a proportion η_B of firms, $\eta_B = 1 - \eta_G$, have the opportunity to sign a *bad* agreement (or type "B" agreement). If the firm signs such an agreement, he receives an extra profit π_B, with $\pi_B > \pi_G$.[5] This agreement has a negative impact on the consumers' welfare and we assume that this negative impact more than compensates for the increase in profits, so that the total impact on social welfare is negative and denoted by $-W_B$ (with $W_B > 0$). If the firm decides not to sign the agreement, there isn't any

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[5]A *bad* agreement is more profitable from the firms' point of view than a *good* one, in the sense that it reduces competition and allows, for example, the firm to charge more easily a price higher than the marginal cost (or to the "competitive" price).

impact on profits and social welfare. We denote by β the proportion of firms that sign a *bad* agreement when they have this opportunity.

2.2 Competition Authority

The unique objective of the Competition Authority, in this model, is to maximize the social welfare. It wants to provide incentives to sign *good* agreements while deterring *bad* deals.[6] To do that, the CA has two different tools at its disposal:

- an *ex-ante* control, or authorization regime, which consists of mandatory notifications. The firms have to notify any agreement and to obtain a positive clearance before implementing it;

- an *ex-post* control, or exception regime, which consists of random investigations, after the contract has been signed and partially implemented. Ex-post audit takes place and the CA can impose fines if it decides to reject the agreement. To simplify, we assume that the collected fines are a simple transfer from the firms to the consumers without any impact on the social welfare and cannot be used by the CA to increase the auditing probabilities.

The CA can decide to use both regimes simultaneously, or to implement only one of them. In either case, the CA, due to limited budget, can only audit $n < 1$ cases (we assume the cost incurred with *ex-ante* and *ex-post* controls is identical).

2.3 Other Relevant Notations

Whatever the chosen regime, the CA has to decide whether to confirm the legality of any investigated agreement. Under the authorization regime, the CA takes the decision to allow the firms to sign the contract or not. Under the exception system, the agreement has partially been implemented and the CA has to decide whether to stop the implementation or not, and, if the case arises, to fine or not.

To take its decision, the CA investigates the case in order to collect information on the impact of the agreement on the social surplus. During the investigation, the CA receives a signal which is imperfectly correlated with the actual type of agreement. We denote by ρ the precision or informativeness of this signal, so that:

$$\Pr(signal = g \mid G) = \Pr(signal = b \mid B) = \rho, \text{ where } \frac{1}{2} \leq \rho \leq 1$$

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[6]We decide here to make no assumption on the welfare function. It could be for example the consumers' surplus, or any weighted average of the consumers' surplus and the firms' profits. The unique condition is that a *good* (resp. *bad*) agreement has a positive (resp. negative) impact on social welfare.

This signal can be seen as the probability that the CA will obtain a formal proof, or at least sufficient clues for strong suspicion, that the agreement is indeed a *good* or a *bad* one for the social welfare.

These two regimes are however different, not only because of the tools (fines are only possible in an *ex-post* regime), but also because of their impact on the Social Welfare and firms' profits.

In the authorization regime, the impact of an agreement is null as long as it has not been allowed by the CA. This is not true anymore in a legal exception world (*ex-post* investigation regime). Indeed, the investigations are made by the CA once the agreements have been signed, and therefore, the firms have been implementing for some periods these agreements. They have thus already improved or damaged the social welfare and the profits.

However, it can be argued that the effect of the agreement is not fully effective, as the CA can intervene to shut it down. We therefore assume that when the CA decides to cancel the agreement after its *ex-post* investigation, a part μ of the agreement has already been implemented (with $0 < \mu < 1$).

If the CA cancels a *good* agreement by mistake or a *bad* one with legitimacy, the impact on welfare and profits are respectively: μW_G, $\mu \pi_G$, $-\mu W_B$ et $\mu \pi_B$.

2.4   The General Timing

Firms decide whether to sign the agreement that is proposed to them, while the CA chooses its budget split between *ex-ante* filing (authorization regime) and *ex-post* investigations (legal exception), and also its fine policy. The game (G) we analyze proceeds as follows:

1. Nature chooses the type of agreement firms will be asked to sign. A proportion η_G (resp. η_B) of firms has the opportunity to sign a *good* (resp. a *bad*) agreement.

2. The Competition Authority publicly announces how its budget will be split, and its fines policy.

3. CA examines the *ex-ante* notifications filing

4. Firms decide whether to sign the agreement proposed to them

5. *Ex-post* investigations occur.

The system that prevailed prior to the reform, even if it was supposed to be a combination of investigations and notifications, looked far more like a pure notification regime because all the budget was serving the *ex-ante* policy, investigations were quasi non existent.

This justifies why we first address the analysis of the two extreme cases: a pure notification regime (all the budget is devoted to *ex-ante* control) and a pure investigation system (the CA devotes all its budget to *ex-post* repression).

3   Authorization Regime

In this section, we analyse the following pure *ex-ante* control game:

1. Nature draws the type of the agreement the firms have the opportunity to sign. This type is observed by the firms, but not by the Competition Authority. All the firms notify the agreement to the CA. In this model, we assume that the firms are obliged to notify in order to be able to implement the agreement, and the notification does not convey any relevant information to the CA.

2. The CA announces its policy towards investigated agrements, and publicly announces the decision contingent to the signal it receives during an investigation. We will hereafter denote by $\delta_N(g)$ (resp. $\delta_N(b)$) the CA's decision when the signal is *good* (resp. *bad*).

3. The CA investigates n cases. During each investigation, the CA receives a signal imperfectly correlated with the type of the agreement (ρ is the accuracy of this signal). The CA takes its decision in accordance with the previously announced policy.

4. The firms whose notification has been investigated, and which have received a positive clearance decision and those whose agreement has not been audited, sign the agreement. The profits and the welfare surplus are then realised.

Notification is compulsory for firms if they want to protect their interests. Indeed, an agreement can be sustained before a Court only if it has been notified first. In a pure notification regime, the fact of assuming that every non-notified agreement is null and void is not really constraining. As relationships between firms turn to conflict, firms can complain before a Court in order to make the contract they signed enforced.

If *ex-ante* firms easily agree on contract terms, then after the transaction has been implemented, one can find it profitable to deviate and not to honor its part of the agreement. As every contract which is not notified will be seen as null and void by Courts, firms will thus be able to insure their legal security only by fulfilling a notification.

The CA can only examine a limited number of files. This implies that some notifications will not be judged. For these files, as the CA will not have any clue that could justify its prohibition, it is thus obliged to let them pass (and the corresponding agreements will not be prohibited)[7]. Conversely, when the agreement is given attention, the CA receives a signal on the type of agreement signed and it can then use it to justify (in fact, to enforce) its decision. We assume the CA can ban an agreement whatever the signal it received $(\delta_N(.) = 0)$[8].

As *ex-post* control and fines do not exist in this setting, the firm will always find it profitable to sign the agreement proposed when it can, that is when the CA has not examined the notification or not forbidden the agreement after its examination.

When the notification file is analyzed, the CA receives a signal that is more or less accurate as to the type of the agreement. This signal allows its to revise its belief about the type of the agreement it faces. Therefore, the CA's decision depends on the expected welfare, which is evaluated using its new beliefs. If it decides to shut down the agreement, when the signal was a *good* one, the resulting welfare is null. If, conversely, it decides to let it pass, the welfare becomes:

$$
\begin{aligned}
\mathbb{E}W(g) &= \Pr(G \mid s = g)W_G - \Pr(B \mid s = g)W_B \\
&= \frac{\rho\eta_G}{\rho\eta_G + (1-\rho)\eta_B}W_G - \frac{(1-\rho)\eta_B}{\rho\eta_G + (1-\rho)\eta_B}W_B
\end{aligned}
$$

The CA authorizes an agreement with a *good* signal only if:

$$
\begin{aligned}
\mathbb{E}W(g) \geq 0 &\Leftrightarrow \rho\eta_G W_G \geq (1-\rho)\eta_B W_B \\
&\Leftrightarrow \omega \geq \widetilde{\omega}_G(\rho) \equiv \tfrac{1-\rho}{\rho}
\end{aligned}
$$

where $\omega = \frac{\eta_G W_G}{\eta_B W_B}$ represents the relative impact of *good* agreements in the economy.

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[7]In reality, firms who do not agree with the EC can go on appeal before the Court of First Instance. Judges then ask to each party to give the clues of the argument they defend. But, if the Commission shuts down an agreement without any analysis, she cannot give any clue. Therefore, a firm which agreement has been refused without any justification could surely turn to the Appeal Court in order to see his agreement approved.

[8]We assume in this case that the information she received will always motivate her first decision before the Appeal Court. Even without any formal proof of the damage of the agreement, she can always plead that the end of the file examination led her to acquire the intim conviction that this agreement is damageable for Society.

The CA's decision when it receives a *good* signal depends on the accuracy of the signal, as well as on the relative impact of *good* agreements in the economy. When the signal is very accurate (ρ close to 1) or when the *good* agreements are numerous in the economy (ω sufficiently high), a *good* signal may highly reflect a *good* agreement. The CA will thus authorize all the agreements that give a *good* signal. Nevertheless, when the signal is not accurate or the proportion of *good* agreements is low ($\omega < \widetilde{\omega}_G(\rho)$), the probability to let a *bad* agreement pass on the basis of the signal becomes too high, and the CA will forbid all agreements, even when the signal is a *good* one.

The CA's optimal policy when the received signal is a *good* one for the authorization regime is such that:

$$
\delta_N^*(b) = \begin{cases} 1 & \text{if } \omega \geq \widetilde{\omega}_G(\rho) \\ 0 & \text{if } \omega < \widetilde{\omega}_G(\rho) \end{cases}
$$

With the same methodology, when the signal is a *bad* one, the critical threshold on ω is:

$$
\widetilde{\omega}_B(\rho) \equiv \frac{\rho}{1-\rho}
$$

and the CA's decision is thus given by:

$$
\delta_N^*(b) = \begin{cases} 1 & \text{if } \omega \geq \widetilde{\omega}_B(\rho) \\ 0 & \text{if } \omega < \widetilde{\omega}_B(\rho) \end{cases}
$$

Proposition 1 *Under the Authorization Regime, all the potential agreements are signed. The optimal policy depends on the* good *agreements' relative impact on social welfare (ω) and on the signal precision (ω):*

1. *If ω is low enough ($0 \leq \omega < \widetilde{\omega}_G(\rho)$) the CA rejects all the investigated agreements ($\delta_N^*(g) = \delta_N^*(b) = 0$). The total welfare is therefore $w_N^* = (1-n)(\omega - 1) < 0$;*

2. *If ω is high enough ($\omega \geq \widetilde{\omega}_B(\rho)$) the CA authorizes all the agreements ($\delta_N^*(g) = \delta_N^*(b) = 1$) and the social welfare is $w_N^* = (\omega - 1)$*

3. *For the intermediate values ($\widetilde{\omega}_G(\rho) \leq \omega < \widetilde{\omega}_B(\rho)$), the CA rejects the investigated agreements for which the signal is* bad *($\delta_N^*(b) = 0$) and authorizes agreement with a* good *signal ($\delta_N^*(g) = 1$). The social welfare is then $w_N^* = \omega - 1 + n(\rho - (1-\rho)\omega)$*

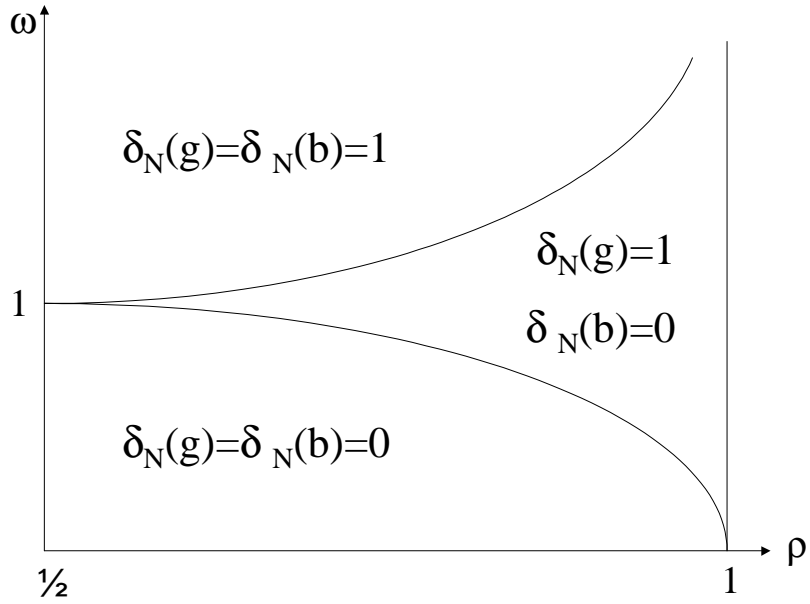These results are summarised in figure 1 :

Figure 1: Optimal Policy under Authorization Regime

The CA's decision depends on two elements: the judgement accuracy while examining the file (represented by ρ, the more accurate is the judgement, the closer to 1 is parameter ρ) and the weight of *good* agreements in the total Welfare relative to *bad* ones. When the accuracy of the CA judgement is low, it decides not to discriminate between agreements rather than to be wrong. As long as *good* agreements are numerous enough, it will decide to let them all pass (whether the signal is *good* or *bad*). It is indeed costly in such an economy to forbid agreements that might be *good*. Besides, when *good* agreements are numerous enough, they will override the damage of *bad* ones. In the same spirit, the CA decides to reject all agreements if the relative weight of *bad* agreements in the economy is too high. This policy is enforced only when the CA can afford it, that is, when it has analyzed the notification files.

However, as long as the CA's judgement is increasing in quality (this means the signal is getting more accurate), it decides to forbid agreements for which the signal is a *bad* one. This is due to the fact that an increasing ρ means the correlation between the agreement's signal and the true type of the agreement is also increasing. It then becomes optimal for the CA to reject agreements with a *bad* signal, because it is very likely that this agreement really is a *bad* agreement, or also because the likelihood of rejecting a *good* agreement is very low.

4 Exception System

In this case, the CA puts all its budget on *ex-post* control. The agreements are a priori accepted and implemented but after an ex post investigation, the CA can decide to stop the agreement. To sum up, let us present the following pure ex post game:

1. Nature draws the type of the agreement the firms have the opportunity to sign. This type is observed by the firms, but not by the Competition Authority.

2. The CA publicly announces its policy, that is if it allows or stops an agreement knowing the received signal and the level of fines. Here, we denote by $\delta_E(g), F(g)$ (resp. $\delta_E(b)$, $F(b)$) the CA's decision and the associated fine when the signal is *good* (resp. *bad*).

3. The firms decide whether to sign the agreement they can agree on.

4. CA investigates n cases and receives a signal imperfectly correlated with the type of the agreement (ρ is the accuracy of this signal). Then, the CA makes its decision in accordance with the previously announced policy.

Moreover, we assume that during the investigation the CA learns with probability equal to 1 and without any cost if an agreement has been signed or not. If an agreement has not been signed, the CA does not apply a fine. In the case of *ex post* control, the policy is divided in two parts. First, the CA has to validate or to stop the agreement. If it decides to stop the agreement, the effect of such an agreement is partially realized. Precisely, we consider that a part μ has already been realized (with $0 \le \mu \le 1$). For a *good* (resp. *bad*) agreement that is stopped, the impacts on the social welfare and on the profit of the firms are μW_G and $\mu \pi_G$ (resp. μW_B and $\mu \pi_B$). Second, it has to determine the level of fines the firm has to paid contingent to the received signal, that is $F(g)$ and $F(b)$.

4.1 Optimal Ex-Post Control

The objective of the CA is to maximize the social welfare. The first best is a competition policy which allows an equilibrium to be achieved, where all the *good* agreements are signed and never stopped (ex post) and all the *bad* agreements are not signed. Contrary to the authorization regime, the CA now uses the fine as a threat. The firms compute their expected profits knowing the announced competition policy (particularly knowing the fines they may eventually pay). If their expected profits are positive, they choose

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to sign the agreement. Otherwise, they prefer not to sign the agreement. Finally, if the received signal is sufficiently accurate, it is possible to achieve an equilibrium where only *good* signals are signed. But, such an equilibrium is costly since *good* agreements have to be stopped. For a less accurate signal, a separating equilibrium does not exist, and the CA chooses between an equilibrium where all agreements are signed and an equilibrium where no agreements are signed.

By resolving the game, we find the following proposition

Proposition 2 *Under the Legal Exception Regime, the CA's optimal policy depends on the* good *agreements' relative impact on the social welfare* (ω) *and on the signal accuracy* (ω):

1. *When the signal is accurate,* $\rho \geq \widehat{\rho}(n)$, *where* $\widehat{\rho}(n) \in \left] \frac{1}{2}; \frac{\pi_B}{\pi_B + \pi_G} \right[$ *is the unique solution to the equation:* $(1 - \rho) \pi_B - \rho \pi_G = (1 - \mu) \rho (1 - \rho) n (\pi_B - \pi_G)$ *and when the* good *agreements' relative impact on social welfare is:*

   (a) *high:* $\omega \geq \widehat{\omega}(\rho, n) \equiv \dfrac{1}{(1 - \mu) n (1 - \rho)}$ *then the CA validates all the agreements. All the firms sign the agreements and the associated social welfare is* $w_E^* = \omega - 1$.

   (b) *low:* $\omega < \widehat{\omega}(\rho, n)$ *then the CA only validates the agreements for which it receives a* good *signal. If it receives a* bad *signal it stops the agreements and the firm is fined. The fine is equal to* $F^*(m)) = \dfrac{(1 - (1 - \mu) \rho n) \pi_B}{\rho n}$. *With that policy, the achieved equilibrium is such that only* good *agreements are signed. The associated social welfare is* $w_E^* = (1 - (1 - \mu)(1 - \rho) n) \omega$.

2. *When the precision of the signal is low,* $\rho < \widehat{\rho}(n)$, *and when the* good *agreements' relative impact on social welfare is:*

   (a) *high:* $\omega \geq \widetilde{\omega}(\rho)$, *then the CA validates all the agreements. All the firms sign the agreements and the associated social welfare is* $w_E^* = \omega - 1$.

   (b) *low:* $\omega < \overline{\omega}(\rho, n) \equiv \dfrac{1 - (1 - \mu) \rho n}{1 - (1 - \mu) n (1 - \rho)}$ *then the AC chooses to stop all agreements whatever the received signal. Moreover it chooses a level of fines sufficiently high such that no agreement is signed at the equilibrium. The associated social welfare is thus equal to 0.*

   (c) *intermediate:* $\overline{\omega}(\rho, n) \leq \omega \leq \widetilde{\omega}(\rho)$. *In that case, the AC validates the agreements when it receives a* good *signal and stops them when it receives a* bad *one. But, it chooses a sufficiently low fine (for example* $F = 0$) *such that all agreements are signed in equilibrium. The associated social welfare is thus* $w_E^* = (\omega - 1) + (1 - \mu) n (\rho - (1 - \rho) \omega)$.

The agreements signed at the equilibrium are summarized in the following figure
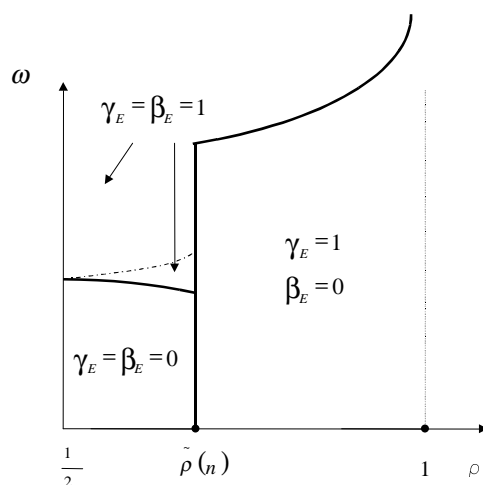

Figure 2: Firms' Optimal Choices under Legal Exception Regime

When the signal is sufficiently accurate, the CA can, by choosing an appropriate level of fine, induce an equilibrium where only *good* agreements are signed. But such a policy is costly in term of social welfare. Indeed, at the equilibrium, the CA has to stop agreements for which it receives a *bad* signal, even if it knows that such agreements are in reality *good*. This is due to the fact that the signal is not perfectly accurate ($\rho \leq 1$). Therefore, when the *good* agreements' relative impact on the social welfare is low, this social cost is also low, and is more than compensated by the fact that no *bad* agreements are signed. Thus, the CA chooses to implement such an equilibrium. On the other hand, when the *good* agreements' relative impact on the social welfare is high, this social cost becomes too important and the CA prefers not to stop any agreements whatever the signal it receives. Thus, at the equilibrium all the agreements are signed.

When the signal is not sufficiently accurate, it becomes impossible to achieve a separating equilibrium where only *good* agreements are signed. If the *good* agreements are signed, the *bad* ones are also signed. Since it is never optimal to achieve an equilibrium where only *bad* agreements are signed, the CA has to choose between an equilibrium where no agreement is signed and an equilibrium where all the agreements are signed. On the one hand, when the *good* agreements' relative impact on the social welfare is low, it becomes more costly in terms of social welfare to allow a *bad* agreement rather than to stop a *good* one. The CA thus chooses a level of fines sufficiently high such that at

the equilibrium no agreement is signed. On the other hand, when the *good* agreements' relative impact on the social welfare is high, the CA chooses a level of fines sufficiently low (for example $F = 0$) such that all the agreements are signed at the equilibrium. It becomes more costly to stop a *good* agreement rather than to allow a *bad* one. But, we have to distinguish between two cases. When the *good* agreements' relative impact on the social welfare is really high, the CA prefers not to intervene and never stop an agreement after an investigation whatever the signal it receives. But, for intermediate values taken by the *good* agreements' relative impact it decides to achieve an equilibrium where all the agreements are signed but it stops agreements when it received a *bad* signal on it after investigation.

4.2 Authorization or Legal Exception Regime

The European Commission wishes to modernize its firms' agreements control system. It proposes to shift from a notification system to a legal exception regime. Thanks to the study of the two pure regimes, we can try to explain this decision. The idea is to compare the efficiency of the two kinds of system, by comparing the social welfare in the two systems. The results are summarized in the following proposition:

Proposition 3 *The choice by the CA of a pure system depends on the* good *agreements' relative impact on the social welfare* (ω) *as well as on the signal accuracy* (ρ).

1. *When the signal is sufficiently high* $\rho \geq \widehat{\rho}(n)$ *and the* good *agreements' relative impact on the social welfare is:*

   (a) *high:* $\omega \geq \widehat{\omega}(\rho, n)$, *both regimes are equivalent.*

   (b) *low:* $\omega < \widehat{\omega}(\rho, n)$, *the legal exception regime is more efficient.*

2. *When the signal is sufficiently low* $\rho < \widehat{\rho}(n)$ *and the* good *agreements' relative impact on the social welfare is:*

   (a) *high:* $\omega \geq \widetilde{\omega}_M(\rho)$, *both regimes are equivalent.*

   (b) *low:* $\omega < \overline{\omega}(\rho, n)$, *the legal exception regime is more efficient.*

   (c) *intermediate:* $\overline{\omega}(\rho, n) < \omega \leq \widetilde{\omega}_M(\rho)$, *the authorization regime is more efficient.*

Proof. see Appendix B. ∎

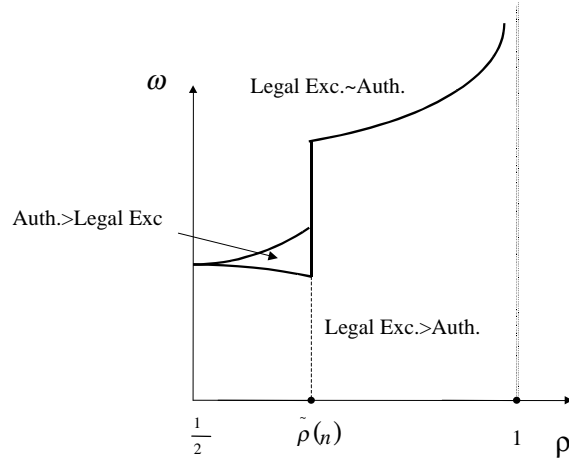Figure 3 summarizes the results of the previous proposition.

Figure 3: Comparison of the Pure Regimes

When the *good* agreements' relative impact on the social welfare is sufficiently high, the CA allows all the agreements whatever the system. Both system are then equivalent since the CA plays no role at all.

When the signal is sufficiently accurate $(\rho \geq \widehat{\rho}(n))$, the comparison between the two pure systems depends on the *good* agreements' relative impact on the social welfare. On the one hand, when this impact is low, all the notifications which are considered are forbidden. But, the agreements whose notification has not been considered are realized which is not welfare improving. On the other hand, with a legal exception regime, by choosing an appropriate level of fine, the CA achieves an equilibrium where only *good* agreements are signed. In such a case, the legal exception regime dominates the authorization regime in term of social welfare.

When the impact of *good* agreements is intermediate, the optimal policy for a notification regime is to prohibit agreements whose signal is *bad* and to allow agreements whose signal is *good*. But, since all the notifications are not considered, agreements whose signal would have been *bad* are not forbidden which is *bad* for the social welfare. On the contrary, with a legal exception regime only *good* agreements are signed. Even if the ex post control is also costly (since *good* agreements whose signal is *bad* are stopped) this cost is more than compensated by the fact that only *good* agreements are signed. Therefore, the legal exception regime dominates the authorization regime.

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When the signal is less accurate ($\rho < \widehat{\rho}(n)$), it is not possible to achieve a separating equilibrium where only *good* agreements are signed. When the *good* agreements' relative impact is low, the social welfare is negative at the equilibrium with a notification regime, since all the notification which are considered are prohibited but not those which cannot be considered. On the other hand, thank to the fine, it is possible to achieve an equilibrium where no agreements are signed with a legal exception regime. Thus, the ex post control system dominates the ex ante one.

Finally, let us consider the case where the *good* agreements' impact is intermediate. In that case the optimal policies are the same with the two systems. Agreements whose signal is *good* are allowed and agreements whose signal is *bad* are prohibited (or stopped). But with an ex post control system, these kind of agreements have been partially realized which is welfare decreasing. Therefore, in such a case, the notification regime dominates the legal exception regime.

5   Optimal Mixed Regime

Up to now, we have only analysed "pure" audit systems. We supposed that the CA had only the choice between two regimes, and had to choose between investigating either before the agreements could be signed (authorization regime), or once they have been signed and partially implemented (exception system).

Even though in practice, the implementation of the regulation 17/62 looks like a pure authorization regime (the European Commission has been quickly overwhelmed by thousands of notifications and only few agreements have been investigated *ex post* since 1962), this regulation is, at least in theory, an intermediate system. The firms are indeed required to notify any agreement, but the European Commission can always, at least in theory, allocate part of its budget to run investigations (*ex post* audits).

The objective of this section is to determine the optimal mixed regime, and, in particular, the optimal budget sharing rule, between *ex ante* and *ex post* control. The general timing of the game (G) is then:

1. Nature determines the type of agreement each firm has the opportunity to sign;

2. The Competition Authority decides its budget allocation and its policy (decisions taken after investigations). During the *ex post* auditing stage, the CA faces three types of firms: those whose notification has been considered and for whose the CA has received a *good* (type g) or a *bad* (type b) signal, and those whose file has not

been investigated *ex ante* (type $\emptyset$). The CA has therefore to decide a type contingent policy, and so announces:

(a) the budget sharing rule between *ex ante* $(1 - \alpha)$ and *ex post* (α) control

(b) the type contingent sharing rule of the budget allocated to *ex post* control

$$\alpha(m) + \alpha(b) + \alpha(\emptyset) = \alpha$$

(c) the decisions taken during the *ex ante* auditing stage as functions of the signal, $\delta_N(g)$ et $\delta_N(b)$

(d) the decisions taken during the *ex post* auditing stage (as functions of the audited firm's type (g, b or $\emptyset$), of the signal (b or g)), and the corresponding fines (only when the decision is to prohibit the agreement), that is $\delta_E(b)$ and $F(b)$, $\delta_E(m)$ and $F(m)$, $\delta_E(\emptyset, b)$ and $F(\emptyset, b)$, $\delta_E(\emptyset, m)$ and $F(\emptyset, m)$

3. The CA considers $(1 - \alpha)n$ notifications. If the CA decides to prohibit a notified agreement ($\delta_N(g$ or $b) = 0$), the firm cannot sign it and its profits are therefore null

4. The firms decide whether to sign the agreement

5. The CA investigates (*ex post)* αn cases (among the three types of firms, according to the policy announced at stage 2). If the CA investigated a notification which has already been considered *ex ante* (type g or b), the signal received during the investigation is perfectly identical to the signal received *ex ante*.[9]

Let us first consider the budget sharing rules $(\alpha(g), \alpha(b), \alpha(\emptyset))$ and the decision to be taken during *ex ante* auditing stage $(\delta_N(g), \delta_N(b))$ as given, and determine the optimal *ex post* decisions (and possible fines).

Type g firms are those whose notification has been investigated *ex ante* and for which the CA received a *good* signal. Since the CA does not receive any new information during the *ex post* audit, it is never possible for the CA to implement a separating policy such that the *good* firms only will actually sign an agreement. Indeed, either the CA decides not to prohibit ($\delta_E(g) = 1$) and both types of firms (*good* and *bad*) sign, or it decides to prohibit the agreement ($\delta_E(g) = 0$) and to impose a fine $F(g)$ when it discovers that an agreement has actually been signed and the firms' profits are thus:

$$
\begin{aligned}
\mathbb{E}\pi_G &= (1 - (1 - \mu)p_E(g))\pi_G - p_E(g)F(g) \\
\mathbb{E}\pi_B &= (1 - (1 - \mu)p_E(g))\pi_B - p_E(g)F(g)
\end{aligned}
$$

---

[9]We assume that the CA does not receive any new information during a second audit. Due to this assumption, we decided to simplify notations and use, for example, $\delta_E(b)$ instead of $\delta_E(b, b)$.

where $p_E(b)$ denotes the probability of being investigated during the *ex post* stage (this is a function of $\alpha(g)$ and of the number of firms which signed an agreement). It is easy to check that, as the fine increases, the *good* firms will be the first to give up the opportunity of signing an agreement. The CA has therefore the choice between two policies: a sufficiently low fine (0 for example) such that both types of firms sign, or a very high fine in order to ensure that none them are willing to sign.[10]

Since the CA does not learn anything during a second investigation, it cannot revise its beliefs. These beliefs are therefore:

$$\Pr(G \mid s = g) = \frac{\rho \eta_G}{\rho \eta_G + (1 - \rho)\eta_B}$$

If the CA decide to authorize this agreement, the expected welfare (induced by this type of firms) is:

$$\mathbb{E}W(g) \simeq \omega - 1$$

On the contrary, if the CA decides to prohibit all the agreements and to impose a sufficiently large fine such that no firm is willing to sign, the welfare is null. Therefore, it is optimal to provide incentives to sign to all the firms (of this category g) or to none of them. It would then be optimal to apply this policy during the *ex ante* stage rather than spending money to investigate the same firm twice. The analysis being identical for the type b firms (those already investigated *ex ante* for which the signal was *bad*), this leads to the following lemma:

Lemma 4 *For a given budget sharing rule $(1 - \alpha, \alpha)$ the optimal policy is:*

1. *the CA never audits* ex post *a firm whose notification has been investigated* ex ante, *i.e. $\alpha(b) = \alpha(m) = 0$,*

2. *the optimal* ex ante *decisions ($\delta_N(g)$ and $\delta_N(b)$) are the same as in the pure authorization regime (i.e. $\alpha = 0$) and are therefore given by proposition 1.*

Let us now consider the firms whose notification has not been analyzed by the CA during the *ex ante* auditing stage. From the Competition Authority's point of view, this situation is identical to the pure exception regime, except that the number of firms involved in the auditing process (but not the proportions of *good* and *bad* firms) and the

---

[10]An intermediate fine would induce a separating equilibrium in which only the bad agreements are signed. In this case, the social welfare is negative and this policy is always dominated by the severe policy ensuring that no firm is willing to sign (leading to a null welfare).

number of cases the CA can investigate are different. To determine the optimal *ex post* decisions, we now have to consider the analysis proposed in the previous section replacing the auditing probability (n) by:

$$p_E(\alpha) = \frac{\alpha n}{1 - n + \alpha n}$$

For a given budget sharing rule $(1 - \alpha, \alpha)$, the optimal decisions $\delta_E(\emptyset, g)$ and $\delta_E(\emptyset, b)$ (and the corresponding fines) are given by the results of the proposition 2, in which it suffices to replace the *ex post* auditing probability (n) by its new value ($p_E(\alpha)$).

It remains to compute the optimal budget sharing rule. The computations lead to the following propositions (the proofs have been relegated to appendix C):

Proposition 5 *When the signal received by the CA during an investigation is sufficiently accurate, $\rho \geq \widehat{\rho}(0) = \frac{\pi_M}{\pi_M + \pi_B}$, and the* good *firms' relative weight is :*

1. *higher than $\frac{1}{(1-\mu)(1-\rho)}$, the CA decides to focus on notifications while keeping a strictly positive probability of* ex post *control, i.e. $\alpha^* = 0^+$.*

2. *less than $\frac{1}{(1-\mu)(1-\rho)}$, the optimal choice is the pure exception regime, i.e. $\alpha^* = 1$.*

When the signal is sufficiently accurate, the CA is able to screen the types of agreements actually signed, whatever the budget sharing rule. In other words, the CA can choose any budget allocation and ensure that only the *good* firms are willing to sign ($\gamma_E(\emptyset) = 1$ and $\beta_E(\emptyset) = 0$). Since the decisions taken during the *ex ante* auditing stage (notifications) do not depend on the share of the budget allocated to this type of control (see lemma 4), the budget choice only affects the *ex post* auditing stage and the type of agreement signed by the firms whose notification has not been investigate *ex ante.*

The CA can always ensure that the *bad* contracts are not signed, but this policy is not costless. The CA has to threaten the firms to prohibit the agreement and impose a large fine when it receives a *bad* signal ($\delta_E(\emptyset, b) = 0$). Since the signal is imperfectly correlated with the true type, a number of the *good* agreements will thus be prohibited. This error has a negative impact on the total welfare as these contracts have only been partially implemented ($\mu < 1$). A way to limit this social cost is to decrease the probability of audit, thereby increasing the associated fine. However, this policy can be welfare enhancing only if the relative weight of the *good* agreements in the economy is sufficiently important, as we also need to take into account the expected welfare effects of the *ex ante* control.

When the relative weight of the *good* agreement is large, the CA never prohibits any agreement *ex ante.* All the notified contracts analysed during this stage are therefore

authorized and signed, and this include a number of *bad* agreements. Nevertheless, since the number of *good* agreements is relatively large, it is preferable to let some *bad* firms sign (those whose notification has been investigated *ex ante,* and have therefore been authorized), rather than prohibiting a large number of *good* agreements during the *ex post* auditing stage. The CA thus decides to focus on the notifications. However, it has to keep a minimum share of the budget available for the *ex post* stage $(\alpha = 0^+)$ in order to maintain a strictly positive probability of audit, so that the threat is still credible. In this case, the CA only exists to let the threat of prohibition hang over the *bad* firms, which eventually decide not to sign, but (almost) never fines.

On the contrary, when the relative weight of the *good* agreement is low, the CA prefers to give the whole budget to the *ex post* control $(\alpha = 1)$. In this case, the notification stage is useless as the CA will never consider any of the files. During the *ex post* auditing, the CA prohibits and fines as long as the signal it receives is *bad*. In this case, both types of firms prefer not to sign, but the loss implied by the absence of *good* agreements is more than compensated by the elimination of any *bad* agreements.

Proposition 6 *When the signal is inaccurate, $\rho < \widehat{\rho}(n)$, and the* good *firms' relative weight is:*

1. *lower than $\widetilde{\omega}_G(\rho)$, the CA is indifferent between any sharing rule as long as the probability of audit* ex post *is strictly positive, i.e. $\alpha^* > 0$.*

2. *higher than $\widetilde{\omega}_B(\rho)$, all the regimes are perfectly equivalent.*

3. *between $\widetilde{\omega}_G(\rho)$ and 1, the CA decides to focus on notifications while keeping a strictly positive probability of* ex post *control, i.e. $\alpha^* = 0^+$.*

4. *between 1 and $\widetilde{\omega}_B(\rho)$, the CA chooses a pure authorization regime, i.e. $\alpha^* = 0$.*

When the signal is inaccurate, the CA can no longer screen the two types of contracts, even if it spends all of the budget on the *ex post* control. The possible continuation equilibria are now pooling situations: both types of contracts or none of them are signed.

When the relative weight of the *good* agreements is low, both types of control have the same effects. Whatever the type of investigation (*ex ante* or *ex post* audit) and whatever the signal received, the CA prohibits the agreements and imposes a large fine, if possible. In this case, no contract is signed. The firms whose notification has been considered *ex ante* are not allowed to sign, and the others prefer not to bear the risk of being fined. The budget allocation between the two types of control does not affect the social welfare, as

long as the threat of fine *ex post* is credible, that is as long as the probability of audit *ex post* remains strictly positive. The CA is therefore indifferent between any allocation such that a strictly positive share of the budget is allocated to the *ex post* control ($\alpha^* > 0$).

On the contrary, when the relative weight of the *good* agreements is sufficiently large, the CA never prohibits any agreement. In this case, all firms sign and the budget sharing rule does not affect the welfare. In this case, the CA plays no role as any investigated agreement obtains clearance.

Finally, for the intermediate values, the situation is quite different. During the *ex ante* auditing stage, the CA prohibits only the agreements for which it receives a *bad* signal. On the other hand, during the *ex post* stage, it can implement two different policies: either it threatens to impose a large fine when the signal is *bad* and all firms prefer not to sign, or it decides to do nothing and let all firms sign. In this last case, we can notice that the budget sharing rule does not affect the welfare created by this *ex post* stage. Since the welfare created by the *ex ante* auditing stage is a strictly increasing function of the probability of audit (and thereby of the budget spend during this stage), it is preferable to focus on the notifications ($\alpha^* = 0$). The analysis is similar in the first case. As the policy implemented *ex post* is such that each firm decides not to sign rather than to bear the risk of being fined, the welfare is not affected by the budget sharing rule as long as the probability of audit *ex post* remains strictly positive (associated with an increasing fine). The welfare created by the *ex ante* auditing stage is again an increasing function of the budget spent on this type of control. The optimal choice is therefore to maximise the share of the budget allocated to the *ex ante* control, while keeping a strictly positive share to maintain a credible threat *ex post* ($\alpha^* = 0^+$).

Proposition 7 *For intermediate values of the signal's accuracy, $\widehat{\rho}(n) \leq \rho < \widehat{\rho}(0)$, and when the* good *firms' relative weight is:*

1. *higher than $\frac{\rho(\pi_B - \pi_G)}{\pi_B - \rho(\pi_B + \pi_G)}$, all the possible regimes are perfectly equivalent.*

2. *lower than $\frac{1}{(1-\mu)(1-\rho)}$, the CA prefers the pure exception regime, i.e. $\alpha^* = 1$.*

3. *between $\frac{1}{(1-\mu)(1-\rho)}$ and $\frac{\rho(\pi_B - \pi_G)}{\pi_B - \rho(\pi_B + \pi_G)}$, the CA chooses a budget sharing rule function of the signal's accuracy such that $\alpha^*(\rho) = \widehat{\alpha}(\rho)$, where*

$$\widehat{\alpha}(\rho) = \frac{(1-n)\left((1-\rho)\pi_B - \rho\pi_G\right)}{\left((1-(1-\mu)\rho)(1-\rho)\pi_B + (1-(1-\mu)(1-\rho))\rho\pi_G\right)n}$$

This last zone is much more complicated as the effect of the budget allocation (α) is twofold: firstly, the choice between *ex ante* and *ex post* control directly affects the welfare

through the auditing and prohibition probabilities. Moreover, the budget allocated to the *ex post* control modifies the choices for the firms whose notification has not been considered by the CA during the *ex ante* auditing stage ($\gamma_E(\emptyset)$ and $\beta_E(\emptyset)$). Contrary to the accurate signal case ($\rho \geq \widehat{\rho}(0)$), it cannot allocate only a small share of the budget to the *ex post* control in order to induce a separating equilibrium. If it has decided to ensure that only the *good* contracts are actually signed, the CA now needs to allocate to the *ex post* control a share of the budget at least equal to $\widehat{\alpha}(\rho)$. In this case, in order to implement a separating equilibrium ($\gamma_E(\emptyset) = 1$ and $\beta_E(\emptyset) = 0$), the CA will have, during the *ex post* auditing stage, to prohibit and fine some of the *good* agreements. It therefore faces a trade-off between ensuring that the *bad* agreements are not signed and limiting the number of *good* agreements that are prohibited.

When the relative weight of the *good* agreements is low, implementing a separating equilibrium is not very costly, and the CA prefers to minimise the number of *bad* agreements actually signed. The optimal choice is therefore to adopt the pure exception regime ($\alpha^* = 1$).

As the relative weight of *good* agreements increases, the social cost of *ex post* prohibition increases and the CA prefers to limit this cost, though it allows some *bad* agreements to be signed. The CA therefore decides to implement a mixed policy, combining *ex ante* and *ex post* investigations. It then sets the respective shares of budget allocated to these two types of control at a minimum level such that, among the firms whose notification has not been investigated *ex ante*, only the *good* sign, that is $\alpha^*(\rho) = \widehat{\alpha}(\rho)$.

Finally, when the relative weight of the *good* agreements is sufficiently high, it is preferable to authorize all the agreements, either *ex ante* or *ex post.* The budget sharing rule does not matter in this case, as the CA plays no role.

The following figure summarizes these results:

6 Conclusions

This paper justifies, from a theoretical point of view, the reform of the regulation 17,defining the rules implementing the European Competition Policy towards agreements between firms. After a learning process allowing the Competition Authority to improve its knowledge and the accuracy of its decisions through a pure authorization regime, as implemented in the European Union since 1962, it becomes optimal to move away from this highly administrative regime to adopt an exception system based on *ex post* control. Moreover, using the tools available to the European Commission, we show that it is optimal to choose a mixed regime combining both types of control, even though, in most cases the optimal
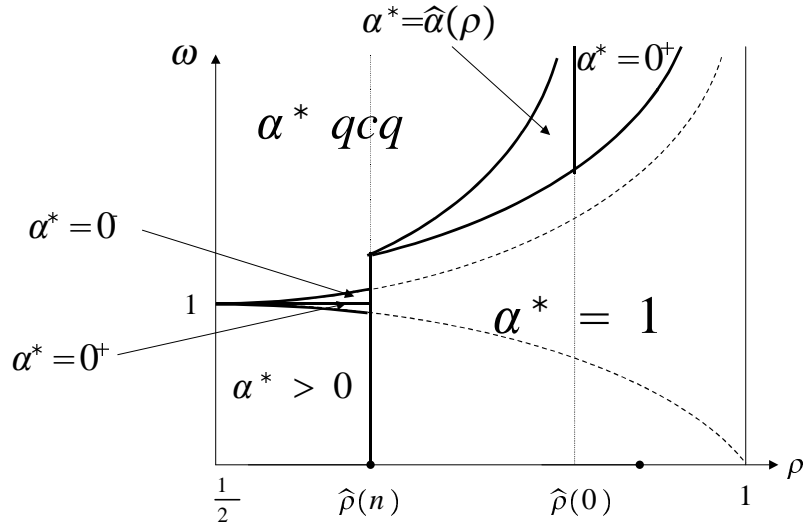
Figure 4: Optimal Budget Sharing Rule

mixed regime looks more like a pure system. This mixed regime allows the Competition Authority to screen the different types of firms more often. To do this, the CA uses the *ex post* control as a threat to give incentives to the firms not to sign a *bad* agreement.

The remaining question relates to the possible generalization of this analysis. Could it be possible to propose a more general policy, possibly using new tools, in order to improve the efficiency of the Competition Authority? It could easily be shown that using our framework, the Competition Authority would be more efficient, if it was allowed to prohibit any agreement during the *ex ante* period, even those which have not been investigated. On the other hand, if the CA was allowed to use fines and bonuses (negative fines) in both types of control, it would always be possible to implement a separating equilibrium. It therefore seems thereby possible to develop the analysis further to look for the optimal competition law. However, this optimal regime needs unrealistic tools, as the Competition Authority needs to be able to prohibit an agreement without any argument (that is without investigating the case) or to subsidize some firms.

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A   Proof of Proposition 2

A.1   Continuation Equilibria

The CA has the choice between four policies.

A.1.1 $\delta_E(g) = \delta_E(b) = 1$

If the CA decides to authorize all the agreements after investigation, all the firms sign and the (normalised) social welfare is then $w_E^1(1,1) = \omega - 1$.

A.1.2 $\delta_E(g) = 0$, $\delta_E(b) = 1$

The CA authorizes all investigated agreements for which it receives a *bad* signal and rejects the other ones. The firms' expected profits (if they decide to sign the agreement) are thus:

$$\mathbb{E}\Pi_B = (1 - (1-\nu)(1-\rho)n)\pi_B - n(1-\rho)F(g)$$

$$\mathbb{E}\Pi_G = (1 - (1-\nu)\rho n)\pi_G - n\rho F(g)$$

and we have:

$$\beta_E = 0 \Leftrightarrow F(g) \geq \frac{(1 - (1-\nu)(1-\rho)n)\pi_B}{n(1-\rho)} \equiv A \tag{1}$$

$$\text{et } \gamma_E = 0 \Leftrightarrow F(g) \geq \frac{(1 - (1-\nu)\rho n)\pi_G}{n\rho} \equiv B \tag{2}$$

Since $\rho \in \left[\frac{1}{2}; 1\right]$, $\rho \geq 1 - \rho$ and as $\pi_B > \pi_G$, $A > B$. Therefore, $\beta_E = 0$ implies $\gamma_E = 0$.

The possible continuation equilibria are then:

1. For any $F(g) > A$, $\gamma_E = \beta_E = 0$

   With sufficiently high fines ($F(g) > A$), the CA ensures that none of the agreements are signed. The social welfare is therefore null: $w_E(0,0) = 0$. This shows that it is always possible for the CA to secure a non negative social surplus (there exist other ways to implement this continuation equilibrium, but in all cases the social surplus is null). For this reason, it is never optimal for the CA to implement continuation equilibria leading to a negative social surplus, such as for example $\gamma_E = 0$ and $\beta_E = 1$. We will therefore never consider this equilibrium in the following analysis.

2. For any $F(g) \leq B$, $\gamma_E = \beta_E = 1$

   With fines low enough ($F(g) < B$), the CA ensures that all the agreements are signed. The social welfare is then:

   $$\begin{aligned} w_E^2(1,1) &= \omega - 1 - (1-\nu)n(\rho\omega - (1-\rho)) \\ &= (1 - (1-\nu)n\rho)(\omega - 1) - (1-\nu)n(2\rho - 1) \end{aligned}$$

We can notice than this case is always dominated. Indeed, if ω is lower than 1, this strategy is dominated by any equilibrium leading to $(\gamma_E, \beta_E) = (0,0)$. If ω is higher than 1, then the CA prefers to authorize all the agreements $(\delta_E(g) = \delta_E(b) = 1)$.

A.1.3 $\quad \delta_E(g) = \delta_E(b) = 0$

In this case, the CA decides to reject and fine all the investigated agreements, and the social welfare is therefore:

$$\mathbb{E}\Pi_B = (1 - (1 - \nu)n)\,\pi_B - n\,(\rho F(b) + (1 - \rho)F(g))$$
$$\mathbb{E}\Pi_G = (1 - (1 - \nu)n)\,\pi_G - n\,((1 - \rho)F(b) + \rho F(g))$$

and we have

$$\beta_E = 0 \Leftrightarrow \rho F(b) + (1 - \rho)F(g) \geq \frac{(1 - (1 - \nu)n)\,\pi_B}{n} \equiv C \tag{3}$$

$$\text{et } \gamma_E = 0 \Leftrightarrow (1 - \rho)F(b) + \rho F(g) \geq \frac{(1 - (1 - \nu)n)\,\pi_G}{n} \equiv D \tag{4}$$

In this case, two types of (relevant) continuation equilibria, $(\gamma_E, \beta_E) = (1,1)$ and $(1,0)$, may arise.

1. $\gamma_E = \beta_E = 1$

   With sufficiently low fines (for example equal to 0), all the agreements are signed. The social welfare is then:

   $$w_E^3(1,1) = [1 - (1 - \nu)n](\omega - 1)$$

   This case is never optimal, since for the same reasons than the previous one, it is always dominated.

2. $\gamma_E = 1$ and $\beta_E = 0$

   In this case, only the *good* agreements are signed. To implement this continuation equilibrium, the CA has to set fines such that

   $$\rho F(b) + (1 - \rho)F(g) \geq C \text{ and } (1 - \rho)F(b) + \rho F(g) \leq D$$

   This is possible if and only if the signal accuracy is high enough:

   $$\rho > \frac{\pi_B}{\pi_B + \pi_G}$$

   In this case, the social welfare is:

   $$w_E^3(1,0) = [1 - (1 - \nu)n]\,\omega > 0$$

A.1.4 $\delta_E(g) = 1$, $\delta_E(b) = 0$

In this fourth case, the CA rejects the investigated agreement when the signal it receives is *bad*. The firms' expected profits are:

$$\mathbb{E}\Pi_B = (1 - (1 - \nu)\rho n)\pi_B - n\rho F(b)$$

$$\mathbb{E}\Pi_G = (1 - (1 - \nu)(1 - \rho)n)\pi_G - n(1 - \rho)F(b)$$

and we have

$$\beta_E = 0 \Leftrightarrow F(b) \geq \frac{(1 - (1 - \nu)\rho n)\pi_B}{n\rho} \equiv E$$

$$\text{et } \gamma_E = 0 \Leftrightarrow F(b) \geq \frac{(1 - (1 - \nu)(1 - \rho)n)\pi_G}{n(1 - \rho)} \equiv G$$

The possible continuation equilibria are thus:

1. If $F(b) < \min(E, G)$, $\gamma_E = \beta_E = 1$

   With sufficiently low fines, the CA ensures that all the agreements are signed. The social welfare is thus:

   $$w_E^4(1,1) = \omega - 1 - (1 - \nu)n((1 - \rho)\omega - \rho)$$

2. If $E < F(b) \leq G$, $\gamma_E = 1$ and $\beta_E = 0$

   This situation exists only if

   $$E < G \Leftrightarrow (1 - \rho)\pi_B - \rho\pi_G < (1 - \nu)\rho(1 - \rho)n(\pi_B - \pi_G)$$

   Denote by $\widehat{\rho}(n)$ the unique solution[11] of the following equation:

   $$(1 - \rho)\pi_B - \rho\pi_G = (1 - \nu)\rho(1 - \rho)n(\pi_B - \pi_G)$$

   Notice that $\widehat{\rho}(n) \in \left]\frac{1}{2}, \frac{\pi_B}{\pi_B + \pi_G}\right[$. In this case, only the *good* agreements are signed (but some are rejected) and the social welfare is:

   $$w_E^4(1,0) = [1 - (1 - \nu)n(1 - \rho)]\omega$$

   It is easy to check that this solution is preferred to $w_E^3(1,0)$, since the *good* agreements are less often rejected and fined.

A.2   Optimal Policy

In order to determine the CA's optimal policy under the exception regime, let us divide the possible values for the signal's accuracy in two intervals.

---

[11] within the interval $\left[\frac{1}{2}; 1\right]$.

A.2.1   High Precision, $\rho \geq \widehat{\rho}(n)$

In this case, we have to compare the following welfare levels: $w_E^1(1,1)$, $w_E^4(1,1)$ and $w_E^4(1,0)$.[12] Let us first notice that

$$w_E^4(1,1) = w_E^4(1,0) - (1 - (1 - \nu)\,n\rho) < w_E^4(1,0)$$

Finally,

$$w_E^1(1,1) \geq w_E^4(1,0) \Leftrightarrow \omega \geq \frac{1}{(1 - \rho)\,(1 - \nu)\,n}$$

Therefore, for any $\rho \geq \widehat{\rho}(n)$, we have:

1. if $\omega \geq \frac{1}{(1-\rho)(1-\nu)n}$, $\delta_E(g) = \delta_E(b) = 1$ and all the agreements are signed.

2. if $\omega \leq \frac{1}{(1-\rho)(1-\nu)n}$, $\delta_E(g) = 1$, $\delta_E(b) = 0$ and $E < F(b) \leq G$. In that case, only the *good* agreements are signed.

A.2.2   Low Precision, $\rho < \widehat{\rho}(n)$

It is now impossible to sustain $w_E^4(1,0)$ at an equilibrium. We thus have to compare 0, $w_E^1(1,1)$, and $w_E^4(1,1)$. Notice first that

$$w_E^1(1,1) \geq w_E^4(1,1) \Leftrightarrow \omega \geq \frac{\rho}{1 - \rho} \geq 1$$

Finally

$$w_E^4(1,1) \geq 0 \Leftrightarrow \omega \geq \frac{1 - (1 - \nu)\,n\rho}{1 - (1 - \nu)\,n\,(1 - \rho)}$$

We have thus for any $\rho \leq \widehat{\rho}(n)$

1. if $\omega \leq \frac{1-(1-\nu)n\rho}{1-(1-\nu)n(1-\rho)}$, $\delta_E(g) = \delta_E(b) = 0$ and the CA sets high fines, so that none of the agreements are signed.

2. if $\frac{1-(1-\nu)n\rho}{1-(1-\nu)n(1-\rho)} \leq \omega \leq \frac{\rho}{1-\rho}$, $\delta_E(g) = 1$ and $\delta_E(b) = 0$, and with low fines, for example $F(b) = 0$, the CA ensures that all the agreements are signed.

3. if $\omega \geq \frac{\rho}{1-\rho}$, then the CA always authorizes the agreement, and all contracts are signed.

---

[12]Up to now, we have only considered pure strategy equilibria ($\gamma_E = \beta_E = 0$ or 1). Nevertheless, it is never optimal for the CA to implement a mixed strategy equilibrium. It is indeed easy to check that for any mixed strategy equilibrium of the form (γ, β), the corresponding social welfare is a linear function of both γ and β. It is thereby optimal to implement a pure strategy equilibrium.

B Proof of Proposition 3

In order to compare the efficiency of the two pure regimes, it is sufficient to compare the different values of the welfare given by the propositions 1 and 2. Denoting by w_N^* (resp. w_E^*), the welfare corresponding to the optimal policy under the authorization (resp. exception) regime, we can now define the following zones:

1. For $\rho < \widehat{\rho}(n)$ and

   (a) $\omega \geq \widetilde{\omega}_B(\rho) : w_N^* = w_E^* = \omega - 1$. The CA is therefore indifferent.

   (b) $\overline{\omega}(\rho, n) < \omega < \widetilde{\omega}_B(\rho) : w_N^* = \omega - 1 + n(\rho - (1 - \rho)\omega)$ and $w_E^* = \omega - 1 + (1 - \mu)n(\rho - (1 - \rho)\omega)$. Since $\omega \leq \frac{1-\rho}{\rho}$, the authorization regime is preferred to the exception system.

   (c) $\widetilde{\omega}_G(\rho) \leq \omega < \overline{\omega}(\rho, n) : w_N^* = \omega - 1 + n(\rho - (1 - \rho)\omega) < 0$ and $w_E^* = 0$. In this case, the exception regime dominates.

   (d) $\omega \leq \widetilde{\omega}_G(\rho) : w_N^* = (1 - n)(\omega - 1) < 0$ and $w_E^* = 0$. Once again, the CA prefers the *ex post* control regime.

2. For $\rho \geq \widehat{\rho}(n)$ and

   (a) $\omega \geq \widehat{\omega}(\rho, n) : w_N^* = w_E^* = \omega - 1$. The CA is indifferent.

   (b) $\widetilde{\omega}_B(\rho) \leq \omega < \widehat{\omega}(\rho, n) : w_N^* = \omega - 1$ and $w_E^* = (1 - (1 - \gamma)(1 - \rho)n)\omega$. Since $\omega < \frac{1}{(1-\gamma)(1-\rho)n}$, the *ex post* control system is preferred to the authorization regime.

   (c) $\widetilde{\omega}_G(\rho) \leq \omega \leq \widetilde{\omega}_B(\rho) : w_N^* = \omega - 1 + n(\rho - (1 - \rho)\omega)$ and $w_E^* = (1 - (1 - \mu)(1 - \rho)n)\omega$. We have therefore $w_N^* < w_E^*$ if and only if

   $$1 - n\rho + n(1 - \rho)\omega > (1 - \mu)(1 - \rho)n\omega \Leftrightarrow \omega > -\frac{1 - n\rho}{\mu(1 - \rho)n}$$

   In this case, the exception regime always dominates.

   (d) $\omega \leq \widetilde{\omega}_G(\rho) : w_N^* = (1 - n)(\omega - 1) < 0$ et $w_E^* = (1 - (1 - \mu)(1 - \rho)n)\omega > 0$. Once again, the CA prefers the *ex post* control regime.

C Proof of Propositions 5, 6 and 7

C.1 Optimal Decisions For a Given Budget Sharing Rule

Using the results of the "pure" regimes (proposition 1, and proposition 2 in which n is replaced by $p_E(\alpha)$), we can, for a given budget sharing rule $(1 - \alpha, \alpha)$ divide the plan

(ρ, ω) in 8 zones:

1. For a sufficiently inaccurate signal, that is for $\rho < \widehat{\rho}\,(p_E(\alpha))$, where $\widehat{\rho}\,(p_E(\alpha))$ is the unique solution of the equation:

$$(1 - \rho)\pi_B - \rho\pi_G = (1 - \mu)\rho(1 - \rho)p_E(\alpha)(\pi_B - \pi_G)$$

   (a) and $\omega \geq \widetilde{\omega}_B(\rho)$: the CA authorizes any agreement (whatever the type of investigation and the signal). All the firms have therefore incentives to sign and the welfare is $w_{1a} = \omega - 1$.

   (b) and $\overline{\omega}(\rho, p_E(\alpha)) \leq \omega \leq \widetilde{\omega}_B(\rho)$: whatever the type of investigation, the CA prohibits any agreement for which the signal is *bad*, but does not impose any fine in the *ex post* control case. On the other hand, the agreements are authorized when the signal is *good*. The fine being sufficiently low all the firms which notification has not been investigated sign and the total welfare is therefore:

$$
\begin{aligned}
w_{1b}(\alpha) &= (1 - \alpha)\,n\,(\rho\omega - 1 + \rho) + (1 - (1 - \alpha)\,n)\left(\omega - 1 - \frac{\alpha n(1 - \mu)\,(\rho\omega - 1 + \rho)}{1 - (1 - \alpha)\,n}\right) \\
&= (1 - n + \alpha n)\,(\omega - 1) + (1 - (2 - \mu)\,\alpha)\,n\,(\rho\omega - 1 + \rho)
\end{aligned}
$$

   In this case, we have:

$$
\begin{aligned}
w'_{1b}(\alpha) &= n\,(\omega - 1 - (2 - \mu)\,(\rho\omega - 1 + \rho)) \\
&= n\,((1 - (2 - \mu)\,\rho)\,\omega - 1 + (2 - \mu)\,(1 - \rho))
\end{aligned}
$$

   (c) and $\widetilde{\omega}_G(\rho) \leq \omega \leq \overline{\omega}(\rho, p_E(\alpha))$: in this zone, only the firms whose notification has been investigated and for which the CA received a *good* signal sign and the welfare is therefore $w_{1c}(\alpha) = (1 - \alpha)\,n\,(\rho\omega - 1 + \rho)$.
   We then have $w'_{1c}(\alpha) = -n\,(\rho\omega - 1 + \rho) < 0$.

   (d) and $\omega \leq \widetilde{\omega}_G(\rho)$: in this case, any investigation leads to a prohibition, combined with a high fine in the *ex post* control case, so that no contract is signed. Le welfare is then $w_{1d} = 0$.

2. For a highly accurate signal, $\rho \geq \widehat{\rho}\,(p_E(\alpha))$

   (a) and $\omega \geq \widehat{\omega}\,(\rho, p_E(\alpha))$: this zone is identical to $(1a)$, that is all the firms actually sign and any agreement is authorized. The welfare is thus $w_{2a} = \omega - 1$.

   (b) and $\widetilde{\omega}_B(\rho) \leq \omega \leq \widehat{\omega}\,(\rho, p_E(\alpha))$: all the investigated notifications are authorized and the corresponding firms therefore sign the agreement. For the other firms

(type $\emptyset$), the CA implements a separating equilibrium by prohibiting (and fining) when the signal received *ex post* is *bad*. The welfare is therefore:

$$
\begin{aligned}
w_{2b}(\alpha) &= (1-\alpha)\, n\, (\omega - 1) + (1 - (1-\alpha)\, n) \left(1 - \frac{(1-\mu)(1-\rho)\, \alpha n}{1 - (1-\alpha)\, n} \right) \omega \\
&= (1 - (1-\mu)(1-\rho)\, \alpha n)\, \omega - (1-\alpha)\, n
\end{aligned}
$$

Notice that we then have:

$$
w'_{2b}(\alpha) = (1 - (1-\mu)(1-\rho)\, \omega)\, n \geq 0 \Leftrightarrow \omega \leq \frac{1}{(1-\mu)(1-\rho)}
$$

(c) and $\widetilde{\omega}_G(\rho) \leq \omega \leq \widetilde{\omega}_B(\rho)$: among the firms whose notification has been considered *ex ante*, only those for which the CA receives a *good* signal are allowed to sign. For type $\emptyset$ firms, the CA's policy is the same in (2*b*), and the welfare is thus:

$$
w_{2c}(\alpha) = (1-\alpha)\, n\, (\rho\omega - 1 + \rho) + (1 - (1-\alpha)\, n) \left(1 - \frac{(1-\mu)(1-\rho)\, \alpha n}{1 - (1-\alpha)\, n} \right) \omega
$$

We then have:

$$
\begin{aligned}
w'_{2c}(\alpha) &= n\, (-\rho\omega + 1 - \rho + \omega - (1-\mu)(1-\rho)\, \omega) \\
&= n\, (1-\rho)\, (\mu\omega + 1) > 0
\end{aligned}
$$

(d) and $\omega \leq \widetilde{\omega}_G(\rho)$: any investigated notification is rejected. For type $\emptyset$ firms, the CA's policy is the same in (2*b*), and the welfare is thus:

$$
\begin{aligned}
w_{2d}(\alpha) &= (1 - (1-\alpha)\, n) \left(1 - \frac{(1-\mu)(1-\rho)\, \alpha n}{1 - (1-\alpha)\, n} \right) \omega \\
&= (1 - n + (1 - (1-\mu)(1-\rho))\, \alpha n)\, \omega
\end{aligned}
$$

In this case, $w'_{2d}(\alpha) = (1 - (1-\mu)(1-\rho))\, n > 0$.

The following figure shows the different zones. Notices that only the limits between zones (1*b*) and (1*c*) $(\overline{\omega}(\rho, p_E(\alpha)))$, (2*a*) and (2*b*) $(\widehat{\omega}\,(\rho, p_E(\alpha)))$ and the boundary $\widehat{\rho}\,(p_E(\alpha))$ depend on α (these boundaries are represented by bold lines):

C.2 Optimal Sharing Rule

To determine the optimal sharing rule, let us divide the set of possible values for the signal's accuracy in three intervals:
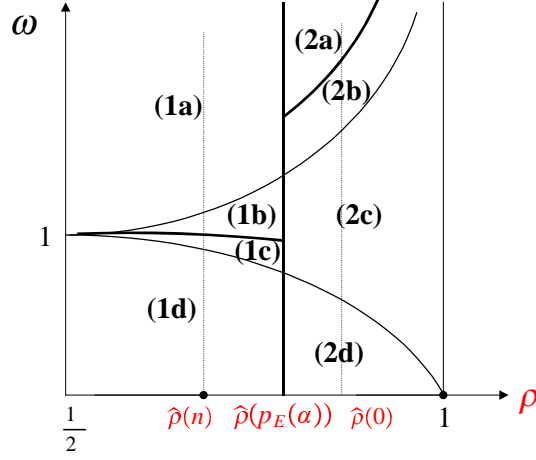
Figure 5:

C.2.1   For $\rho \geq \widehat{\rho}(0) = \dfrac{\pi_B}{\pi_B + \pi_G}$ (Proposition 5)

For these values, we are in one of the zones labelled 2 [(2a) to (2d)], for any value of α.

- In the zones (2c) and (2d), the welfare is a strictly increasing function of α and the boundaries do not depend on α. The optimal choice is therefore $\alpha = 1$.

- For $\widetilde{\omega}_B(\rho) \leq \omega \leq \frac{1}{(1-\mu)(1-\rho)}$, the situation is identical to the previous one and the optimal choice is $\alpha = 1$.

- Finally, for $\omega \geq \frac{1}{(1-\mu)(1-\rho)}$, the welfare is a strictly decreasing function of α in (2b) and does not depend on α in (2a). However, the limit $\widehat{\omega}(\rho, p_E(\alpha))$ between these two zones is a strictly decreasing function of α and tends to infinity when α goes to 0. Nevertheless, notice that the *ex post* control policy imposes to have a strictly positive probability of audit ($\Leftrightarrow \alpha > 0$) in (2c). Moreover, we have

$$w_{2b}(0^+) = \omega - n > w_{2a} = \omega - 1$$

The optimal choice is thus $\alpha = 0^+$.

C.2.2   For $\rho < \widehat{\rho}(n)$ (Proposition 6)

For these values of the accuracy, we are in one of the zones labelled 1 [(1a) to (1d)], for any budget sharing rule.

- For $\omega \geq \widetilde{\omega}_B(\rho)$: in this case, the welfare is independent of α. Since the CA never prohibits any agreement, it plays no role and all budget sharing rules lead to the same result.

- For $\omega \leq \widetilde{\omega}_G(\rho)$: once again, the welfare does not depend on α. However, for the threat of *ex post* control (associated with a high fine when the signal is *bad*) to be credible, the CA needs to maintain a strictly positive probability of audit. The CA is therefore indifferent between any strictly positive α.

- For $\widetilde{\omega}_G(\rho) \leq \omega \leq 1$: in (1c), the welfare is a strictly decreasing function of α and $w_{1c}(0^+) = n(\rho\omega - 1 + \rho)$. Notice that the boundary $\overline{\omega}(\rho, p_E(\alpha))$ between (1b) and (1c) is a strictly decreasing function of α, and goes to 1 when α tends to 0. However, for any $\widetilde{\omega}_G(\rho) \leq \omega \leq 1$, $w_{1c}(0^+) = n(\rho\omega - 1 + \rho) \geq w_{1b}(\alpha)$.

  This implies that for any $\omega \leq 1$, the optimal choice is $\alpha = 0^+$ (the CA needs a strictly positive probability of audit *ex post* to maintain a credible threat).

- Finally, for values of ω between 1 and $\widetilde{\omega}_B(\rho)$, notice that $w'_{1b}(\alpha)$ is a linear function of ω, such that

$$
\begin{aligned}
w'_{1b}(\alpha)_{\omega=1} &= -n(2-\mu)(1-\rho) < 0 \\
w'_{1b}(\alpha)_{\omega=\frac{\rho}{1-\rho}} &= -n\left(\frac{2\rho-1}{\rho}\right)(1-\mu) < 0
\end{aligned}
$$

  The welfare is thus a strictly decreasing function of $\alpha = 0$. In this case, the CA never prohibits an agreement after an *ex post* investigation and therefore, it does not need to allocate a minimal share of the budget to this type of control.

C.2.3 For $\widehat{\rho}(n) \leq \rho < \widehat{\rho}(0)$ (Proposition 7)

This intermediate zone is more complicated since the limit $\widehat{\rho}(p_E(\alpha))$ between zones labelled 1 and 2 varies with α. Moreover, this boundary is a decreasing function of α varying between $\widehat{\rho}(0)$ and $\widehat{\rho}(n)$.

- For $\omega \leq \frac{1}{(1-\mu)(1-\rho)}$:

  For $\rho \geq \widehat{\rho}(p_E(\alpha))$, the *ex post* control policy is such that only the *good* firms actually sign the agreements and always dominates the other possible choices (pooling continuation equilibria). Since the *ex ante* control policy does not depend on α, it is always optimal in this case to set α such that $\rho \geq \widehat{\rho}(p_E(\alpha))$. The welfare being strictly increasing in α, the optimal choice is therefore $\alpha = 1$.

- For $\omega \geq \frac{1}{(1-\mu)(1-\rho)}$:

In this case, the CA has two possible choices: either setting α such that $\rho < \widehat{\rho}(p_E(\alpha))$ or $\rho \geq \widehat{\rho}(p_E(\alpha))$ and $\omega \geq \widehat{\omega}(\rho, p_E(\alpha))$ to stay in the zones $(1a)$ or $(2a)$ and the welfare is $w_{1a} = \omega - 1$, or setting it such that $\rho \geq \widehat{\rho}(p_E(\alpha))$ and $\omega \leq \widehat{\omega}(\rho, p_E(\alpha))$ and, in this case, the welfare is $w_{1b}(\alpha)$.

For any (ω, ρ), it exists a unique value $\widehat{\alpha}(\rho)$ such that

$$(1 - \rho)\pi_B - \rho\pi_G = (1 - \mu)\rho(1 - \rho)p_E(\widehat{\alpha}(\rho))(\pi_B - \pi_G)$$

that is $\widehat{\alpha}(\rho)$ solution of:

$$\frac{\alpha n}{1 - (1 - \alpha) n} = \frac{\pi_B - \rho(\pi_B + \pi_G)}{(1 - \mu)\rho(1 - \rho)(\pi_B - \pi_G)}$$

or

$$\widehat{\alpha}(\rho) = \frac{(1 - n)((1 - \rho)\pi_B - \rho\pi_G)}{((1 - (1 - \mu)\rho)(1 - \rho)\pi_B + (1 - (1 - \mu)(1 - \rho))\rho\pi_G) n}$$

We then have:

- If $\alpha < \widehat{\alpha}(\rho)$, $(\rho, \omega) \in (1a)$ and the welfare is $w(\alpha) = w_{1a} = \omega - 1$.

- If $\alpha \geq \widehat{\alpha}(\rho)$, $(\rho, \omega) \in (2a) \cup (2b)$. However, $w_{2b}(\alpha)$ is a strictly decreasing function of α and

$$w_{2b}(1) = w_{1a} + 1 - (1 - \mu)(1 - \rho) n\omega > w_{1a}$$

since $\omega < \frac{1}{(1-\mu)(1-\rho)n}$. It is therefore never optimal to increase α above $\widehat{\alpha}(\rho)$, as this would only decrease $w_{2b}(\alpha)$ which is always strictly larger than w_{2a}.

Finally, we have:

- if $\omega \geq \frac{\rho(\pi_B - \pi_G)}{\pi_B - \rho(\pi_B + \pi_G)}$, we are always in $(1a)$ or $(2a)$ and the CA is indifferent between any value of α.

- if $\frac{1}{(1-\mu)(1-\rho)} \leq \omega \leq \frac{\rho(\pi_B - \pi_G)}{\pi_B - \rho(\pi_B + \pi_G)}$, the optimal choice depends on the signal's accuracy and is $\widehat{\alpha}(\rho)$.