Module 2: Introduction to Quantitative Data Analysis

Antony Fielding
University of Birmingham & Centre for Multilevel Modelling

Rebecca Pillinger
Centre for Multilevel Modelling

Contents

Introduction ........................................................................................................................................... 1

C2.1 Univariate Data Summary ........................................................................................................... 2
  C2.1.1 Frequency distributions ........................................................................................................ 2
  C2.1.2 Summary statistics of key features of distributions ............................................................. 9

C2.2 Comparisons and relationships: the role of variability ........................................................... 13
  C2.2.1 Comparing subgroups as a form of relationship ................................................................. 13
  C2.2.2 Relationships and their direction ....................................................................................... 14
  C2.2.3 The role of variability ......................................................................................................... 16
  C2.2.4 Homoscedasticity and heteroscedasticity ......................................................................... 17

C2.3 Some other examples of relationships and the role of explained variability .................... 19
  C2.3.1 Extending to two categorical explanators for a continuous response: the possibility of interactions ................................................................................................................................. 19
  C2.3.2 Variability of categorical responses .................................................................................... 24
  C2.3.3 Where both response and explanatory variables are continuous .................................. 26
  C2.3.4 Patterns other than straight lines ..................................................................................... 29
  C2.3.5 Relationship between a continuous response and combinations of categorical and continuous explanators: Progress of students in schools .................................................. 30

C2.4 Working towards the idea of a formal statistical model...................................................... 38

C2.5 Comments on statistical inference; uncertainty, estimation and hypothesis testing 42
  C2.5.1 Estimation .......................................................................................................................... 42
  C2.5.2 Confidence Intervals ......................................................................................................... 45
  C2.5.3 Testing hypotheses ............................................................................................................ 45

1 With contributed material from Kelvyn Jones and Fiona Steele and extensive comment by Harvey Goldstein.
Some of the sections within this module have online quizzes for you to test your understanding. To find the quizzes:

EXAMPLE

From within the LEMMA learning environment
- Go down to the Lesson for Module 2: Introduction to Quantitative Data Analysis
- Click “2.1 Univariate Data Summary” to open Lesson 2.1
- Click Q1(i) to open the first question

Introduction

The aim of this module is to give an overall view of some the principles of effective data analysis. The focus is on how we summarise data to uncover patterns and relationships between variables, and how these relationships can begin to explain the values of the variables that we observe. Some key statistical vocabulary is introduced and concepts are illustrated by example. You will learn about the following:

- Ways of summarising the shape and pattern of the values of a single variable, at different levels of measurement.
- Understanding the role of variability in comparative analysis and the study of relationships.
- The elaboration of relationships, the importance of control variables, and the concepts of confounding, suppression and interaction.
- The essential parts of a statistical model: pattern and residual variation.
- Key concepts in inference from samples.
C2.1 Univariate Data Summary

Before we decide on appropriate methods of analysis to address our research question we will usually want to ‘get to know’ our data. At the very minimum we will want to establish that the variables we plan to use do indeed vary. We will also want to establish an initial picture of the nature of that variation, since it is the study of such variation that will form the basis of further analysis. Thus the first step in any quantitative investigation is to separately summarise each particular variable in various informative ways. Since these initial summaries deal with only one variable they will be called univariate analyses. For instance in our research we might be contemplating using the variable Years of Education, from the European Social Survey data introduced in Module 1. It will be initially useful to know something about the distribution of respondents over the values of this variable. What is the general level of education? How widely dispersed are the values? Is there a concentration of a large number of units (individuals) at particular points? Are there units with extreme values in our data set? Are there curious or inexplicable values which may lead us to suspect errors? In the case of nominal variables, such as Marital Status or Ethnic Group, or of ordinal variables, such as Income measured by twelve ranges, how many units fall into each category? The answers to these questions will affect the analyses we perform.

We should also re-emphasise the major point discussed in Module 1 that the level of measurement of a variable may restrict what summaries are appropriate.

C2.1.1 Frequency distributions

*Frequency distributions* are perhaps the most commonly used initial summaries. They count how many in the set of units under consideration have different values, or groups of values, of the variable. Frequency distributions can usually be employed whatever the level of measurement. However, there are a number of conventions which influence different ways of grouping values.

**C2.1.1.1 Categorical data**

Table 2.1 and Table 2.2 represent typical summaries for the categories of a nominal variable, Marital Status, and an ordered variable, Education Level (defined as the highest level of education attained).

<table>
<thead>
<tr>
<th>Number</th>
<th>% of all cases</th>
<th>% of valid cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Married</td>
<td>22974</td>
<td>54.2</td>
</tr>
<tr>
<td>Separated</td>
<td>627</td>
<td>1.5</td>
</tr>
<tr>
<td>Divorced</td>
<td>2758</td>
<td>6.5</td>
</tr>
<tr>
<td>Widowed</td>
<td>3836</td>
<td>9.1</td>
</tr>
<tr>
<td>Never married</td>
<td>11946</td>
<td>28.2</td>
</tr>
<tr>
<td>Refusal</td>
<td>102</td>
<td>0.2</td>
</tr>
</tbody>
</table>
These tables show the number of respondents in a subsample from the European Social Survey (ESS) data that fall into each of the categories of the variable.\(^2\) There are also a number of categories relating to different sorts of ‘missing data’. Handling such missing data is a complex problem beyond the scope of this module, but it is important to recognise their existence. When they are removed we have what are usually termed valid cases for further analysis. For the rest of this section we will focus only on summarising valid cases. The tables also show percentage distributions. Percentages are usually easier to interpret because of their familiar feel, and because they show us the relative frequency of the values of the variable in the data. This enables us to better judge the relative importance of the values. In Table 2.1 we can see, for instance, that over half of our subsample is married. In Table 2.2 we can see that there are very few cases in the first and last valid categories. Since the ordering of the categories has meaning here, we note that the extremes of Education Level contain relatively small numbers. The most frequently

\(^2\)This is the subsample of cases made available on the ESS website for on-line analysis at http://essedunet.nsd.uib.no/cms/topics/1/.
occurring categories, ‘married’ in the first case and ‘upper secondary’ in the second, are called the *modes* of the distributions. There is an extra column called the *cumulative frequency* in Table 2.2 that is appropriate for ordinal (or grouped ratio or interval level) variables, and which gives the percentage of units with a value of the variable less than or equal to the value of the row in question.

So, for example, 72.8% achieve secondary education or lower or, to look at it another way, 27.2% go beyond secondary education. A cumulative summary in the opposite direction would produce figures like the latter\(^3\), showing the percentage of units with a value of the variable greater than or equal to the value of the row in question.

It is often easier to pick out key features of the data by visualising them. Figure 2.1 is a *pie-chart* for the distribution of marital status where the angles (and thus the sizes) of the slices of the pie are in proportion to the relative frequencies in the last column of Table 2.1. The pie chart is perhaps the most common way of visualising distributions for categorical data but *bar charts* are sometimes also used.

\(^3\)but shifted down a row.

---

**C2.1.1.2 Continuous data**

There are a few new issues connected with frequency distributions for continuous variables since to display patterns we must often group values in a meaningful way. In the ESS there is a variable defined by answers to the question ‘How long have you lived in this area?’, which we will call Years of Residence. Data are recorded to the nearest whole year and the set of possible values are integers between zero and ninety eight, so only a small percentage of units will take each particular value. A full frequency distribution with ninety nine groups would be too detailed, patterns would be difficult to discern and the table would not be very informative. To avoid these problems the values must be grouped into classes or intervals. In determining the appropriate number of intervals and their widths we need to strike a balance...
between too many intervals (which leads to groups with relatively few cases in each) or too few intervals (which results in too great a loss of information). There are no hard and fast rules for this. One possible set of intervals for Years of Residence is shown in Table 2.3. Here the intervals are of an equal width of ten years, and are perhaps easier to handle when this is the case, but as will be seen in another example later in this section there is no necessity that this should be so.

Table 2.3. Grouped frequency distribution of approximate Years of Residence in area at interview (valid cases only)

<table>
<thead>
<tr>
<th>Whole year range</th>
<th>Number of valid cases</th>
<th>Percentage</th>
<th>Cumulative Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-9</td>
<td>11606</td>
<td>27.8</td>
<td>27.8</td>
</tr>
<tr>
<td>10-19</td>
<td>8667</td>
<td>20.7</td>
<td>48.5</td>
</tr>
<tr>
<td>20-29</td>
<td>7470</td>
<td>17.9</td>
<td>66.4</td>
</tr>
<tr>
<td>30-39</td>
<td>5400</td>
<td>12.9</td>
<td>79.3</td>
</tr>
<tr>
<td>40-49</td>
<td>3567</td>
<td>8.5</td>
<td>87.8</td>
</tr>
<tr>
<td>50-59</td>
<td>2369</td>
<td>5.7</td>
<td>93.5</td>
</tr>
<tr>
<td>60-69</td>
<td>1381</td>
<td>3.3</td>
<td>96.8</td>
</tr>
<tr>
<td>70-79</td>
<td>987</td>
<td>2.4</td>
<td>99.2</td>
</tr>
<tr>
<td>80-89</td>
<td>247</td>
<td>0.6</td>
<td>99.8</td>
</tr>
<tr>
<td>90-99</td>
<td>80</td>
<td>0.2</td>
<td>100.0</td>
</tr>
<tr>
<td>Total</td>
<td>41774</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
This document is only the first few pages of the full version. 
To see the complete document please go to learning materials and register: 
http://www.cmm.bris.ac.uk/lemma

The course is completely free. We ask for a few details about yourself for our research purposes only. We will not give any details to any other organisation unless it is with your express permission.